# Bloch-Horowitz Schemes

## A Few-Body Application

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T. Luu

Los Alamos National Laboratory

# Outline. . .

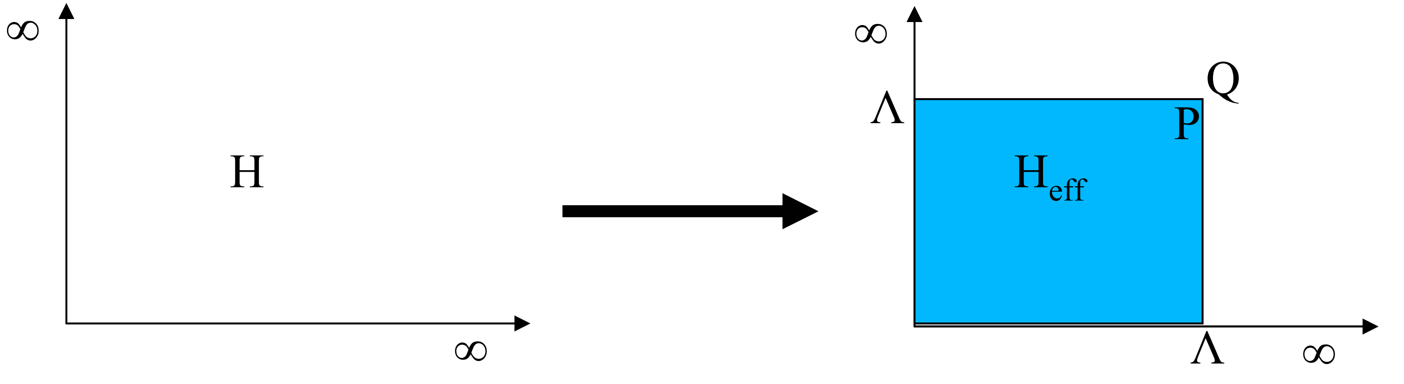
- Bloch-Horowitz primer
- Solving BH "exactly" for s-shell nuclei
  - Non-perturbative method
  - Perturbative method
- Applying LO BH term to p-shell nuclei
  - Above and beyond the G-matrix
- Applying BH as a cluster expansion
  - Separation of A-body forces
- Conclusions

# Bloch-Horowitz Equation

$$
\begin{aligned}
H &= \sum_{i<j}^{A} (T_{ij} + V_{ij}) \\
&= \sum_{i}^{A} T_i + \sum_{i<j}^{A} V_{ij} - \frac{P_{CM}^2}{2M_A}.
\end{aligned}
$$

Non-Rel. *Intrinsic*

Hamiltonian




$$
H_{eff}(E) = P\{H + H \frac{1}{E - QH} QH\} P,
$$

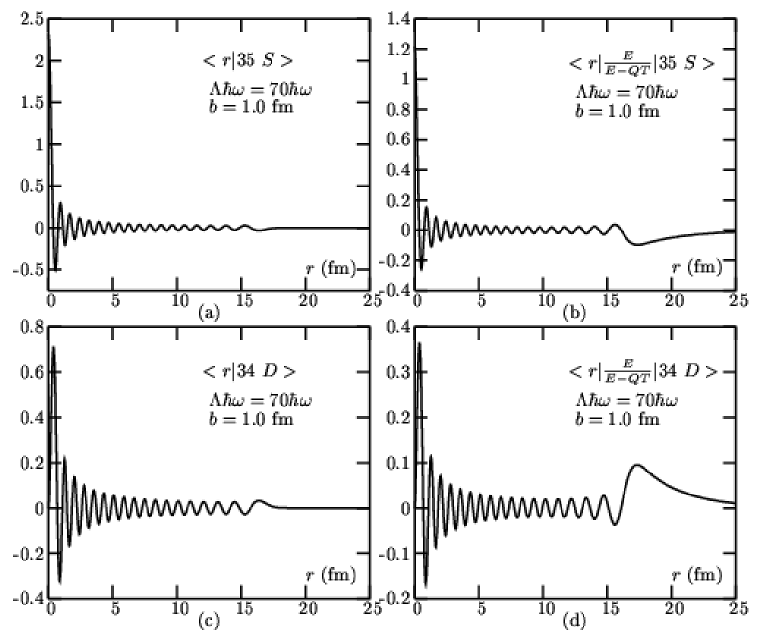
# First Step: Curing the Asymptotic behavior

