Skuller: A volumetric shape registration algorithm for modeling skull deformities

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Problem Statement

- ✓ Medical objective
  - ✓ Quantify the deviation of given skull from a healthy one.

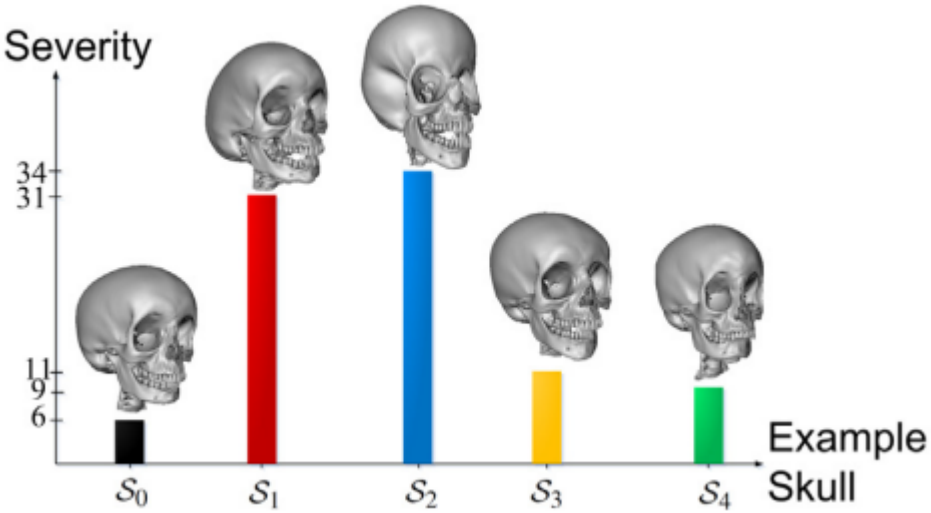
- ✓ Geometric objective:
  - ✓ Develop an algorithm for *volumetric* registration of 3D shapes.

Applications

✓ Shape registration:

✓ Statistical shape analysis:

✓ Objective quantification of medical condition(s):



Contributions

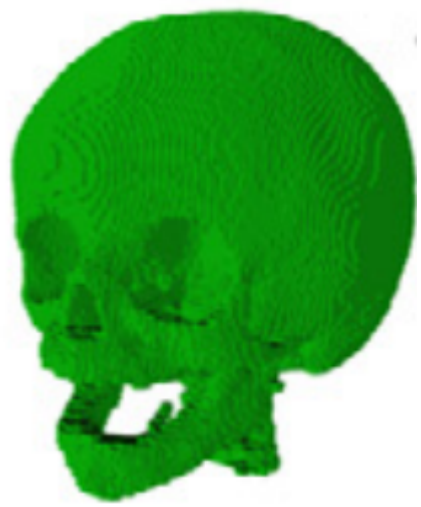
- ✓ A fast and robust solution to a new version of the well-known shape registration problem, namely the registration of two volumetric shapes.
  - ✓ Source: volumetric tetrahedral mesh.
  - ✓ Target: volumetric shape defined by voxels.

- ✓ In contrast to point- and surface-based registration techniques, our method better captures volumetric nature of the data, such as bone thickness.

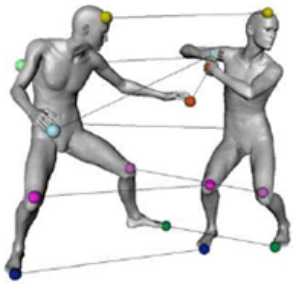
- ✓ ICP with adaptive scaling.
- ✓ Multi-initialization ICP.

Scope

✓ Input: A tetrahedral mesh & A voxelized object (CT scan).



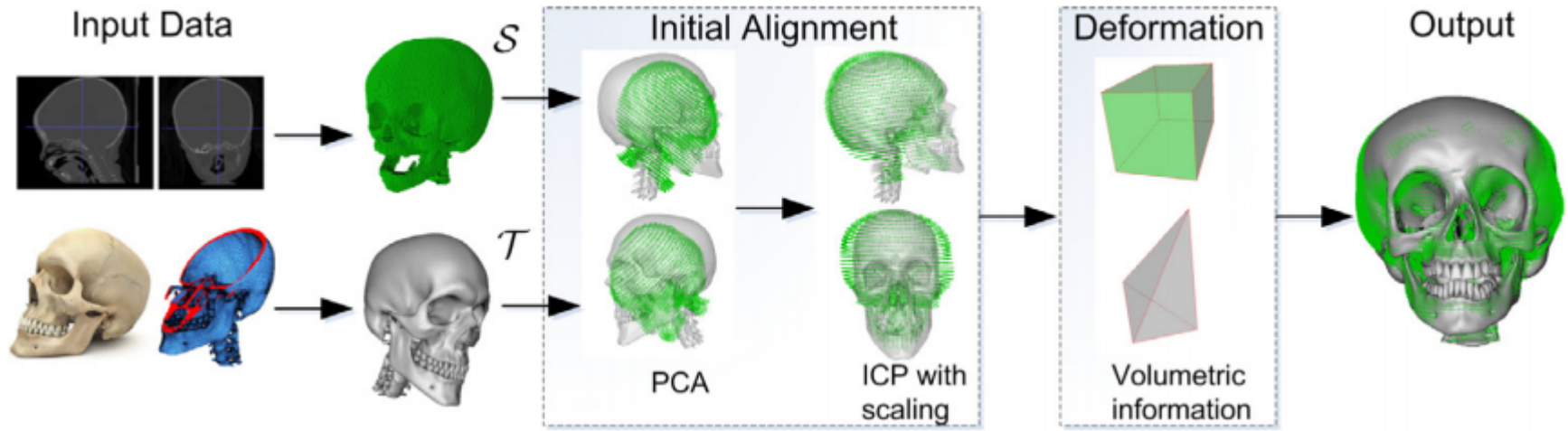
✓Input pair differs by non-isometric/non-rigid transformation.



