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# Formal Specifications of Complex Systems

מפרטים פורמליים למערכות מורכבות

236368

מרצה: פרופ' שמואל כ"ץ

# Technical details

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- **Lecturer: Prof. Shmuel Katz**

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**reception hour: Monday 9:30-11:00**

- **Metargelim: Nimrod Saban-Partosh ( [nimi@cs.technion.ac.il](mailto:nimi@cs.technion.ac.il) )**
- **Cynthia Disenfeld ( [cdisenfe@cs.technion.ac.il](mailto:cdisenfe@cs.technion.ac.il) )**
- **Background materials: from the webpage**
- **Lecture/exercise slides + Book chapters**
- **The lectures have material not on the slides (and the exam can include that material).**
- **Grade:**

**if examgrade < 51,**

**then finalgrade=examgrade**

**else finalgrade= .75\*examgrade + .25\*homework**

# Basic principles

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- **Use precise notations to describe real requirements of complex systems**
- **Allows formal analysis using tools**
- **Clarifies many issues before they become implementation problems**
- **Helps explain concepts like “refinement”, “concurrency”, “closed/open” systems, “fault tolerance”, “real time”**
- **Gives criteria for evaluating spec. methods**
- **Links SE to CS theory**

# What are specifications?

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- **Requirements specification: a contract between the client and the developers; Includes:**
  - > **The operations and their meaning**
  - > **External constraints and assumptions**
  - > **Descriptions of results/output**
  - > **Interactions/synchronization/interference among operations**
  - > **Forbidden situations, Guaranteed situations**
- **Design specification: a contract among the developers**
  - > **internal divisions, interfaces, assumptions and guarantees, plus the above**
- **Formal Specifications (usually) deal with functional requirements, and the behavior of the desired system**

# What are we specifying?

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- **The collection of possible computations with the (usually nonexistent) system**
  - > Emphasizing the permitted, forbidden, and required **states** (Bank balance is never negative)
  - > Emphasizing the permitted, forbidden, and required **sequences of operations** (Need Initialize before Add)
- **Terminating systems with a “final result”**
  - > Mathematical functions (matrix multiplication)
  - > Compilers that produce assembly code
- **Reactive systems: react to requests/input, ongoing**
  - > Operating systems
  - > Email systems
  - > Control systems (air traffic, elevators, nuclear reactors..)
  - > Telephone switches

# Advantages of precise specifications

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- **Understand open questions/contradictions at an early stage of development**
- **Prevent misunderstandings**
- **Essential input to tools**
  - > **For prototyping and simulation**
  - > **For detecting inconsistencies, gaps**
  - > **For testing and verifying implementations**
  - > **For detecting reuse possibilities**

# Evaluating specification methods--

## A good method should be:

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- **Precise** (= unambiguous)
- **Accessible** (=easy to understand and explain)
- **Expressive** (=can write all requirements )
- **Modular** (=can easily add new requirements and extend old ones)
- **Hierarchical** (=“zoom” capabilities to consider different levels of detail)
- **Executable** (=tools for simulation/prototyping)
- **Minimal** (=does not overly restrict the implementation)

# Kinds of Properties

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- **Safety:** properties maintained in the history of the computation so far
  - > Which states (and histories) are allowed and which are forbidden
  - > Doing nothing satisfies all safety properties!
- **Liveness:** properties on guaranteed progress in computations
  - > Which states must occur
  - > Can be replaced by real-time properties



# Examples

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- **Mutual exclusion of critical sections?**
- **No deadlock (some operation is always possible)?**
- **No starvation (every active process actually executes)?**
- **Termination (computations reach halt)?**
- **Partial correctness?**
- **Responsiveness (every request is answered)?**

# Why?

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- **Division to kinds of properties (with more divisions given later) indicates which specification tool is appropriate**
- **Different kinds of properties have different verification and testing techniques**
- **Connect to extensions (we will see) for**
  - > **Real time specification**
  - > **Fault tolerance in computer networks**

# Three kinds of specifications

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- **Data and transition modeling:**
  - > For individual operations or procedures
  - > Hoare logic/annotations: I/O with logic
  - > OCL for object-oriented systems and classes
  - > Z- system specification based on sets; each operation relative to a system state .
  - > Larch- specification based on word algebra: relations among operations....event based
  - > Emphasizes safety properties (plus termination)

# Kinds of Specifications (cont.)

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- **Control: concurrency, overlap, synchronization**
  - > State machines, transition systems
  - > Statecharts –hierarchical, concurrent automata
  - > LOTOS – a calculus of communicating processes (also CCS, CSP, IP,...)
- **Global liveness (and safety too):**
  - > Temporal logic in many versions (CTL, linear, anchored, future/past)

# Theory or Practice?

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- (Almost) all methods in the course have associated **tools**, and have been used on real, large-scale systems
- Some connect with techniques for **OO modeling (UML)** and hardware/firmware verification
- Some are part of **industry standards** for communication or mobile
- But...the methods are not typically used for “regular” software specification
- Will see plusses and minuses of methods, gain insight into what we mean by specification, modularity, refinement, implementation,..