$$H_{eff} = P\left\{\frac{E}{E-TQ}[T_{eff}+V_{eff}]\frac{E}{E-QT}\right\}P,$$

$$T_{eff} = T + T\frac{-1}{E}QT,$$
$$V_{eff} = V + V\frac{1}{E-QH}QV.$$

Define new states $|\widetilde{\Omega}> = \frac{E}{E-QT}P|\Omega>$ $\Longrightarrow$ $<\Omega_f|H_{eff}|\Omega_i> = <\widetilde{\Omega}_f|T_{eff}+V_{eff}|\widetilde{\Omega}_i>.$

New states have nice asymptotic behavior $|\widetilde{\Omega}> \sim \mathrm{Exp}(-\gamma r)/\gamma r$



# Applying BH to s-shell nuclei

Deuteron: Can solve for $V_{eff}$ in terms of included-space HO overlaps

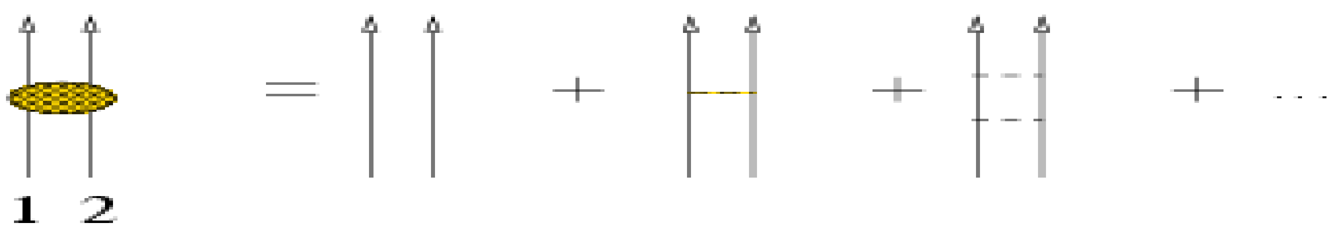
$$V_{eff} = t_{12} - t_{12} G_0 \left[ \Gamma_0 + \Gamma_{\infty} \right]^{-1} G_0 t_{12},$$

