Modeling Uncertainty

in the earth sciences

or, why I fear the priors

and nonlinearities

and model coupling

and numerical error propagation

Based on Jef Caers book with same title

http://uncertaintyES.stanford.edu

This tutorial is more about questions than answers

How can we possibly know what we don't know?

Why would one model of uncertainty be any better than another?

What types of uncertainty are there?

What are ways in which people try to get anywhere?

And many other unanswered questions

Uncertainty quantification: a philosophy and a thorny challenging issue particularly for the nonlinear problems we encounter in practice

Why I am interested in uncertainty

Performance prediction and optimization of reservoir processes:

Highly nonlinear processes

Multi-scale, with iffy subgrid scale models that are calibrated

Multi-objectives

sensitivity of objectives to data

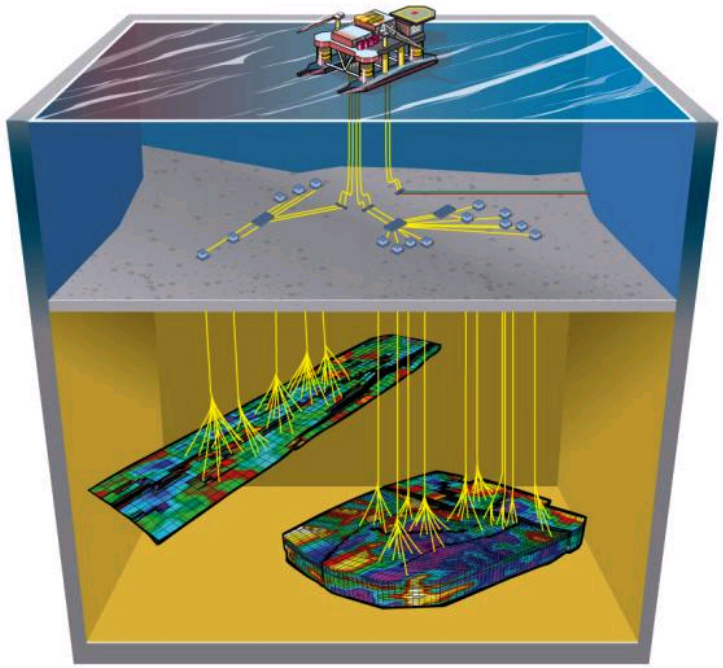
Data and model uncertainties

Various classes of observations

Numerical models prone to error

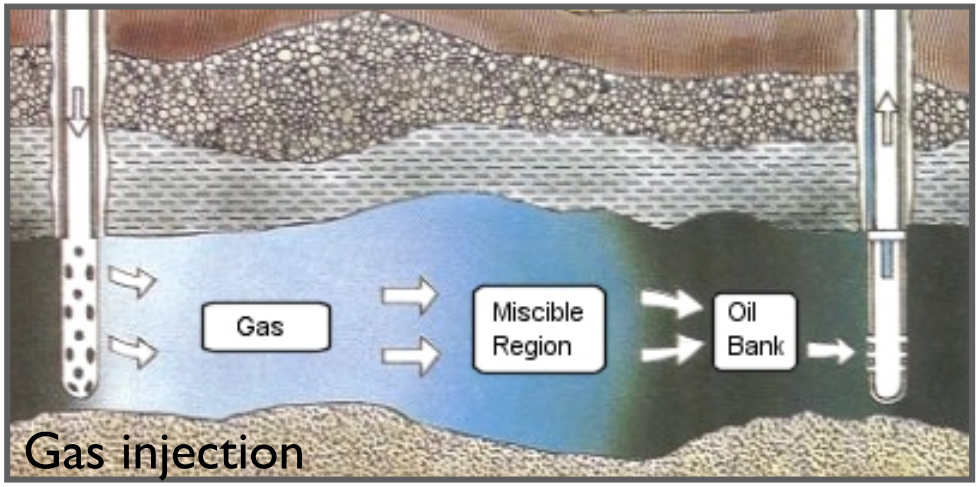
Error behavior not always known

Unknown error propagation in model

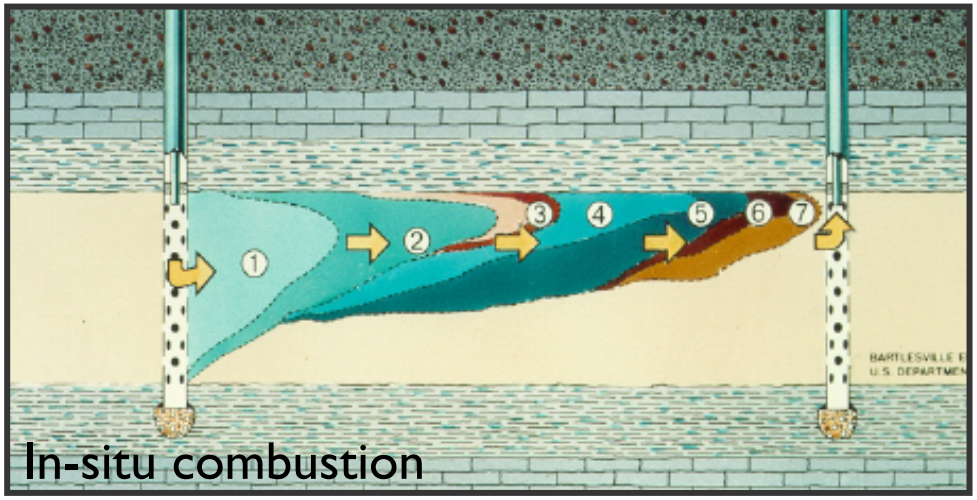


Highly desirable to do more than just guess what decisions are best

Even deterministic models are challenging



Gas injection



In-situ combustion

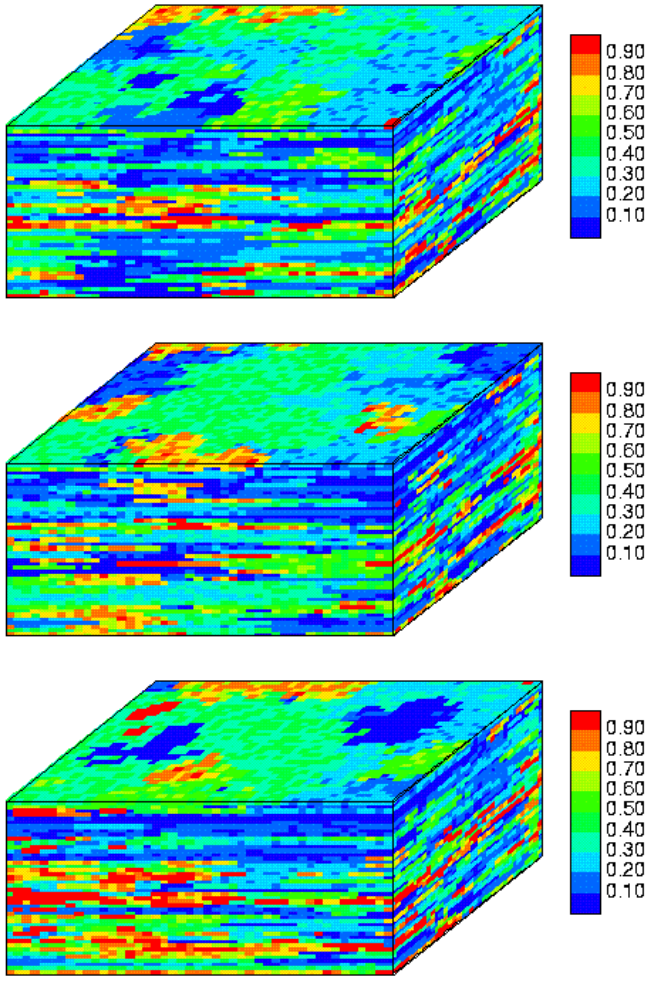
Multi-scale
Strongly nonlinear
Complex domains

Unstructured data
Ill-conditioned matrices

$O(10^7 - 10^8)$ unknowns
not uncommon

So models are:
- Large
- Nonlinear
- Ill-behaved
- highly sensitive to perturbations

A SOURCE OF UNCERTAINTY: GEOLOGY



Have some prior knowledge of structure
Create realizations that fit the observations

Make decisions under these and other uncertainties

Uncertainty – some philosophizing

Uncertainty: incomplete understanding of what we like to quantify

How do we quantify what we do not know?