//an isometric pair

Algorithm

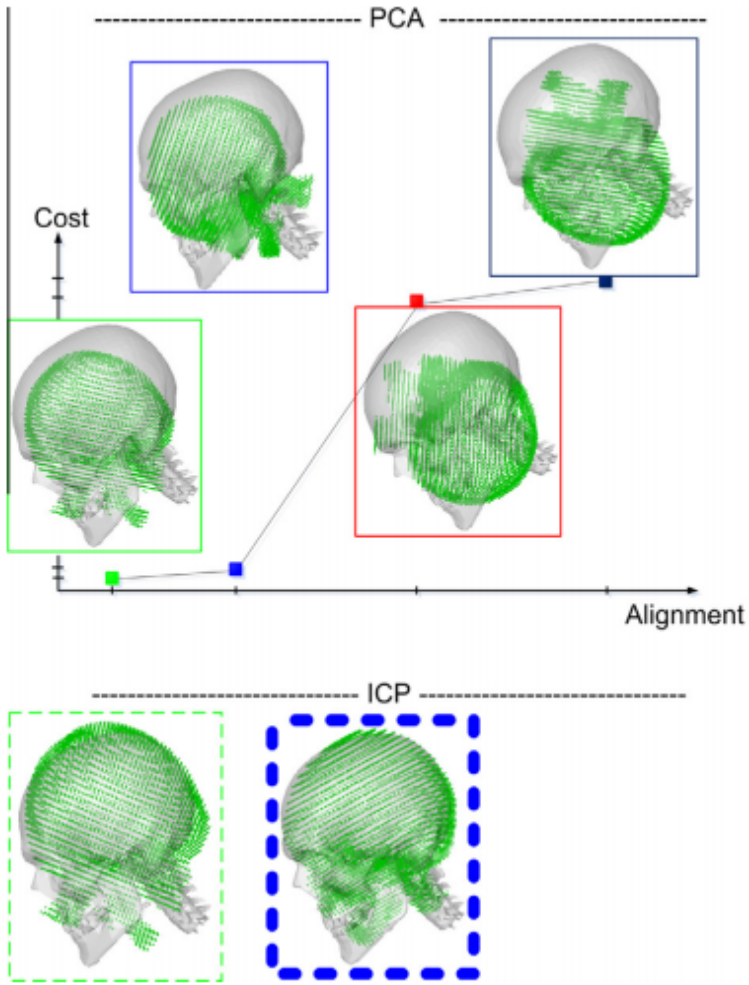
✓ Overview.



Algorithm

- ✓ Initial alignment.
- ✓ A global rigid alignment.
  - ✓ Rotation, translation, uniform scale.

- ✓ Treat input pair as 3D point clouds.
- ✓ PCA-based alignment for multiple ICP initialization.

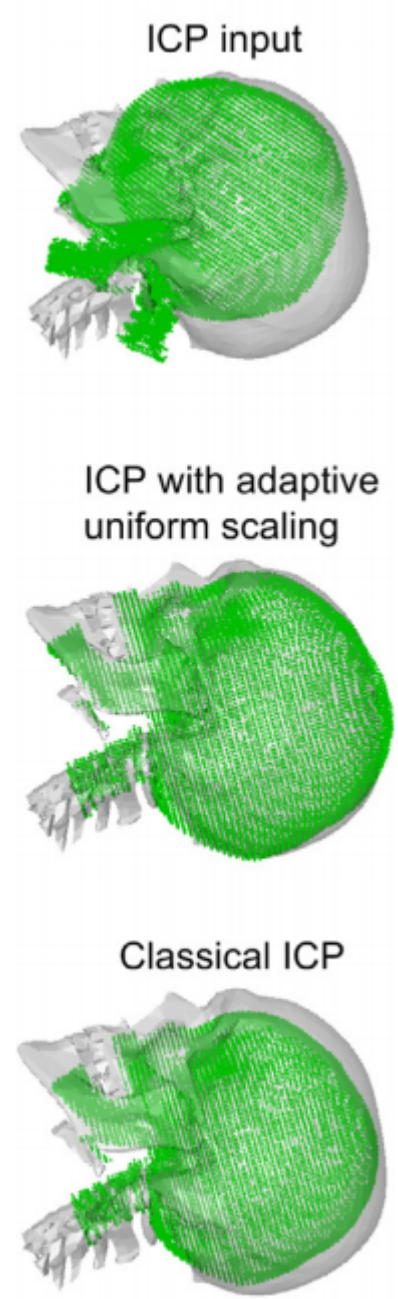


Algorithm

- ✓ Initial alignment.
- ✓ A global rigid alignment.
  - ✓ Rotation, translation, uniform scale.