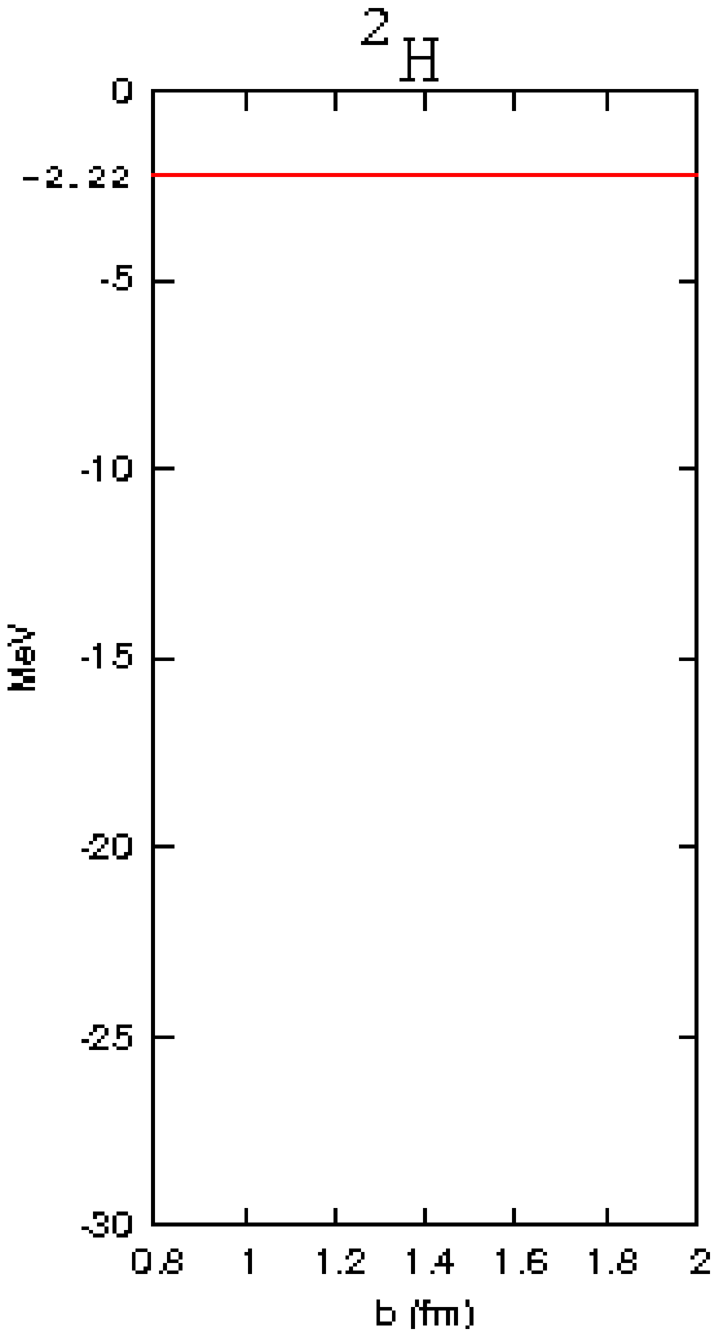
$$G_0 = \frac{1}{E - T},$$

$$\Gamma_0^{-1} = \left\{ P \frac{1}{E - T} P \right\}^{-1}$$

$$t_{12} = V_{12} + V_{12} G_0 t_{12}$$

$$\Gamma_{\infty} = P G_0 t_{12} G_0 P,$$






Exact effective interaction

has the following properties:

1. Gives the correct binding energy
2. Binding energy is independent of $\Lambda$
3. Binding energy is independent of oscillator parameter b

# Three-body system

$$V^3_{eff} = V + V\frac{1}{E - QT}QV^3_{eff}.$$

$V_{eff}$ in integral form

$$|\,\widetilde{\Psi_\sigma} >_{12} = V_{12}|\,\widetilde{\Omega_\sigma} > + V_{12}\frac{1}{E - QT}Q|\,\widetilde{\Psi_\sigma} >,$$

$$|\,\widetilde{\Psi_\sigma} >_{23} = V_{23}|\,\widetilde{\Omega_\sigma} > + V_{23}\frac{1}{E - QT}Q|\,\widetilde{\Psi_\sigma} >,$$

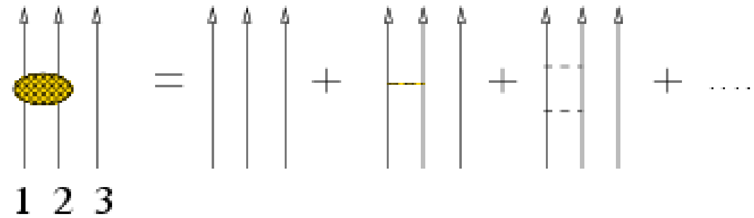
$$|\,\widetilde{\Psi_\sigma} >_{31} = V_{31}|\,\widetilde{\Omega_\sigma} > + V_{31}\frac{1}{E - QT}Q|\,\widetilde{\Psi_\sigma} >,$$

Invoke Faddeev decompositions

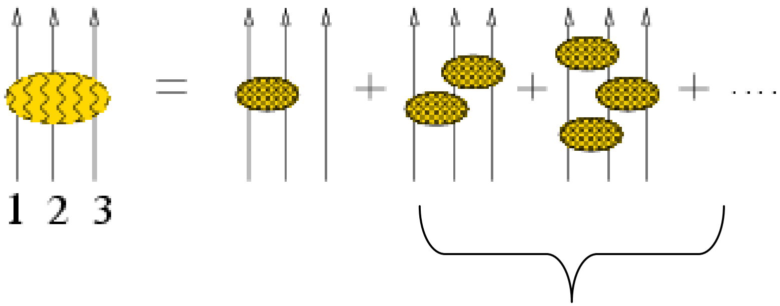
$$|\,\widetilde{\Psi_\sigma} >_{12} = \left(V^{(2+1)}_{12,eff} + V^{(3+0)}_{12,eff}\right)|\,\widetilde{\Omega_\sigma} >.$$

