Vector Expected Utility and Attitudes towards Variation

Marciano Siniscalchi

Northwestern University

ESSM, Pittsburgh, June 2008

Vector Expected Utility: Summary

State space Ω. Outcomes X. Acts $h : \Omega \to X$. Utility $u : X \to \mathbb{R}$.

Vector Expected Utility: Summary

State space Ω. Outcomes X. Acts $h : \Omega \to X$. Utility $u : X \to \mathbb{R}$.

$$V(h) = \mathrm{E}_p[u \circ h] + A\left(\mathrm{E}_p[\zeta_0 \cdot u \circ h], \ldots, \mathrm{E}_p[\zeta_{n-1} \cdot u \circ h]\right).$$

Vector Expected Utility: Summary

State space Ω. Outcomes X. Acts $h : \Omega \to X$. Utility $u : X \to \mathbb{R}$.

$$V(h) = \mathrm{E}_p[u \circ h] + A\left(\mathrm{E}_p[\zeta_0 \cdot u \circ h], \ldots, \mathrm{E}_p[\zeta_{n-1} \cdot u \circ h]\right).$$

Einhorn and Hogarth (1985): Anchoring and Adjustment as response to ambiguity

Vector Expected Utility: Summary

State space Ω. Outcomes X. Acts $h : \Omega \to X$. Utility $u : X \to \mathbb{R}$.

$$V(h) = \mathrm{E}_p[u \circ h] + A\left(\mathrm{E}_p[\zeta_0 \cdot u \circ h], \ldots, \mathrm{E}_p[\zeta_{n-1} \cdot u \circ h]\right).$$

Einhorn and Hogarth (1985): Anchoring and Adjustment as response to ambiguity

Vector Expected Utility: Summary

State space Ω. Outcomes X. Acts $h : \Omega \to X$. Utility $u : X \to \mathbb{R}$.

$$V(h) = \mathrm{E}_p[u \circ h] + A\left(\mathrm{E}_p[\zeta_0 \cdot u \circ h], \ldots, \mathrm{E}_p[\zeta_{n-1} \cdot u \circ h]\right).$$

Baseline prior $p \in \Delta(\Omega)$: uniquely identified from preferences

Vector Expected Utility: Summary

State space Ω. Outcomes X. Acts $h : \Omega \to X$. Utility $u : X \to \mathbb{R}$.

$$V(h) = \mathrm{E}_p[u \circ h] + A\left(\mathrm{E}_p[\zeta_0 \cdot u \circ h], \ldots, \mathrm{E}_p[\zeta_{n-1} \cdot u \circ h]\right).$$

Adjustment Factors: bounded r.v.'s ζ_i on Ω with $\mathrm{E}_p[\zeta_i] = 0$.
Complementarities among ambiguous events (Epstein and Zhang, 2001). Note $n \leq \infty$, finite if Ω finite.

Vector Expected Utility: Summary

State space Ω. Outcomes X. Acts $h : \Omega \to X$. Utility $u : X \to \mathbb{R}$.

$$V(h) = \mathrm{E}_p[u \circ h] + A\left(\mathrm{E}_p[\zeta_0 \cdot u \circ h], \ldots, \mathrm{E}_p[\zeta_{n-1} \cdot u \circ h]\right).$$

Adjustment function $A : \mathbb{R}^n \to \mathbb{R}$: attitudes towards variability, hedging. Aversion, appeal, stake-dependence...

Vector Expected Utility: Summary

State space Ω. Outcomes X. Acts $h : \Omega \to X$. Utility $u : X \to \mathbb{R}$.

$$V(h) = \mathrm{E}_p[u \circ h] + A\left(\mathrm{E}_p[\zeta_0 \cdot u \circ h], \ldots, \mathrm{E}_p[\zeta_{n-1} \cdot u \circ h]\right).$$

Adjustments depend upon variance of $u \circ h$ and correlation with ζ_i:

$$\mathrm{E}_p[\zeta_i \cdot u \circ h] = \mathrm{Cov}_p[\zeta_i, u \circ h] = \mathrm{Corr}_p[\zeta_i, u \circ h] \cdot \sigma_p(\zeta_i) \cdot \sigma_p(u \circ h).$$

Utility smoothing, hedging = variability reduction: key effect in applications of ambiguity-sensitive prefs.

Vector Expected Utility: Summary

State space Ω. Outcomes X. Acts $h : \Omega \to X$. Utility $u : X \to \mathbb{R}$.

$$V(h) = \mathrm{E}_p[u \circ h] + A\left(\mathrm{E}_p[\zeta_0 \cdot u \circ h], \ldots, \mathrm{E}_p[\zeta_{n-1} \cdot u \circ h]\right).$$

Cute names: CEU, MEU, PEU, RDEU, SEU, SOSEU, SOCEU,

Vector Expected Utility: Summary

State space Ω. Outcomes X. Acts $h : \Omega \to X$. Utility $u : X \to \mathbb{R}$.

$$V(h) = \mathrm{E}_p[u \circ h] + A\left(\mathrm{E}_p[\zeta_0 \cdot u \circ h], \ldots, \mathrm{E}_p[\zeta_{n-1} \cdot u \circ h]\right).$$

Cute names: CEU, MEU, PEU, RDEU, SEU, SOSEU, SOCEU, and VEU

Vector Expected Utility: Summary

State space Ω. Outcomes X. Acts $h : \Omega \to X$. Utility $u : X \to \mathbb{R}$.

$$V(h) = \mathrm{E}_p[u \circ h] + A\left(\mathrm{E}_p[\zeta_0 \cdot u \circ h], \ldots, \mathrm{E}_p[\zeta_{n-1} \cdot u \circ h]\right).$$

Main results:

1. Behavioral Characterization of VEU preferences
2. Characterizing p, ζ, n, ambiguity attitudes
3. Updating and Sophisticated choice

Ellsberg: Ambiguity and Complementarity

90 balls: 30 red, 60 green or blue. $\Omega = \{r, g, b\}$. $X = \mathbb{R}$.

