



# Recursive Similarity-Based Algorithm for Deep Learning

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Outline

Main idea

SBL

Deep Learning

RSBL

Results

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# Outline

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- Main idea
- Similarity-Based Learning (SBL)
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# Main idea

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- Classification is one of the most important area of machine learning.
- SB methods (incl. many variants of kNN) the most popular and simplest.
- Advantages:
  - easy handling of unlimited number of classes,
  - stability of solutions against small perturbations of data.
- Their applications are limited, because  $O(n^2)$ .
- Calculations @ time of actual classification instead of training.
- Real-time decisions - too slow.



# Main idea

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- TRN of all SBM (incl. kernel-based SVM) suffers from the same quadratic scaling problem.
- Fast methods for finding approx. neighbours can reduce time to  $O(\log n)$ .
- Simple ML methods seems to reach their limits.
- Future belongs to techniques that autom. compose many transf. as it is done in:
  - Meta-Learning based on search in the model space,
  - learning based on generation of novel features,
  - Deep Learning.



# Main idea

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- RSBL inspired by success of DL.
- Kernel methods make only one step, replacing orig. features with SB features and performing LD in this space.
- DL in NN is based on learning in new feature spaces created by adding many network layers in essence performing recursive transf.
- Instead of seq. performing I/O transf., RSBL systematically expands the feature space using info from all previous stages of data transf.
- Only transf. based on similarities to the nearest  $k$ -samples scaled by Gaussian kernel features.
- Other similarity measures may be used in the same way.
- In essence this connects SBM with DL techniques, creating higher-order kNN method with kernel features.



# Similarity-Based Learning

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- Categorization based on distance to points in TRN dataset is a simple and effective way of classification.
- Many parameters and procedures can be included in the data models  $M$  based on similarity.
- $M$  optimized to calculate posterior probability  $p(C_i|\mathbf{x}; M)$  that a vector  $\mathbf{x}$  belongs to class  $C_i$ .
- Optimization includes:
  - type of distance functions, or type of kernel  $D(\mathbf{x}, \mathbf{y})$ ,
  - selection of reference instances,
  - weighting of their influence.



# Similarity-Based Learning

The most common distance functions:

- Minkowski's metric  $D(\mathbf{x}, \mathbf{y})^\alpha = \sum_{i=1}^d |x_i - y_i|^\alpha$ , becoming Euclidean metric for  $\alpha = 2$ , the city block metric for  $\alpha = 1$  and the Chebychev metric for  $\alpha = \infty$ .

- Mahalanobis distance  $D(\mathbf{x}, \mathbf{y}) = \sqrt{(\mathbf{x} - \mathbf{y})' \mathbf{C}^{-1} (\mathbf{x} - \mathbf{y})}$  where  $\mathbf{C}$  is the covariance matrix, taking into account scaling and rotation of data clusters.

- Cosine distance, equal to the normalized dot product  $D(\mathbf{x}, \mathbf{y}) = \mathbf{x} \cdot \mathbf{y} / \|\mathbf{x}\| \|\mathbf{y}\|$ .

- Hamming distance is used for binary features  $D(\mathbf{x}, \mathbf{y}) = \#(x_i \neq y_i) / d$ .

- Correlation distance is also often used:

$$D(\mathbf{x}, \mathbf{y}) = \frac{\sum_{i=1}^d (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^d (x_i - \bar{x})^2 \sum_{i=1}^d (y_i - \bar{y})^2}}$$



# Deep Learning

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- Start from rich information –  $\rightarrow$  series of transformations  
–  $\rightarrow$  output sufficient for high-level decision.
- This information compression process can be presented as a network, a flow graph where each node = elementary data transformation.
- Flow graphs have different depth.
- Popular classifiers - low depth (SVM, RBF, kNN) = 2 (kernel/distance, LD or selection of nearest neighbours). MLP depth depends on no. hidden layers.
- Low depth, but universal approx (can represent arbitrary function to a given target acc).
- Bengio shows examples of functions that can be represented in a simple way with deep archit., but a shallow one may require exp large no. of nodes in the flow graph and difficult to optimize.