Can solve for Faddeev state

$$V^{(2+1)}_{12,\mathrm{eff}} =$$




$$= t_{12} - t_{12} G_0 \left[\Gamma_0 + \Gamma_\infty^{(2+1)}\right]^{-1} G_0 t_{12},$$

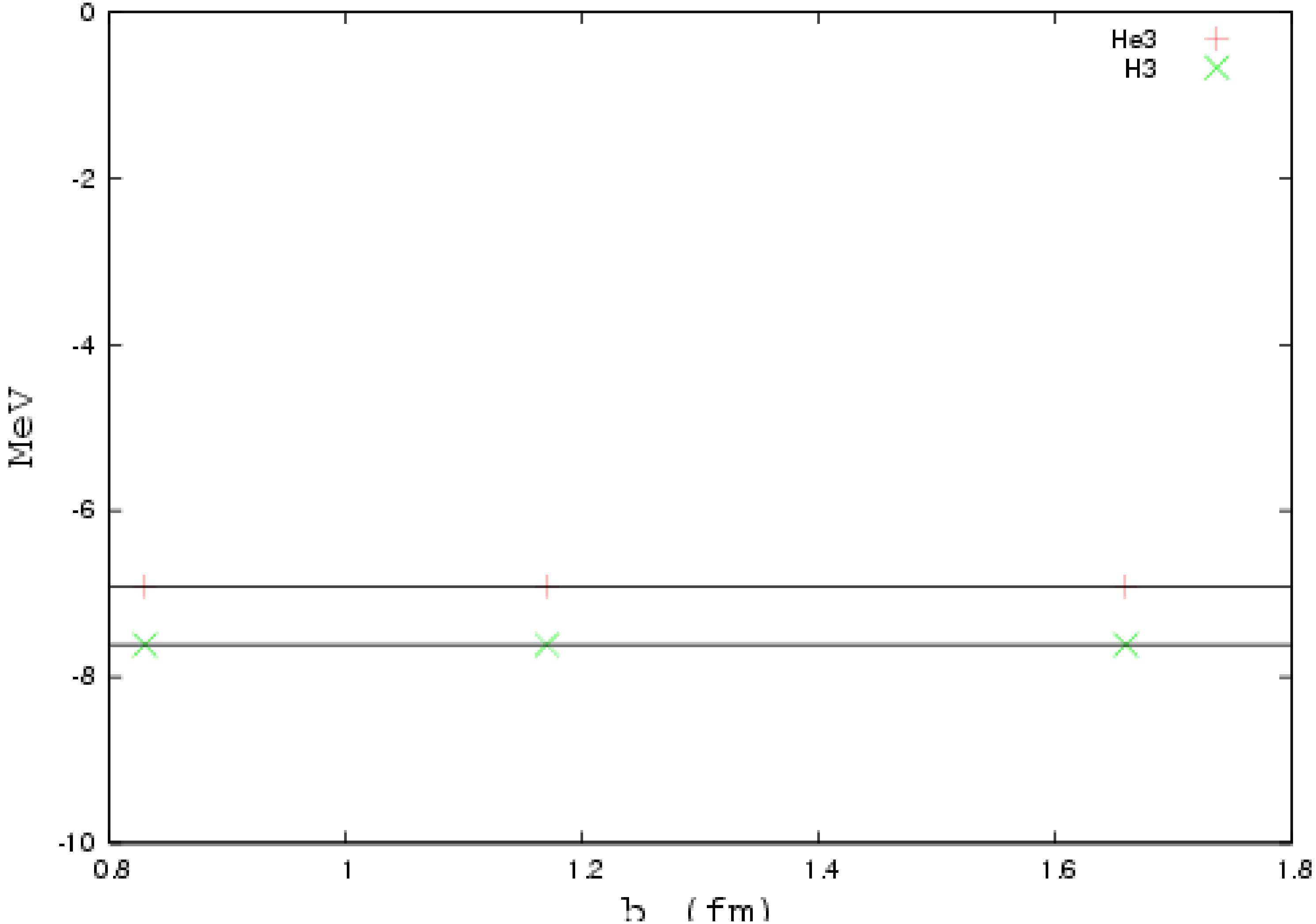



$$V^{(3+0)}_{12,\mathrm{eff}} = V^{(2+1)}_{12,eff} \frac{1}{E - QT - Q\Pi V^{(2+1)}_{12,eff}} Q\Pi V^{(2+1)}_{12,eff}.$$

What is actually solved:

$$V_{\mathrm{eff}} = V^{(2+1)}_{12,\mathrm{eff}} + V^{(2+1)}_{12,\mathrm{eff}} \frac{1}{E - QT} Q\Pi V_{\mathrm{eff}}$$

