Control of a Ground Vehicle Using Quadratic Programming Based Control Allocation

John Plumlee

David M. Bevly

A. Scottedward Hodel

Auburn University

8/5/2004

Outline

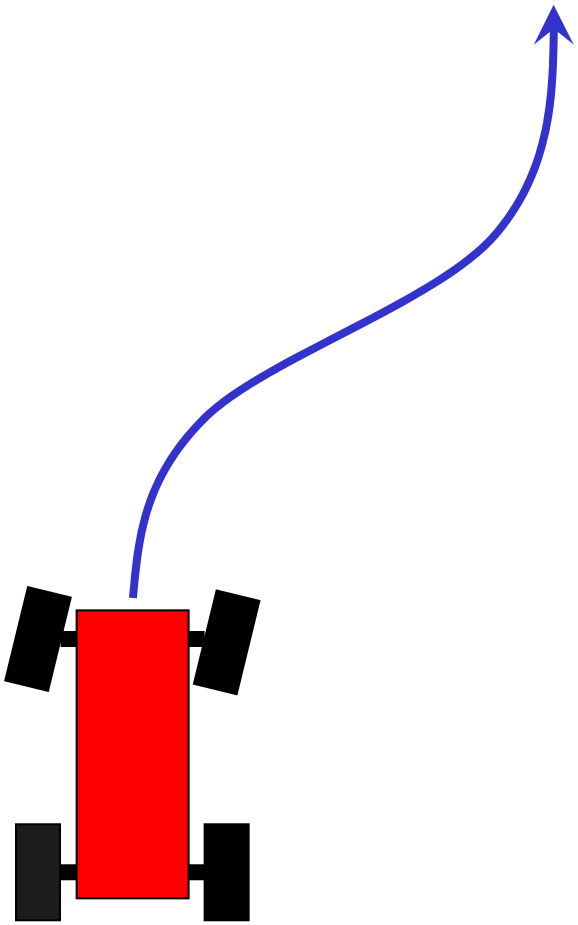
- The Problem and Objectives
- Control Allocation
- Vehicle Model
- Controller Design
- Simulation Results
- Conclusions

Problem and Objectives

We want to track a yaw rate trajectory with a 4 wheeled ground vehicle while minimizing the sideslip angle.

Question: But if there are multiple inputs available, How do you choose which to use and how much?

Answer: Control Allocation!!



Control Allocation (CA)

- CA involves generating a set of effector commands that produce a desired control effect while minimizing the control effort.
- A CA approach is generally used for a redundant set of effectors.
- CA allows for reconfiguration in the event of an effector failure.

Vehicle Model (nonlinear)

Assumptions made:

- The vehicle runs at constant velocity
- No longitudinal weight transfer (pitch is neglected)
- A Pacejka tire model was used to represent nonlinear tire behavior.

Vehicle Model (linear)

The linear vehicle model in state space form:

$$\begin{bmatrix} \dot{\beta} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \frac{-C_o}{mV} & \frac{-C_1}{mV^2} - 1 \\ \frac{-C_1}{I_z} & \frac{-C_2}{VI_z} \end{bmatrix} \begin{bmatrix} \beta \\ r \end{bmatrix} + \begin{bmatrix} \frac{C_{\alpha f}}{mV} & 0 & 0 \\ a\frac{C_{\alpha f}}{I_z} & \frac{T_f}{2I_z} & \frac{T_r}{2I_z} \end{bmatrix} \begin{bmatrix} \delta \\ \Delta F_{xf} \\ \Delta F_{xr} \end{bmatrix}$$