	r	g	b
1_r	1	0	0
1_g	0	1	0
1_{rb}	1	0	1
1_{gb}	0	1	1

Ellsberg: Ambiguity and Complementarity

90 balls: 30 red, 60 green or blue. $\Omega = \{r, g, b\}$. $X = \mathbb{R}$.

	r	g	b
1_r	1	0	0
1_g	0	1	0
1_{rb}	1	0	1
1_{gb}	0	1	1

Modal prefs are $1_r \succ 1_g$, $1_{rb} \prec 1_{gb}$: **not EU (or PS)**

Ellsberg: Ambiguity and Complementarity

90 balls: 30 red, 60 green or blue. $\Omega = \{r, g, b\}$. $X = \mathbb{R}$.

	r	g	b
1_r	1	0	0
1_g	0	1	0
1_{rb}	1	0	1
1_{gb}	0	1	1

Modal prefs are $1_r \succ 1_g$, $1_{rb} \prec 1_{gb}$: **not EU (or PS)**

Epstein and Zhang (2001): *"[t]he intuition for this reversal is the complementarity between g and b—there is imprecision regarding the likelihood of g, whereas $\{b, g\}$ has precise probability $\frac{2}{3}$"*

VEU Representation of Complementarities

- Baseline prior: p uniform

VEU Representation of Complementarities

- Baseline prior: p uniform
- Adjustment function: $A(\varphi) = -|\varphi|$.

VEU Representation of Complementarities

- Baseline prior: p uniform
- Adjustment function: $A(\varphi) = -|\varphi|$.
- Adjustment factor: $n = 1$ random variable $\zeta : \Omega \to \mathbb{R}$ with

$$\zeta(r) = 0, \quad \zeta(g) = 1, \quad \zeta(b) = -1.$$

VEU Representation of Complementarities

- Baseline prior: p uniform
- Adjustment function: $A(\varphi) = -|\varphi|$.
- Adjustment factor: $n = 1$ random variable $\zeta : \Omega \to \mathbb{R}$ with

$$\zeta(r) = 0, \quad \zeta(g) = 1, \quad \zeta(b) = -1.$$

$$V(h) = \mathrm{E}_p[u \circ h] + A\left(\mathrm{E}_p[\zeta \cdot u \circ h]\right)$$

VEU Representation of Complementarities

- Baseline prior: p uniform
- Adjustment function: $A(\varphi) = -|\varphi|$.
- Adjustment factor: $n = 1$ random variable $\zeta : \Omega \to \mathbb{R}$ with

$$\zeta(r) = 0, \quad \zeta(g) = 1, \quad \zeta(b) = -1.$$

$$V(h) = \mathrm{E}_p[u \circ h] + A\left(\mathrm{E}_p[\zeta \cdot u \circ h]\right)$$

Then easy to see that:

- $V(1_r) = \frac{1}{3} - |0| = \frac{1}{3}$ and $V(1_g) = \frac{1}{3} - |\frac{1}{3}| = 0$

VEU Representation of Complementarities

- Baseline prior: p uniform
- Adjustment function: $A(\varphi) = -|\varphi|$.
- Adjustment factor: $n = 1$ random variable $\zeta : \Omega \to \mathbb{R}$ with

$$\zeta(r) = 0, \quad \zeta(g) = 1, \quad \zeta(b) = -1.$$

$$V(h) = \mathrm{E}_p[u \circ h] + A\left(\mathrm{E}_p[\zeta \cdot u \circ h]\right)$$

Then easy to see that:

- $V(1_r) = \frac{1}{3} - |0| = \frac{1}{3}$ and $V(1_g) = \frac{1}{3} - |\frac{1}{3}| = 0$
- $V(1_{rb}) = \frac{2}{3} - |0 - \frac{1}{3}| = \frac{1}{3}$ and $V(1_{gb}) = \frac{2}{3} - |\frac{1}{3} - \frac{1}{3}| = \frac{2}{3}$

Some MEU Examples

- $\Omega = \{\omega_0, \ldots, \omega_k\}$, $p \in \Delta(\Omega)$, $u(x) = x$.

Some MEU Examples

- $\Omega = \{\omega_0, \ldots, \omega_k\}$, $p \in \Delta(\Omega)$, $u(x) = x$.
- Mean-Std Deviation:

$$V(h) = \mathrm{E}_p(h) - \epsilon\sigma_p(h)$$

Some MEU Examples

- $\Omega = \{\omega_0, \ldots, \omega_k\}$, $p \in \Delta(\Omega)$, $u(x) = x$.
- Mean-Std Deviation:

$$V(h) = \mathrm{E}_p(h) - \epsilon\sigma_p(h)$$

- Mean-Gini (generalizes to arbitrary Ω): for $\theta \in [0, 1]$,

$$V(h) = \mathrm{E}_p[h] - \frac{1}{2}\theta \iint |h(\omega) - h(\omega')| \, p(d\omega') \, p(d\omega).$$

Some MEU Examples

- $\Omega = \{\omega_0, \ldots, \omega_k\}$, $p \in \Delta(\Omega)$, $u(x) = x$.
- Mean-Std Deviation:

$$V(h) = \mathrm{E}_p(h) - \epsilon \sigma_p(h)$$

- Mean-Gini (generalizes to arbitrary Ω): for $\theta \in [0, 1]$,

$$V(h) = \mathrm{E}_p[h] - \frac{1}{2}\theta \iint |h(\omega) - h(\omega')| \, p(d\omega') \, p(d\omega).$$

- "Euclidean robustness":

$$V(h) = \sum_{i=0}^{k} h(\omega_i) p_i - \epsilon \sqrt{\sum_{i=0}^{k} \left(h(\omega_i) - \frac{1}{k+1} \sum_{j=0}^{k} h(\omega_j) \right)^2}.$$

Some MEU Examples

- $\Omega = \{\omega_0, \ldots, \omega_k\}$, $p \in \Delta(\Omega)$, $u(x) = x$.
- Mean-Std Deviation:

$$V(h) = \mathrm{E}_p(h) - \epsilon\sigma_p(h)$$

- Mean-Gini (generalizes to arbitrary Ω): for $\theta \in [0, 1]$,

$$V(h) = \mathrm{E}_p[h] - \frac{1}{2}\theta \iint |h(\omega) - h(\omega')|\, p(d\omega')\, p(d\omega).$$

- "Euclidean robustness":

$$V(h) = \sum_{i=0}^{k} h(\omega_i)p_i - \epsilon\sqrt{\sum_{i=0}^{k}\left(h(\omega_i) - \frac{1}{k+1}\sum_{j=0}^{k} h(\omega_j)\right)^2}.$$

Result:
$V(h) = \min\{\int h\, dq : q \in \Delta(\Omega), \sum_i [q(\omega_i) - p(\omega_i)]^2 \leq \epsilon^2\}$.

