Heat Kernel Based Community Detection

Joint with
David F. Gleich,
(Purdue), supported by
NSF CAREER
1149756-CCF

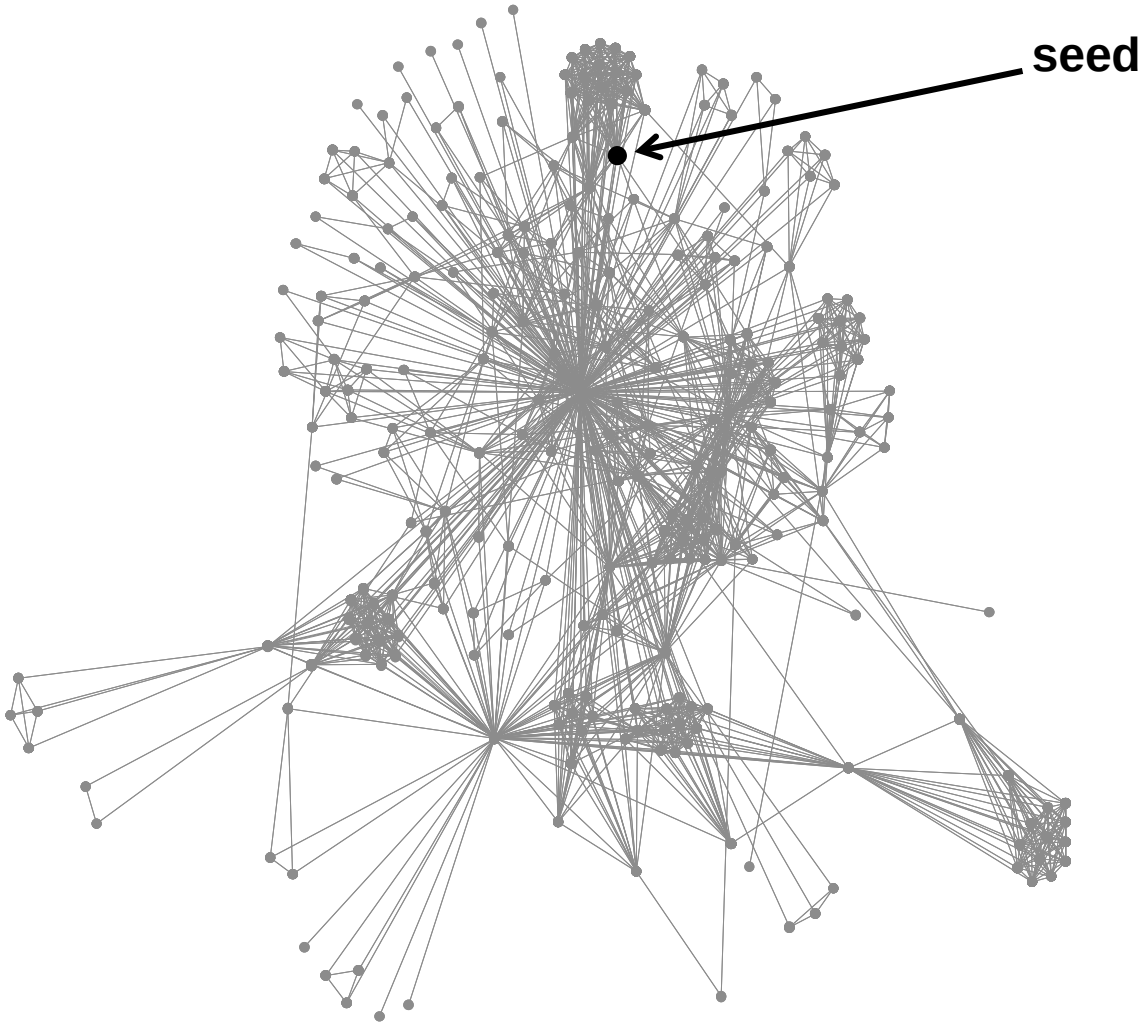
PURDUE UNIVERSITY

Kyle Kloster

Purdue University

Local Community Detection

Given seed(s) *S* in *G*, find a community that contains *S*.



“Community” ?

Local Community Detection

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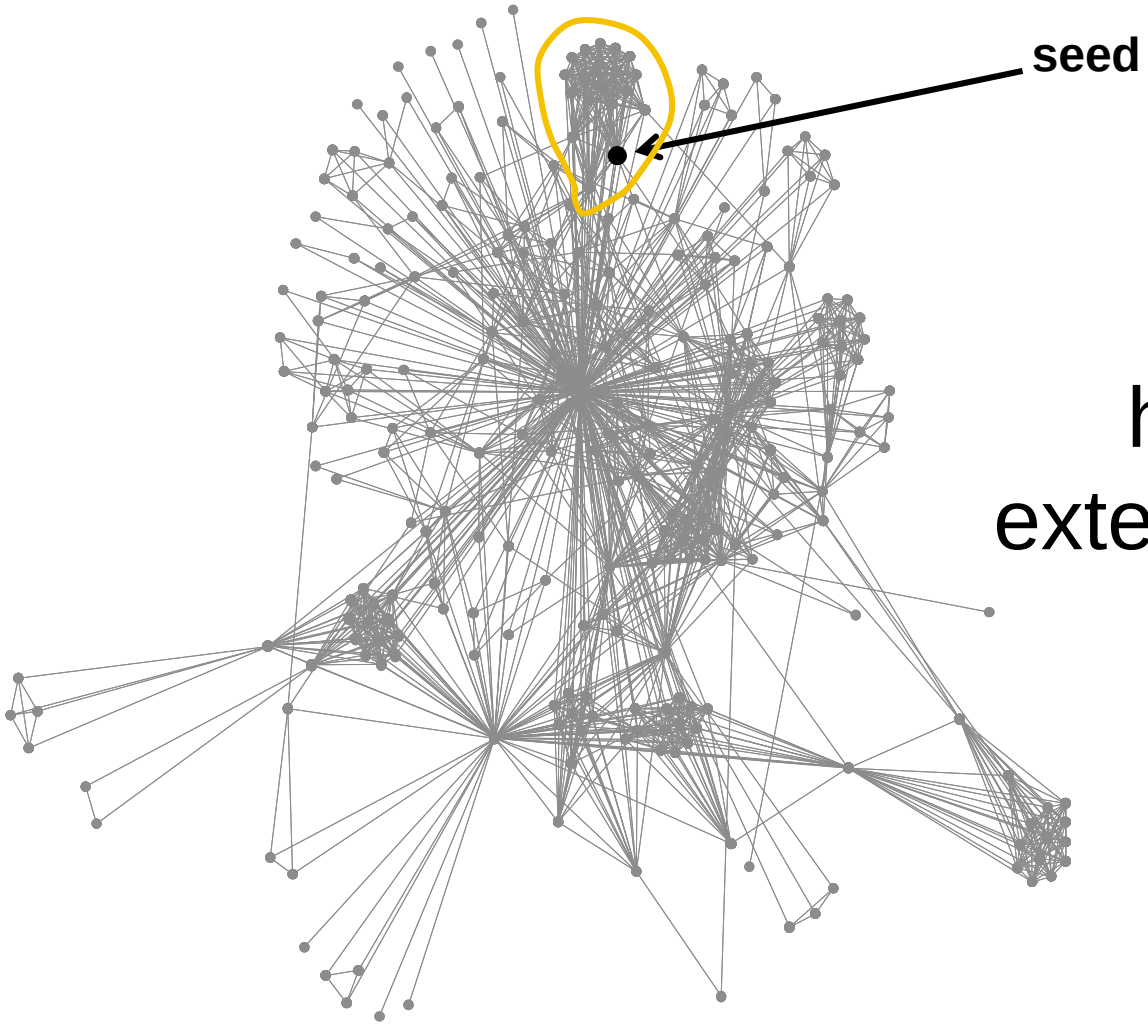


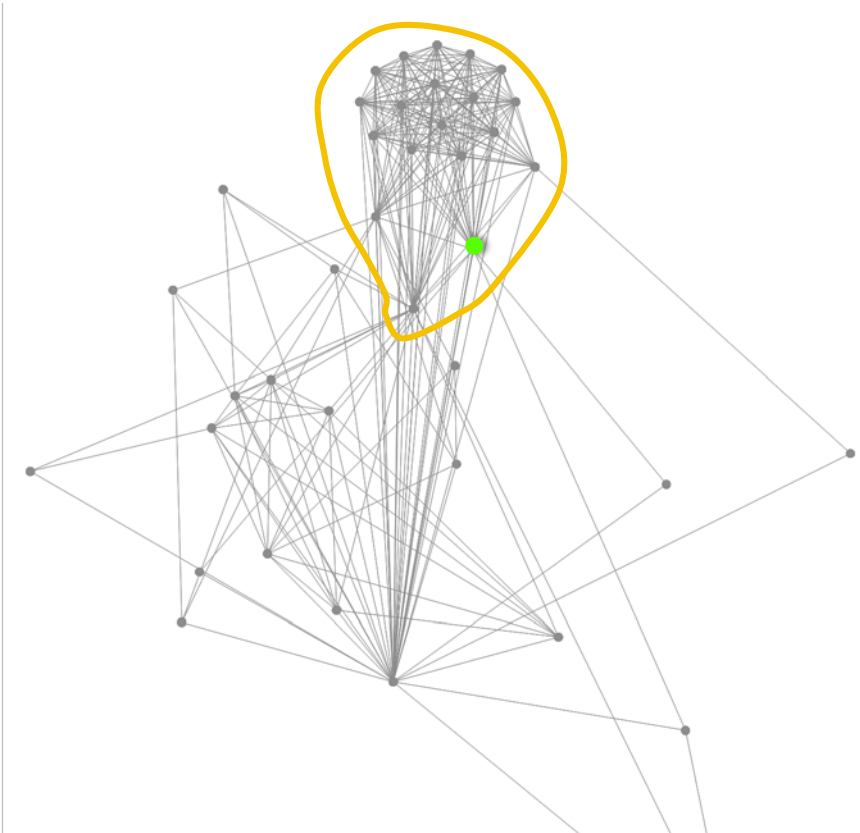
“Community” ?