# Deep Learning

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- Motivation for DL - comes from signal processing by the brain, for ex. image processing - done in many areas, each extracts some features from input and communicates results to the next level. Each level represents the input at a different level of abstraction with more abstract features further up in the hierarchy.
- People organize ideas and concepts hierarchically, learning first simpler concepts and then composing them to represent more abstract ones.
- Engineers break-up solutions into multiple levels of abstraction and processing using divide-and-conquer at many levels.
- RSBL is inspired by such observations.



# Recursive Similarity-Based Learning

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- DL combined with distance-based and Gaussian kernel features - recursive supervised algorithm to create new features; used to provide optimal feature space for any classifier.
- RSBL - Euclidean distance + Gaussian kernel features ( $\sigma=0.1$ ) -  $>$  new feature spaces @ each depth level.
- Classification done by SVM (  $C=2^5$  ) or 1NN.
- In each case  $k_{\max} = 20$  and  $\alpha = 5$ .
- In essence RSBL at each level of depth transforms the actual feature space into the extended feature space, discovering useful info by creating new redundant features.
- Original features are available at each depth.



# Recursive Similarity-Based Learning

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- The final analysis in the extended space (and optimization of parameters at each level of RSBL, including feature selection) may be done by various ML methods.
- Emphasis on generation of new features using DL rather than optimization of learning.
- RSBL may be presented as a constructive algorithm, with new layers representing transformations and procedures to extract and add to the overall pool more features, and a final layer analyzing the image of data in the enhanced feature space.



# Recursive Similarity-Based Learning

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## Algorithm 1. Recursive similarity-based learning

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**Require:** Fix the values of internal parameters:  $k_{\max}$ , maximum depth  $\alpha$ , and  $\sigma$  (dispersion).

- 1: Standardize the dataset,  $n$  vectors,  $d$  features.
  - 2: Set the initial space  $\mathcal{H}^{(0)}$  using input features  $x_{ij}$ ,  $i = 1..n$  vectors and  $j = 1..d$  features.
  - 3: Set the current number of features  $d(0) = d$ .
  - 4: **for**  $m = 1$  to  $\alpha$  **do**
  - 5:     **for**  $k = 1$  to  $k_{\max}$  **do**
  - 6:         For every training vector  $\mathbf{x}_i$  find  $k$  nearest neighbors  $\mathbf{x}_{j,i}$  in the  $\mathcal{H}^{(m-1)}$  space.
  - 7:         Create  $nk$  new kernel features  $z_{j,i}(\mathbf{x}) = K(\mathbf{x}, \mathbf{x}_{j,i})$ ,  $j = 1..k$ ;  $i = 1..n$  for all vectors using kernel functions as new features.
  - 8:         Add new  $nk$  features to the  $\mathcal{H}^{(m-1)}$  space, creating temporary  $\mathcal{H}^{(m,k)}$  space.
  - 9:         Estimate error  $E(m, k)$  in the  $\mathcal{H}^{(m,k)}$  space on the training or validation set.
  - 10:     **end for**
  - 11:     Choose  $k'$  that minimizes  $E(m, k')$  error and retain  $\mathcal{H}^{(m,k')}$  space as the new  $\mathcal{H}^{(m)}$  space.
  - 12: **end for**
  - 13: Build the final model in the enhanced feature space  $\mathcal{H}^{(\alpha)}$ .
  - 14: Classify test data mapped into the enhanced space.
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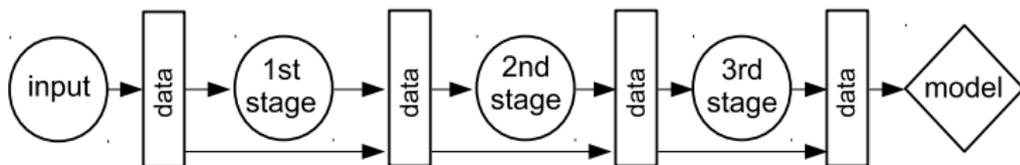


Figure: RSBL method presented in graphical form for depth equal three.



# Results

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- Many sophisticated ML methods introduced every year, tested on trivial problems from UCI.
- Most of them easy: simple and fast algorithms with  $O(nd)$  - results s.s. not worse than those obtained by the best known alg.
- Some benchmark problems are not trivial - require complicated decision borders and may be handled only using sophisticated techniques.
- To distinguish dataset trivial or not - simple methods with  $O(nd)$  complexity have been compared with the optimized SVMG.
- New methods should improve results of simple ML methods in non-trivial cases.