δ = steering angle

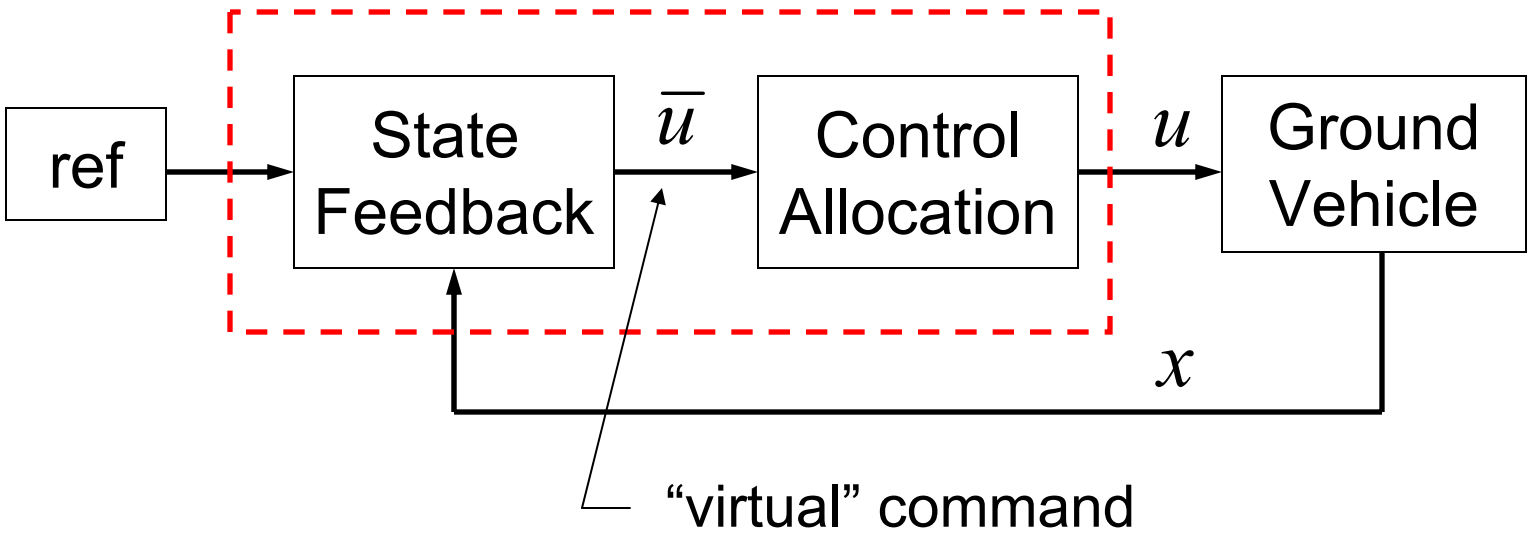
ΔF_{xf} = differential braking force on front axle

ΔF_{xr} = differential braking force on rear axle

Controller Design Approach

Controller has 2 main tasks:

1. Generate Control Effort ⟶ State Feedback
2. Generate Effector Commands ⟶ Control Allocation



Control Effort

LQR gains are designed for the following modified system:

$$\begin{bmatrix} \dot{\beta} \\ \dot{r} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \frac{-C_o}{mV} & \frac{-C_1}{mV^2} - 1 & 0 \\ \frac{-C_1}{I_z} & \frac{-C_2}{VI_z} & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \beta \\ r \\ \psi \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{u}_\beta \\ \bar{u}_r \end{bmatrix}$$

This model is used solely for generating the virtual command effort.

The gain matrix K is applied to the error vector to produce the overall desired effect.

$$\begin{bmatrix} \bar{u}_\beta \\ \bar{u}_r \end{bmatrix} = -K_{2x3} \begin{bmatrix} \beta_{error} \\ r_{error} \\ \psi_{error} \end{bmatrix}$$

Command Generation

Physical effector commands are generated by the Control Allocation routine.

Quadratic Programming (QP)

Given the virtual command $\bar{u}$, solve for the effector commands u:

$$\min_u: \quad \frac{1}{2} u^T Q u + c^T u$$

subject to:

$$Bu = \bar{u}$$

$$u^- \leq u \leq u^+$$

Command Generation: slack variables 1

The Sign Preserving Quadratic Programming (SPQP) problem set up as follows:

$$\min u,\sigma: \quad \tfrac{1}{2}u^T Q_u u + c^T u + \tfrac{1}{2}Q_\sigma(1-\sigma_\beta)^2 + \tfrac{1}{2}Q_\sigma(1-\sigma_r)^2$$

subject to: $Bu - \Sigma\overline{u} = 0$

where

$$\Sigma = \begin{bmatrix} \sigma_\beta & 0 \\ 0 & \sigma_r \end{bmatrix}$$

Ignore virtual command $\rightarrow$

$$\begin{bmatrix} u^- \\ 0 \\ 0 \end{bmatrix} \leq \begin{bmatrix} u \\ \sigma_\beta \\ \sigma_r \end{bmatrix} \leq \begin{bmatrix} u^+ \\ 1 \\ 1 \end{bmatrix}$$

$\leftarrow$ achieve virtual command

Command Generation: slack variables 2

Due to the structure of the input matrix *B*, it is necessary to add a 4th "pseudo" input *v*.

$$B = \begin{bmatrix} \frac{C_{\alpha f}}{mV} & 0 & 0 & 1 \\ a\frac{C_{\alpha f}}{I_z} & \frac{T_f}{2I_z} & \frac{T_r}{2I_z} & 0 \end{bmatrix} \qquad u = \begin{bmatrix} \delta \\ \Delta F_{xf} \\ \Delta F_{xr} \\ v \end{bmatrix} \qquad Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & Q_v \end{bmatrix}$$