Examples: Ambiguity and Outcome Size (1)

Einhorn-Hogarth (1986); Koch-Schunk (2007): stakes affect ambiguity attitudes:

- Ambiguity-averse for large stakes;
- Ambiguity-neutral or -seeking for small stakes.

Cf. Prelec-Lowenstein (1991), “peanuts effect.”

Examples: Ambiguity and Outcome Size (1)

Einhorn-Hogarth (1986); Koch-Schunk (2007): stakes affect ambiguity attitudes:

- Ambiguity-averse for large stakes;
- Ambiguity-neutral or -seeking for small stakes.

Cf. Prelec-Lowenstein (1991), "peanuts effect."

Match this with differentiable VEU prefs.

$\Omega = \{\omega, \omega'\}$; p uniform, u linear; ζ single adjustment function.

Parameter $t > 1$.

Examples: Ambiguity and Outcome Size (1)

Einhorn-Hogarth (1986); Koch-Schunk (2007): stakes affect ambiguity attitudes:

- Ambiguity-averse for large stakes;
- Ambiguity-neutral or -seeking for small stakes.

Cf. Prelec-Lowenstein (1991), "peanuts effect."

Match this with differentiable VEU prefs.

$\Omega = \{\omega, \omega'\}$; p uniform, u linear; ζ single adjustment function.

Parameter $t > 1$.

$A(\varphi) = \log\left(1 + t^2\right) - \frac{1}{2}\log\left(1 + (\varphi + t)^2\right) - \frac{1}{2}\log\left(1 + (\varphi - t)^2\right)$.

Examples: Ambiguity and Outcome Size (2)

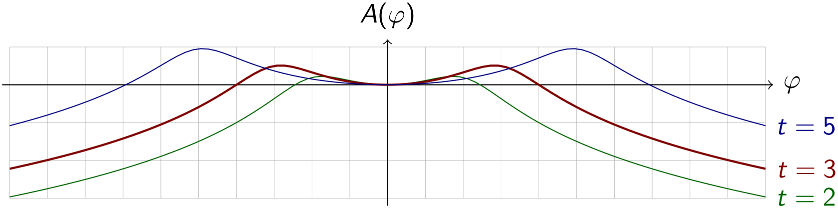


Figure: The Adjustment Function $A(\varphi)$

Examples: Ambiguity and Outcome Size (2)

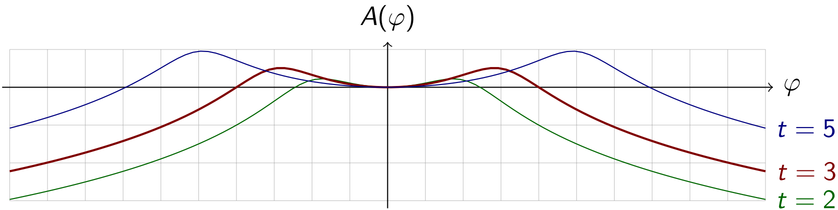


Figure: The Adjustment Function $A(\varphi)$

- Let $t = 3$ and $\zeta(\omega) = -\zeta(\omega') = 0.4$

Examples: Ambiguity and Outcome Size (2)

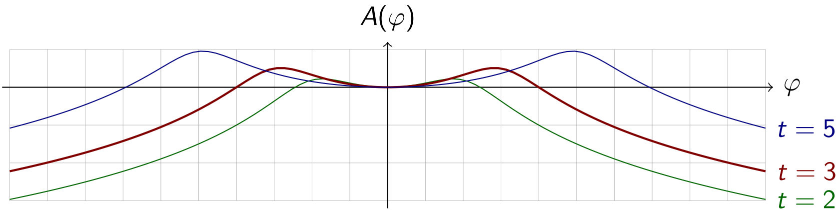


Figure: The Adjustment Function $A(\varphi)$

- Let $t = 3$ and $\zeta(\omega) = -\zeta(\omega') = 0.4$
- Small bet: $f(\omega) = 10,\ f(\omega') = 0$

Examples: Ambiguity and Outcome Size (2)

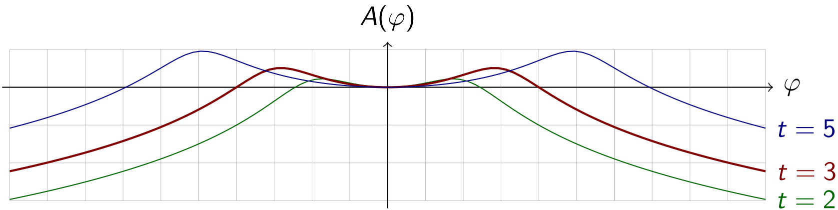


Figure: The Adjustment Function $A(\varphi)$

- Let $t = 3$ and $\zeta(\omega) = -\zeta(\omega') = 0.4$
- Small bet: $f(\omega) = 10$, $f(\omega') = 0$
- Large bet: $g(\omega) = 1000$, $g(\omega') = 0$

Examples: Ambiguity and Outcome Size (2)

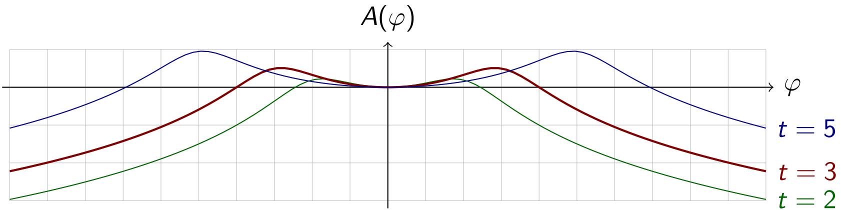


Figure: The Adjustment Function $A(\varphi)$

- Let $t = 3$ and $\zeta(\omega) = -\zeta(\omega') = 0.4$
- Small bet: $f(\omega) = 10,\ f(\omega') = 0$
- Large bet: $g(\omega) = 1000,\ g(\omega') = 0$
- Baseline certainty equivalents: $\int f\, dp = 5$ and $\int g\, dp = 500$.