# Results

Table: Summary of datasets used in experiments.

Dataset	#Vectors	#Features	#Classes
ionosphere	351	34	2
monks-1	556	6	2
monks-2	601	6	2
parkinsons	195	22	2
sonar	208	60	2

Duch W., Jankowski N., Maszczyk T.: **Make it cheap: learning with  $O(nd)$  complexity.** In: Proceedings of the IEEE World Congress on Computational Intelligence, Brisbane, Australia (2012), pp. 132-135

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# Which dataset from UCI is Trivial

(our previous work)

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Data	Trivial	MajorityClass	INP	MLC	LVQ	NaiveBayes	K2MLP	SVML	SVMG
arrhythmia	+	29.43±7.82	44.00±27.42	29.79±14.76	<b>63.69±77.46</b>	10.19±9.80	58.00±16.63	50.92±17.31	43.36±21.47
autos	+	28.18±3.14	56.83±10.39	62.23±11.97	30.23±10.18	<b>63.32±10.28</b>	<b>70.44±10.31</b>	54.48±13.75	74.29±12.58
balance-scale	+	45.37±0.55	70.07±6.19	53.80±5.56	89.88±9.10	90.80±1.40	<b>91.15±4.03</b>	84.47±3.17	89.83±2.09
blood-transfusion-service-center	+	<b>76.20±0.57</b>	69.13±3.74	60.28±3.08	76.20±0.57	75.01±3.02	79.59±8.35	76.20±0.48	79.14±4.57
breast-cancer-wisconsin-diagnostic	+	62.74±0.73	93.08±3.47	91.78±3.44	93.04±3.47	93.18±3.47	<b>97.00±2.33</b>	97.36±2.51	97.54±2.50
breast-cancer-wisconsin-original	+	65.01±0.83	96.44±2.22	94.46±2.62	95.83±2.22	96.24±2.30	<b>96.72±2.17</b>	96.48±2.49	96.77±2.48
breast-cancer-wisconsin-prognostic	+	<b>76.34±3.02</b>	62.64±10.02	74.17±8.28	76.34±3.02	66.39±8.83	73.00±9.92	77.84±8.03	76.86±8.20
breast-tissue	+	17.68±3.64	61.73±11.70	<b>68.34±11.45</b>	15.85±6.55	64.03±12.78	63.04±11.17	53.00±10.87	63.27±7.59
car-valuation	-	70.02±0.16	73.22±2.90	84.08±2.50	73.59±3.65	3.76±0.33	<b>91.13±2.54</b>	69.57±2.03	98.84±0.77
cardiotocography-1	+	27.23±0.16	54.80±3.02	70.98±2.46	27.23±0.16	72.57±2.28	<b>77.88±2.83</b>	57.90±4.12	80.43±2.79
cardiotocography-2	+	77.85±0.31	76.57±1.78	73.72±2.21	77.85±0.31	82.54±1.86	<b>87.32±2.69</b>	87.53±1.48	92.09±2.01
chess-king-rook-vs-king-pawn	-	52.22±0.12	86.25±1.33	83.86±1.68	61.40±15.17	67.27±1.81	<b>90.90±3.29</b>	96.21±1.38	99.28±0.36
CMC (Contraceptive Methods)	+	42.70±0.23	46.03±3.52	47.89±3.62	22.61±0.34	<b>49.64±3.96</b>	48.82±3.40	19.14±2.14	34.09±3.67
congressional-voting-records	+	53.46±2.26	89.86±5.40	94.73±4.37	89.69±5.27	94.25±5.11	<b>94.95±3.94</b>	94.48±3.62	92.65±4.11
connectionist-bench-sonar	+	53.35±2.26	69.65±7.49	70.62±5.99	71.67±7.44	69.03±8.68	<b>76.73±8.05</b>	75.47±8.26	85.52±5.28
connectionist-bench-vowel	-	7.58±0.06	51.98±6.61	52.00±5.99	9.09±1.36	67.53±6.28	<b>80.95±8.32</b>	25.76±5.01	96.77±2.20
cylinder-bands	+	64.25±1.17	68.97±8.43	38.68±7.02	64.47±4.26	<b>74.07±7.51</b>	71.35±5.02	74.58±5.23	76.89±7.57
dermatology	+	31.01±0.97	<b>96.87±3.15</b>	88.40±4.55	91.30±3.79	90.13±4.52	94.88±3.84	94.01±3.54	94.49±3.88
ecoli	+	42.57±1.58	81.38±5.76	77.13±12.09	78.50±9.39	70.76±20.46	<b>83.85±6.04</b>	78.45±5.90	84.17±5.82
glass	+	35.54±2.56	48.82±9.88	48.67±6.25	34.79±4.52	43.34±8.44	<b>59.78±8.84</b>	42.61±10.05	62.43±8.70