high internal, low external connectivity

Low conductance sets are communities

$$\text{conductance}(T) = \frac{\#\text{ edges leaving } T}{\#\text{ edge endpoints in } T}$$

= “ chance a random step exits T ”



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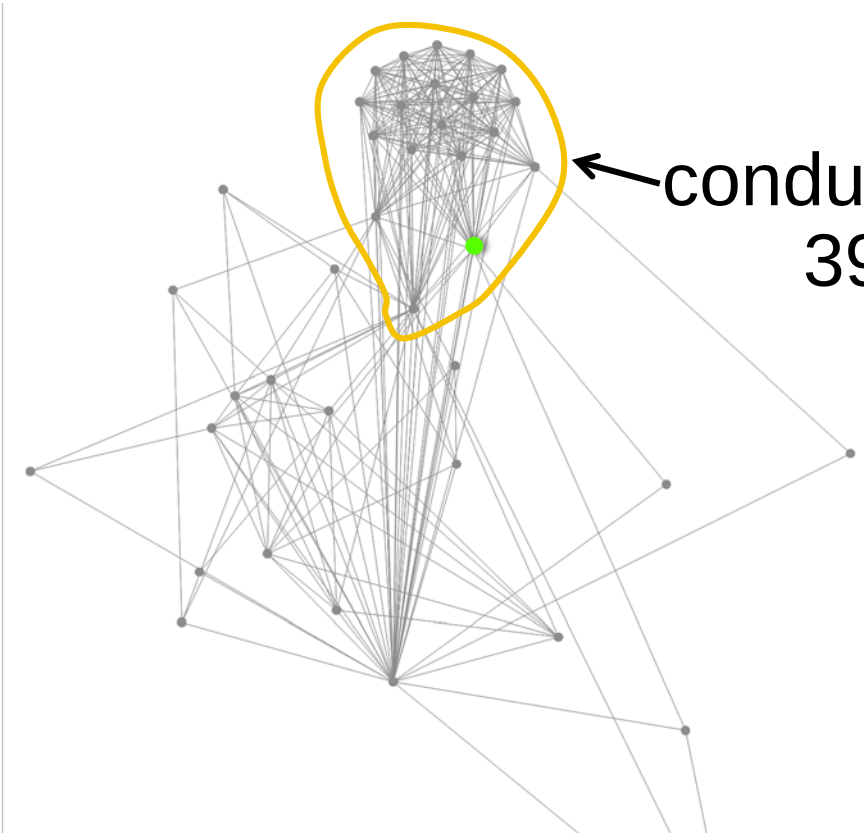
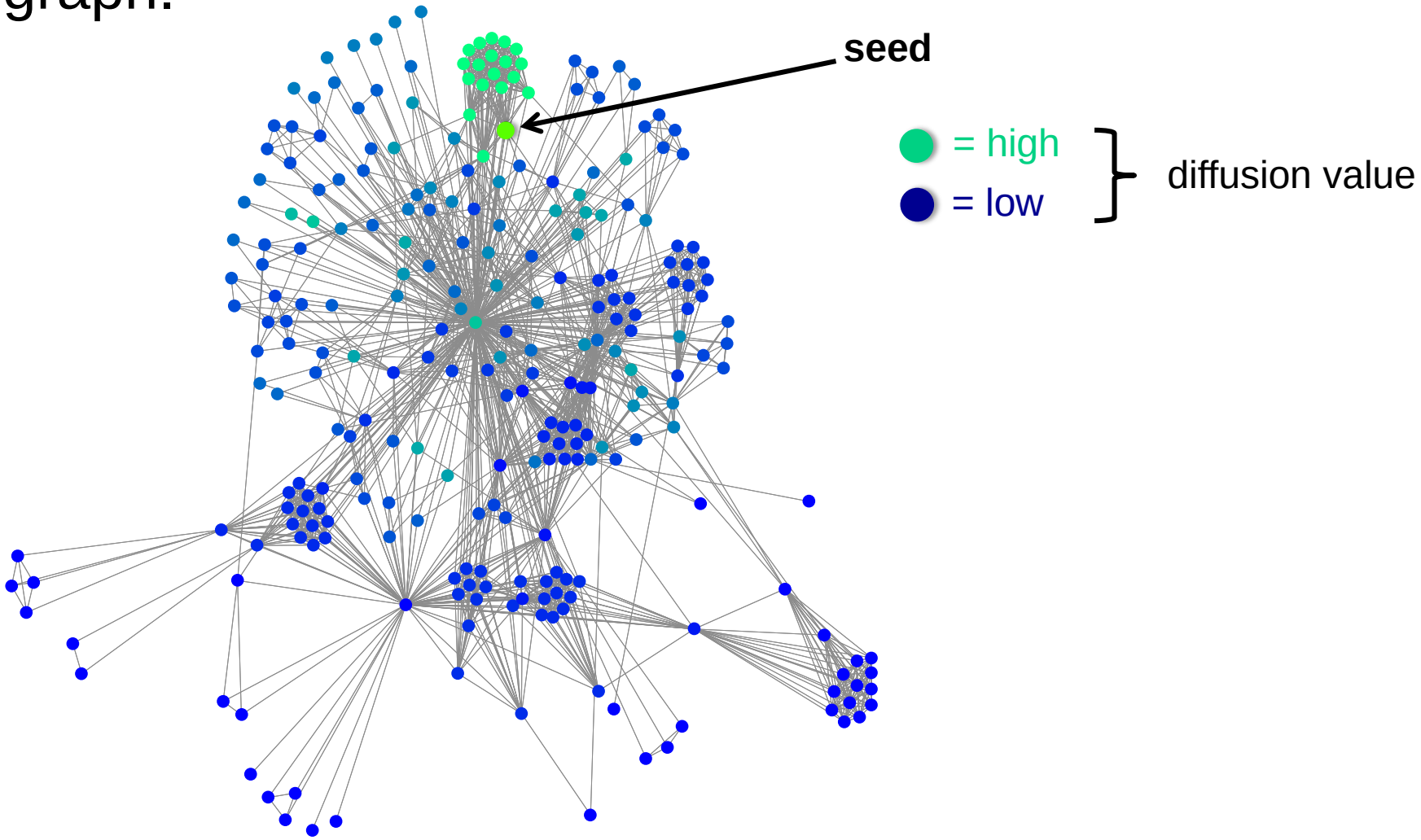


conductance(comm) =
39/381 = .102

How to find these ?

Graph diffusions find low conductance sets

A diffusion propagates “rank” from a seed across a graph.



Graph diffusions find low-conductance sets

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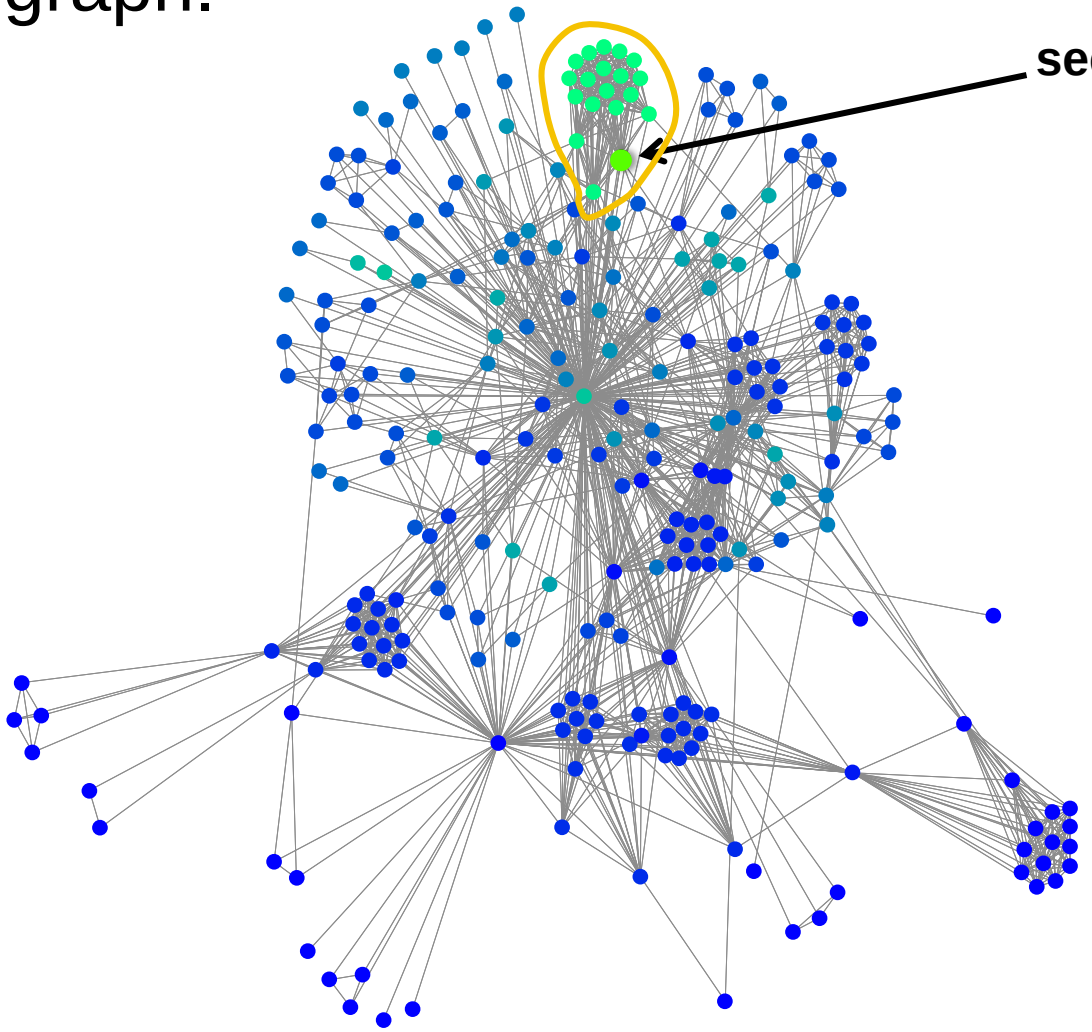
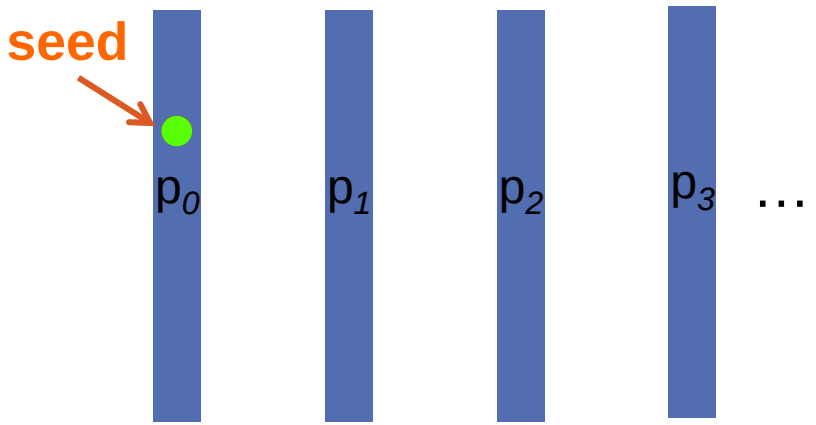