- ✓ Treat input pair as 3D point clouds.
- ✓ Solve for uniform scale u in addition to the translation d once you have the ICP-rotated points $\mathbf{p}$.

$$f(\mathbf{p}) = \sum_i \|u\mathbf{p}_i + \mathbf{d} - \mathbf{q}_i\|^2$$



Algorithm

- ✓ Volumetric registration.
- ✓ A non-rigid alignment.
  - ✓ Bending/warping allowed.

- ✓ Seek **t** that minimizes the following energy:
  - ✓ $$E_{\text{def}}(\mathbf{t}) = E_{\text{corr}}(\mathbf{t}) + \alpha E_{\text{Dirichlet}}(\mathbf{t}) + \beta E_{\text{Tikhonov}}(\mathbf{t})$$

Algorithm

- ✓ Volumetric registration.
- ✓ A non-rigid alignment.
  - ✓ Bending/warping allowed.

- ✓ Correspondence/matching term.
  - ✓ $E_{\text{corr}}(\mathbf{t}) = ||\mathbf{t} - \mathbf{P}\mathbf{s}||^2$
  - ✓ Brings transformed vertices **t** to the scan voxels **s** as close as possible.
  - ✓ Permutation matrix **P** defines closest-point correspondences.

Algorithm

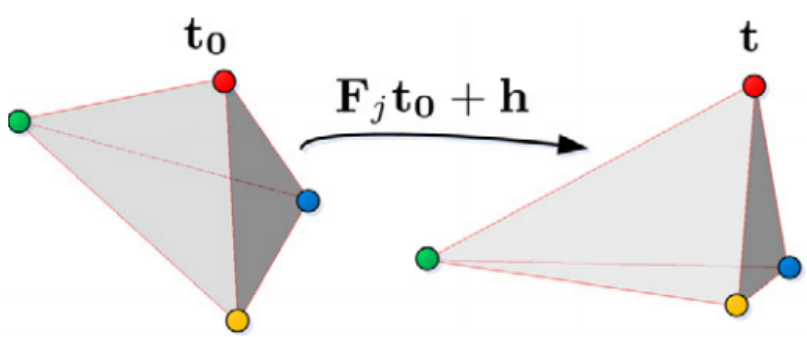
- ✓ Volumetric registration.
- ✓ A non-rigid alignment.
  - ✓ Bending/warping allowed.

- ✓ Regularization term: Volumetric Dirichlet energy.
  - ✓ $E_{\text{Dirichlet}}(\mathbf{t}) = (\mathbf{t} - \mathbf{t}_0)^T \mathbf{L} (\mathbf{t} - \mathbf{t}_0)$
  - ✓ Preserves rest-pose coordinates $\mathbf{t_0}$, hence the good shape, as much as possible.

Algorithm

- ✓ Volumetric registration.
- ✓ A non-rigid alignment.
  - ✓ Bending/warping allowed.

- ✓ Regularization term: Volumetric Dirichlet energy.
  - ✓ $E_{\text{Dirichlet}}(\mathbf{t}) = (\mathbf{t} - \mathbf{t}_0)^T \mathbf{L} (\mathbf{t} - \mathbf{t}_0)$
  - ✓ Measures deviation of tetrahedra's deformation gradients **F** from **I** using the mesh Laplacian **L**.
  - ✓



Algorithm

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  - ✓ Measures deviation of tetrahedra's deformation gradients **F** from **I** using the mesh Laplacian **L**.
    - ✓ $\mathbf{L} = \sum_{j \in \mathcal{T}} \lambda_j \mathbf{G}_j^T \mathbf{G}_j$, where $\mathbf{G}_j$ extracts vectorized gradient $\mathbf{F}_j$ when multiplied with all vertices, i.e., $\text{vec}(\mathbf{F}_j) = \mathbf{G}_j(\mathbf{t} - \mathbf{t}_0)$

Algorithm

- ✓ Volumetric registration.
- ✓ A non-rigid alignment.
  - ✓ Bending/warping allowed.

- ✓ Regularization term: Tiknohonov energy.
  - ✓ $E_{\text{Tikhonov}}(\mathbf{t}) = ||\mathbf{t} - \mathbf{t}_{\text{prev}}||^2$
  - ✓ Keeps the amount of displacement from the previous step's coordinates $\mathbf{t}_{\mathbf{prev}}$ as small as possible, i.e., no jumps.

Algorithm

- ✓ Volumetric registration.
- ✓ A non-rigid alignment.
  - ✓ Bending/warping allowed.