habermans-survival	+	73.54±1.86	74.45±7.19	71.21±7.65	73.19±4.07	<b>74.83±5.58</b>	64.34±14.53	73.52±1.86	71.55±8.42
hepatitis	+	83.75±5.76	82.50±13.06	90.38±10.18	83.75±5.76	<b>91.25±9.15</b>	84.20±12.32	83.25±11.54	84.87±11.98
ionosphere	-	64.10±1.43	81.14±6.41	59.23±6.24	83.72±5.34	84.24±6.15	<b>86.46±5.48</b>	87.72±4.63	94.61±3.68
iris	+	33.33±0.00	85.80±8.67	94.60±5.42	85.67±8.50	95.40±5.42	<b>95.60±4.76</b>	72.20±7.59	94.86±5.75
libras-movement	-	4.64±1.31	56.86±6.22	51.92±7.07	6.67±1.48	<b>65.50±6.57</b>	52.67±8.17	49.16±5.24	84.44±6.02
liver-disorders	+	57.96±1.62	57.60±8.18	<b>65.12±7.97</b>	57.85±3.43	66.28±7.93	62.61±8.29	68.46±7.36	70.30±7.90
lymph	+	54.71±4.52	<b>86.43±8.61</b>	78.79±9.36	82.49±9.35	81.18±8.95	82.67±9.24	81.26±9.79	83.61±9.82
monks-problems-1	+	49.46±0.60	74.64±4.18	74.64±4.18	70.58±10.05	52.28±2.05	<b>83.07±3.03</b>	65.81±6.50	99.82±0.56
monks-problems-2	-	65.72±0.84	54.90±5.85	53.85±6.19	65.54±1.28	54.50±4.21	<b>74.76±5.14</b>	65.72±0.82	84.86±9.91
monks-problems-3	+	51.99±0.86	96.39±2.17	96.39±2.17	96.39±2.17	94.78±5.29	<b>98.37±1.69</b>	80.13±4.91	96.75±2.22
parkinsons	-	75.44±3.19	73.55±8.71	78.22±8.46	77.76±6.89	69.83±9.09	<b>85.64±7.57</b>	86.26±10.17	93.26±5.61
pima-indians-diabetes	+	65.10±0.54	72.72±4.84	68.63±4.66	75.02±4.50	<b>75.30±4.39</b>	73.82±5.05	77.08±4.20	77.04±3.69
sonar	+	53.35±2.26	69.65±7.49	70.62±5.99	71.67±7.44	69.03±8.68	<b>76.73±8.05</b>	73.71±9.62	86.42±7.65
spambase	+	60.60±1.10	89.50±1.31	87.41±1.44	82.79±11.23	81.78±1.52	<b>91.58±1.57</b>	92.96±1.30	93.69±1.04
SPECT-heart	+	79.42±2.05	72.15±8.20	<b>83.64±8.80</b>	79.42±2.05	72.19±6.99	77.37±7.75	82.72±4.73	83.50±6.62
SPECTF-heart	+	<b>79.42±2.05</b>	66.48±8.43	79.31±2.06	79.42±2.05	67.68±8.35	72.89±11.01	78.61±9.73	80.18±3.74
statlog-australian-credit	+	55.51±0.67	<b>84.45±4.35</b>	79.55±4.81	83.13±7.53	79.54±4.40	82.43±4.69	85.50±3.86	84.49±3.48
statlog-german-credit-numeric	+	70.00±0.00	72.81±4.26	67.55±4.73	72.09±3.38	72.84±4.33	<b>72.93±4.61</b>	77.50±2.32	76.01±3.16
statlog-heart	+	55.56±0.00	84.63±6.08	82.41±7.41	<b>85.07±6.24</b>	84.22±7.13	82.44±7.00	82.96±7.65	81.48±4.61
statlog-vehicle-silhouettes	+	25.12±0.54	45.33±4.57	52.96±4.20	25.77±0.86	45.69±3.98	<b>72.85±4.65</b>	69.86±2.74	79.78±2.66
teaching-assistant-evaluation	+	34.45±2.74	50.85±12.71	48.58±12.82	33.05±3.60	24.32±8.88	<b>52.09±12.00</b>	13.25±9.94	42.37±9.44
thyroid-disease	-	92.58±0.09	71.07±1.84	86.56±1.44	92.58±0.09	36.55±3.36	<b>94.74±2.17</b>	93.76±0.47	97.47±0.66
vote	+	53.46±2.26	89.86±5.40	94.73±4.37	89.69±5.27	91.93±4.98	<b>95.29±4.04</b>	96.12±3.85	96.89±3.11
wine	+	39.91±1.85	97.25±3.94	96.84±3.72	97.25±3.94	<b>97.30±3.80</b>	96.18±3.91	97.71±2.95	97.15±4.08
ZOO	+	40.63±3.19	<b>91.55±6.91</b>	86.34±8.54	83.45±7.11	86.82±8.58	83.25±8.42	91.61±6.67	93.27±7.53
wins / ties / losses		31 / 64 / 130	75 / 73 / 177	73 / 64 / 188	80 / 59 / 186	81 / 63 / 181	137 / 61 / 127		