Okay…
how does this work?

Graph Diffusion

A diffusion models how a mass (green dye, money, popularity) spreads from a seed across a network.



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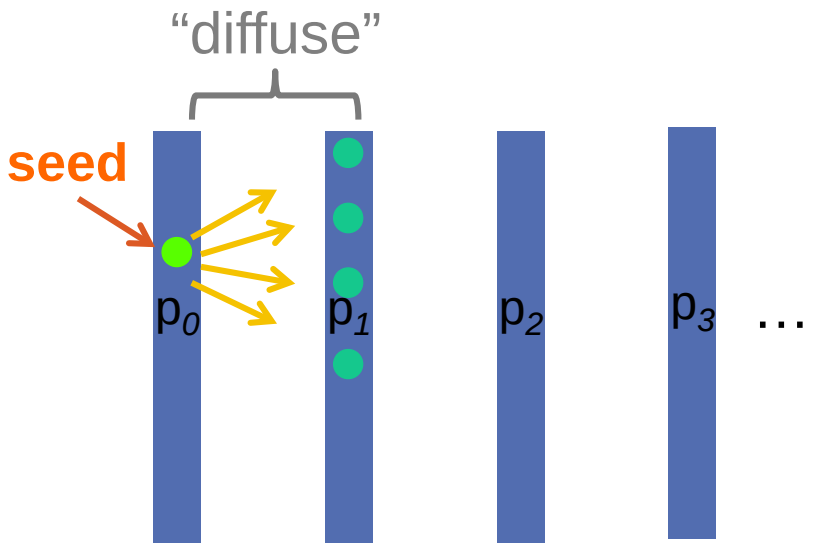

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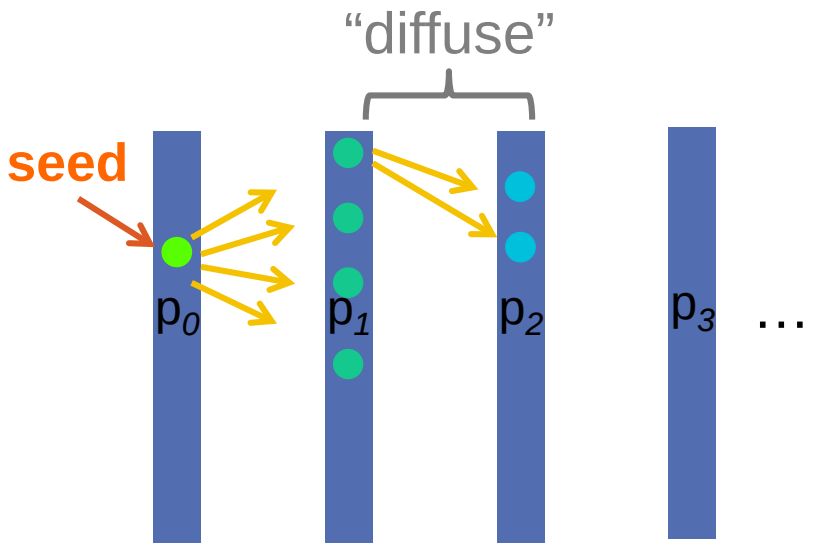

Once mass reaches a node, it propagates to the neighbors, with some decay.

“decay”: dye dilutes, money is taxed, popularity fades

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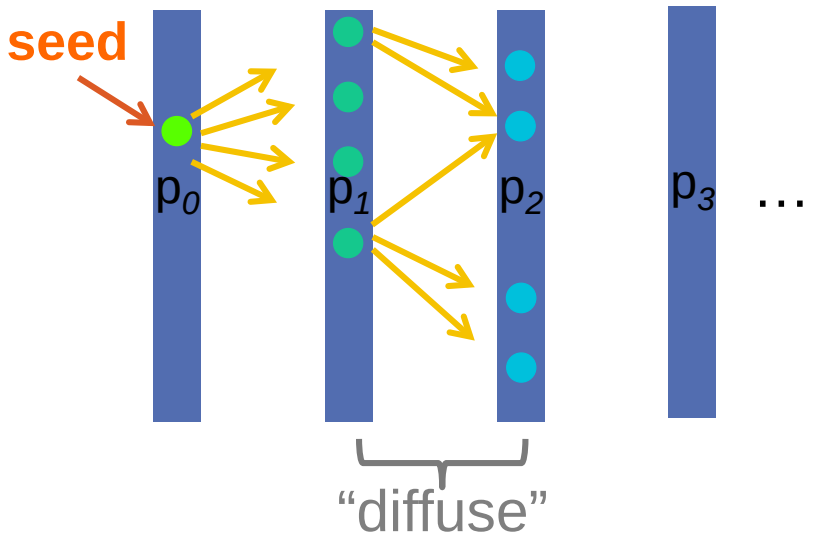

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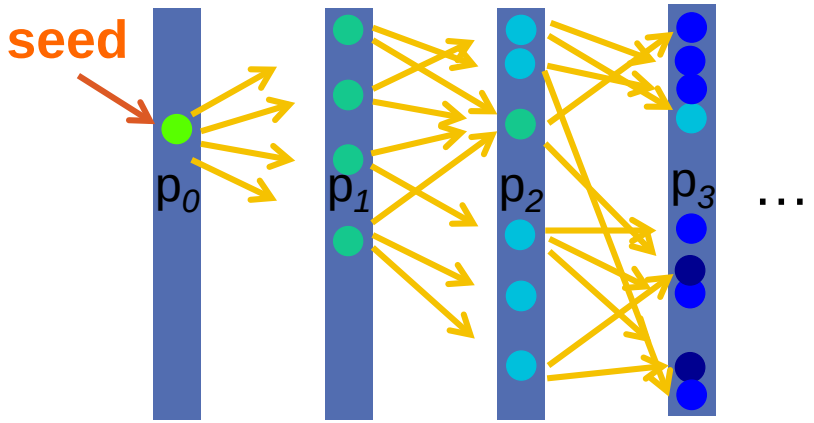

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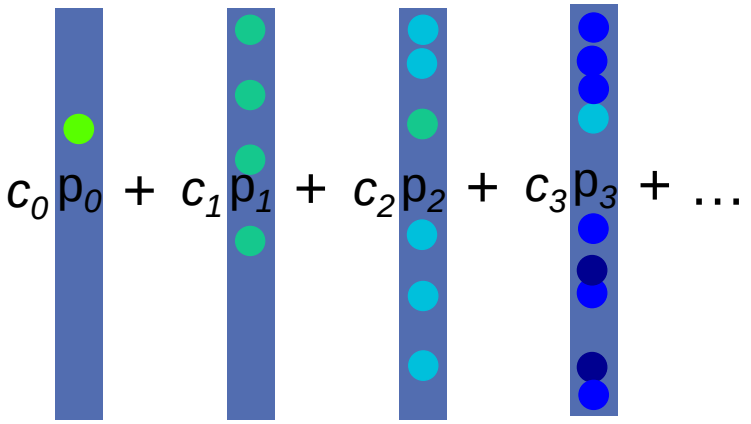
Diffusion score

“diffusion score” of a node = weighted sum of the mass at that node during different stages.