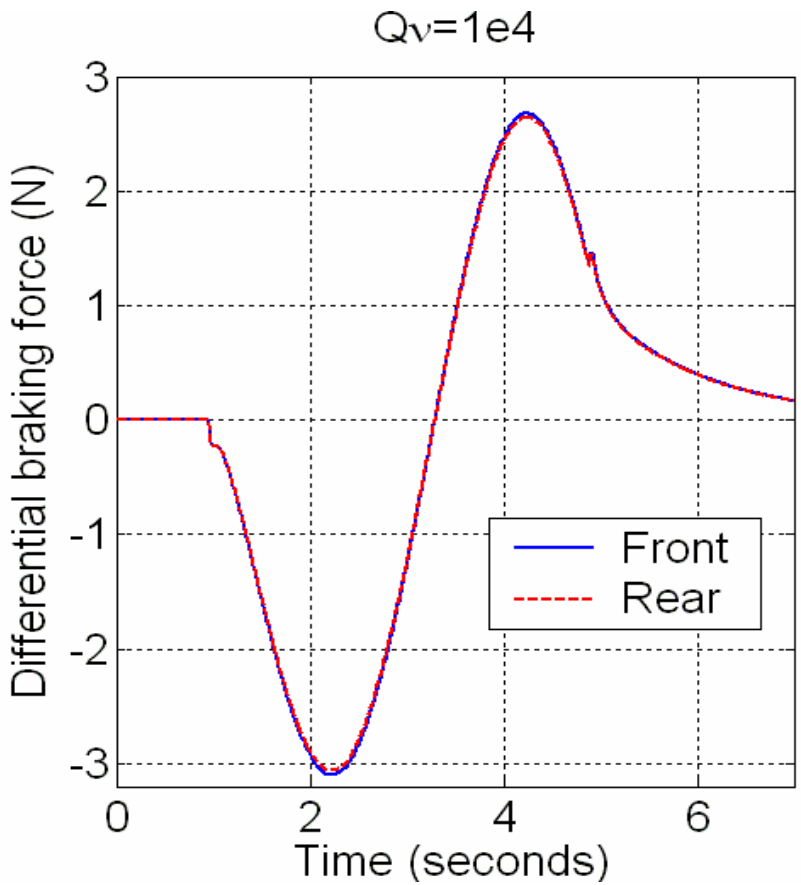
This relaxes the equality constraint placed on the sideslip angle.

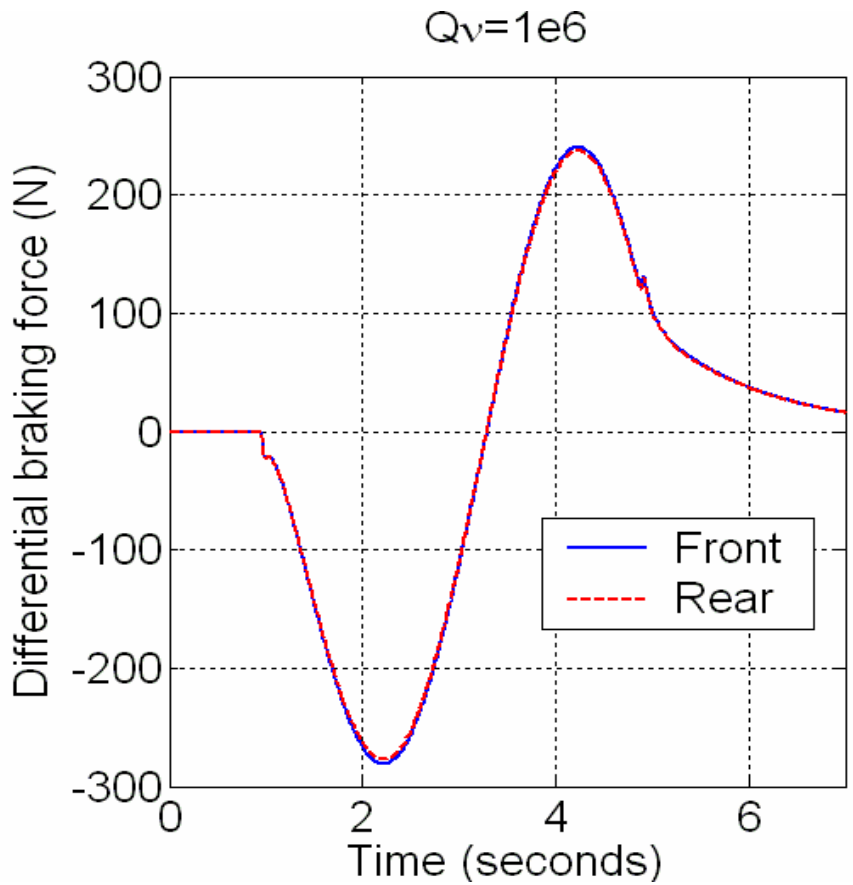
Simulation

- The vehicle is simulated traveling at constant velocities of 45, 55, and 65mph.
- Scenarios examined: nominal, brake failure, and steering failure.
- Penalty on pseudo effector: $Q_v = 10^4, 10^6$.
- The desired yaw rate trajectory simulates a double lane change maneuver.
- Effector limitations: steering angle= $\pm 0.5 rad$