# Three-body results



# Four-body system

Similar procedure:

$$V_{eff} = V_{12,eff}^{(2+2)} + V_{\alpha,eff} + V_{\beta,eff}$$

Invoke Faddeev-Yakubovsky decompositions

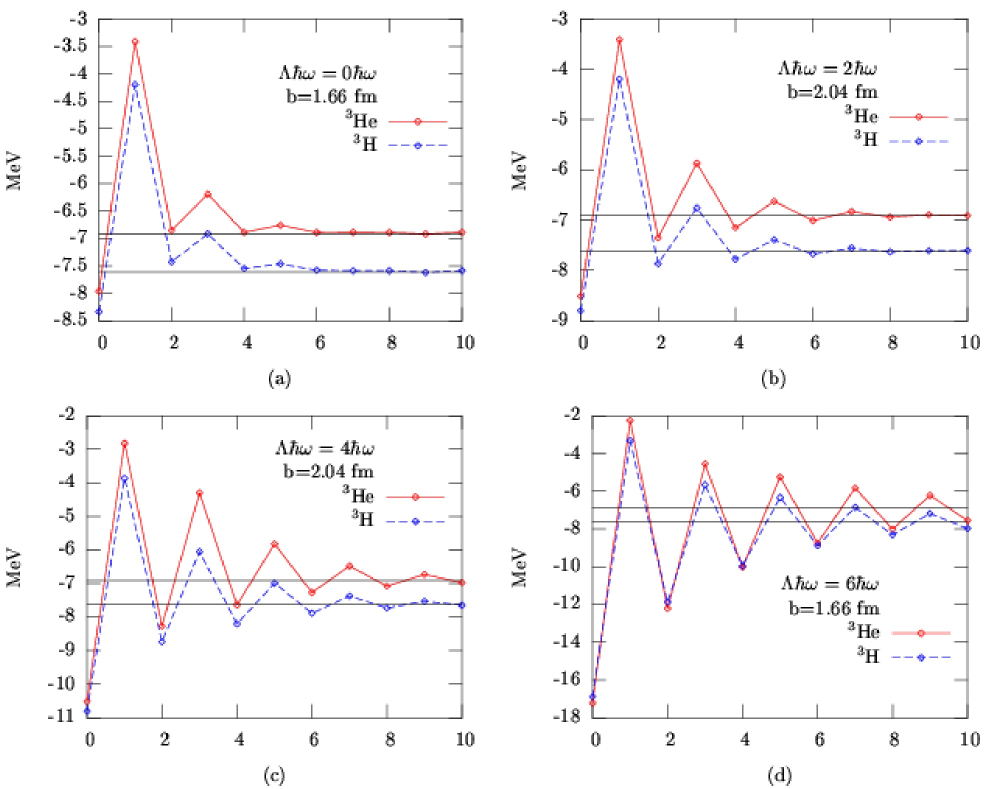
$$V_{\alpha,eff} = \overbrace{T_{12,eff}^{(3+1)} \frac{1}{E-QT} Q\Pi V_{12,eff}^{(2+2)}}^{\text{3-body eff.}} + T_{12,eff}^{(3+1)} \frac{1}{E-QT} Q\Pi(-\Pi_{34} V_{\alpha,eff} + V_{\beta,eff})$$