Quantifying what we do not know is subjective and can not be tested

There is no true uncertainty

There are only models of uncertainty

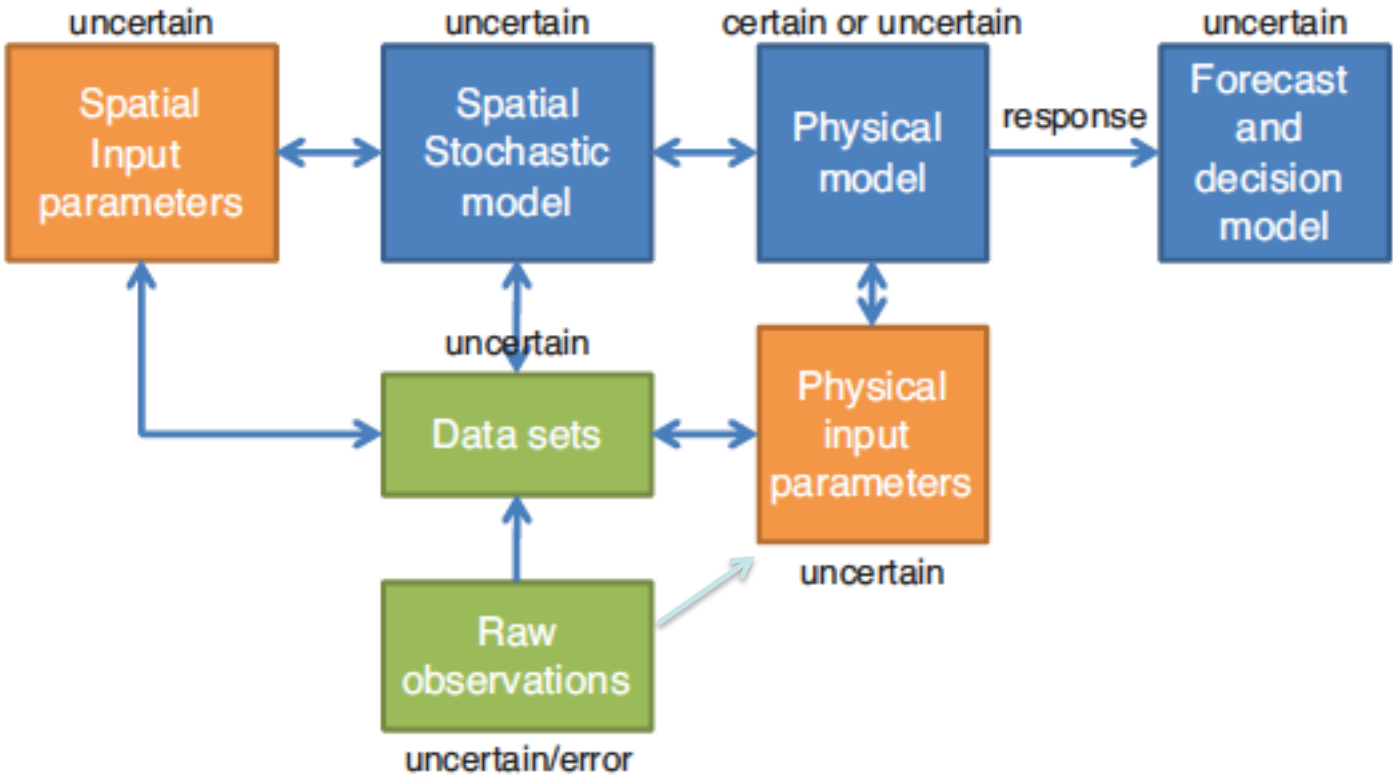
And ultimately they are only as good as the intuition of the designer

Sources of uncertainty

Typically, two sources of uncertainty are considered:

1. As a result of process randomness (inherent randomness in nature)
2. As a result of limited understanding
   - Measurement errors
   - Insufficient data to understand behavior (miss out on features)
   - Subjective and different interpretations
   - Unknown processes, both physical and computational
   - Unmeasurable data

The big picture



Information and knowledge (experts) informs all of the above

Interaction/symbiosis, always, between data and models:

You process data in a way that reflects your knowledge

You create a model in a way that reflects your knowledge

Bayes can provide a framework

Given a model A of reality, what is the probability of data B (data)?

In other words, what is the *likelihood* $P(B \mid A)$ of B?

Given data B, what is the probability $P(A \mid B)$ of a model A (inverse modeling)?

Using that $P(A \, and \, B) = P(A \mid B)P(B) = P(B \mid A)P(A)$

we get $P(A \mid B) = \dfrac{P(B \mid A)\,P(A)}{P(B)}$

posterior likelihood priors

Example

A = profitable gas extraction from a shale reservoir

Suppose we know (from global data) that P(A) = 0.1, so P(~A) = 0.9

Let B = permeability > 0.5 Darcy

We know also that P(B|A) = 0.8, and P(B|~A) = 0.4

What is P(A|B)?

Here, P(A)=0.1, P(B|A)=0.8 and P(B) = P(B|A)P(A)+P(B|~A)P(~A)=

0.8*0.1 + 0.4*0.9

So, P(A|B) = 0.18

Bayes for complex models

A = earth model we are after (permeability, production numbers, ….)

B = all available and used data (in sets Bi)

P(A|B) is the desired uncertainty. It depends on some prior uncertainty P(A),

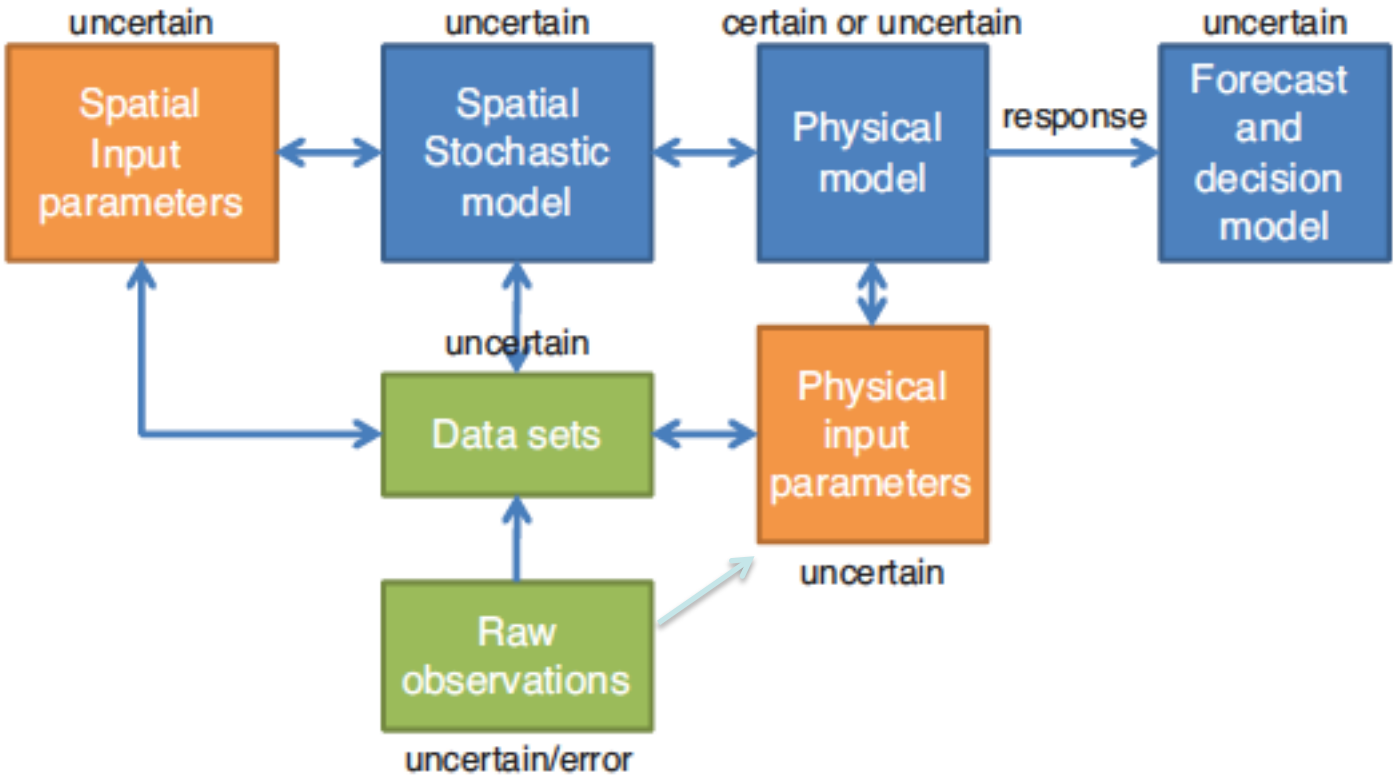
as well as the likelihood probability

Prior selection critical and completely up to the imagination of the modeler

Likelihood can often be determined from models

Choice of Bi typically done through sensitivity analyses

The big picture



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Falsification, not validation

Deterministic model matches data? No guarantee of truth

Maybe we could say that a model is "valid" if

- we cannot detect apparent flaws
- we have (some) consistency between model and observed data

Karl Popper: "We can never prove a model is correct,
we can only ever prove that it is false"

The prior consists of all possibilities imagined by human modeler + computer

The posterior includes only those possibilities that can not be falsified with data as modeled in the likelihood

Many more questions than answers

How to select priors?