Examples: Ambiguity and Outcome Size (2)

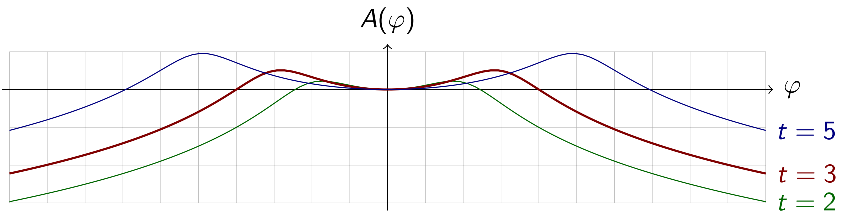


Figure: The Adjustment Function $A(\varphi)$

- Let $t = 3$ and $\zeta(\omega) = -\zeta(\omega') = 0.4$
- Small bet: $f(\omega) = 10$, $f(\omega') = 0$
- Large bet: $g(\omega) = 1000$, $g(\omega') = 0$
- Baseline certainty equivalents: $\int f \, dp = 5$ and $\int g \, dp = 500$.
- Actual certainty equivalents: $\mathrm{CE}[f] = 5.33$, $\mathrm{CE}[g] = 471.91$.

Axioms and Characterization

Basic axioms: Maccheroni, Marinacci and Rustichini (2006) minus Ambiguity Aversion

Axioms and Characterization

Basic axioms: Maccheroni, Marinacci and Rustichini (2006) minus Ambiguity Aversion

Key axiom: Complementary Independence (later)

Axioms and Characterization

Basic axioms: Maccheroni, Marinacci and Rustichini (2006) minus Ambiguity Aversion

Key axiom: Complementary Independence (later)

Additional axiom: Complementary Translation Invariance, Monotone Continuity.

Axioms and Characterization

Basic axioms: Maccheroni, Marinacci and Rustichini (2006) minus Ambiguity Aversion

Key axiom: Complementary Independence (later)

Additional axiom: Complementary Translation Invariance, Monotone Continuity.

Theorem

Axioms if and only if representation.

Axioms and Characterization

Basic axioms: Maccheroni, Marinacci and Rustichini (2006) minus Ambiguity Aversion

Key axiom: Complementary Independence (later)

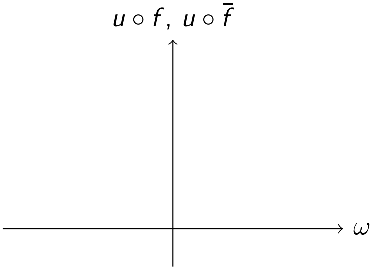
Additional axiom: Complementary Translation Invariance, Monotone Continuity.

Theorem

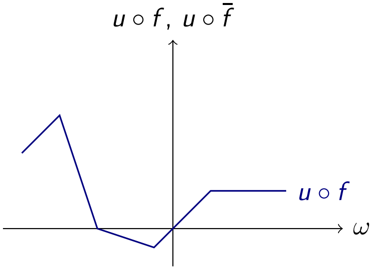
Axioms if and only if representation.

p unique, u cardinally unique, minimal n unique; A, ζ_i unique up to linear bijection that preserves adjustments.

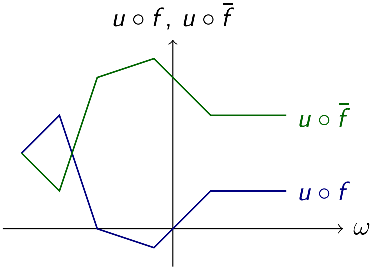
Complementary Acts



Complementary Acts

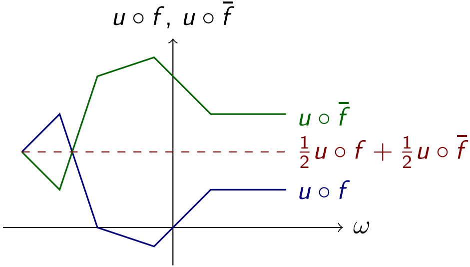


Complementary Acts



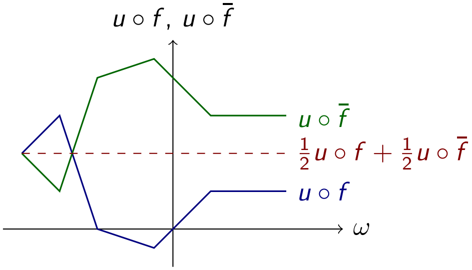
"$\bar{f}$ = constant − f". Same "variability"

Complementary Acts



Therefore, $\frac{1}{2} f + \frac{1}{2} \bar{f}$ has constant utility

Complementary Acts

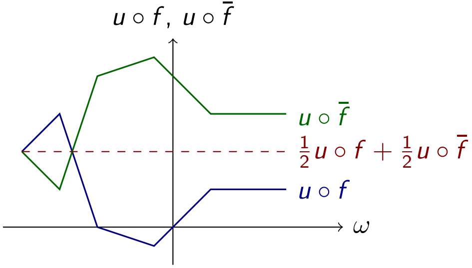


Definition

Two acts $f, \bar{f}$ are complementary if, for all $\omega, \omega' \in \Omega$,

$$\frac{1}{2} f(\omega) + \frac{1}{2} \bar{f}(\omega) \sim \frac{1}{2} f(\omega') + \frac{1}{2} \bar{f}(\omega').$$

Complementary Acts



Definition

Two acts $f, \bar{f}$ are complementary if, for all $\omega, \omega' \in \Omega$,

$$\frac{1}{2} f(\omega) + \frac{1}{2} \bar{f}(\omega) \sim \frac{1}{2} f(\omega') + \frac{1}{2} \bar{f}(\omega').$$

$f, \bar{f}$ and $g, \bar{g}$ compl. $\Rightarrow \alpha f + (1-\alpha) g,\ \alpha \bar{f} + (1-\alpha)\bar{g}$ compl.