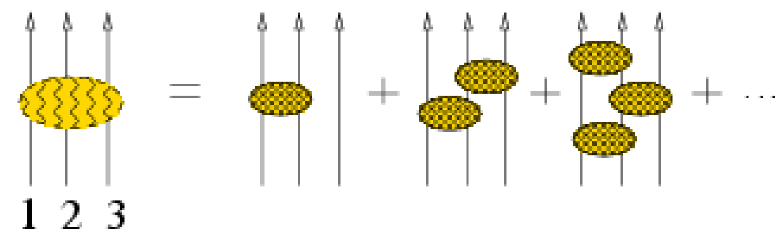
$$V_{\beta,eff} = \underbrace{\bar{T}_{12,eff}^{(2+2)} \frac{1}{E-QT} Q\bar{\Pi} V_{12,eff}^{(2+2)} + \bar{T}_{12,eff}^{(2+2)} \frac{1}{E-QT} Q\bar{\Pi}(1-\Pi_{34}) V_{\alpha,eff}}_{\text{4-body eff.}}$$

One day, with help from Andreas. . .

# Is the few-body problem perturbative?

For certain ranges of b, solving the integral equations (i.e. summing diagrams to all orders) is overkill



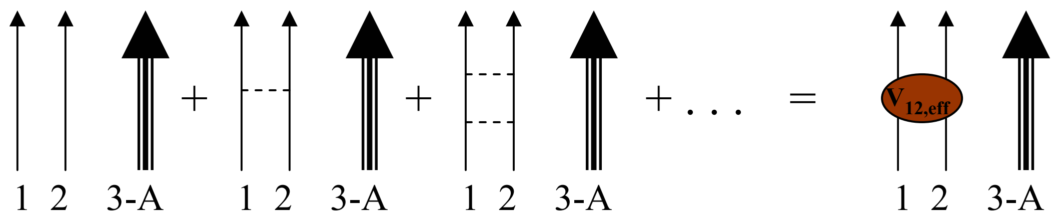




$0^{th}$ $1^{st}$ $2^{nd}$

$V_{12,eff}$

How does $0^{th}$ order work on heavier nuclei?

# How does $V_{12,eff}$ compare to G?




$$= \quad \frac{1}{E-QT}Q + \frac{1}{E-QT}QV_{12}\frac{1}{E-QT}Q + \frac{1}{E-QT}QV_{12}\frac{1}{E-QT}QV_{12}\frac{1}{E-QT}Q\ldots$$

Both represent a ladder summation, however. . .

- There is no ‘starting energy’—E is the self-consistent energy
- Q is the many-body Pauli operator => $V_{12,eff}$ is multi-valued, i.e. depends on $\Lambda$ of spectator nucleons
- T represents the many-body kinetic operator ($T=\sum T_{ij}$) => cluster recoil is incorporated

=> $V_{12,eff}$ is an A-body operator

# Dealing with $V_{12,eff}$

$$V_{12,eff} = t_{12} - t_{12} G_0 \left[ \Gamma_0 + \Gamma_\infty \right]^{-1} G_0 t_{12},$$

$$G_0 = \frac{1}{E - T},$$

$$\Gamma_0 = \left\{ P \frac{1}{E - T} P \right\}$$

$$t_{12} = V_{12} + V_{12} G_0 t_{12}$$

$$\Gamma_\infty = P G_0 t_{12} G_0 P,$$

$t_{12}(p`,p;\ E-\sum q_i^2/2\mu)$

Matrix elements of $t_{12}$ involve A-dimensional nested integrals?

Fortunately, the answer is NO!