Based on prior expertise modeler. How to quantify intuition/expertise?

Easy when systems are linear and data Gaussian

What data to be included?

Based on sensitivity analysis using prior models. Linearization dangerous

How do we ensure consistency between models and data?

Particularly if different experts generate different parts.

What does it mean to mismatch data?

Inherently, the overall design process is iterative, or should be

Musings on climate models

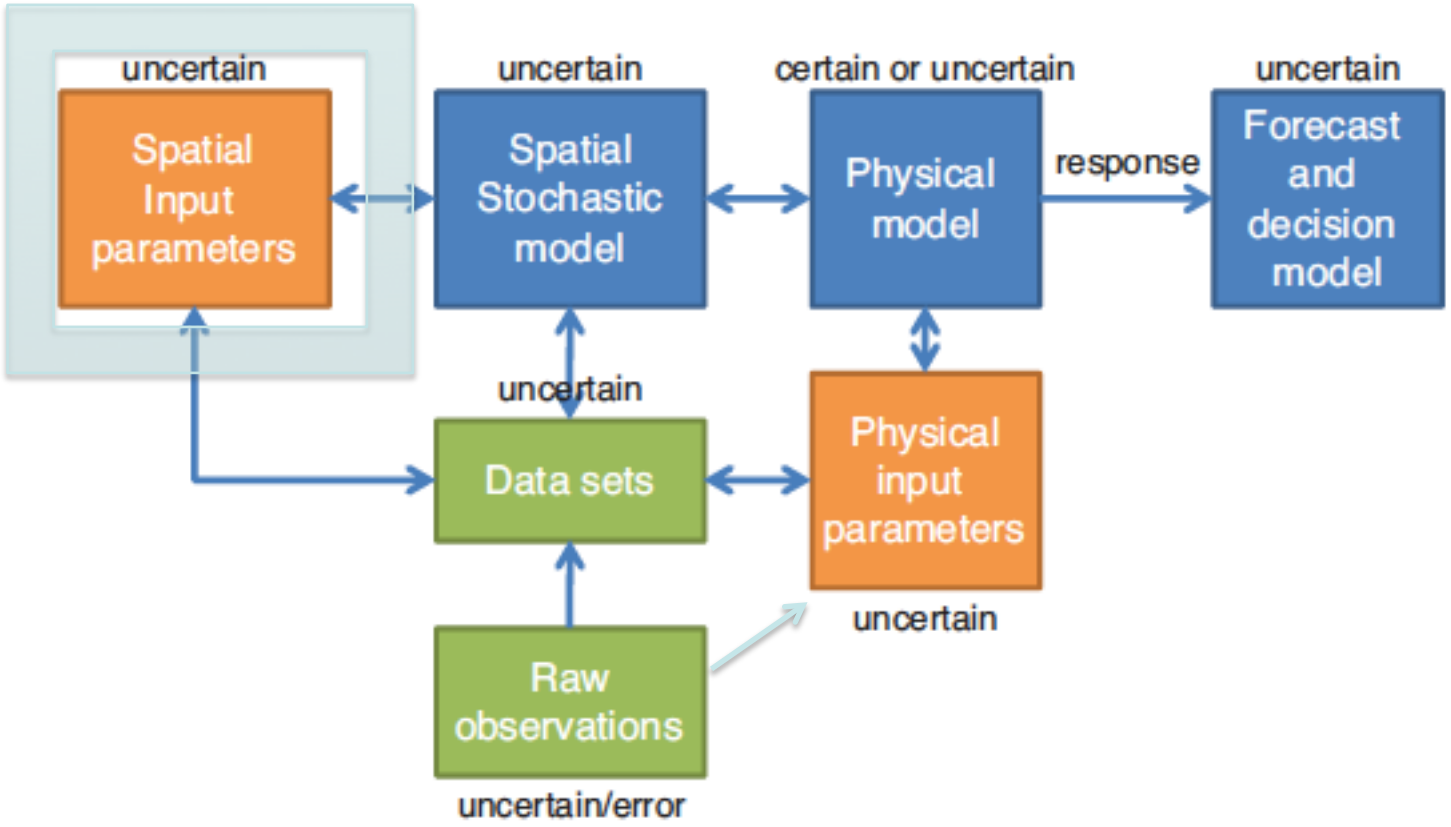
- Large, but still relatively coarse. Subgrid modeling key
- Coupling of O(10) models
- Raw observations require intense filtering and processing
- Subgrid models by definition local and imprecise (calibration)
- Numerical modeling very challenging (Skamarock)

How complex should we make it? As computer power grows, tendency is to include more and more models (physics)

Simple models are more easily falsifiable

Complex models include more parameters that all introduce uncertainty and stronger dependence on the priors