- ✓ Differentiating overall energy w.r.t. **t** gives:

  - ✓ $$\begin{aligned}\frac{\partial E_{\text{def}}}{\partial \mathbf{t}} &= \frac{\partial E_{\text{corr}}}{\partial \mathbf{t}} + \alpha \frac{\partial E_{\text{Dirichlet}}}{\partial \mathbf{t}} + \beta \frac{\partial E_{\text{Tikhonov}}}{\partial \mathbf{t}} \\ &= 2(\mathbf{t} - \mathbf{P}\mathbf{s} + \alpha \mathbf{L}(\mathbf{t} - \mathbf{t}_0) + \beta(\mathbf{t} - \mathbf{t}_{\text{prev}}))\end{aligned}$$

- ✓ Setting this derivative to 0 gives a sparse linear sys to solve:

  - ✓ $$(\alpha \mathbf{L} + (\beta + 1)\mathbf{I})\mathbf{t} = \mathbf{P}\mathbf{s} + \alpha \mathbf{L}\mathbf{t}_0 + \beta \mathbf{t}_{\text{prev}}$$

Algorithm

- ✓ Volumetric registration.
- ✓ A non-rigid alignment.
  - ✓ Bending/warping allowed.

- ✓ Tricks to make this process work well.
  - ✓ Adaptive weighting.
  - ✓ Initially high \alpha \beta as the closest-point correspondences in **P**

    are not so reliable.
    - ✓ Regularization (\alpha \beta) prevents a potential damage from $\mathbf{E}_{\mathbf{corr}}$.
  - ✓ Reduce regularization strength when **t** is very close to $\mathbf{t}_{\mathbf{prev}}$, i.e., little/no progress.
    - ✓ $\mathbf{E}_{\mathbf{corr}}$ becomes more reliable thanks to better correspondences.

Algorithm

- ✓ Volumetric registration.
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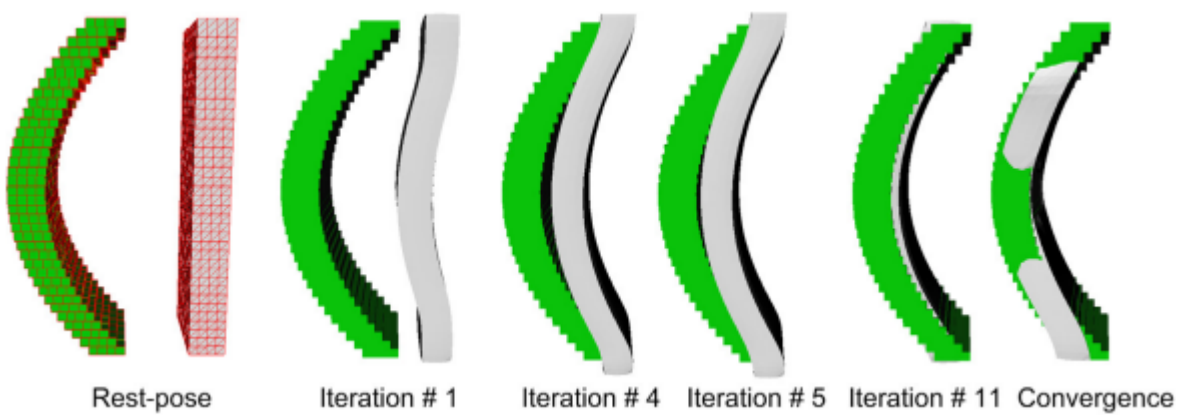
  - ✓ Bending/warping allowed.

- ✓ Tricks to make this process work well.
  - ✓ Adaptive weighting.

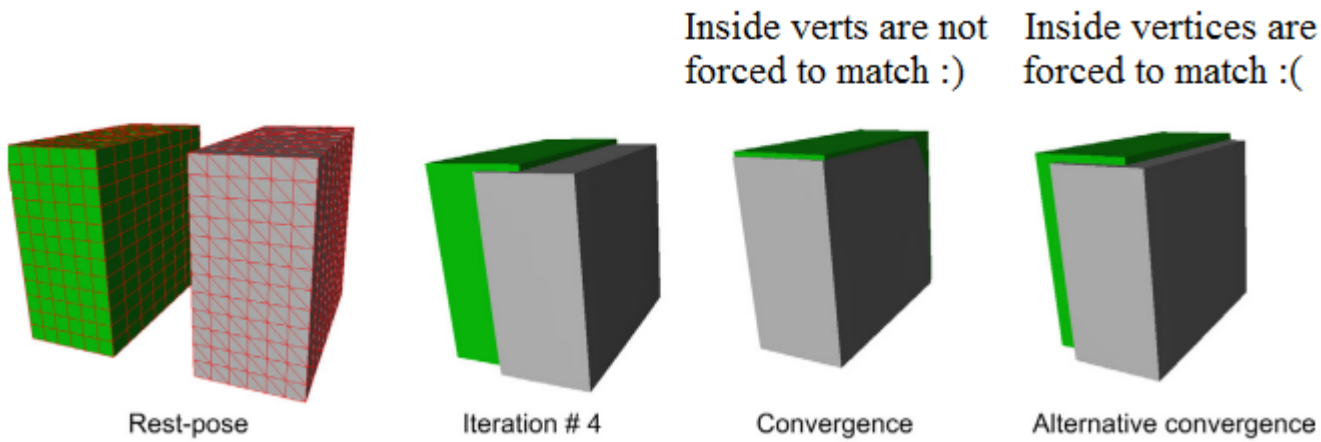


  - ✓ Little displacement b/w iter 4 & 5 triggers adaptive weighting.

Algorithm

- ✓ Volumetric registration.
- ✓ A non-rigid alignment.
  - ✓ Bending/warping allowed.