The Key Axiom

Axiom (Complementary Independence)

If $f, \bar{f}$ and $g, \bar{g}$ are complementary pairs and $f \succcurlyeq \bar{f}$, $g \succcurlyeq \bar{g}$, then $\alpha f + (1-\alpha)g \succcurlyeq \alpha \bar{f} + (1-\alpha)\bar{g}$ for all $\alpha \in (0,1)$.

The Key Axiom

Axiom (Complementary Independence)

If $f, \bar{f}$ and $g, \bar{g}$ are complementary pairs and $f \succcurlyeq \bar{f}$, $g \succcurlyeq \bar{g}$, then $\alpha f + (1-\alpha)g \succcurlyeq \alpha \bar{f} + (1-\alpha)\bar{g}$ for all $\alpha \in (0,1)$.

- AA Independence $\Leftrightarrow$ above property for all $f, \bar{f}, g, \bar{g}$.

The Key Axiom

Axiom (Complementary Independence)

If $f, \bar{f}$ and $g, \bar{g}$ are complementary pairs and $f \succcurlyeq \bar{f}$, $g \succcurlyeq \bar{g}$, then $\alpha f + (1-\alpha)g \succcurlyeq \alpha \bar{f} + (1-\alpha)\bar{g}$ for all $\alpha \in (0,1)$.

- AA Independence $\Leftrightarrow$ above property for all $f, \bar{f}, g, \bar{g}$.
- VEU: rank acts using baseline EU evaluation and adjustment

The Key Axiom

Axiom (Complementary Independence)

If $f, \bar{f}$ and $g, \bar{g}$ are complementary pairs and $f \succcurlyeq \bar{f}$, $g \succcurlyeq \bar{g}$, then $\alpha f + (1-\alpha)g \succcurlyeq \alpha \bar{f} + (1-\alpha)\bar{g}$ for all $\alpha \in (0,1)$.

- AA Independence $\Leftrightarrow$ above property for all $f, \bar{f}, g, \bar{g}$.
- VEU: rank acts using baseline EU evaluation and adjustment
- Adjustment depends upon statewise variation around baseline

The Key Axiom

Axiom (Complementary Independence)

If $f, \bar{f}$ and $g, \bar{g}$ are complementary pairs and $f \succcurlyeq \bar{f}$, $g \succcurlyeq \bar{g}$, then $\alpha f + (1-\alpha) g \succcurlyeq \alpha \bar{f} + (1-\alpha)\bar{g}$ for all $\alpha \in (0,1)$.

- AA Independence $\Leftrightarrow$ above property for all $f, \bar{f}, g, \bar{g}$.
- VEU: rank acts using baseline EU evaluation and adjustment
- Adjustment depends upon statewise variation around baseline
- Complementary acts: same variation, hence same adjustment

The Key Axiom

Axiom (Complementary Independence)

If $f, \bar{f}$ and $g, \bar{g}$ are complementary pairs and $f \succcurlyeq \bar{f}$, $g \succcurlyeq \bar{g}$, then $\alpha f + (1-\alpha) g \succcurlyeq \alpha \bar{f} + (1-\alpha)\bar{g}$ for all $\alpha \in (0,1)$.

- AA Independence $\Leftrightarrow$ above property for all $f, \bar{f}, g, \bar{g}$.
- VEU: rank acts using baseline EU evaluation and adjustment
- Adjustment depends upon statewise variation around baseline
- Complementary acts: same variation, hence same adjustment
- Therefore prefs driven by baseline evaluations, assumed EU

The Key Axiom

Axiom (Complementary Independence)

If $f, \bar{f}$ and $g, \bar{g}$ are complementary pairs and $f \succcurlyeq \bar{f}$, $g \succcurlyeq \bar{g}$, then $\alpha f + (1-\alpha) g \succcurlyeq \alpha \bar{f} + (1-\alpha)\bar{g}$ for all $\alpha \in (0,1)$.

- AA Independence $\Leftrightarrow$ above property for all $f, \bar{f}, g, \bar{g}$.
- VEU: rank acts using baseline EU evaluation and adjustment
- Adjustment depends upon statewise variation around baseline
- Complementary acts: same variation, hence same adjustment
- Therefore prefs driven by baseline evaluations, assumed EU
- So, Independence should hold for complementary acts.

The Key Axiom

Axiom (Complementary Independence)

If $f, \bar{f}$ and $g, \bar{g}$ are complementary pairs and $f \succcurlyeq \bar{f}$, $g \succcurlyeq \bar{g}$, then $\alpha f + (1-\alpha)g \succcurlyeq \alpha\bar{f} + (1-\alpha)\bar{g}$ for all $\alpha \in (0,1)$.

- AA Independence $\Leftrightarrow$ above property for all $f, \bar{f}, g, \bar{g}$.
- VEU: rank acts using baseline EU evaluation and adjustment
- Adjustment depends upon statewise variation around baseline
- Complementary acts: same variation, hence same adjustment
- Therefore prefs driven by baseline evaluations, assumed EU
- So, Independence should hold for complementary acts.

Baseline prior has clear behavioral interpretation: yields EU representation of preferences over complementary acts

Ambiguity Aversion: Classical Definition

Begin with the "classical" notion due to Schmeidler (1989)

Axiom (Ambiguity Aversion)

For all acts $f, g \in L_0$ and $\alpha \in (0,1)$: $f \sim g$ implies $\alpha f + (1-\alpha)g \succcurlyeq g$.

Ambiguity Aversion: Classical Definition

Begin with the "classical" notion due to Schmeidler (1989)

Axiom (Ambiguity Aversion)

For all acts $f, g \in L_0$ and $\alpha \in (0,1)$: $f \sim g$ implies $\alpha f + (1-\alpha)g \succcurlyeq g$.