Let's have a look at spatial models

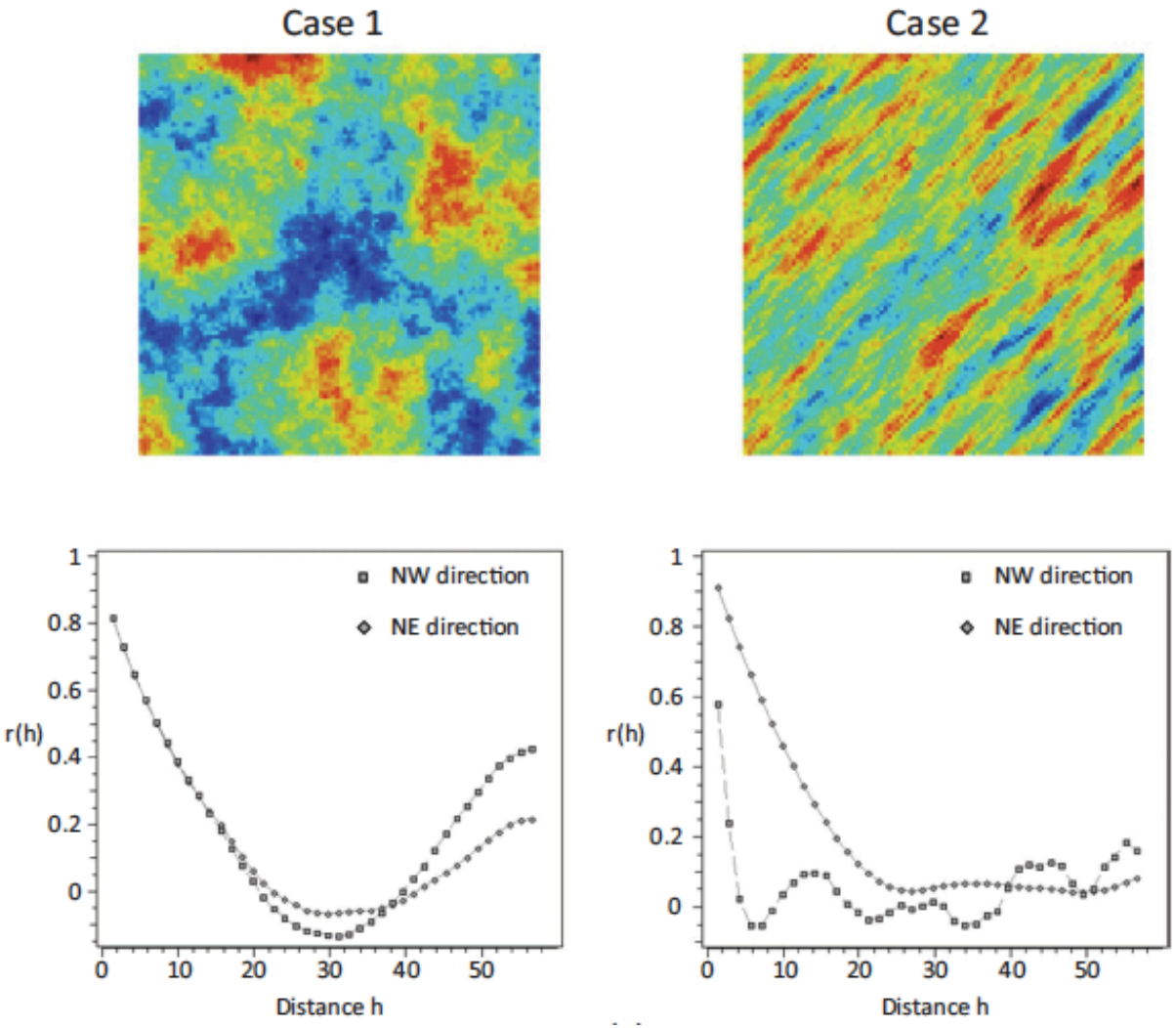


Critical for earth models

Modeling spatial continuity

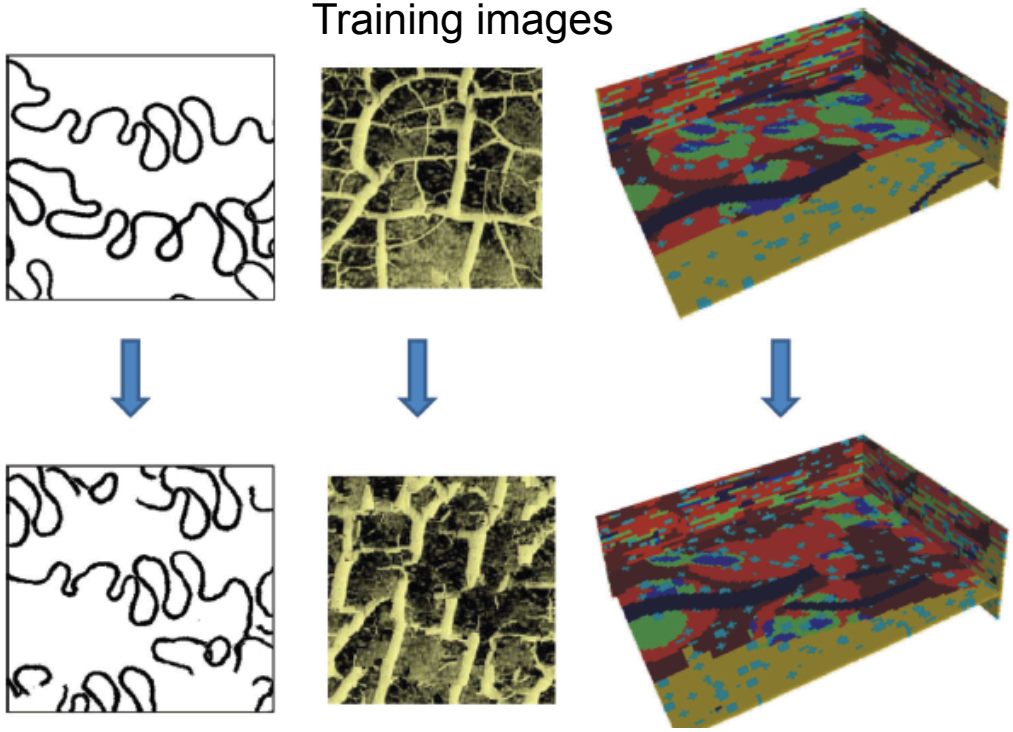
- Variograms – crude, takes samples values 2 at a time only
- Object models – introduce realistic shapes (objects) given by a set of parameters
- 3D training images – conceptual model rendering of major variations

Variograms



Measures geological distance between 2 points in n-dimensions (crude)

An Earth sciences approach: 3D training images



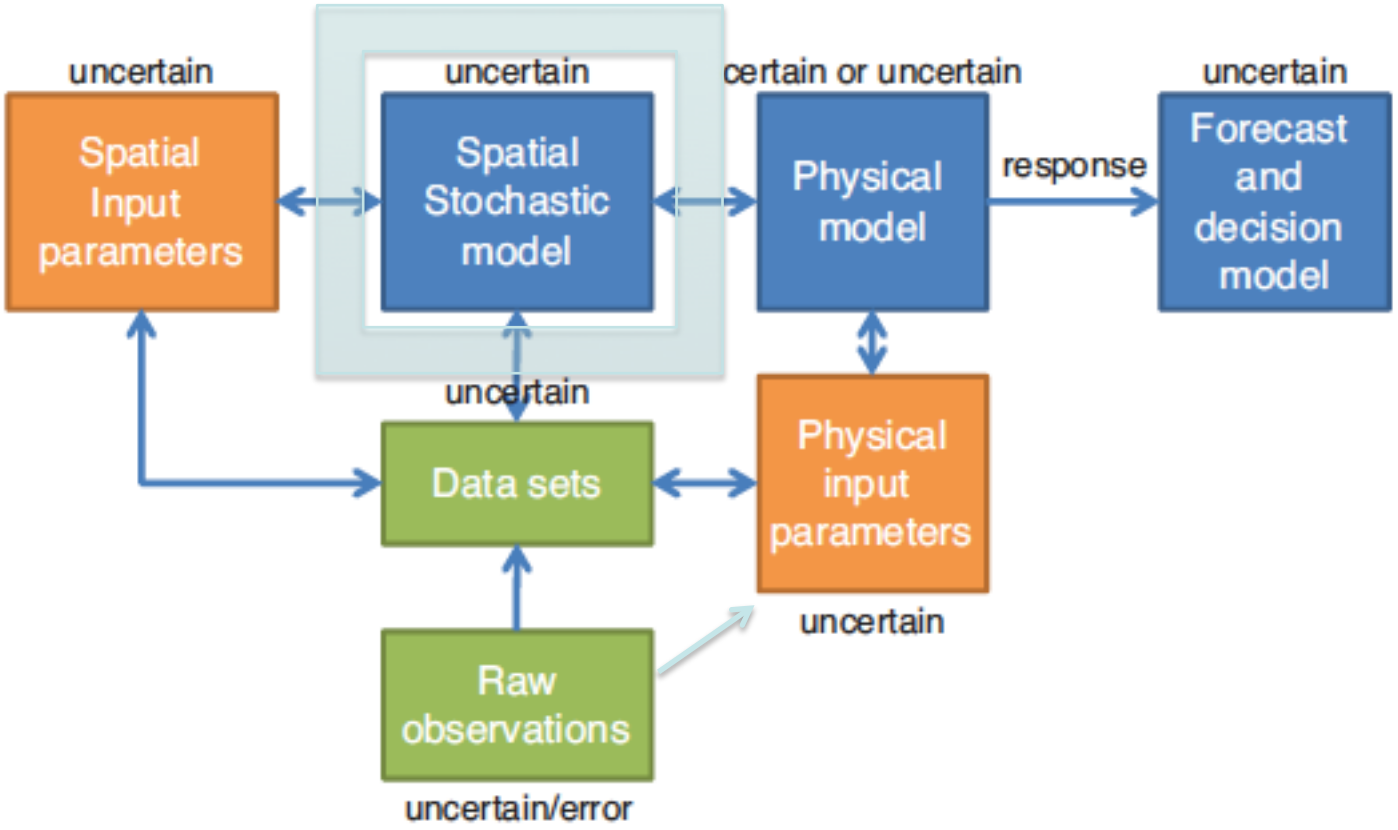
Earth models that mimic training images but are constrained to data

Explicit representation of what you expect. No mathematical formulaes

Training images derived from stratigraphy, for example

Uncertainty manifests itself in multiple training images and multiple realizations that match data

LET'S HAVE A LOOK AT SPATIAL MODELS



Critical for earth models.

Basic thought: there can be multiple models that satisfy the data

We look for a probability distribution of such models

Constraining to training images to construct priors

A training image

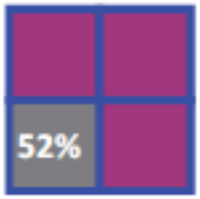
A case with a 2x2 grid

Another possible model

For another choice for first cell

Step 1.
Pick a cell

Step 2.
Assign probability



Step 3.
Assign color

Using random draw based on given prob.