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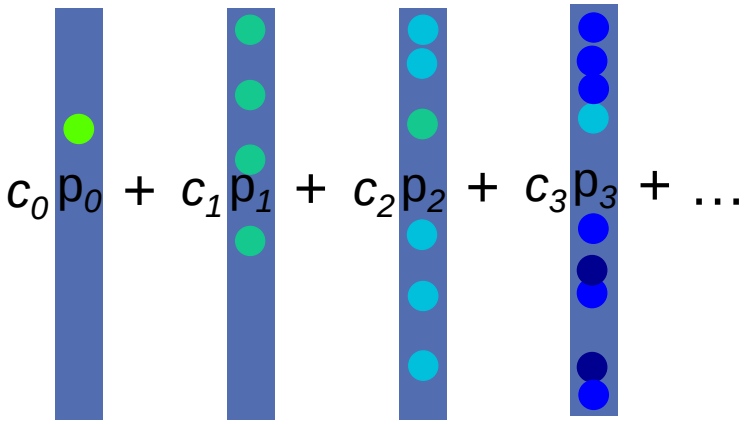


diffusion score vector = f

$$\mathbf{f} = \sum_{k=0}^{\infty} c_k \boldsymbol{P}^k \mathbf{s}$$

$\boldsymbol{P} =$ random-walk transition matrix

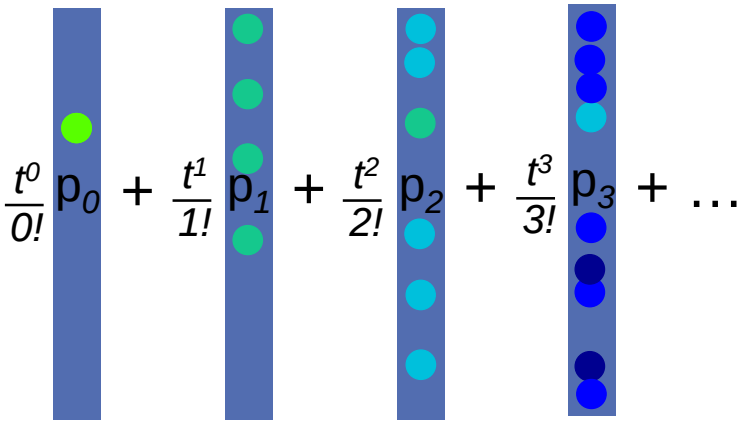
$\mathbf{s} =$ normalized seed vector

$c_k =$ weight on stage *k*

Heat Kernel vs. PageRank Diffusions

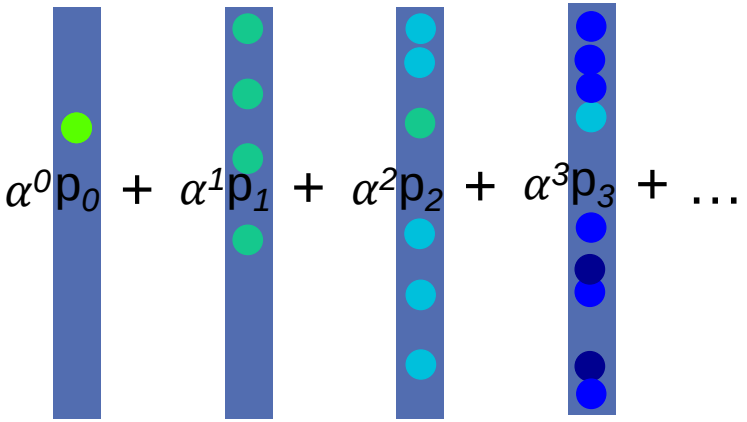
Heat Kernel uses $t^k/k!$

Our work is new analysis for this diffusion.



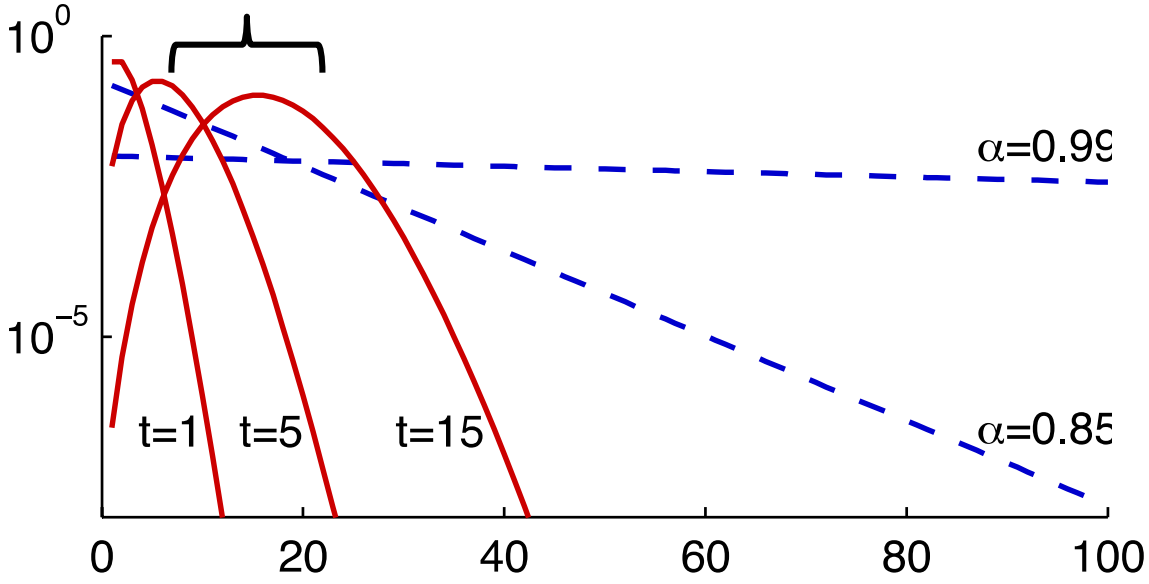
PageRank uses α^k at stage k.

Standard, widely-used diffusion we use for comparison.



Heat Kernel vs. PageRank Behavior

HK emphasizes earlier stages of diffusion.



PR, α^k

HK, $t^k/k!$

→ involve shorter walks from seed,

→ so HK looks at smaller sets than PR

Heat Kernel vs. PageRank Theory

	good conductance	fast algorithm
PR	Local Cheeger Inequality: "PR finds set of near-optimal conductance"	"PPR-push" is $O(1/(\varepsilon(1-\alpha)))$ in theory, fast in practice [Andersen Chung Lang 06]
HK		

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HK	Local Cheeger Inequality [Chung 07]	

Heat Kernel vs. PageRank Theory

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HK	Local Cheeger Inequality [Chung 07]	Our work

Our work on Heat Kernel: theory

THEOREM Our algorithm for a relative ε-accuracy in a degree-weighted norm has

$$\text{runtime} <= O(e^{t}(\log(1/\varepsilon) + \log(t)) / \varepsilon)$$

(which is constant, regardless of graph size)

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COROLLARY HK is local!

(O(1) runtime → diffusion vector has O(1) entries)

Our work on Heat Kernel: results

First efficient, deterministic HK algorithm.
Deterministic is important to be able to compare the behaviors of HK and PR *experimentally*:

Our key findings

- HK more accurately describes ground-truth communities in real-world networks
- identifies smaller sets → better precision
- speed & conductance comparable with PR

Twitter graph

41.6 M nodes

2.4 B edges

Python demo

un-optimized Python code

on a laptop

Available for download:

`https://gist.github.com/dgleich/cf170a226aa848240cf4`

Algorithm Outline

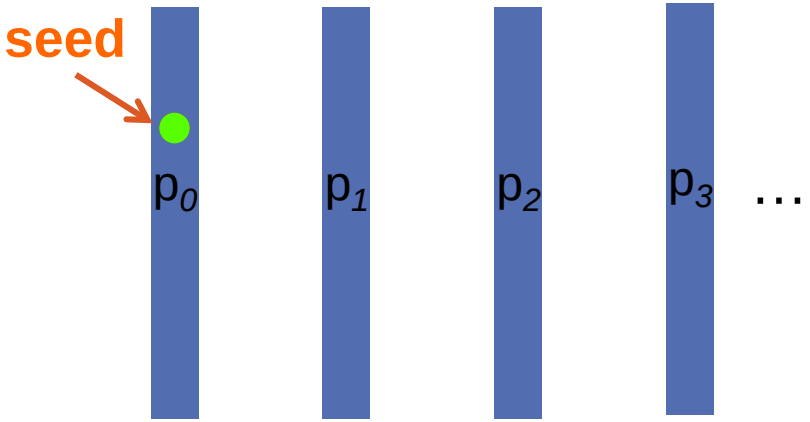
Computing HK

1. Pre-compute “push” thresholds
2. Do “push” on all entries above threshold

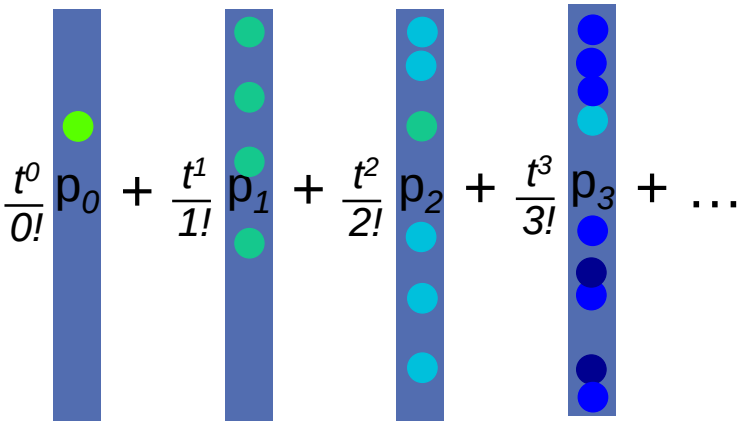
Algorithm Intuition

Computing HK given parameters t, ε, seed s

Starting from here…



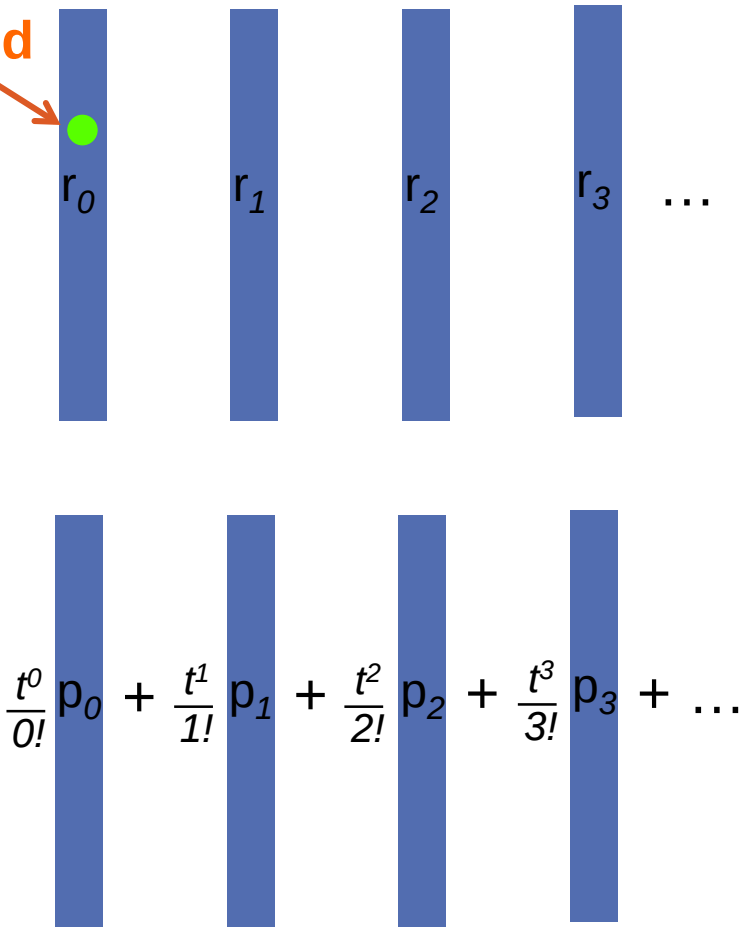
How to end up here?



Algorithm Intuition

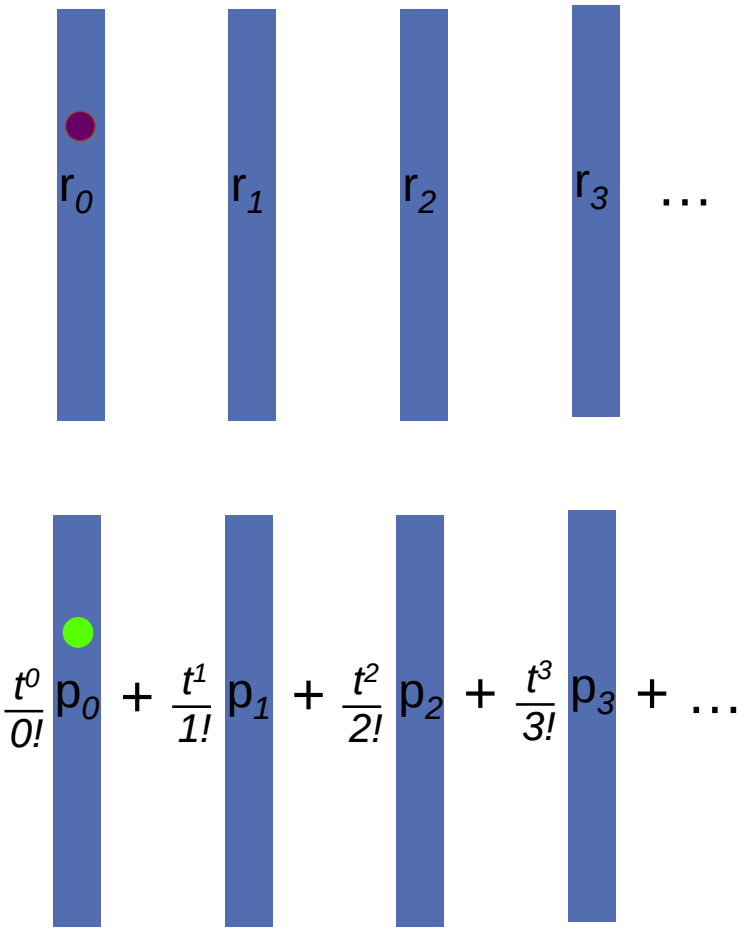
Begin with mass at seed(s) in a “residual” staging area, r_0

The residuals r_k hold mass that is unprocessed – it’s like *error*



Push Operation

push – (1) remove entry in r_k,
(2) put in p,



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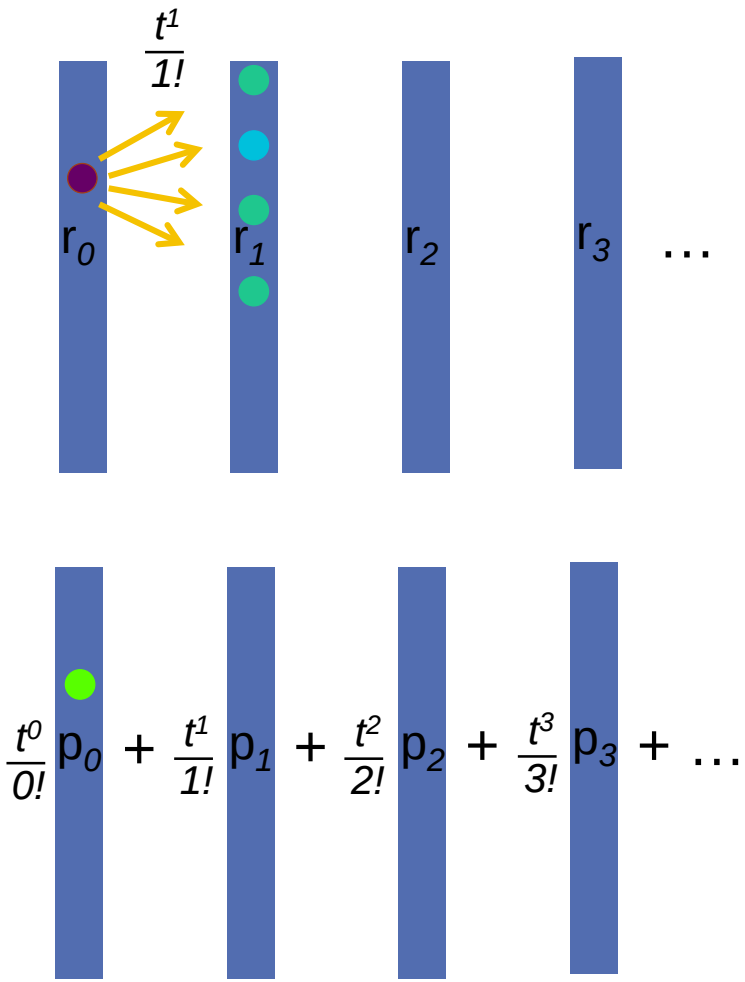

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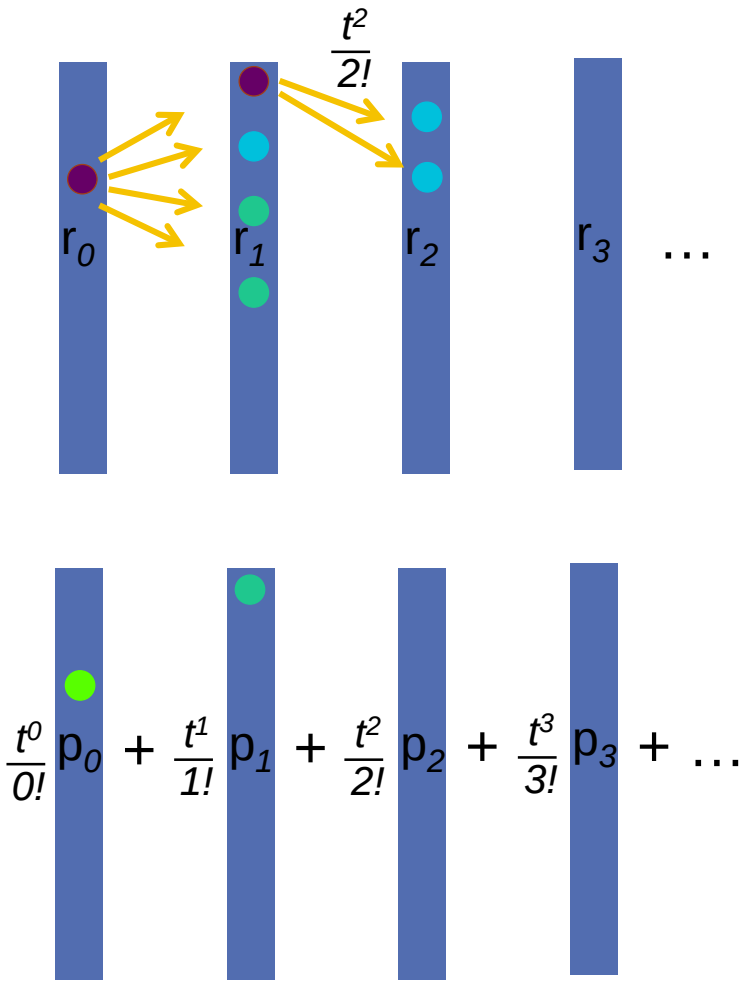