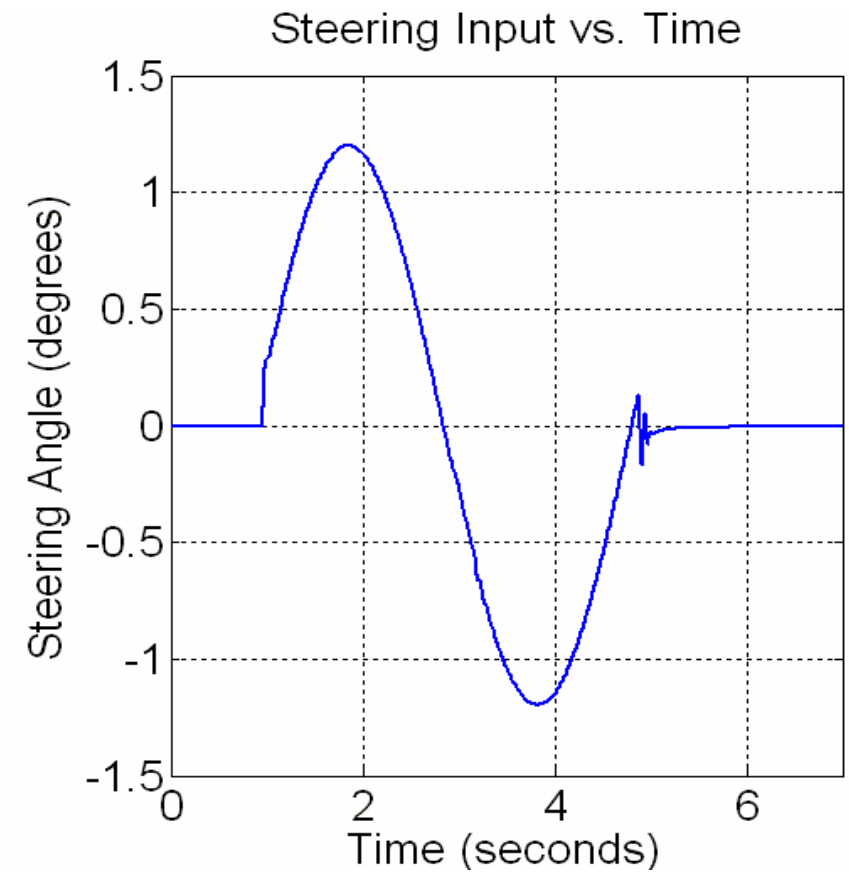
  braking= $0.75 * Fzi$

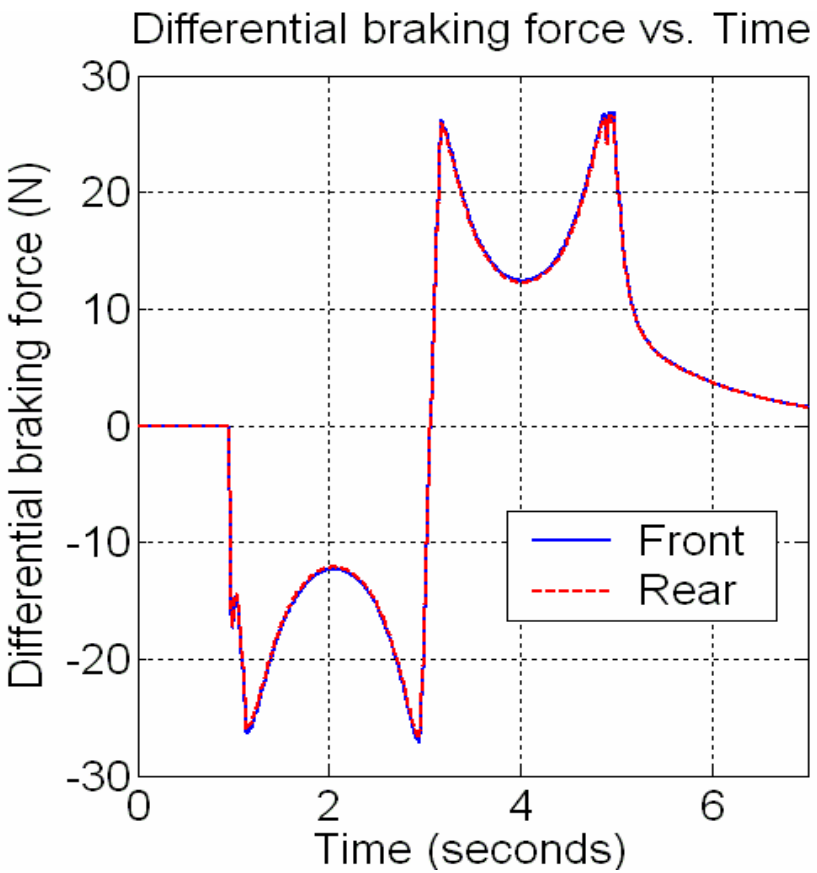
Results (pseudo effector) Nominal , 55mph, QP



