

# A Large Scale Study of the Small Sample Performance of Random Coefficient Models of Demand

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# Introduction

# Objectives

This talk's objectives:

- ▶ Discuss Monte Carlo experiments to characterize properties of Berry, Levinsohn, and Pakes (1995) (BLP) estimator:
  - ▶ 'BLP' characteristics IV vs. cost shifter IV
  - ▶ Asymptotics as  $J \rightarrow \infty$  and  $T \rightarrow \infty$
  - ▶ Finite sample bias
  - ▶ Bias of different quadrature methods
- ▶ Demonstrate power of modern software engineering tools to answer practical econometric questions, such as behavior of an estimator:
  - ▶ PADS cluster + parameter sweep
  - ▶ C++ and Eigen for implementing high performance code
  - ▶ State of the art BLP implementation
- ▶ Generate data from structural model

# Overview

Introduction

Estimation Infrastructure

Data Generation

Experiments & Results

# Estimation Infrastructure

# Overview of Infrastructure

This project depends heavily on modern software engineering and numerical methods:

- ▶ Robust and speedy implementation of BLP estimation code
- ▶ Robust and speedy implementation of code to generate data
- ▶ PADS Cluster
- ▶ Data analysis scripts (R, Python, BASH)

# A Robust BLP Implementation

Uses current best practice to create a robust BLP implementation:

- ▶ Best optimization strategy: MPEC (Su & Judd, 2011)
- ▶ Best quadrature rules: SGI (Skrainka & Judd, 2011)
- ▶ Modern solver: SNOPT (Gill, Murray, & Saunders, 2002)
- ▶ Numerically robust:
  - ▶ C++
  - ▶ Eigen, a cutting edge template library for linear algebra – at least as fast as Intel MKL!
  - ▶ Higher precision arithmetic (long double)
  - ▶ Analytic derivatives

# Finding a Global Optimum

Even with MPEC, BLP is a difficult problem to solve reliably:

- ▶ Often very flat – perhaps even non-convex!
- ▶ Used 50 starts per replication:
  - ▶ Some did not converge, especially for larger  $T$  and  $J$
  - ▶ Some did not satisfy feasibility conditions, especially for larger  $T$  and  $J$ , despite generating initial guesses which satisfied constraints
- ▶ Restarted every successful start to make sure it converged to the same point
- ▶ Performed for both BLP and cost shifter IV



# PADS Cluster

PADS cluster provides High Throughput Computing (HTC):

- ▶ PBS Job Manager facilitates parameter sweeps, an easy technique for parallelizing work which is independent
- ▶ Uses scripts to generate data or estimate code for come chunk of runs (1 to 50) per task
- ▶ Chunk jobs together for shorter jobs to spread scheduler overhead across more jobs
- ▶ Could never estimate BLP  $> 300,000$  times on my laptop!

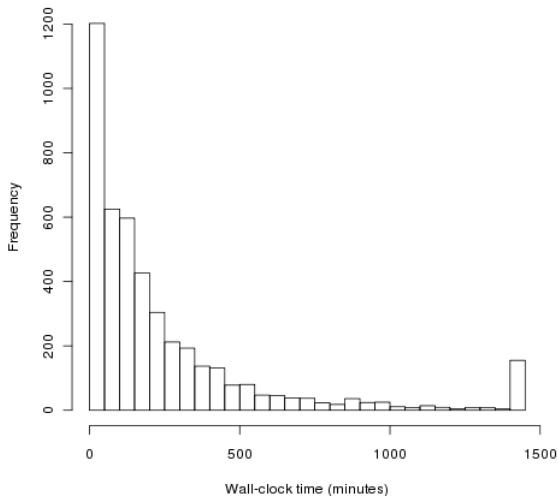
# Parallelization

*Parameter Sweep* provides easy parallelization:

- ▶ Each job:
  - ▶ Estimates one or more replication and starting value
  - ▶ Short runs are chunked to minimize scheduler overhead
  - ▶ Independent of all other jobs
  - ▶ Identified by an index it receives from *Job Manager* → use to determine which starts to run
  - ▶ Writes results to several output files
- ▶ Job manager logs whatever the job writes to standard output and standard error to .o and a .e files
- ▶ A separate program computes bias, RMSE, and other statistics from the output files
- ▶ Impose time limit to terminate slow or runaway jobs

# Job Times

Distribution of Runtimes for T=50 and J=100 with BLP IV



# Computational Cost

Some statistics about these experiments:

- ▶ > 85,656 CPU-hours
- ▶ > 27,969 jobs
- ▶ 16 experiments  $\times$  100 replications  $\times$  50 starts  $\times$  2 restarts  $\times$  2 IV types = 320,000 estimations of BLP!