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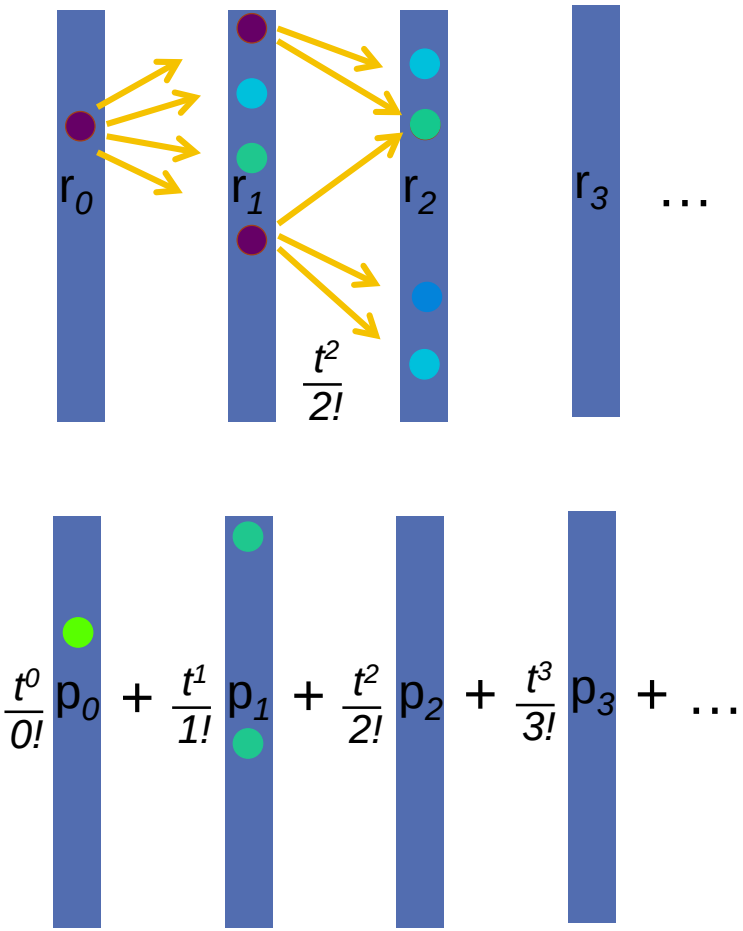

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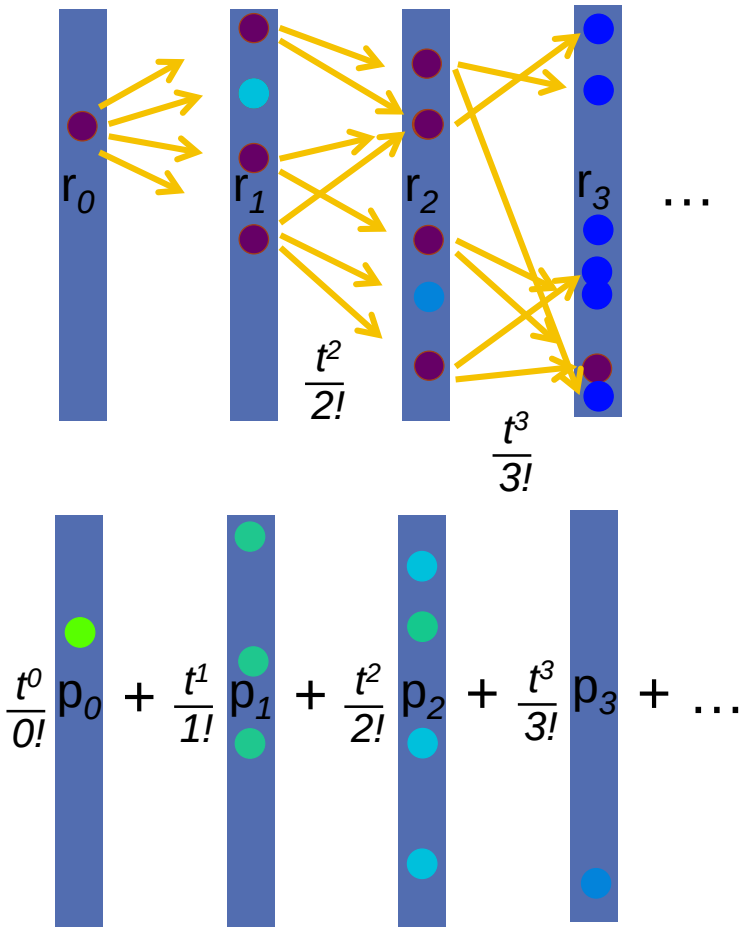
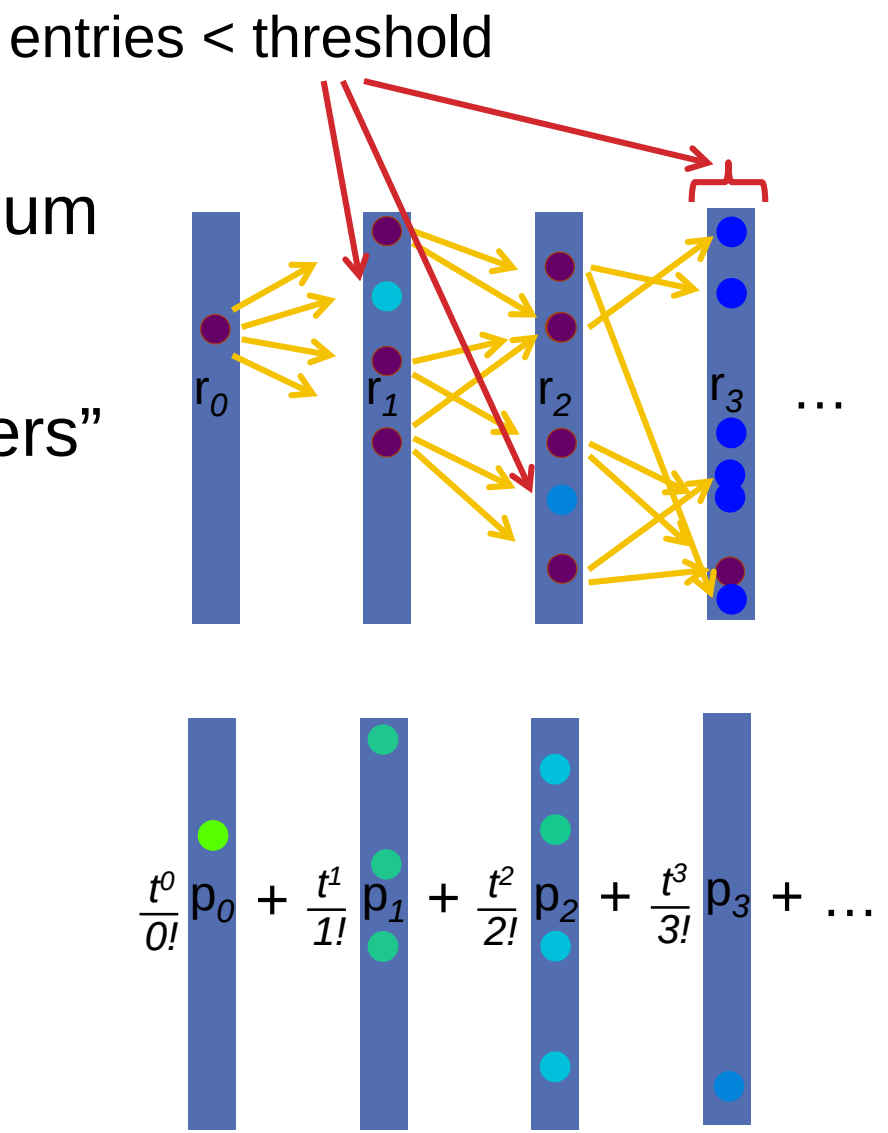


Thresholds

ERROR equals weighted sum of entries left in r_k

→ Set threshold so “leftovers” sum to < ε



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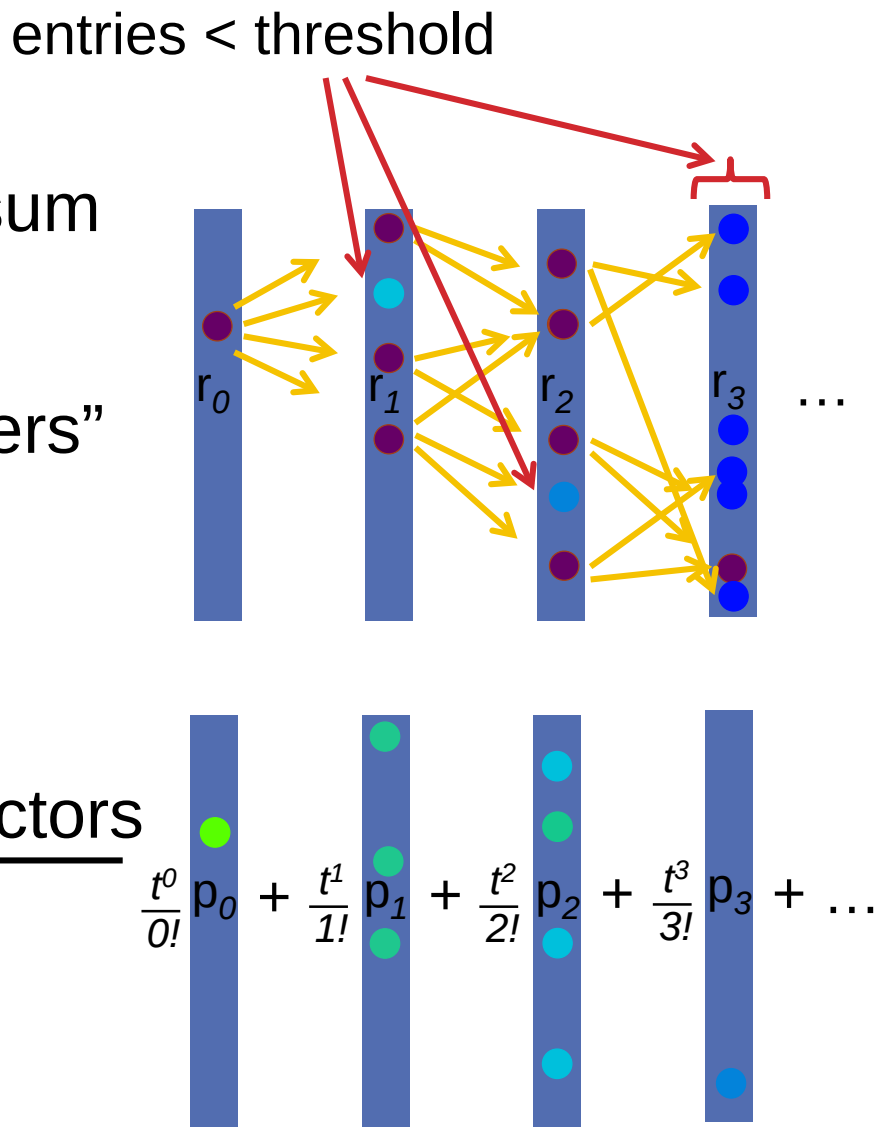
Threshold for stage r_k is