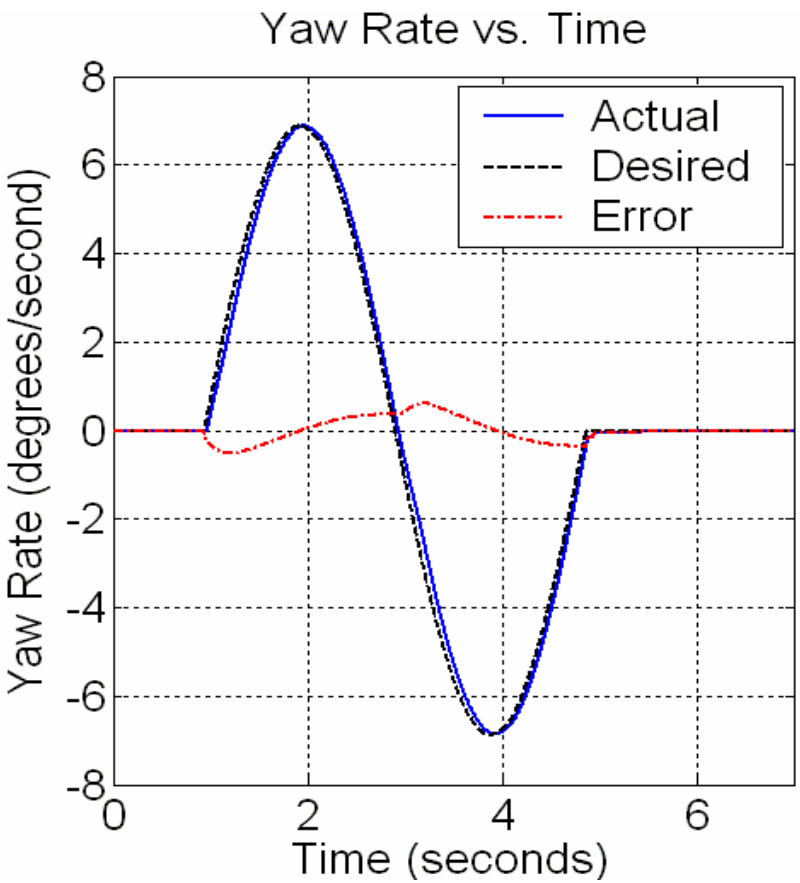


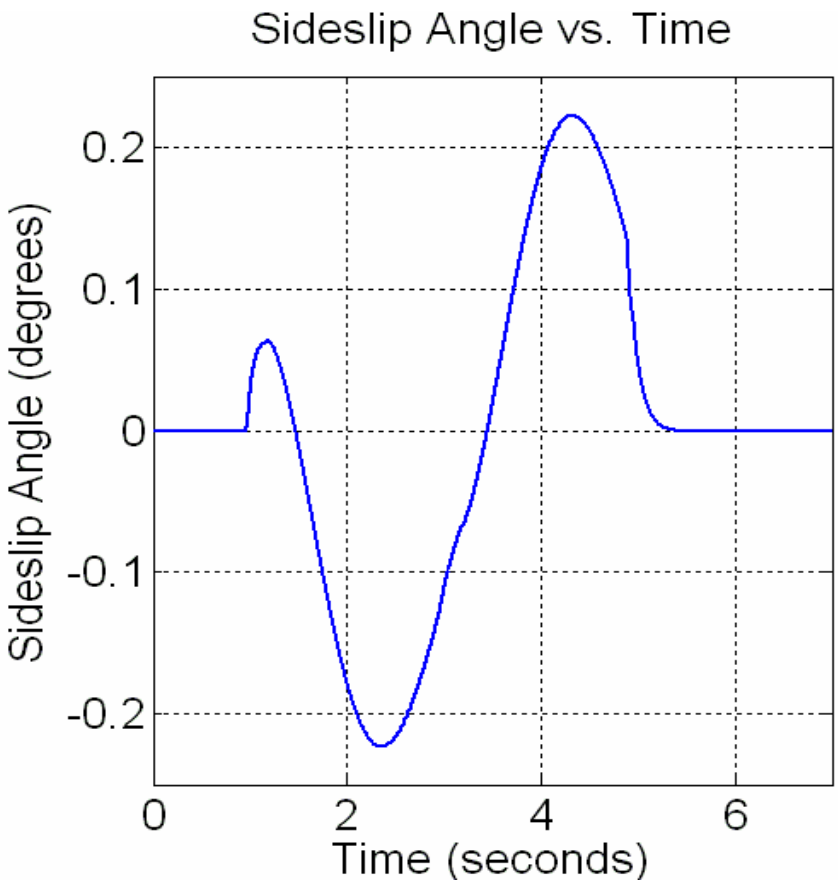
Results – Nominal, 45mph SPQP



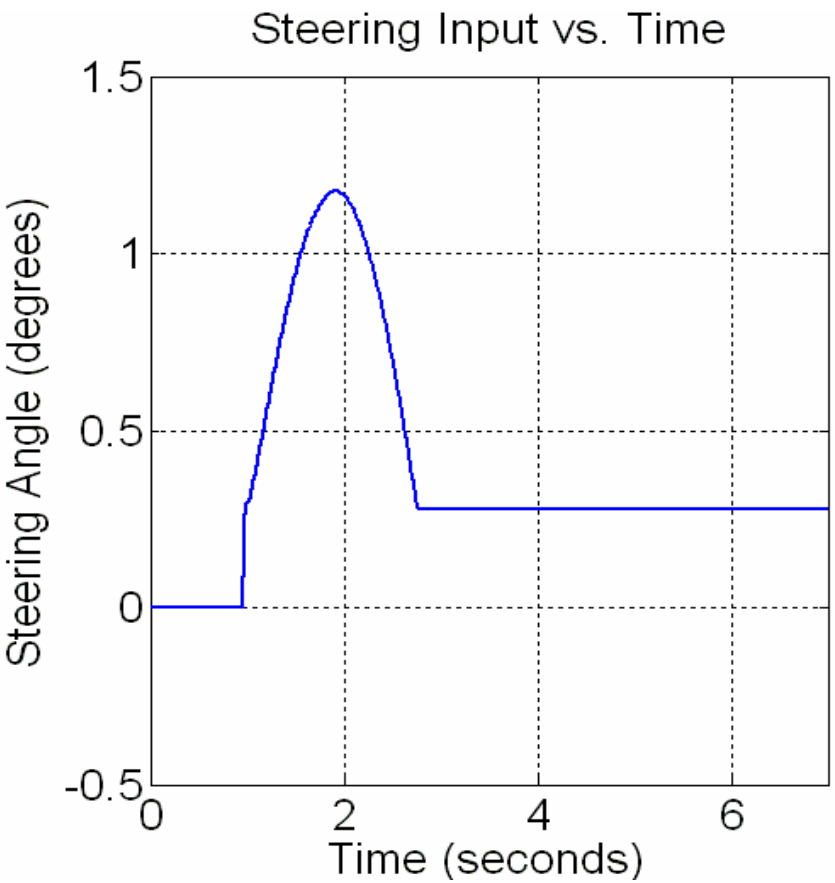


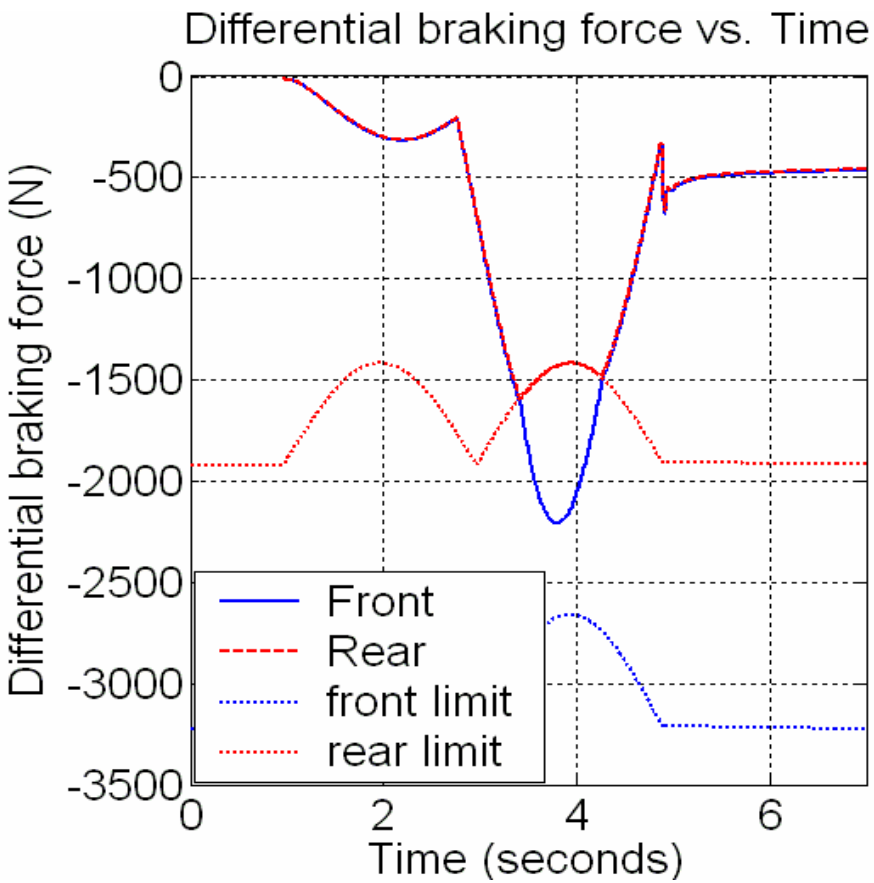
Results – Nominal, 45mph SPQP