Corollary

$\succcurlyeq$ satisfies Ambiguity Aversion if and only if A is non-positive and concave.

Alternative notions of Ambiguity Aversion

GMM's comparative definition:

Definition

$\succcurlyeq_1$ is **comparatively ambiguity-averse** iff $\succcurlyeq_2$ conforms to EU and, for all $f \in L_0$ and $x \in X$, $f \succcurlyeq_1 x$ implies $f \succcurlyeq_2 x$.

Alternative notions of Ambiguity Aversion

GMM's comparative definition:

Definition

$\succcurlyeq_1$ is **comparatively ambiguity-averse** iff $\succcurlyeq_2$ conforms to EU and, for all $f \in L_0$ and $x \in X$, $f \succcurlyeq_1 x$ implies $f \succcurlyeq_2 x$.

Two more quasiconcavity axioms:

Axiom (Simple Ambiguity Aversion)

For all complementary pairs $(f, \bar{f})$ and prizes $x, \bar{x} \in X$ such that $f \sim x$ and $\bar{f} \sim \bar{x}$: $\frac{1}{2}f + \frac{1}{2}\bar{f} \succcurlyeq \frac{1}{2}x + \frac{1}{2}\bar{x}$.

Alternative notions of Ambiguity Aversion

GMM's comparative definition:

Definition

$\succcurlyeq_1$ is **comparatively ambiguity-averse** iff $\succcurlyeq_2$ conforms to EU and, for all $f \in L_0$ and $x \in X$, $f \succcurlyeq_1 x$ implies $f \succcurlyeq_2 x$.

Two more quasiconcavity axioms:

Axiom (Simple Ambiguity Aversion)

For all complementary pairs $(f, \bar{f})$ and prizes $x, \bar{x} \in X$ such that $f \sim x$ and $\bar{f} \sim \bar{x}$: $\frac{1}{2}f + \frac{1}{2}\bar{f} \succcurlyeq \frac{1}{2}x + \frac{1}{2}\bar{x}$.

Axiom (Minimal Ambiguity Aversion)

For all complementary pairs $(f, \bar{f})$ with $f \sim \bar{f}$, $\frac{1}{2}f + \frac{1}{2}\bar{f} \succcurlyeq f$.

Ambiguity Aversion: Further Characterization

Proposition

Let $\succcurlyeq$ be a preference with VEU representation (u, p, n, m, A). Then the following are equivalent:

1. $\succcurlyeq$ *is comparatively ambiguity-averse.*
2. $\succcurlyeq$ *satisfies Simple Ambiguity Aversion.*
3. $A \leq 0$.

If $u(X)$ is unbounded above or below, or if $\succcurlyeq$ satisfies C-Independence, then 1–3 are equivalent to

4. $\succcurlyeq$ *satisfies Minimal Ambiguity Aversion.*

Updating VEU Preferences: Overview

Behaviorally based updating rule for VEU prefs.

Updating VEU Preferences: Overview

Behaviorally based updating rule for VEU prefs.

Bayesian updating for p. But how about adjustments ζ?

Updating VEU Preferences: Overview

Behaviorally based updating rule for VEU prefs.

Bayesian updating for p. But how about adjustments ζ?

Idea: from $\mathrm{E}_p[u \circ h] + A(\mathrm{Cov}_p[\zeta \cdot u \circ h])$...

...to $\mathrm{E}_p[u \circ h | E] + A(p(E) \cdot \mathrm{Cov}_p[\zeta \cdot u \circ h | E])$

where $p(E)$ in $A(\cdot)$ ensures monotonicity.

Updating VEU Preferences: Overview

Behaviorally based updating rule for VEU prefs.

Bayesian updating for p. But how about adjustments ζ?

Idea: from $\mathrm{E}_p[u \circ h] + A(\mathrm{Cov}_p[\zeta \cdot u \circ h])$...

...to $\mathrm{E}_p[u \circ h|E] + A(p(E) \cdot \mathrm{Cov}_p[\zeta \cdot u \circ h|E])$

where $p(E)$ in $A(\cdot)$ ensures monotonicity.

"Rescaling and recentering"

$$\zeta_E = p(E) \cdot \left[\zeta - \mathrm{E}_p[\zeta|E]\right].$$

Updating VEU Preferences: Overview

Behaviorally based updating rule for VEU prefs.

Bayesian updating for p. But how about adjustments ζ?

Idea: from $\mathrm{E}_p[u \circ h] + A(\mathrm{Cov}_p[\zeta \cdot u \circ h])$...

...to $\mathrm{E}_p[u \circ h | E] + A(p(E) \cdot \mathrm{Cov}_p[\zeta \cdot u \circ h | E])$

where $p(E)$ in $A(\cdot)$ ensures monotonicity.

"Rescaling and recentering"

$$\zeta_E = p(E) \cdot \left[\zeta - \mathrm{E}_p[\zeta | E]\right].$$

Facilitates recursive analysis of sophisticated choice.

Baseline-Variation Consistency: Intuition

- VEU: ex-ante and E-contingent evaluation of act h may change because:

Baseline-Variation Consistency: Intuition

- VEU: ex-ante and E-contingent evaluation of act h may change because:
  - The baseline EU evaluation is updated

Baseline-Variation Consistency: Intuition

- VEU: ex-ante and E-contingent evaluation of act h may change because:
  - The baseline EU evaluation is updated
  - Variability on $\Omega \setminus E$ no longer matters.

Baseline-Variation Consistency: Intuition

- VEU: ex-ante and E-contingent evaluation of act h may change because:
  - The baseline EU evaluation is updated
  - Variability on $\Omega \setminus E$ no longer matters.
- Conversely, expect no change for acts that have:
  - Same baseline EU evaluation ex-ante and given E

Baseline-Variation Consistency: Intuition

- VEU: ex-ante and E-contingent evaluation of act h may change because:
  - The baseline EU evaluation is updated
  - Variability on $\Omega \setminus E$ no longer matters.
- Conversely, expect no change for acts that have:
  - Same baseline EU evaluation ex-ante and given E
  - No variation away from baseline outside E.