$$\frac{\text{sum of remaining scale factors}}{\text{prior scaling factor}}$$



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Communities in Real-world Networks

Given a seed in an unidentified real-world community, how well can HK and PR describe that community?

Measure quality using F_1-measure.

Graph	$\lvert V\rvert$	$\lvert E\rvert$
amazon	330 K	930 K
dblp	320 K	1 M
youtube	1.1 M	3 M
lj	4 M	35 M
orkut	3.1 M	120 M
friendster	66 M	1.8 B

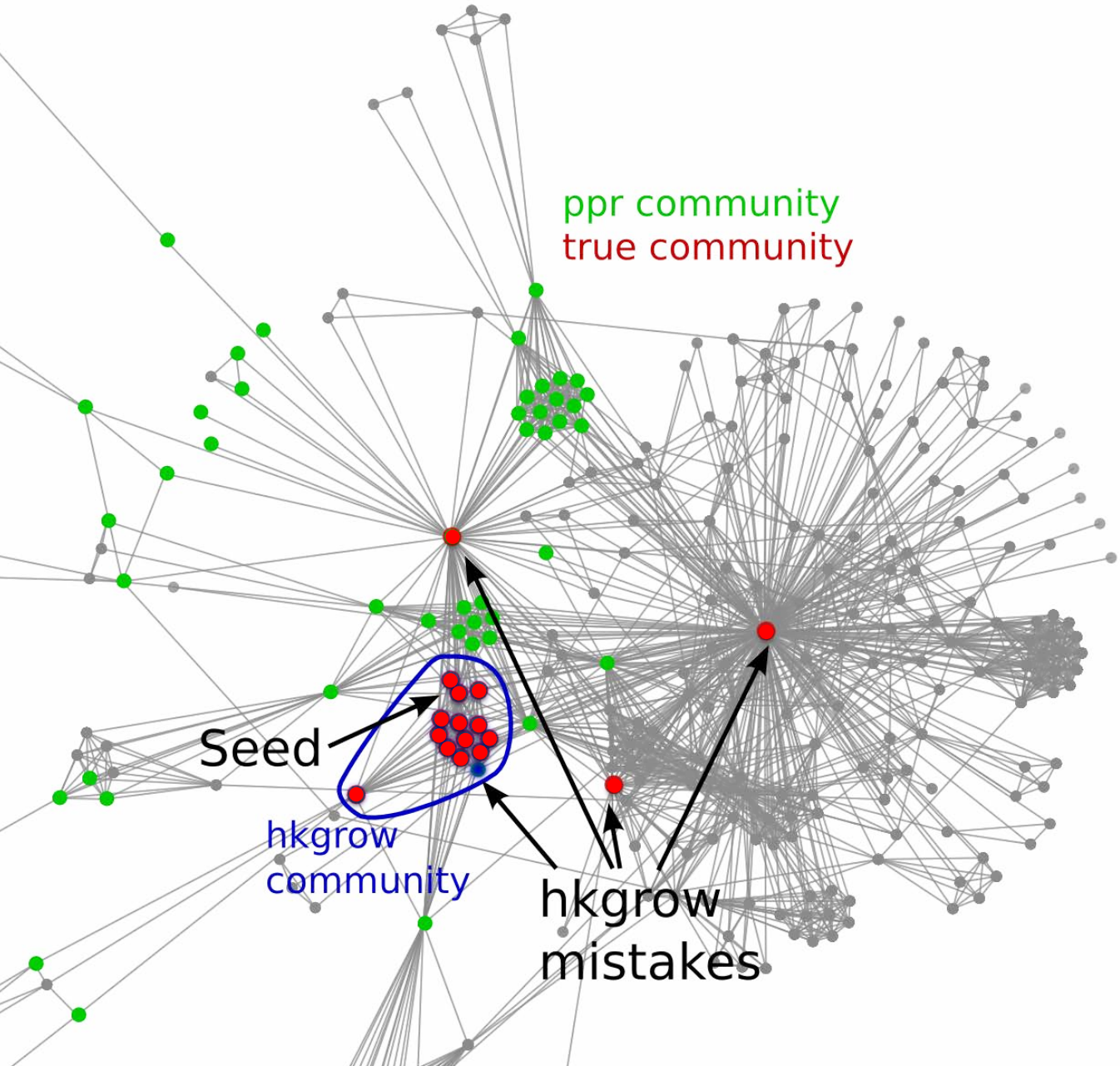
Datasets from SNAP collection [Leskovec]

F_1-measure is the harmonic mean of

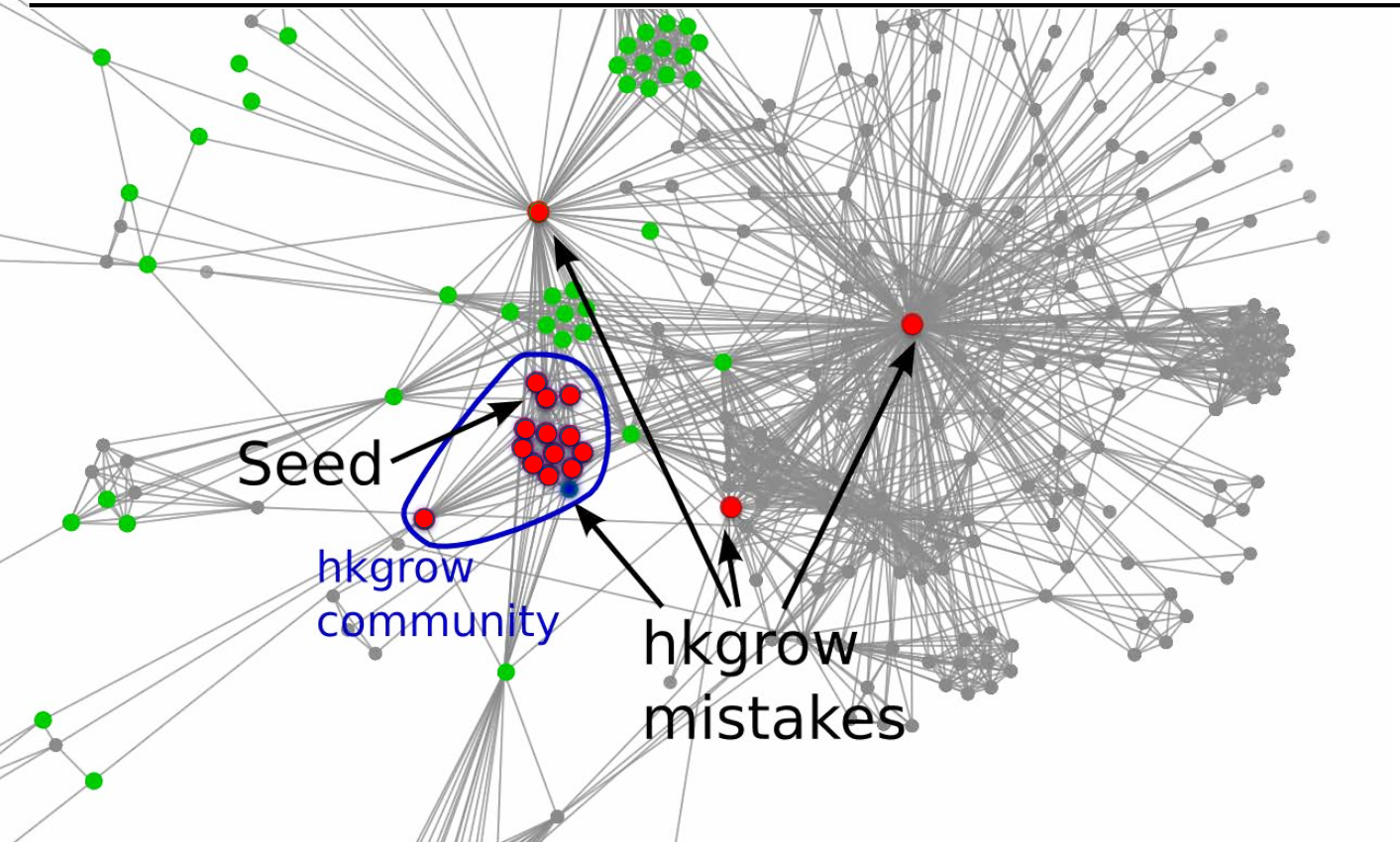
$$\text{precision} = \frac{\text{\# correct guesses}}{\text{\# total guesses}}$$

and

$$\text{recall} = \frac{\text{\# answers you get}}{\text{\# answers there are}}$$


ppr community
true community
Seed
hkgrow
community
hkgrow
mistakes

data	F_1		precision		set size		comm
	HK	PR	HK	PR	HK	PR	size
amazon	0.325	0.140	0.244	0.107	193	15293	495