Step 4.
Pick a cell

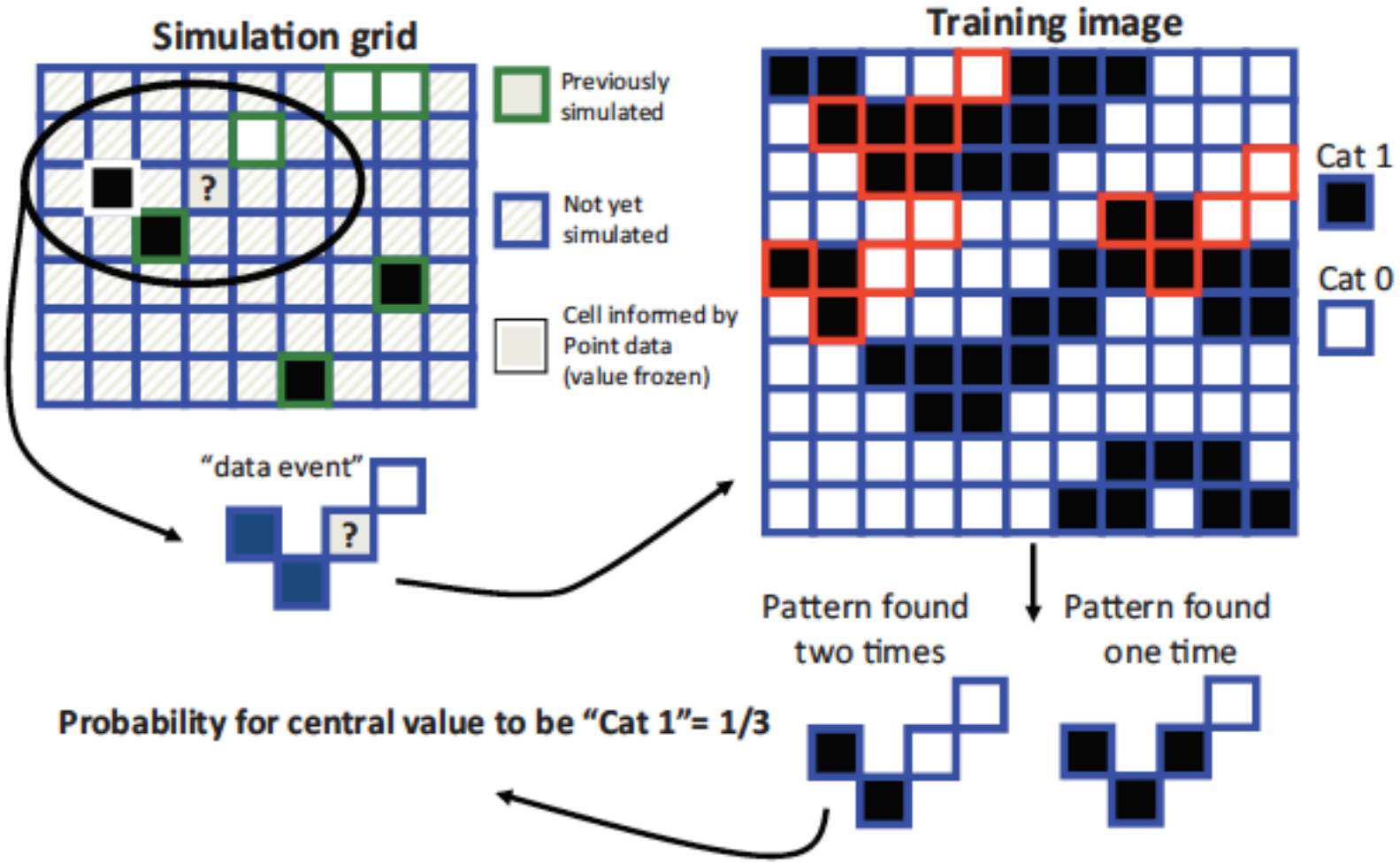
Step 5.
Assign color

Based on previously assigned color and training model.

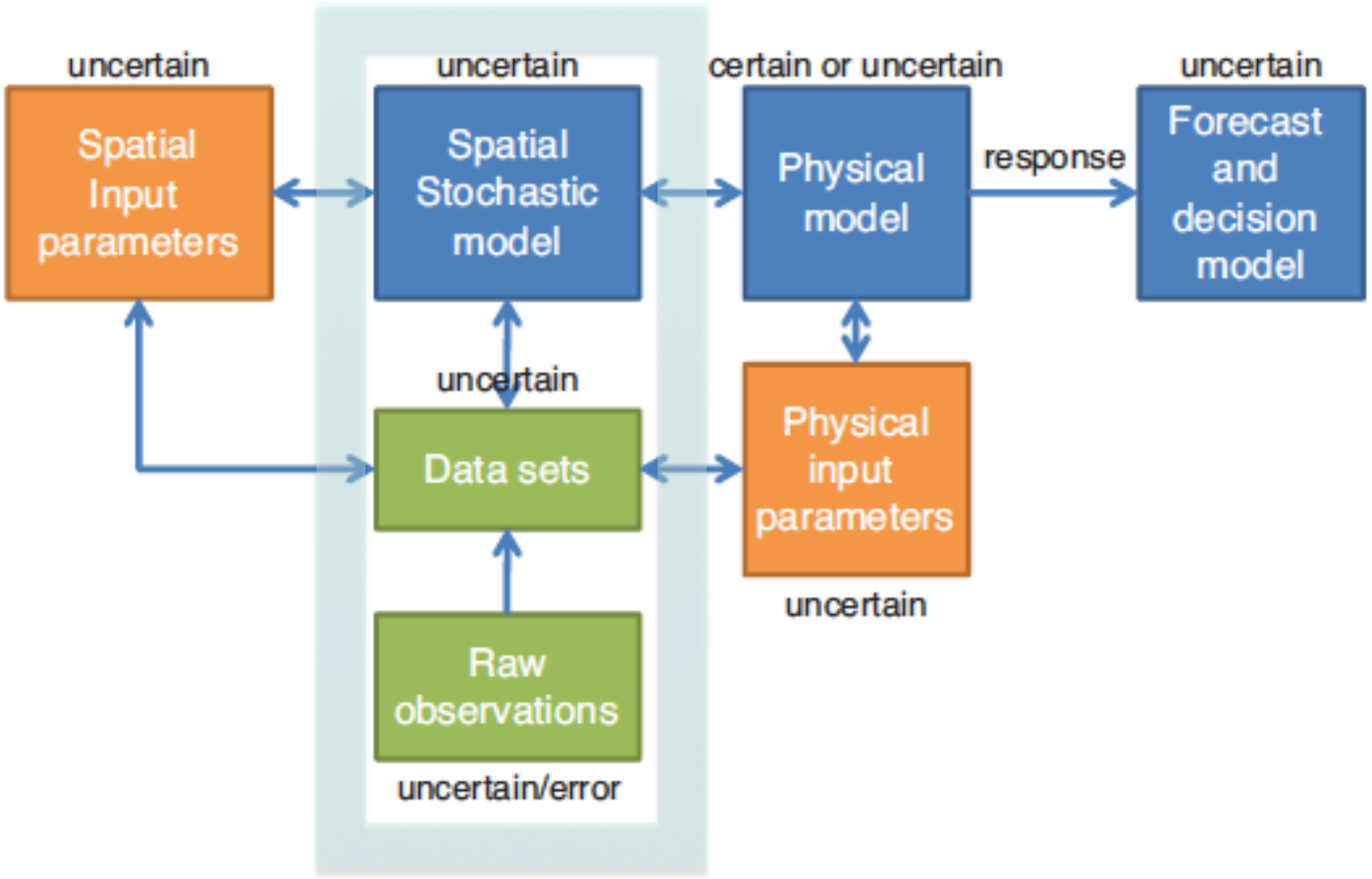
Step 6.
Final result

For this choice of first cell

Constraining priors to cell data



Constraining spatial uncertainty models with *multiple* data



Different data at different scales

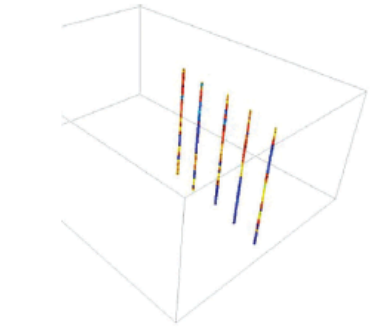
In earth sciences, we typically have

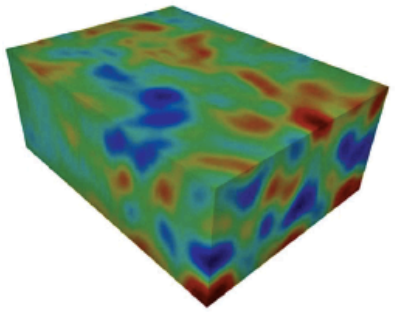
- hard data or samples

  detailed info on small scale

- soft information, e.g., remote sensing

  data that give indication only





How can soft info be used to reduce uncertainty?

Both types of data are "partial". How do we combine these?

Here, we look at a possible approach

Calibration

First we need to assess what is the information content of a source.

Example: how much does seismic impedance (B) tell us about porosity (A)?

Fit a calibration function F to pairs (ai, bi), that is, find

$$F(s,t) = P(\text{porosity} < t \mid s < \text{impedance} < s + ds)$$

Assuming we have several P(A | Bi) for i=1,…, n. What is P(A | B1, B2, ….) ?

Calibration approach unreasonable for multiple data (higher dimensions)
unless behavior linear and interactions are non-existing

Integration example

Will it rain tomorrow (event A)?