# Course outline

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- Using assertions in **logic** and **OCL for OO systems**
- **Logic and Set theory in Z** to describe properties, and for high level modeling of systems and their operations
- Modelling control with **state machines and statechart diagrams**
- **Temporal logics** for global properties
- **LOTOS** for coordination and synchronization in parallel processes—a process algebra
- **Algebraic specifications** showing relations among operations
- **Realtime and fault tolerance**
- Using formal specifications in practical tools

# Extending Logic for assertions and modelling

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- **Classic predicate calculus can express many properties used in specification**
- **Problem #1: Does it really capture our intentions?**  
$$\forall i \exists j: a[j] = a[0][i] \wedge \forall i: 1 \leq i < n. a[i] \leq a[i + 1]$$
- **Problem #2: many assertions natural for modelling and requirements are hard to write in logic**
  - > **Push on to a stack**
  - > **Traverse a tree**

# Input/output specifications

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- **R. Floyd, 1967. For flowcharts**
- **C.A.R. Hoare, 1969. For while programs**
- **Assumptions:**
  - > **a single sequential module**
  - > **computes a function of the input**
- **Uses first-order predicate calculus over program variables, with standard mathematics.**



# Standard Mathematic Symbols

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- **inequalities:**  $<$ ,  $\cdot$ ,  $=$ ,  $>$ ,  $\geq$ ,  $\neq$
- **operations:**  $+$ ,  $-$ ,  $*$
- **aggregations:**  $\Sigma$ ,  $\Pi$ ,  $n!$
- **standard data types:** *integer(x)*, *real(y)*
- **range:**  $1..n$
- **$x_0$**  for the initial value of  $x$
- *Sometimes,  $x'$*  for the value of  $x$  after an operation
- **can use logical 'one-time variables'**
- *Problem: logic might not really express what we intend---will see throughout the course*

# Hoare logic notation

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$\{P\} S \{Q\}$

- Interpretation: **if** predicate  $P$  is true of the state before the code  $S$  is executed, and the code executes and terminates, **then** the predicate  $Q$  will be true of the resultant state.
- The idea: a program logic, sound and complete under this interpretation.

# Originally: mixes code and logic

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- **Claim:** due to the assumptions, only the initial and final states are significant.
- *P Precondition:* required inputs or states.
- *Q Postcondition:* relates inputs and outputs.
- A partial correctness assertion
- Is it safety or liveness?
- **Total correctness: partial correctness + termination**

# Hoare Logic Deduction Rules

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- **Example: conditional statement**

$$\{ P \wedge B \} S1 \{ Q \}$$
$$\{ P \wedge \neg B \} S2 \{ Q \}$$

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$$\{ P \} \text{ if } B \text{ then } S1 \text{ else } S2 \{ Q \}$$

Used by Hoare to prove partial correctness — can be seen as the **specification** of the conditional statement, to be satisfied by any translation to assembly code

# While loop Hoare logic rule

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$$\{ P \wedge B \} S \{ P \}$$
$$(P \wedge \neg B) \Rightarrow Q$$

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$$\{ P \} \text{ while } B \text{ do } S \{ Q \}$$

**The purpose of a loop: maintain the loop invariant while taking a step towards the **termination condition** of the loop**

# Annotation

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- an assertion between, before, or after statements of code.
- Examples

$\{ a[1..n] : \text{integer} \}$

**S**

$\{ \forall i \exists j: a[j] = a[i] \wedge \forall i: 1 \leq i < n. a[i] \leq a[i + 1] \}$

What is specified? Is it our intention?

$\{ \text{integer}(x) \wedge x \geq 1 \}$

**T**

$\{ \text{integer}(y) \wedge y^2 \leq x \wedge (y + 1)^2 > x \}$

Can we write it differently?

# Invariant

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- An assertion true whenever the control is where the assertion appears (also called a **local invariant**).
- The intended interpretation of an annotation is as an invariant.
- Input/output assertions should be invariants.
- Can annotate unwritten code, as its specification.
- **Global invariant**: true throughout every execution.

# An Annotated Linear Search

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```
{  
    }  
i := 1; found := false ;  
{  
    }  
while (( i ≤ n ) and not found )  
do  
{  
    }  
if a[i] = x  
    then found := true  
    else i := i + 1  
od  
{  
    }
```



# Another possible view

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```
{ n ≥ 1 }  
i := ? ; found := ? ;  
{ n ≥ 1 ∧ i = 1 ∧ found = false }  
while (( i ≤ n ) and not found )  
do  
{ ∃ j. 1 ≤ j < i. a[j] = x ...  
  }  
  ????  
od  
{ ( found ⇒ a[i] = x ) ∧ ( ¬ found ⇒ ∃ j. 1 ≤ j ≤ n. a[j] = x ) }
```

# Annotations as specifications

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- Can use the pre- and post conditions as the design specification, and the annotations as restrictions on the implementation
- Have annotations with holes (= program skeleton)
- (The purpose of a loop: maintain the loop invariant while taking a step towards the **termination condition** of the loop)

# Tricks

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- **Logical/rigid/specification variables:** to "remember" values before a code segment

$\{y = Y\} y := y - 7 \{y < Y\}$

- **Auxiliary variables** and programs: adding variables and statements just for expressing specifications

$c := 0;$

$x := \dots$ \*/ a key statement ;  $c := c + 1;$

$y := \dots$ \*/ another ;  $c := c + 1;$

...

$\{c < K\}$  “there are less than  $K$  key statements”

- **Problem:** formalizing exactly what it means...