# Data Generation

# Data Generation

Data must be generated from a structural model:

- ▶ Armstrong (2011):
  - ▶ Proves general result that for logit, nested logit, random coefficients, BLP, etc., these models are only identified as  $J \rightarrow \infty$  with cost shifters.
  - ▶ I.e., BLP is unidentified with BLP instruments in large markets!
  - ▶ Corrects Berry, Linton, Pakes (2004)
  - ▶ Shows that you must generate data from a structural model or the data will not behave correctly asymptotically
- ▶ Note: each firm must produce at least two products to use BLP instruments

# Intuition

Intuition comes from logit:

- ▶ FOC:  $0 = s_j + (p_j - c_j) \frac{\partial s_j}{\partial p_j}$  or  $p_j = c_j - \frac{s_j}{\partial s_j / \partial p_j}$
- ▶ This simplifies to:  $p_j = c_j + \frac{1}{\alpha_{price} (1 - s_j)}$
- ▶ As  $J \rightarrow \infty$ ,  $s_j \rightarrow 0$  so product characteristics drop out of pricing equation!

# Implementation

Generating synthetic data is more difficult than estimating BLP:

- ▶ Must generate from a structural model (Armstrong, 2011)
- ▶ Used same software technologies (C++, Eigen, higher precision arithmetic, C++ Standard Library) as BLP code
- ▶ Used PATH (Ferris, Kanzow, & Munson, 1999) to solve for Bertrand-Nash price equilibrium
  - ▶ Hard for large  $J$  because dense
  - ▶ Hard to solve because BLP FOCs are highly non-linear
  - ▶ Gaussian root finding is  $O(N^3) \Rightarrow$  root finding is slow
  - ▶ Divided FOCs by market shares to facilitate convergence



# Experiments & Results

# Experiments

The study performs the following experiments:

- ▶ Asymptotics
- ▶ Finite sample bias
- ▶ Bias of different quadrature methods

# Design

Experiments consist of:

- ▶ Fixed DGP parameters  $(\beta, \Sigma)$  for all experiments
- ▶  $T = \{1, 10, 25, 50\}$
- ▶  $J = \{12, 24, 48, 100\}$
- ▶ 100 replications per experiment
- ▶ Two instrumentation strategies (BLP, Cost)
- ▶ Estimation time ranges from seconds to more than 24 hours

## Results: Overview

Bottom line: there is pronounced and persistent finite sample bias:

- ▶ Traditional BLP instruments:
  - ▶ Biased point estimates and elasticities
  - ▶ Bias always in one direction!
  - ▶  $T$  and  $J$  not yet large enough for asymptotics to work
- ▶ Cost shifter instruments: better than BLP instruments but finite sample bias still present for most parameters
- ▶ Numerical problems increase with  $T$  and  $J$
- ▶ pMC is more biased than SGI quadrature
- ▶ Fundamental problem: 'a few, weak instruments'

## Results: Price Parameter $\hat{\theta}_{13}$ – BLP IV

T	J	Bias	Mean Abs Dev	RMSE	!CI <sup>95</sup>
1	12	-2	3	5.7	0
1	24	-0.72	1.9	3.2	0
1	48	-0.52	1.9	3	0
1	100	-0.57	1.7	2.3	0
10	12	-1.7	2.6	6	1
10	24	-0.65	2	3.6	0
10	48	-0.64	1.9	3.2	0
10	100	-0.83	2	3.9	0
25	12	-0.62	1.9	3.1	3
25	24	-0.96	2.3	3.7	1
25	48	-1.3	2.8	7.6	0
25	100	-0.95	2.1	3.7	0
50	12	-0.39	1.6	2.7	1
50	24	-1.2	2.5	5.4	1
50	48	-1.2	2.2	5.2	0
50	100	-0.63	1.9	3	0

## Results: Price Parameter $\hat{\theta}_{13}$ – Cost IV

T	J	Bias	Mean Abs Dev	RMSE	$!CI^{95}$
1	12	-0.38		1.5	1
1	24	-0.05		1.3	0
1	48	0.012	0.99	1.2	2
1	100	0.057	0.72	0.88	0
10	12	-0.62	1.3	2	0
10	24	-0.18	0.8	1.3	0
10	48	-0.15	0.62	0.86	0
10	100	-0.027	0.39	0.52	1
25	12	-0.38	1	1.6	0
25	24	-0.3	0.73	0.98	0
25	48	-0.11	0.45	0.63	0
25	100	-0.033	0.25	0.33	0
50	12	-0.081	0.79	1.1	0
50	24	-0.22	0.55	1	0
50	48	-0.026	0.28	0.4	0
50	100	0.003	0.19	0.26	0

# Results: Scale of Product Characteristic $\hat{\theta}_{21}$ – BLP IV

T	J	Bias	Mean Abs Dev	RMSE	$!CI^{95}$
1	12	3.1	3.9	7.3	0
1	24	4.8	5.3	10	0
1	48	5.7	6.5	23	0
1	100	2.1	2.7	5.2	0
10	12	3.5	4.1	8.1	0
10	24	2.9	3.3	7.1	1
10	48	4.7	5.1	9.9	0
10	100	1.7	2.2	6.7	0
25	12	3.6	4.1	7	0
25	24	3.3	3.6	7.2	0
25	48	2.9	3.3	7.4	0
25	100	2.2	2.7	6.7	0
50	12	2.5	3	5.6	0
50	24	4.1	4.5	11	0
50	48	1.5	2	3.6	0
50	100	2.7	3.1	7.4	0