Assume we have B1 and B2, with P(A|B1)=0.7 and P(A|B2) = 0.6

We must *model* redundancy: here assume that B2 contributes regardless of B1

$$b_1 = \frac{1 - P(A|B_1)}{P(A|B_1)}, \quad b_2 = \frac{1 - P(A|B_2)}{P(A|B_2)}, \quad a = \frac{1 - P(A)}{P(A)}$$

We'd like to know $$x = \frac{1 - P(A|B_1, B_2)}{P(A|B_1, B_2)} \Rightarrow P(A|B_1, B_2) = \frac{1}{1 + x}$$

$$\frac{b_2 - a}{a} = \text{the relative contribution of information source } B_2 \text{ when not having } B_1$$

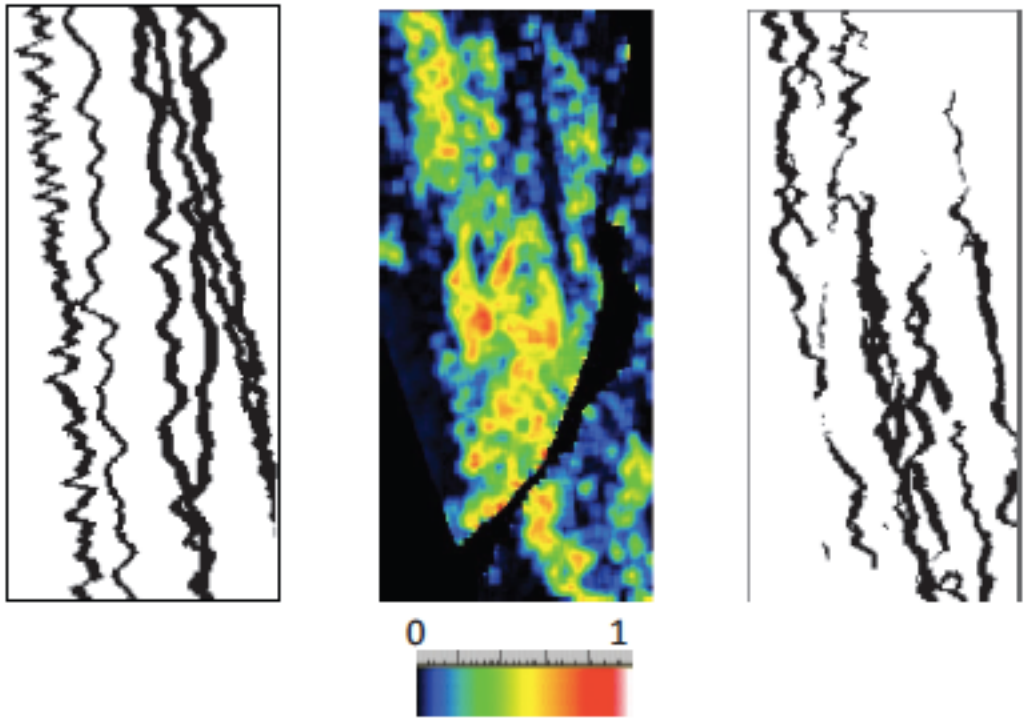
$$\frac{x - b_1}{b_1} = \text{the relative contribution of information source } B_2 \text{ when } B_1 \text{ is available}$$

$$\frac{b_2 - a}{a} = \frac{x - b_1}{b_1} \Rightarrow x = \frac{b_1 b_2}{a} \quad \text{or} \quad P(A|B_1, B_2) = \frac{a}{a + b_1 b_2}$$

$$P(A|B_1, B_2) = 0.91$$

Now, a more complicated example

Presence or absence of sand at each grid block. We are given probability derived from seismic. We'd like more sand channels where sand prob is higher



Training image (left), probability derived from seismic (middle), Earth model (right).

A POSSIBLE PROCESS

1. Assign hard data to the earth model grid
2. Define a random path that loops over all grid cells
3. At each grid cell:
   - Determine P(A|B1) by scanning
   - Find P(A|B2) (sand probability)
   - Use previous equation to find P(A|B1, B2)
   - Sample a value from this distribution function and assign to grid cell

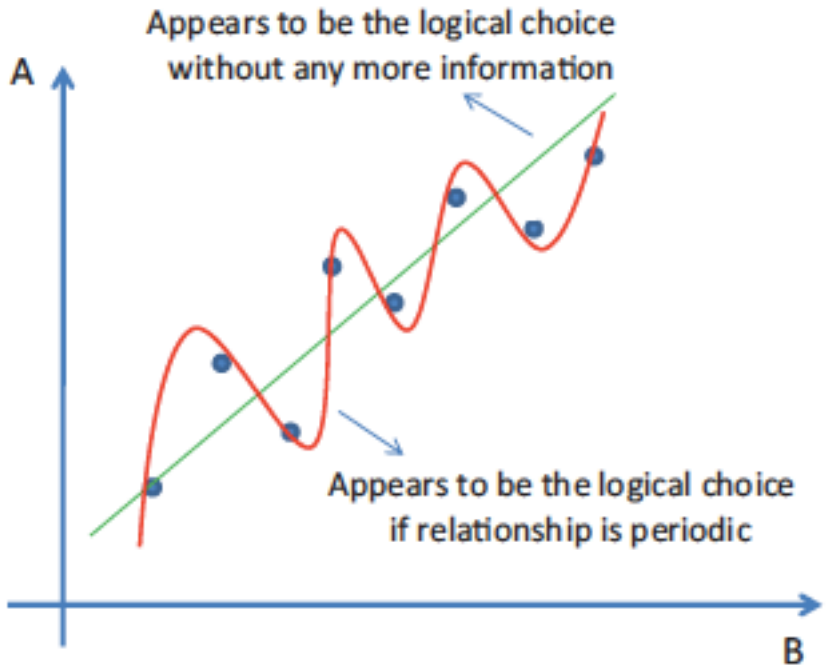
Probability is often not the only info

In previous approach, we need conditional probabilities modeled from calibration data sets

They are not always available, and not always good

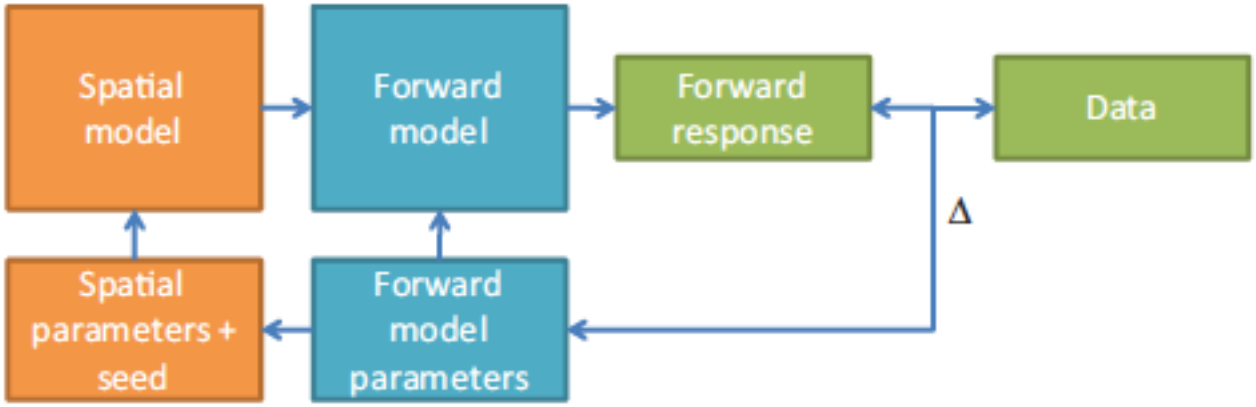
What if we knew that the data, or part of the data, were physical responses?

How to take that into account?



Inverse modeling takes into account physical data responses

If unacceptable mismatch between response and observed data, then adjust input parameters for earth or forward model, or physical laws



What to adjust?

Adjust parameters/models that impact data fit

and/or

Adjust parameters/models that impact decision questions

Example

M has 10 possible outcomes. A prior distribution is known

m	Prior probability P(M = m)
0	0
10	5
20	20
30	30
40	25
50	10
60	10
>60	0 in %

m	g(m)
0	1
10	1
20	1
30	2
40	2
50	2
60	3
>60	4

A physical law g(M) gives forward response g(m)

If the observed data for this forward model is D=1, then m=0, 10 or 20.

What are the probabilities of these outcomes?

m	Prior probability P(M = m)
0	0
10	5
20	20
30	30
40	25
50	10
60	10
>60	0 in %

m	g(m)
0	1
10	1
20	1
30	2
40	2
50	2
60	3
>60	4

We have prior for M

Do we have prior for D? Nope, but probs for all M must add to one:

$$P(M = 10|D = 1) = 20\%$$
$$P(M = 20|D = 1) = 80\%$$
$$P(D = d) = \sum_{m=0}^{100} P(D = d|M = m)\,P(M = m)$$
$$P(D = 1) = 100\% \times 0\% + 100\% \times 5\% + 100\% \times 20\% = 25\%$$

What if there are measurement errors?