- ✓ Tricks to make this process work well.
  - ✓ $\mathbf{E}_{\mathbf{corr}}$ disabling.
  - ✓ A vertex that is already inside the volume should not be affected by $\mathbf{E}_{\mathbf{corr}}$ as it is not possible to establish meaningful matches now.
  - ✓ Instead it should be free to move as dictated by regularization.



Algorithm

- ✓ Computational complexity.
- ✓ n tetrahedral mesh vertices, m scan voxels.
- ✓ Initial global alignment.
  - ✓ $O(n\log m)$ due to closest-point search.
- ✓ Volumetric registrations.
  - ✓ $O(n\log m)$ due to closest-point search.
  - ✓ $O(n + w)$ Cholesky factorization performed i times, where i = # deformation iterations is about 20 in our runs. w high constant.

Algorithm

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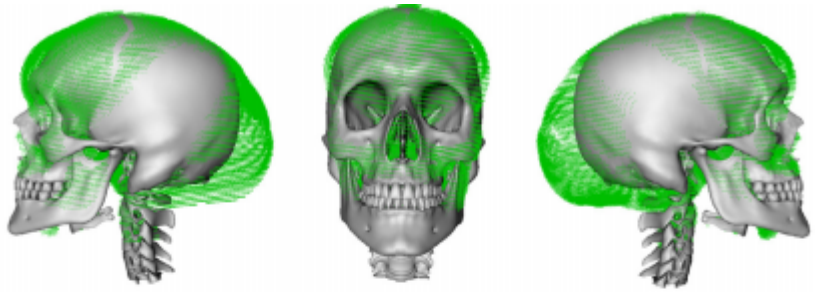
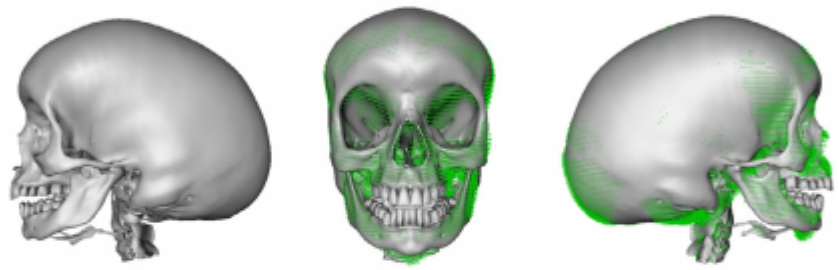
- ✓ Volumetric registrations.
  - ✓ $O(n \log m)$ due to closest-point search.
  - ✓ $O(n + w)$ Cholesky factorization performed i times, where i = # deformation iterations is about 20 in our runs. w high constant.
- ✓ Execution time on a 2.2 GHz PC.
  - ✓ Input tetrahedral mesh: 160K vertices, 560K tetrahedra.
  - ✓ Input CT scan: 190K voxels.
  - ✓ 20 minutes.
    - ✓ 133 secs initial alignment, 1 sec (closest-point) + 49 sec (Cholesky) per deformation iteration.

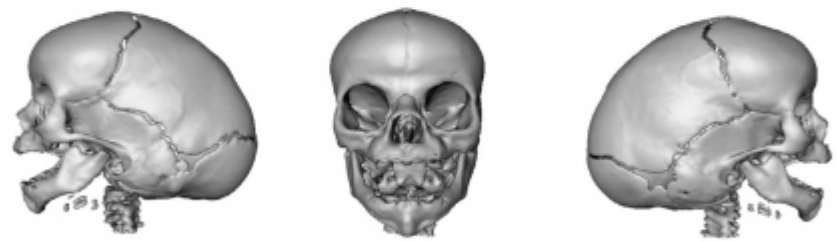
Results



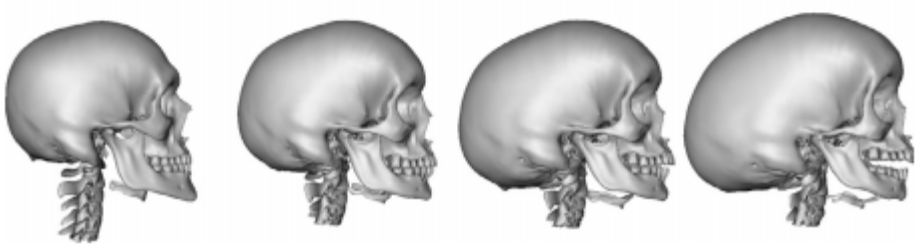
//Initial global alignment.



//Resulting registered model.