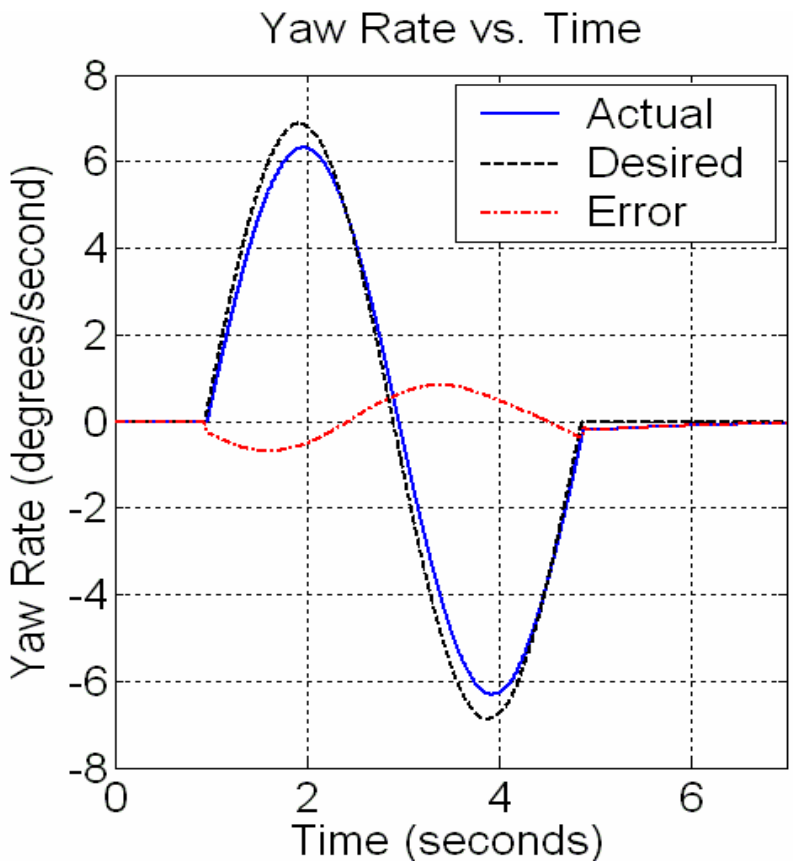


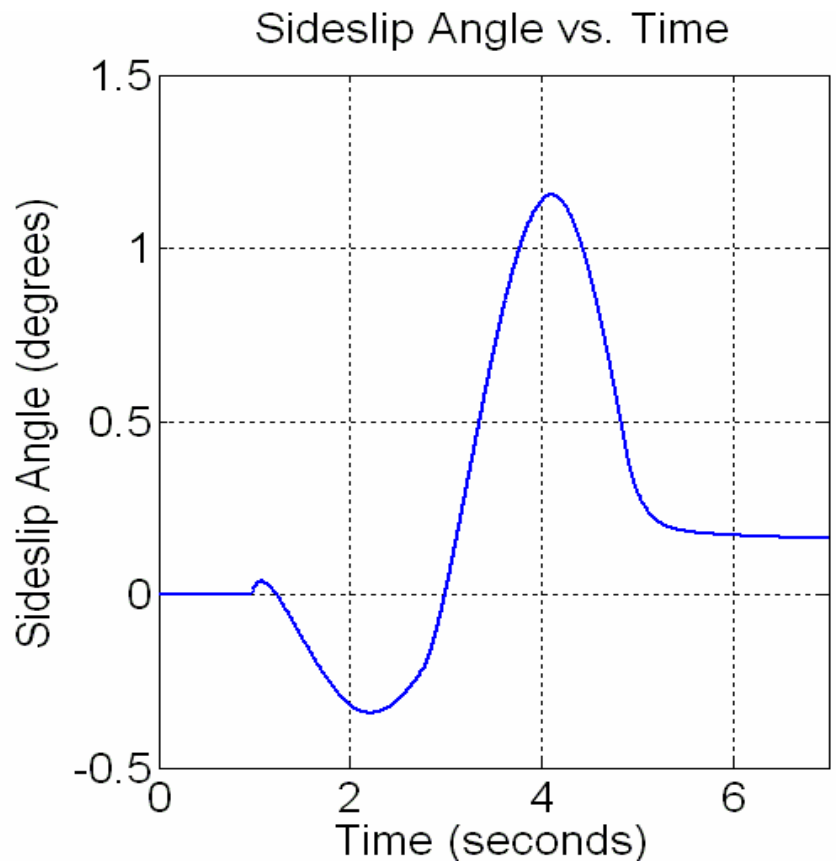
Results – Steering failure, 55mph QP





Results – Steering failure, 55mph QP





Conclusions

- QP provides an intelligent reconfiguration of control effort in the event of a failure and also obeys effector limits.
- SPQP gives the best yaw rate tracking but standard QP provides the best sideslip minimization.
- The quadratic penalty placed on the pseudo effector significantly affects the results.