We model it somehow. Say

$$\mathrm{P}(\mathsf{D} = 1|\mathsf{M} = 10) = 75\% \text{ and } \mathrm{P}(\mathsf{D} = 1|\mathsf{M} = 20) = 25\%$$

Now, Bayes gives

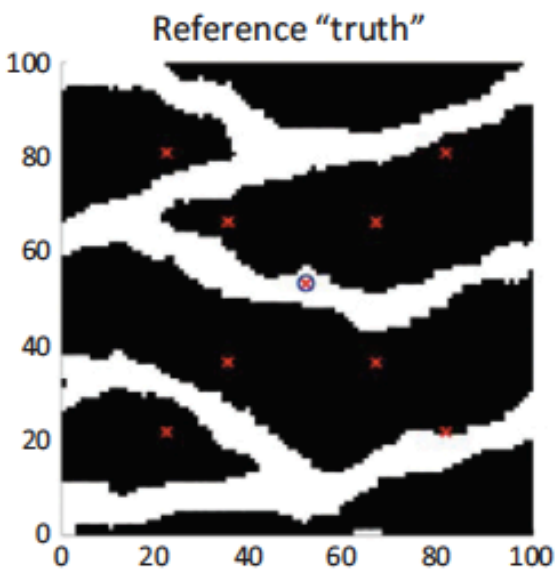
$$\mathrm{P}(\mathsf{M} = 10|\mathsf{D} = 1) = \frac{75 \times 5}{75 \times 5 + 25 \times 20} = 0.428$$

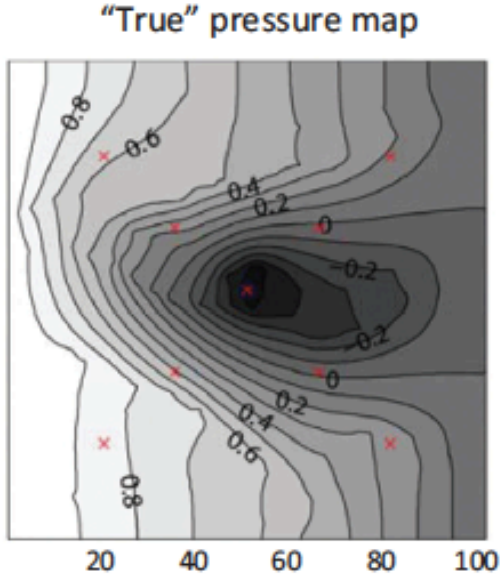
$$\mathrm{P}(\mathsf{M} = 20|\mathsf{D} = 1) = \frac{25 \times 20}{75 \times 5 + 25 \times 20} = 0.572$$

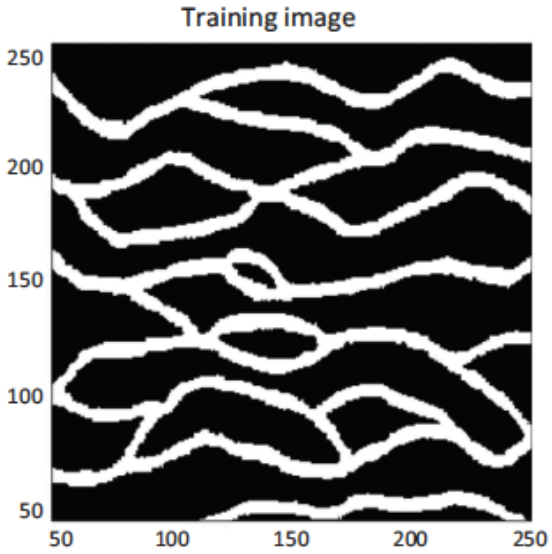
Example

Want to find a model for hydraulic conductivity: either in a channel or not

Style of channeling is available in training model







Model: vector of 100x100 binary variables

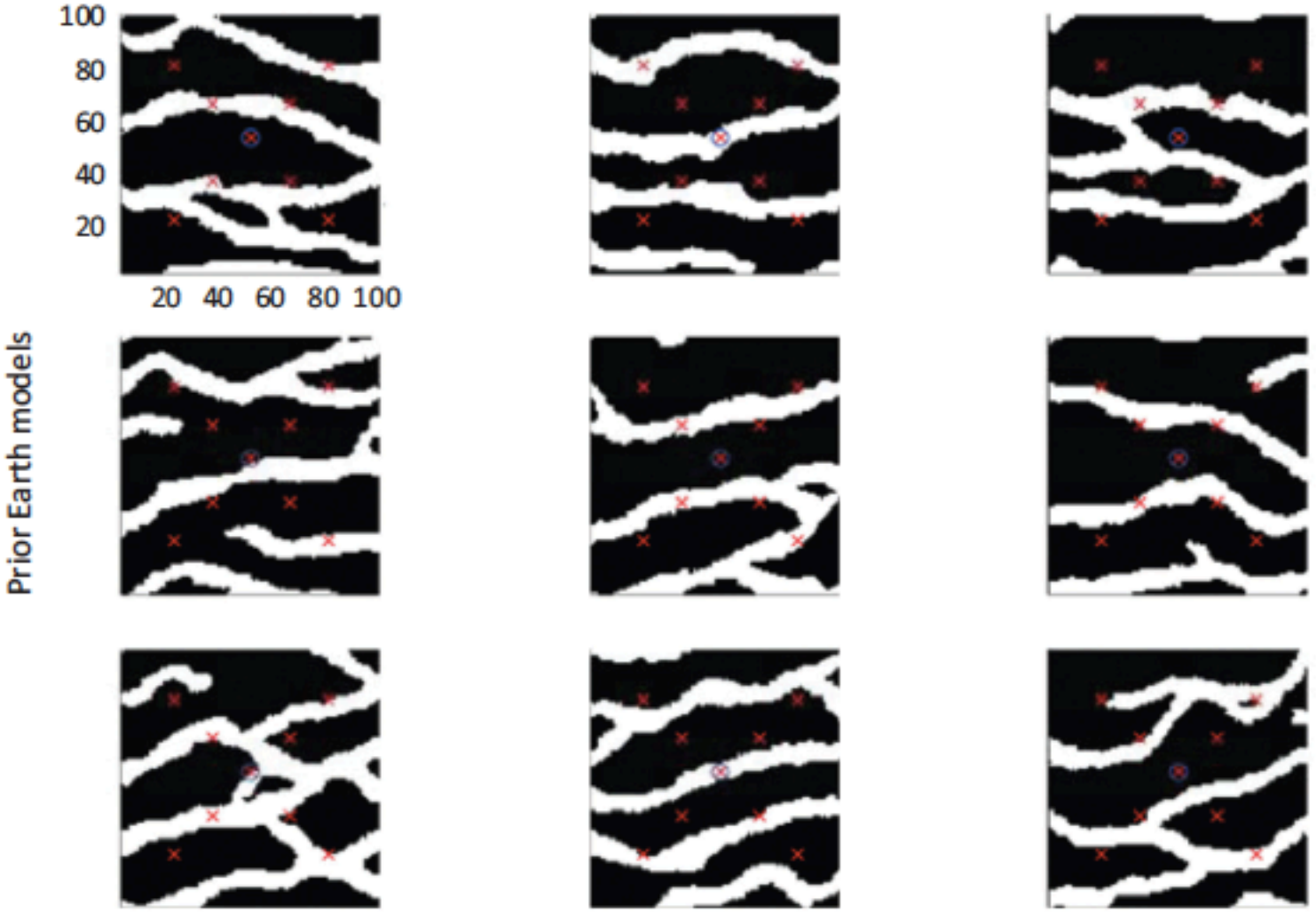
We have pressure data at 9 locations

Pressure not a direct parameter of spatial model: it results from forward model

What model matches those values?