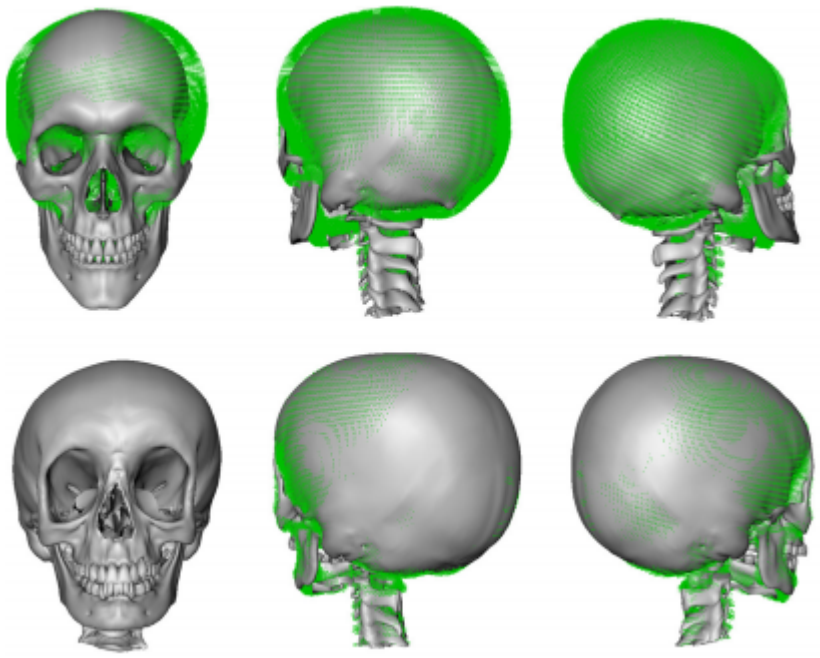


//Underlying ground-truth surf.
//(extracted by Marching Cubes)



//Deformation process.

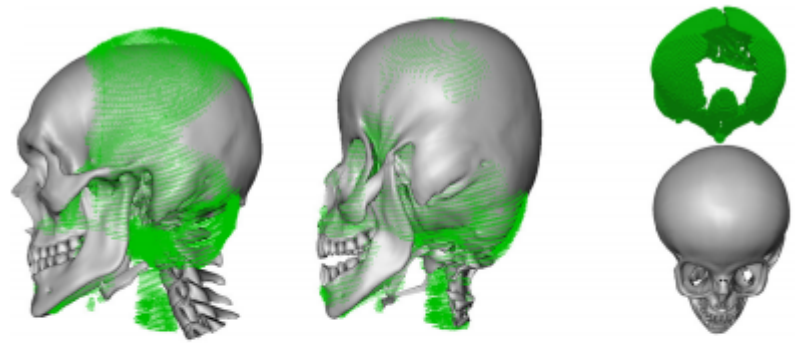
Results



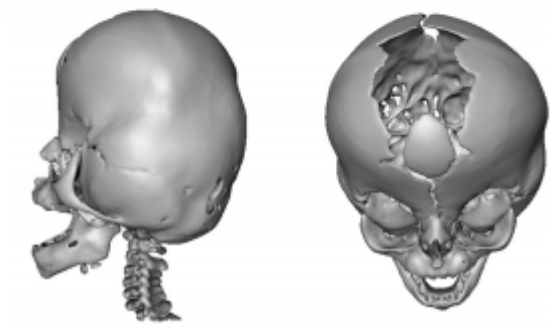
//Initial global alignment.

//Resulting registered model.

Results

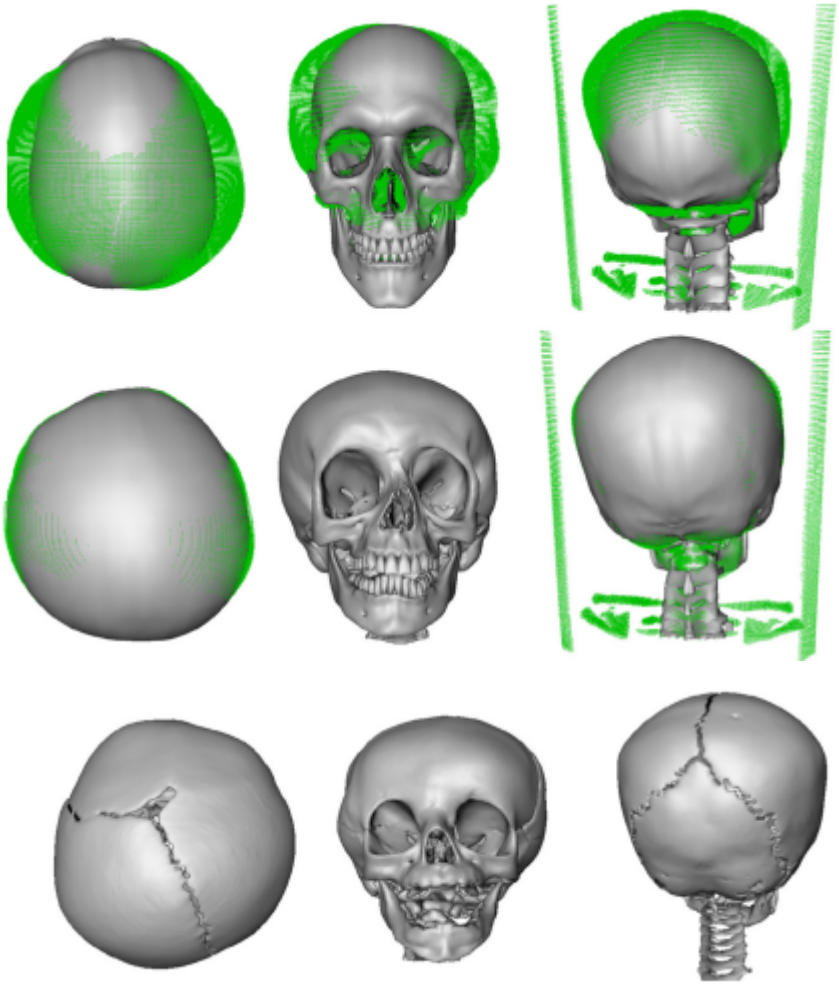


//Initial global alignment and
//resulting registered model.