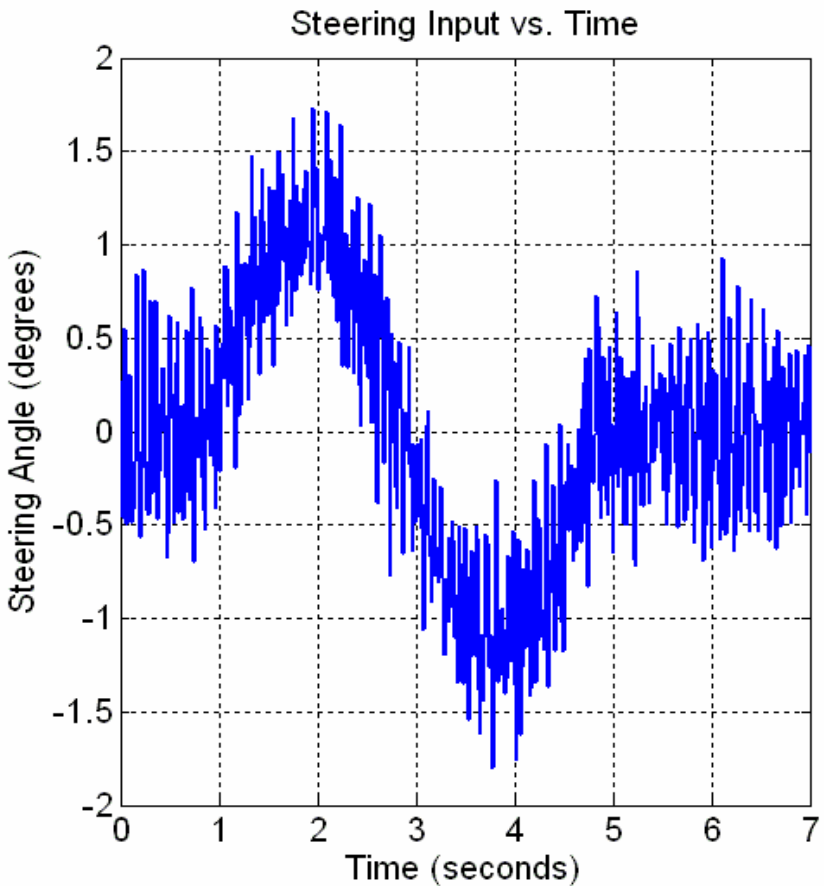
Current Work

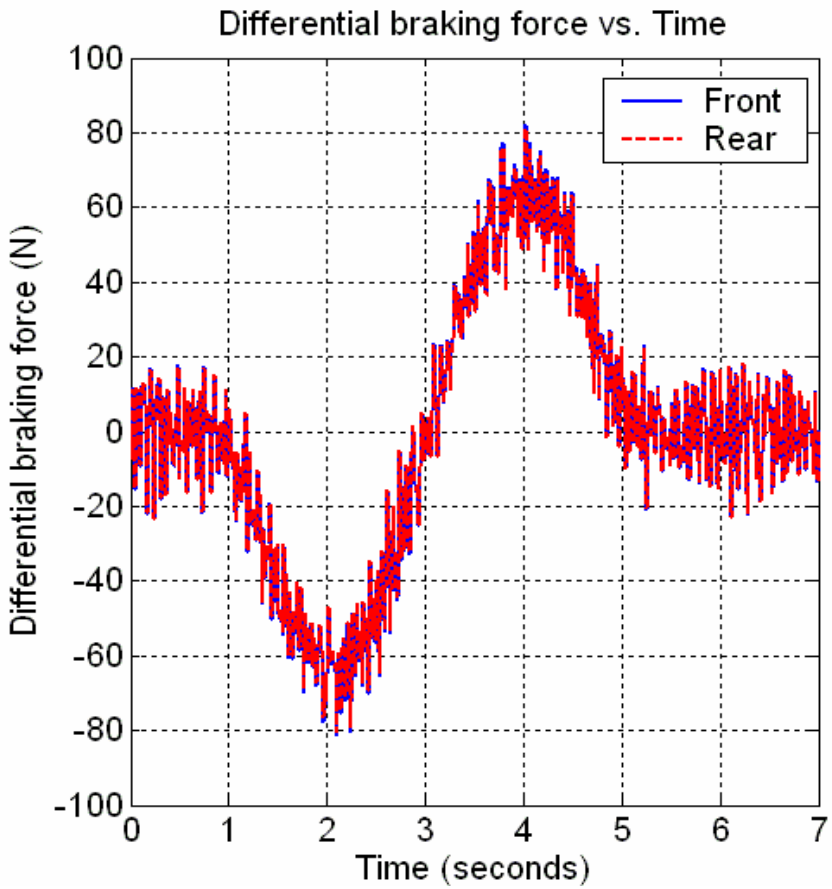
- Gain insight for controller tuning through further investigation of the effects of the pseudo effector weight Q_v.
- Address differential braking saturation limits. Better estimate of limits using the friction circle.
- Study the effects of additional inputs such as rear steering and individual torque control of each wheel.

Questions

???

Answers – Commands w/ noisy state estimates





Answers – tracking w/ noise