Baseline-Variation Consistency: Intuition

- VEU: ex-ante and E-contingent evaluation of act h may change because:
  - The baseline EU evaluation is updated
  - Variability on $\Omega \setminus E$ no longer matters.
- Conversely, expect no change for acts that have:
  - Same baseline EU evaluation ex-ante and given E
  - No variation away from baseline outside E.
- Axiom requires this. Thus, need to characterize such acts.

Baseline-Variation Consistency: Intuition

- VEU: ex-ante and E-contingent evaluation of act h may change because:
  - The baseline EU evaluation is updated
  - Variability on $\Omega \setminus E$ no longer matters.
- Conversely, expect no change for acts that have:
  - Same baseline EU evaluation ex-ante and given E
  - No variation away from baseline outside E.
- Axiom requires this. Thus, need to characterize such acts.
- Key idea: if $h, \bar{h}$ complementary, constant on $\Omega \setminus E$, and

$$\forall \omega \in \Omega \setminus E, \quad \frac{1}{2}h + \frac{1}{2}\bar{h}(\omega) \sim \frac{1}{2}\bar{h} + \frac{1}{2}h(\omega)$$

then $u(h(\omega)) = \mathrm{E}_p[u \circ h] = \mathrm{E}_p[u \circ h | E]$ for $\omega \in \Omega \setminus E$

Baseline-Variation Consistency

Axiom (Baseline-Variation Consistency)

For all complementary pairs $(f, \bar{f})$ and $(g, \bar{g})$ such that:

- $f, \bar{f}, g, \bar{g}$ are constant on $\Omega \setminus E$

Baseline-Variation Consistency

Axiom (Baseline-Variation Consistency)

For all complementary pairs $(f, \bar{f})$ and $(g, \bar{g})$ such that:

- $f, \bar{f}, g, \bar{g}$ are constant on $\Omega \setminus E$
- for every $\omega \in \Omega \setminus E$, $\frac{1}{2}f + \frac{1}{2}\bar{f}(\omega) \sim \frac{1}{2}\bar{f} + \frac{1}{2}f(\omega)$

Baseline-Variation Consistency

Axiom (Baseline-Variation Consistency)

For all complementary pairs $(f, \bar{f})$ and $(g, \bar{g})$ such that:

- $f, \bar{f}, g, \bar{g}$ are constant on $\Omega \setminus E$
- for every $\omega \in \Omega \setminus E$, $\frac{1}{2}f + \frac{1}{2}\bar{f}(\omega) \sim \frac{1}{2}\bar{f} + \frac{1}{2}f(\omega)$
- for every $\omega \in \Omega \setminus E$, $\frac{1}{2}g + \frac{1}{2}\bar{g}(\omega) \sim \frac{1}{2}\bar{g} + \frac{1}{2}g(\omega)$:

Baseline-Variation Consistency

Axiom (Baseline-Variation Consistency)

For all complementary pairs $(f, \bar{f})$ and $(g, \bar{g})$ such that:

- $f, \bar{f}, g, \bar{g}$ are constant on $\Omega \setminus E$
- for every $\omega \in \Omega \setminus E$, $\frac{1}{2}f + \frac{1}{2}\bar{f}(\omega) \sim \frac{1}{2}\bar{f} + \frac{1}{2}f(\omega)$
- for every $\omega \in \Omega \setminus E$, $\frac{1}{2}g + \frac{1}{2}\bar{g}(\omega) \sim \frac{1}{2}\bar{g} + \frac{1}{2}g(\omega)$:

$f \succcurlyeq_E g$ if and only if $f \succcurlyeq g$.

Baseline-Variation Consistency

Axiom (Baseline-Variation Consistency)

For all complementary pairs $(f, \bar{f})$ and $(g, \bar{g})$ such that:

- $f, \bar{f}, g, \bar{g}$ are constant on $\Omega \setminus E$
- for every $\omega \in \Omega \setminus E$, $\frac{1}{2}f + \frac{1}{2}\bar{f}(\omega) \sim \frac{1}{2}\bar{f} + \frac{1}{2}f(\omega)$
- for every $\omega \in \Omega \setminus E$, $\frac{1}{2}g + \frac{1}{2}\bar{g}(\omega) \sim \frac{1}{2}\bar{g} + \frac{1}{2}g(\omega)$:

$f \succcurlyeq_E g$ if and only if $f \succcurlyeq g$.

Proposition

Bayesian updating plus "Rescaling and recentering" if and only if axioms hold.

One-Slide Analysis of the Equity Premium Puzzle

- $\max_\xi U(c_t, c_{t+1})$ s. to $c_t = e_t - p_t\xi$, $c_{t+1} = e_{t+1} + \xi x_{t+1}$.

One-Slide Analysis of the Equity Premium Puzzle

- $\max_\xi U(c_t, c_{t+1})$ s. to $c_t = e_t - p_t\xi$, $c_{t+1} = e_{t+1} + \xi x_{t+1}$.
- Stochastic Discount Factor: $p_t = \mathrm{E}[m x_{t+1}]$

One-Slide Analysis of the Equity Premium Puzzle

- $\max_\xi U(c_t, c_{t+1})$ s. to $c_t = e_t - p_t\xi$, $c_{t+1} = e_{t+1} + \xi x_{t+1}$.
- Stochastic Discount Factor: $p_t = \mathrm{E}[m x_{t+1}]$
- $U(c_t, c_{t+1}) = u(c_t) + \beta \mathrm{E}_p[u(c_{t+1})]$, $m = \frac{\beta u'(c_{t+1})}{u'(c_t)}$.