//Underlying ground-truth surf.

Results

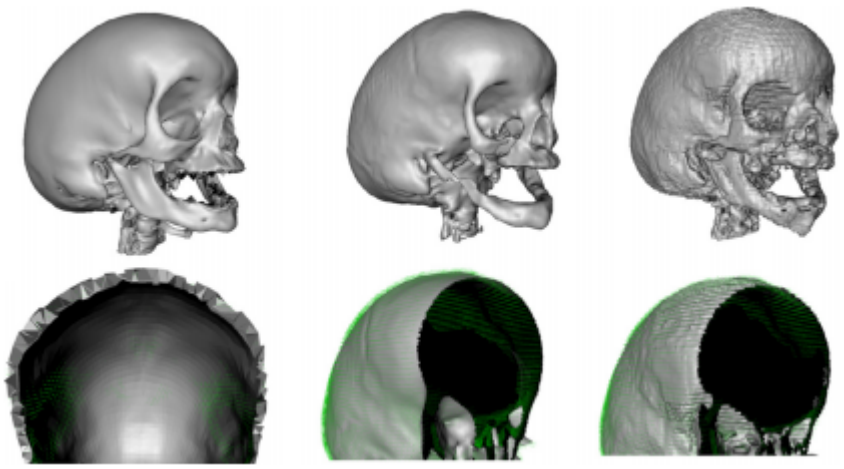


//Initial global alignment.

//Resulting registered model.

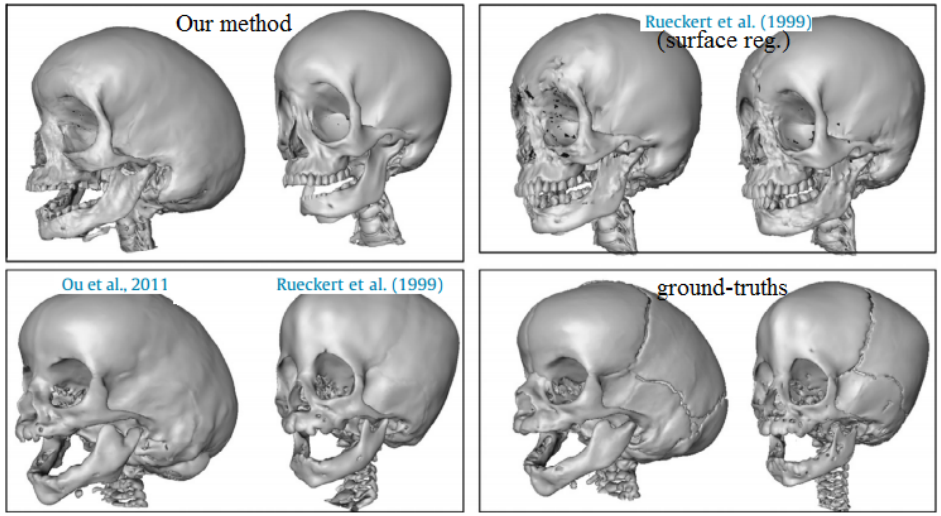
//Underlying ground-truth surf.

Results



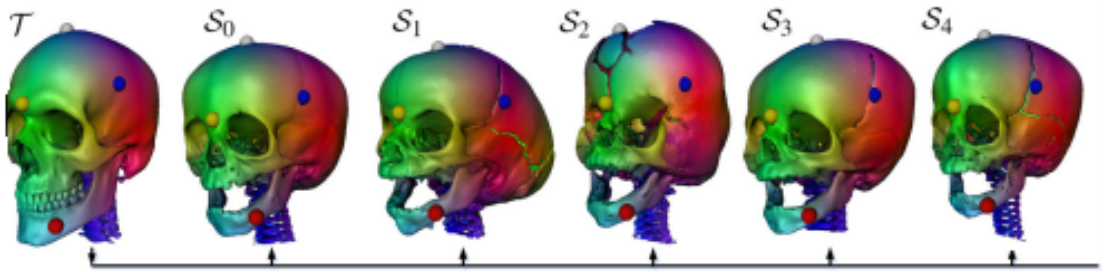
//Our ***volumetric*** registration
//that respects bone thickness
//(left column) is favorable over
//unscreened and screened
//Poisson ***surface*** registrations.