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Table: 10 × 10 crossvalidation accuracy and standard deviation for RSBL combined with SVM.

Dataset	Method						
	ORG	RSBL(1)	RSBL(2)	RSBL(3)	RSBL(4)	RSBL(5)	SVMG
ionosphere	88.2±6.4	92.3±3.8	94.0±3.9	94.0±3.9	94.0±3.9	94.0±3.9	94.6±3.7
monks-1	74.6±4.6	100±0.0	100±0.0	100±0.0	100±0.0	100±0.0	99.8±0.6
monks-2	65.7±0.6	79.6±3.1	84.9±4.1	85.7±4.2	85.7±4.2	85.7±4.2	84.9±4.9
parkinsons	88.7±7.8	89.3±5.4	93.3±4.9	91.3±6.0	89.2±5.1	87.7±5.4	93.2±5.6
sonar	74.9±9.5	82.2±7.9	85.1±4.6	86.6±7.0	87.4±7.7	87.9±7.3	86.4±7.6



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Table: 10 × 10 crossvalidation accuracy and standard deviation for RSBL combined with 1NN.

Dataset	Method					
	ORG	RSBL(1)	RSBL(2)	RSBL(3)	RSBL(4)	RSBL(5)
ionosphere	87.1±5.2	87.4±4.8	87.8±4.9	87.8±4.9	87.8±4.9	87.8±4.9
monks-1	100±0.0	99.9±0.1	99.4±1.2	99.3±1.2	99.4±1.1	99.4±1.0
monks-2	68.8±6.2	69.2±8.7	71.6±6.2	71.6±6.2	71.6±6.2	71.8±6.2
parkinsons	93.8±5.4	92.8±6.6	91.7±6.1	91.7±6.1	91.7±6.1	91.7±6.1
sonar	85.0±5.8	85.5±6.8	86.0±6.6	87.9±6.5	87.9±6.5	87.9±6.5



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- The most important goal of CI - create methods that can automatically discover the best models for a given data.
- Simple methods - no; therefore such techniques like DL, meta-L or feature construction should be used.
- RSBL focused on hierarchical heneration of new distance-based and kernel-based features rather than improvement in optimization and classification alg.
- Finding interesting views on the data by systematic addition of novel features is very important, because combination of such transformation-based systems should bring us closer to the practical applications that automatically create the best data models for any data.



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- Results on several non-trivial problems shows that RSBL creates explicitly feature spaces in which linear methods reach results that are at least as good as optimized SVMG.
- Further improvements:
  - different distance measures,
  - fast approximate neighbors,
  - feature selection and global optimization of the whole procedure.



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Thank you for your attention.