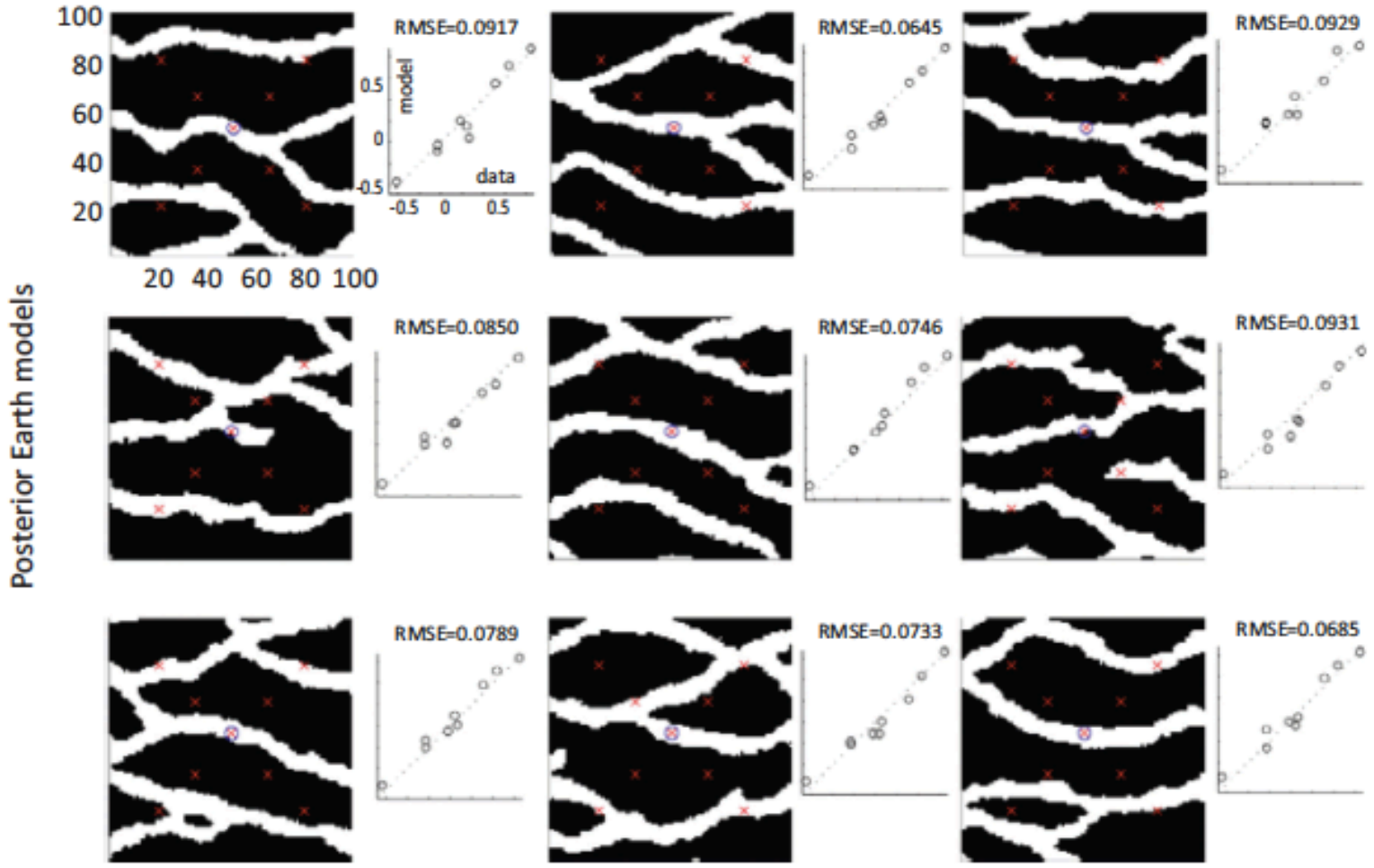
Find prior earth models that fit training images

Nine possibilities (but there are many more):



Amongst the prior models, search for those that match the data

If no models can be found, it may be that there is inconsistency between priors and the forward model (developed by different experts)



Amongst the prior models, search for those that match the data. HOW?

A simple approach: rejection sampling (Popper would like this)

1. Generating a (prior) Earth model $\mathbf{m}$ consistent with any prior information.
2. Evaluating $g(\mathbf{m})$.
3. If $g(\mathbf{m}) = \mathbf{d}$, then keeping that model; if not, reject it.
4. Go to step 1 until a desired amount of models has been found.

We could also accept the model with some defined probability, rather than requiring an exact match

Or, we could use Metropolis sampling instead

But, we can also use optimization methods such as gradient-based optimization, or Ensemble Kalman filtering

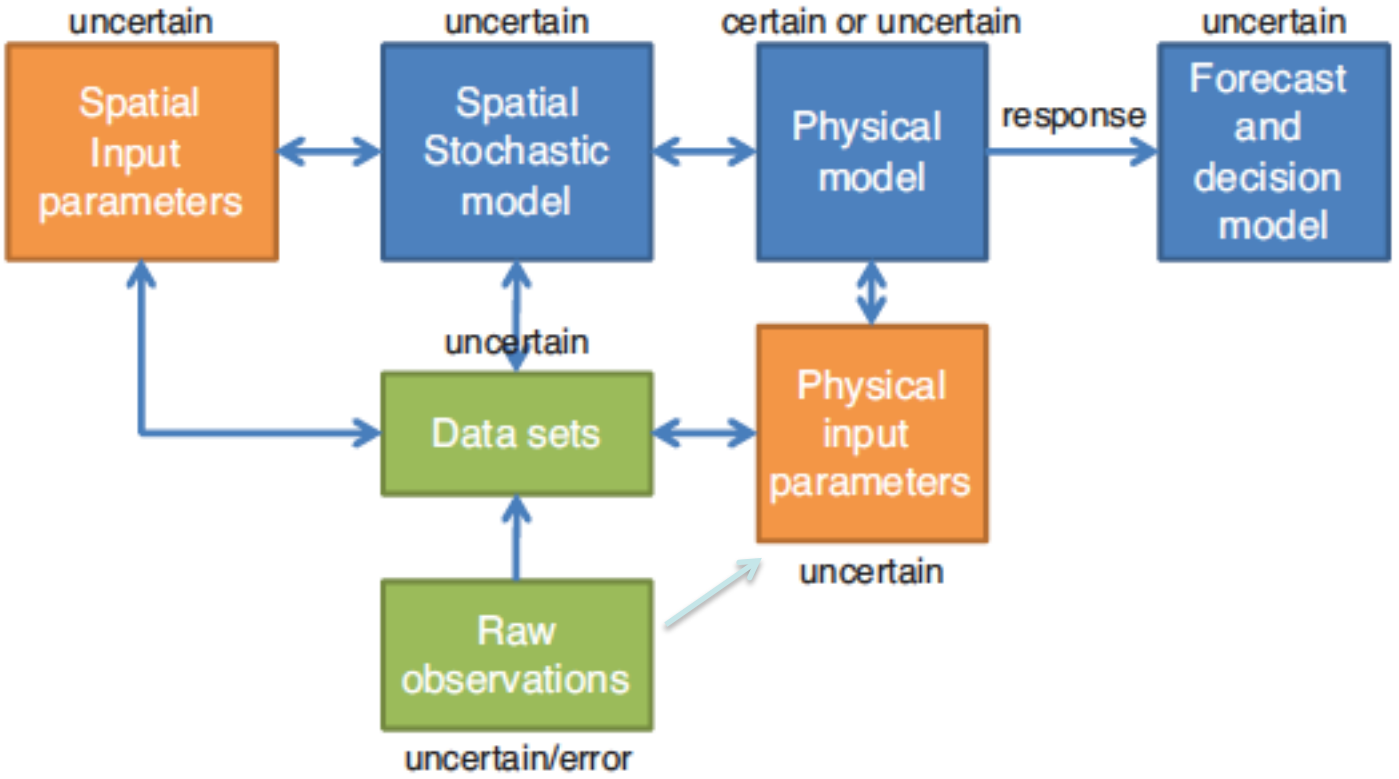
Here, primarily looking to find an optimal fit to the data

In sampling, m is changed consistent with prior information

In optimization methods, it may happen that models are created that are not part of the initial prior set

Be very careful when choosing such methods!

Other components: response and decisions model



How about the response?

- Surrogate model (proxy) can be used if simulation model too expensive
- Is OK typically for ranking exercises
- Experimental design -> create response surfaces
- Sampling from data (Monte Carlo) too expensive: be smarter
- Response surface: typically smoothed out. How well does it capture nonlinearities? Scenarios difficult to test with experimental design
- Need to come up with quantitative measures for scenarios (eg realizations)

Decision making under uncertainty

- Should really happen over extended time period
- Identification of realistic alternative decisions dependent again on expertise
- Multiple objectives in different "currencies"
- Once you have alternatives and objectives you estimate payoff for objective j is we use alternative i. In payoff matrix include those objectives that distinguish amongst alternatives
- Then we evaluate preference for objectives relative to each other. May be nonlinear and are subjective
- Combine in some structured way (eg decision tree). Conflicting objectives? Then evaluate tradeoffs: typically trade-off in risk and return ("are you willing to trade-off more risk for more return?")