One-Slide Analysis of the Equity Premium Puzzle

- $\max_{\xi} U(c_t, c_{t+1})$ s. to $c_t = e_t - p_t\xi$, $c_{t+1} = e_{t+1} + \xi x_{t+1}$.
- Stochastic Discount Factor: $p_t = \mathrm{E}[m x_{t+1}]$
- $U(c_t, c_{t+1}) = u(c_t) + \beta \mathrm{E}_p[u(c_{t+1})]$, $m = \frac{\beta u'(c_{t+1})}{u'(c_t)}$.
- $U(c_t, c_{t+1}) = u(c_t) + \beta\{\mathrm{E}_p[u(c_{t+1})] + A(\mathrm{E}_p[\zeta_{t+1} u(c_{t+1})])\}$:

$$m = \frac{\beta u'(c_{t+1})[1 + \zeta_{t+1} A'(\mathrm{E}_p[\zeta_{t+1} u(c_{t+1})])]}{u'(c_t)}$$

One-Slide Analysis of the Equity Premium Puzzle

- $\max_{\xi} U(c_t, c_{t+1})$ s. to $c_t = e_t - p_t\xi$, $c_{t+1} = e_{t+1} + \xi x_{t+1}$.
- Stochastic Discount Factor: $p_t = \mathrm{E}[m x_{t+1}]$
- $U(c_t, c_{t+1}) = u(c_t) + \beta \mathrm{E}_p[u(c_{t+1})]$, $m = \frac{\beta u'(c_{t+1})}{u'(c_t)}$.
- $U(c_t, c_{t+1}) = u(c_t) + \beta\{\mathrm{E}_p[u(c_{t+1})] + A(\mathrm{E}_p[\zeta_{t+1} u(c_{t+1})])\}$:

$$m = \frac{\beta u'(c_{t+1})[1 + \zeta_{t+1} A'(\mathrm{E}_p[\zeta_{t+1} u(c_{t+1})])]}{u'(c_t)}$$

- HJ bounds: slope of efficient frontier $\frac{\sigma(m)}{\mathrm{E}_p[m]}$.

One-Slide Analysis of the Equity Premium Puzzle

- $\max_\xi U(c_t, c_{t+1})$ s. to $c_t = e_t - p_t\xi$, $c_{t+1} = e_{t+1} + \xi x_{t+1}$.
- Stochastic Discount Factor: $p_t = \mathrm{E}[m x_{t+1}]$
- $U(c_t, c_{t+1}) = u(c_t) + \beta \mathrm{E}_p[u(c_{t+1})]$, $m = \frac{\beta u'(c_{t+1})}{u'(c_t)}$.
- $U(c_t, c_{t+1}) = u(c_t) + \beta\{\mathrm{E}_p[u(c_{t+1})] + A(\mathrm{E}_p[\zeta_{t+1} u(c_{t+1})])\}$:

$$m = \frac{\beta u'(c_{t+1})[1 + \zeta_{t+1} A'(\mathrm{E}_p[\zeta_{t+1} u(c_{t+1})])]}{u'(c_t)}$$

- HJ bounds: slope of efficient frontier $\frac{\sigma(m)}{\mathrm{E}_p[m]}$.
- Power utility $u'(c) = c^{-\gamma}$, $\ln c_{t+1} - \ln c_t \sim \mathrm{N}(\mu, \sigma^2)$.
  Puzzle: slope ≈ 0.50, requires $\gamma \approx 50$; also, r_f too high.

One-Slide Analysis of the Equity Premium Puzzle

- $\max_\xi U(c_t, c_{t+1})$ s. to $c_t = e_t - p_t\xi$, $c_{t+1} = e_{t+1} + \xi x_{t+1}$.
- Stochastic Discount Factor: $p_t = \mathrm{E}[m x_{t+1}]$
- $U(c_t, c_{t+1}) = u(c_t) + \beta \mathrm{E}_p[u(c_{t+1})]$, $m = \frac{\beta u'(c_{t+1})}{u'(c_t)}$.
- $U(c_t, c_{t+1}) = u(c_t) + \beta\{\mathrm{E}_p[u(c_{t+1})] + A(\mathrm{E}_p[\zeta_{t+1} u(c_{t+1})])\}$:

$$m = \frac{\beta u'(c_{t+1})[1 + \zeta_{t+1} A'(\mathrm{E}_p[\zeta_{t+1} u(c_{t+1})])]}{u'(c_t)}$$

- HJ bounds: slope of efficient frontier $\frac{\sigma(m)}{\mathrm{E}_p[m]}$.
- Power utility $u'(c) = c^{-\gamma}$, $\ln c_{t+1} - \ln c_t \sim \mathrm{N}(\mu, \sigma^2)$.
  Puzzle: slope ≈ 0.50, requires $\gamma \approx 50$; also, r_f too high.
- $\zeta_{t+1} = \mathrm{sign}(\ln c_{t+1} - \ln c_t)$, $A(\varphi) = -\theta|\varphi|$.

One-Slide Analysis of the Equity Premium Puzzle

- $\max_{\xi} U(c_t, c_{t+1})$ s. to $c_t = e_t - p_t\xi$, $c_{t+1} = e_{t+1} + \xi x_{t+1}$.
- Stochastic Discount Factor: $p_t = \mathrm{E}[m x_{t+1}]$
- $U(c_t, c_{t+1}) = u(c_t) + \beta \mathrm{E}_p[u(c_{t+1})]$, $m = \frac{\beta u'(c_{t+1})}{u'(c_t)}$.
- $U(c_t, c_{t+1}) = u(c_t) + \beta\{\mathrm{E}_p[u(c_{t+1})] + A(\mathrm{E}_p[\zeta_{t+1} u(c_{t+1})])\}$:

$$m = \frac{\beta u'(c_{t+1})[1 + \zeta_{t+1} A'(\mathrm{E}_p[\zeta_{t+1} u(c_{t+1})])]}{u'(c_t)}$$

- HJ bounds: slope of efficient frontier $\frac{\sigma(m)}{\mathrm{E}_p[m]}$.
- Power utility $u'(c) = c^{-\gamma}$, $\ln c_{t+1} - \ln c_t \sim \mathrm{N}(\mu, \sigma^2)$.
  Puzzle: slope ≈ 0.50, requires $\gamma \approx 50$; also, r_f too high.
- $\zeta_{t+1} = \mathrm{sign}(\ln c_{t+1} - \ln c_t)$, $A(\varphi) = -\theta|\varphi|$.
- With VEU, can fit with $\theta = 0.5$, $\gamma = 2$, and better r_f, too.