Results



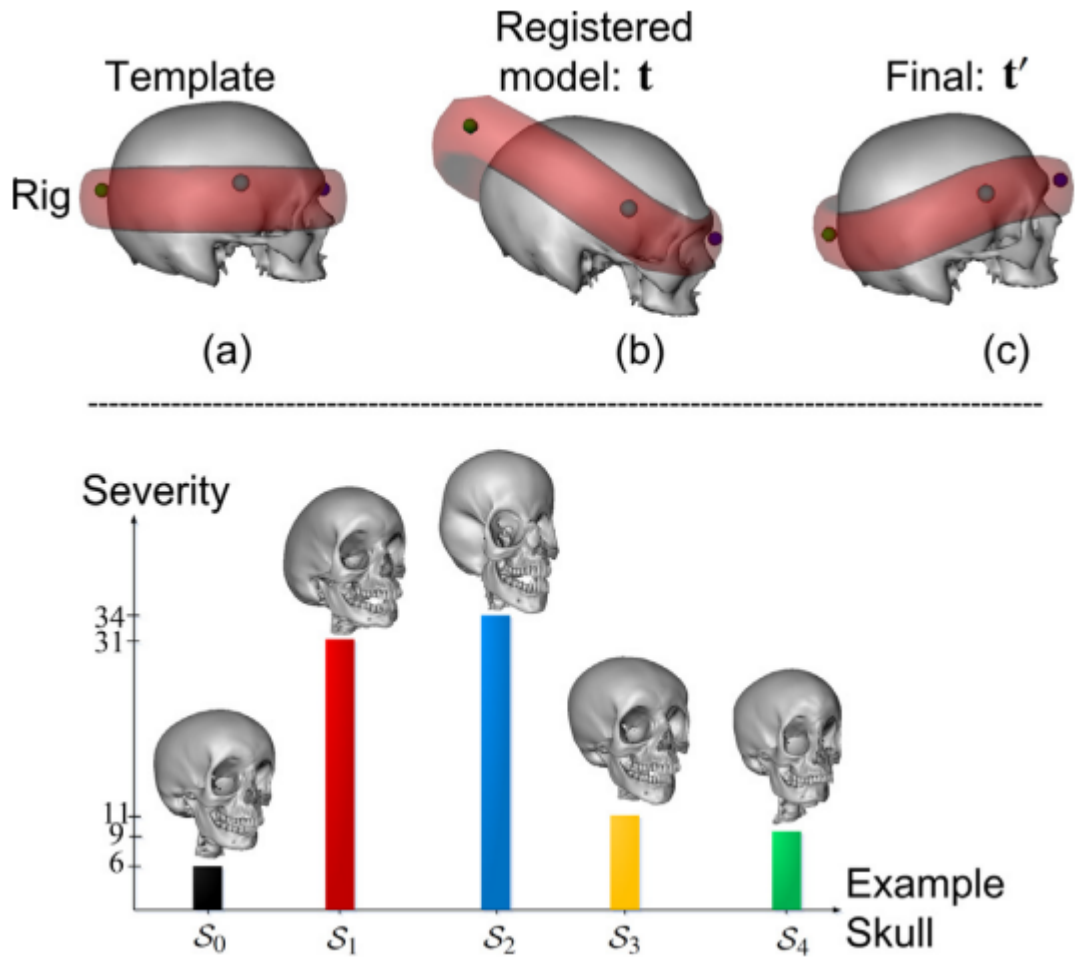
//Our volumetric registration
//in comparison with volumetric
//image registration techniques.

Applications



//Transfer of colors (or any
//other attributes) through the
//***dense correspondence***
//achieved b/w the template and
//deformed/registered pose, and
//effectively between ground-
//truth geometries of all the
//example skulls

Applications



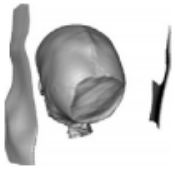
//Objective quantification of
//severity of skull deformities.
//Rig designed specifically for
//the template mesh (top)
//enables interactive
//deformation of the registered
//model using ARAP energy.
//The effort of the doctor during
//this session, i.e., the
//deformation energy, is used as
//the objective quantification
//(bottom).

Future Work

- ✓ Identify most efficient user interfaces for quantification.
- ✓ Data-driven statistical models for quantification.
- ✓ Remeshing algorithms to handle the missing data (teeth) or the finest details (cracks) in the CT scans.

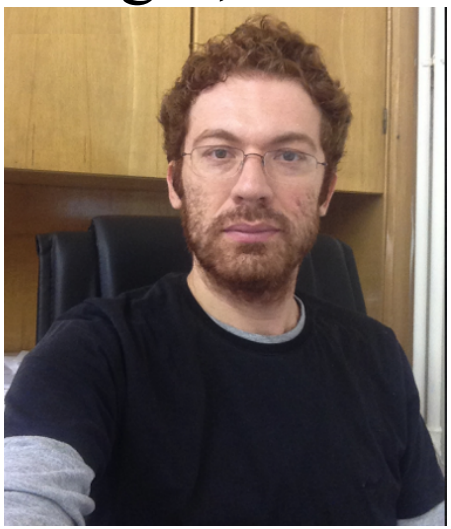
Conclusion

- ✓ Fast and robust solution to volumetric registration of 3D solid shapes.
- ✓ Volumetric deformation energy plus additional tricks such as disabling of correspondence term and adaptive weighting.
- ✓ Faster than registering volume images as mostly done in the medical imaging community.
  - ✓ Also using a template mesh makes it robust to outliers.



- ✓ More accurate than surface registration as mostly done in computer graphics community.
- ✓ Side contributions for ICP (uniform scale, multi-init).
- ✓ Cool apps.
  - ✓ Dense correspondence and objective quantification.

People

- ✓ Asst. Prof. Yusuf Sahillioğlu, METU, Turkey.
- ✓ 
- ✓ Asst. Prof. Ladislav Kavan, UPenn, USA.
- ✓