# Results: Scale of Product Characteristic $\hat{\theta}_{21}$ – Cost IV

T	J	Bias	Mean Abs Dev	RMSE	$!CI^{95}$
1	12	7.4	8.2	13	0
1	24	8.4	8.8	14	0
1	48	7.2	8.1	13	0
1	100	6.2	7.1	12	1
10	12	0.8	1.8	2.7	0
10	24	4	4.9	11	1
10	48	2.9	3.8	6.6	0
10	100	5.9	6.8	11	0
25	12	1.5	2.3	3.4	0
25	24	3.6	4.4	7.7	0
25	48	3.7	4.6	7	1
25	100	6.2	7	11	0
50	12	0.97	2	3.1	0
50	24	3.9	4.6	12	0
50	48	3.6	4.2	6.3	1
50	100	5.9	6.6	12	0



## Results: Elasticities – BLP IV

T	J	Bias	Mean Abs Dev	Med Abs Dev	RMSE
1	12	-0.77	2.2	0.94	4.9
1	24	-0.095	1.5	0.77	3.3
1	48	-0.082	1.6	0.91	2.7
1	100	-0.39	1.5	0.98	2.5
10	12	-0.5	1.7	0.81	3.3
10	24	-0.57	1.7	0.83	3.3
10	48	-0.16	1.5	0.97	2.2
10	100	-0.53	1.7	0.93	3.3
25	12	-0.3	1.4	0.94	2.7
25	24	-0.72	1.8	1.1	3
25	48	-0.87	2.2	1.1	4.9
25	100	-0.61	1.7	0.97	2.7
50	12	-0.43	1.5	0.94	2.6
50	24	-0.77	1.9	0.91	3.8
50	48	-0.97	1.9	1.1	4
50	100	-0.56	1.8	1.1	2.9

## Results: Elasticities – Cost IV

T	J	Bias	Mean Abs Dev	Med Abs Dev	RMSE
1	12	0.059	0.86	0.52	1.4
1	24	0.17	0.83	0.55	1.3
1	48	0.11	0.85	0.6	1.3
1	100	-0.59	1.3	0.43	60
10	12	-0.098	0.69	0.48	1
10	24	-0.095	0.52	0.33	0.82
10	48	-0.15	0.48	0.28	4.2
10	100	-0.072	0.3	0.19	0.54
25	12	-0.23	0.56	0.38	0.83
25	24	-0.22	0.48	0.34	0.69
25	48	-0.062	0.3	0.19	0.45
25	100	-0.16	0.3	0.13	0.68
50	12	-0.27	0.54	0.32	0.92
50	24	-0.32	0.46	0.22	1
50	48	-0.1	0.2	0.12	0.33
50	100	-0.15	0.24	0.098	0.57

## Results: Solver Convergence

SNOPT has increasing difficulty finding an optimum as the number of markets and products increase:

- ▶ Most common problem: cannot find a feasible point
- ▶ Other problems:
  - ▶ Hits iteration limit
  - ▶ Not enough real storage
  - ▶ Singular basis

## Results: pMC vs SGI

	Bias		Mean Abs Dev		RMSE	
	SGI	pMC	SGI	pMC	SGI	pMC
$\theta_{11}$	0.96	12.34	2.29	13.25	4.00	28.92
$\theta_{12}$	0.02	-0.13	0.52	0.38	0.94	0.48
$\theta_{13}$	-0.28	-0.38	1.47	1.21	3.01	1.51
$\theta_{21}$	22.57	128.22	23.01	128.24	81.76	253.87
$\theta_{22}$	0.02	-0.04	0.12	0.16	0.19	0.20
$\theta_{23}$	0.08	0.64	0.36	0.75	0.75	0.90

**Table:** Comparison of bias in point estimates : SGI vs. pMC for T=2 markets and J=24 products with 165 nodes.

## Next Steps

This infrastructure can be used to solve several related problems:

- ▶ Rerun experiments in Skrainka & Judd (2011) on a larger scale and compute bias for different rules
- ▶ Evaluate sensitivity of results to DGP
- ▶ Evaluate impact of strong and weak instruments
- ▶ Bootstrap BLP to study where asymptotic GMM standard errors are valid
- ▶ Evaluate other estimation approaches such as Empirical Likelihood (Conlon, 2010)
- ▶ Compute with (approximations to) optimal instruments (Reynaert & Verboven, 2012)

# Conclusion

Developed infrastructure to test BLP estimator:

- ▶ Characterize estimator's bias for a range of markets and number of products
- ▶ Computed bias for BLP and Cost IV
- ▶ Demonstrated power of modern HTC + Monte Carlo experiments to answer questions where (econometric) theory has failed to produce an answer.
- ▶ Shown that these resources are easily accessible to economists