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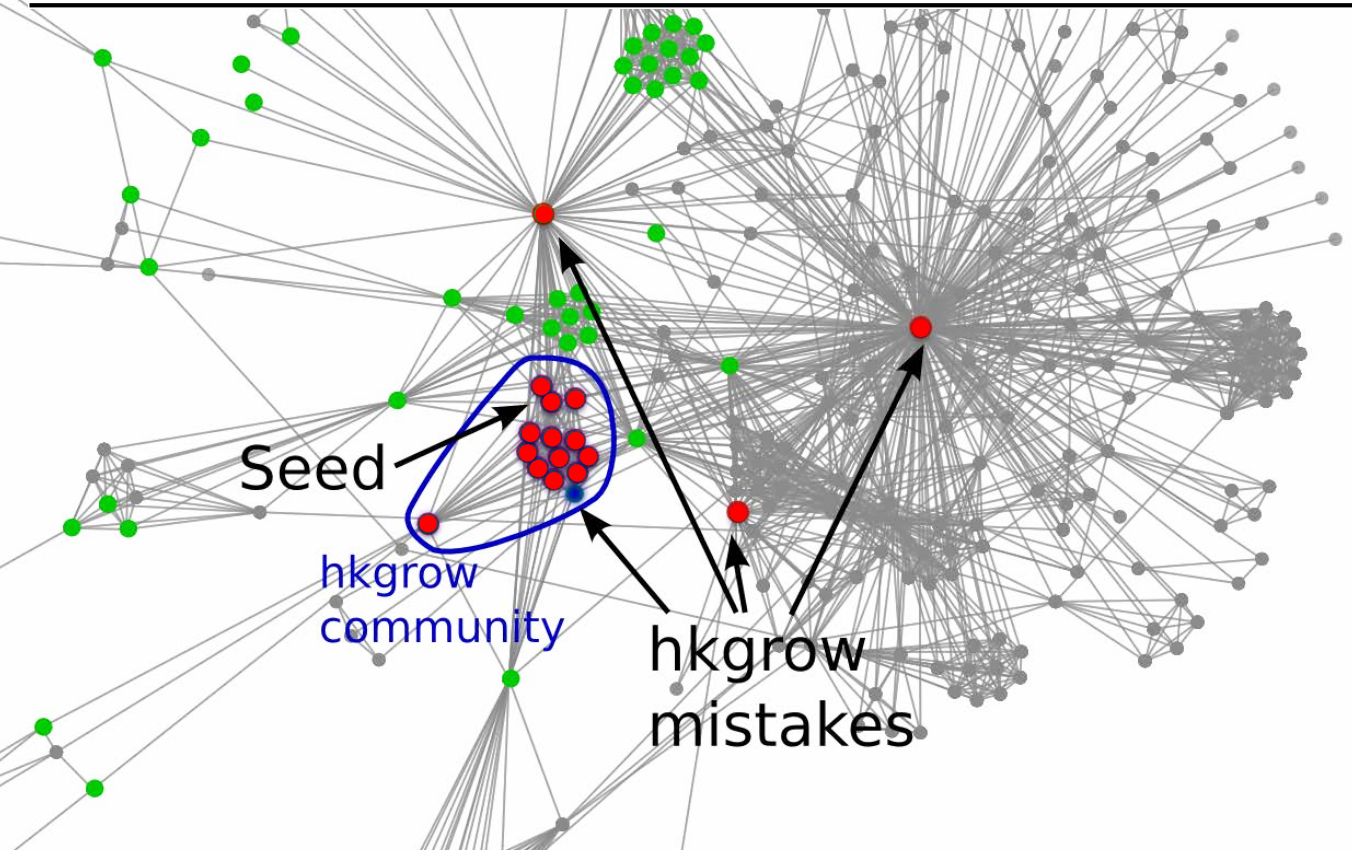
dblp	0.257	0.115	0.208	0.081	44	16026	1429
youtube	0.177	0.136	0.135	0.098	1010	6079	1615
lj	0.131	0.107	0.102	0.086	283	738	662
orkut	0.055	0.044	0.036	0.031	537	1989	4526
friendster	0.078	0.090	0.066	0.075	229	333	724



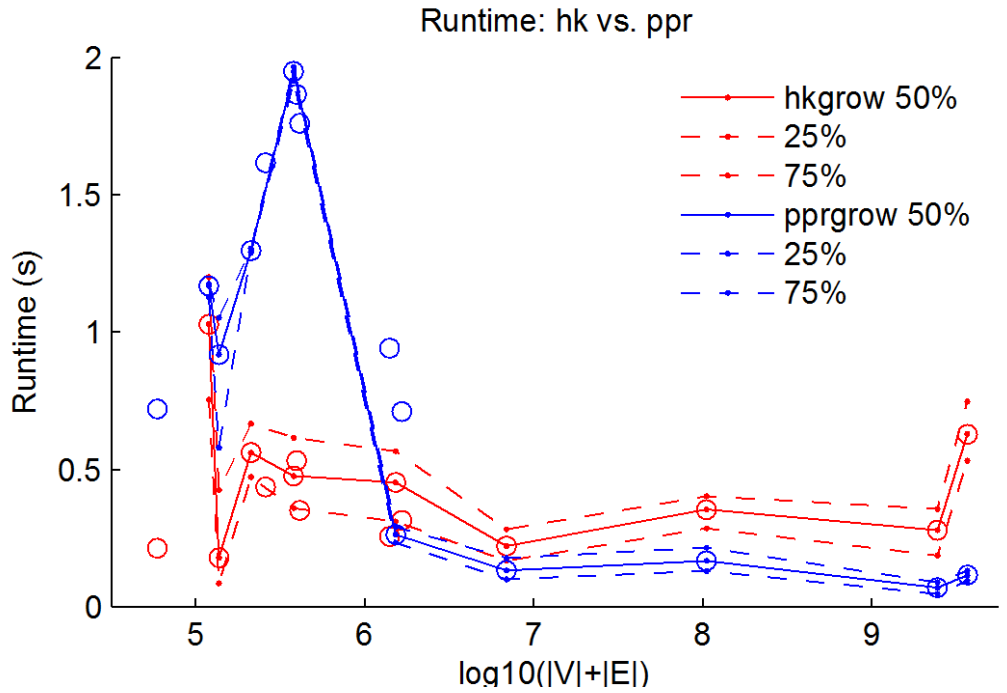
PR achieves high recall by "guessing" a huge set

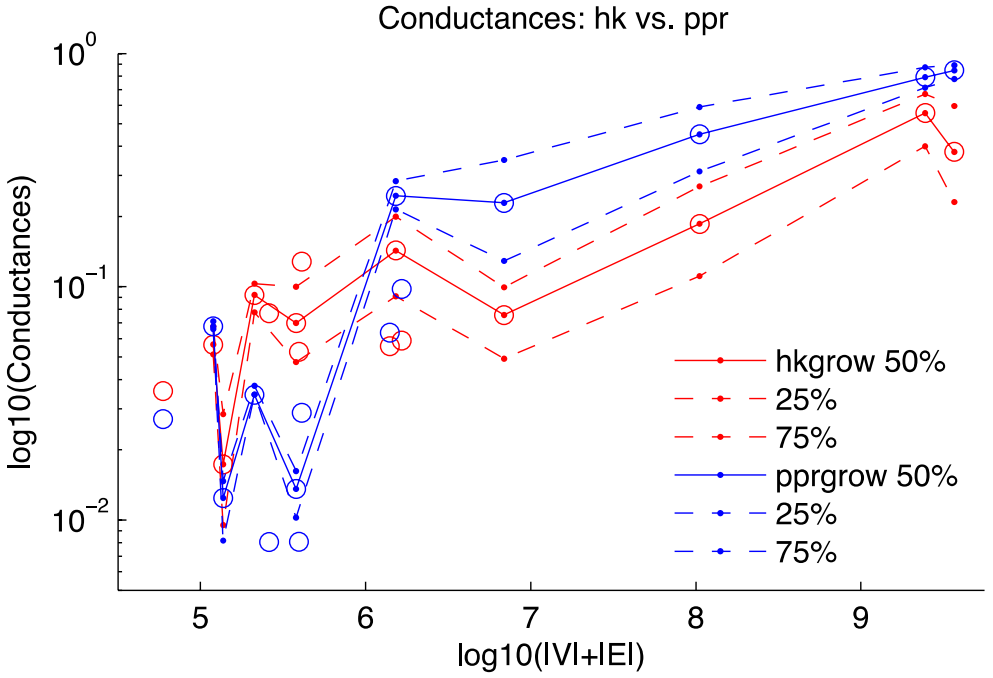
HK identifies a tighter cluster, so attains better precision

Runtime & Conductance

HK is comparable in runtime and conductance.

As graphs scale, the diffusions' performance becomes even more similar.





Code, references, future work

Code available at

`http://www.cs.purdue.edu/homes/dgleich/codes/hkgrow`

Ongoing work

- generalizing to other diffusions
- simultaneously compute multiple diffusions

Questions or suggestions? Email Kyle Kloster at `kkloste-at-purdue-dot-edu`