# Dealing with $t_{12}$

$$\int dq_1 \cdots dq_{A-2} q_1^2 \cdots q_{A-2}^2 R_{n_1' l_1}(q_1) R_{n_1 l_1}(q_1) \cdots R_{n'_{A-2} l_{A-2}}(q_{A-2}) R_{n_{A-2} l_{A-2}}(q_{A-2}) t_{12}^{l' l}(p', p; E - \sum_{i=1}^{A-2} \frac{q_i^2}{2\mu}) =$$

$$(-1)^{n_1+n_1'+\cdots+n_{A-2}+n'_{A-2}} \sqrt{(\cdots\cdots)} \times \sum_{m_1,m_1'=0}^{n_1,n_1'} (\cdots) \cdots \sum_{m_{A-2},m'_{A-2}=0}^{n_{A-2},n'_{A-2}} (\cdots) \times \prod_{i=2}^{A-2} [\cdots]$$

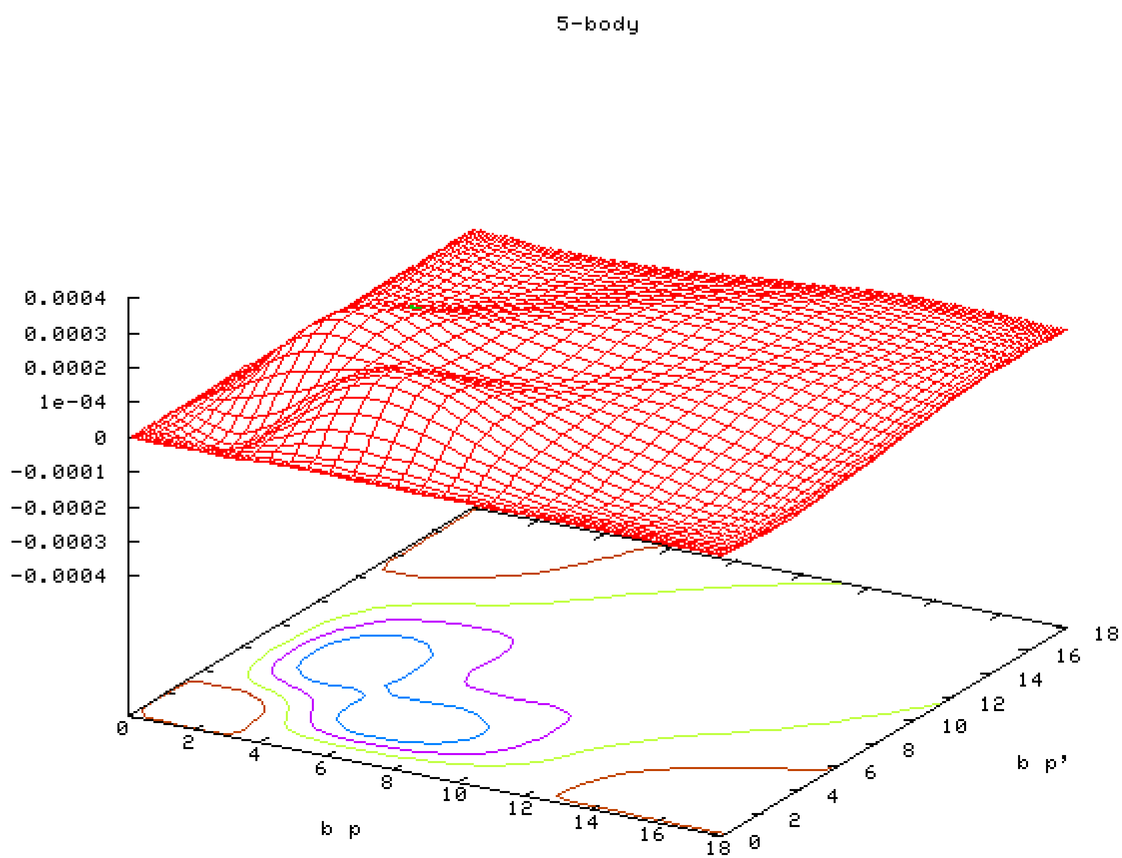
$$\times \left[ \int_0^\infty d\rho \; \rho^{A-3+2\sum_{i=1}^{A-2}(l_i+m_i+m_i'+1)} e^{-\rho^2} t_{12}^{l' l}(p', p; E - \frac{\rho^2}{2\mu b^2}) \right]$$

$$= \quad \overline{t_{12}^{l' l}}(p', p; E)$$

"Mean-field" interaction

# What does $\overline{t^{\nu l}_{12}}(p', p; E)$ look like?

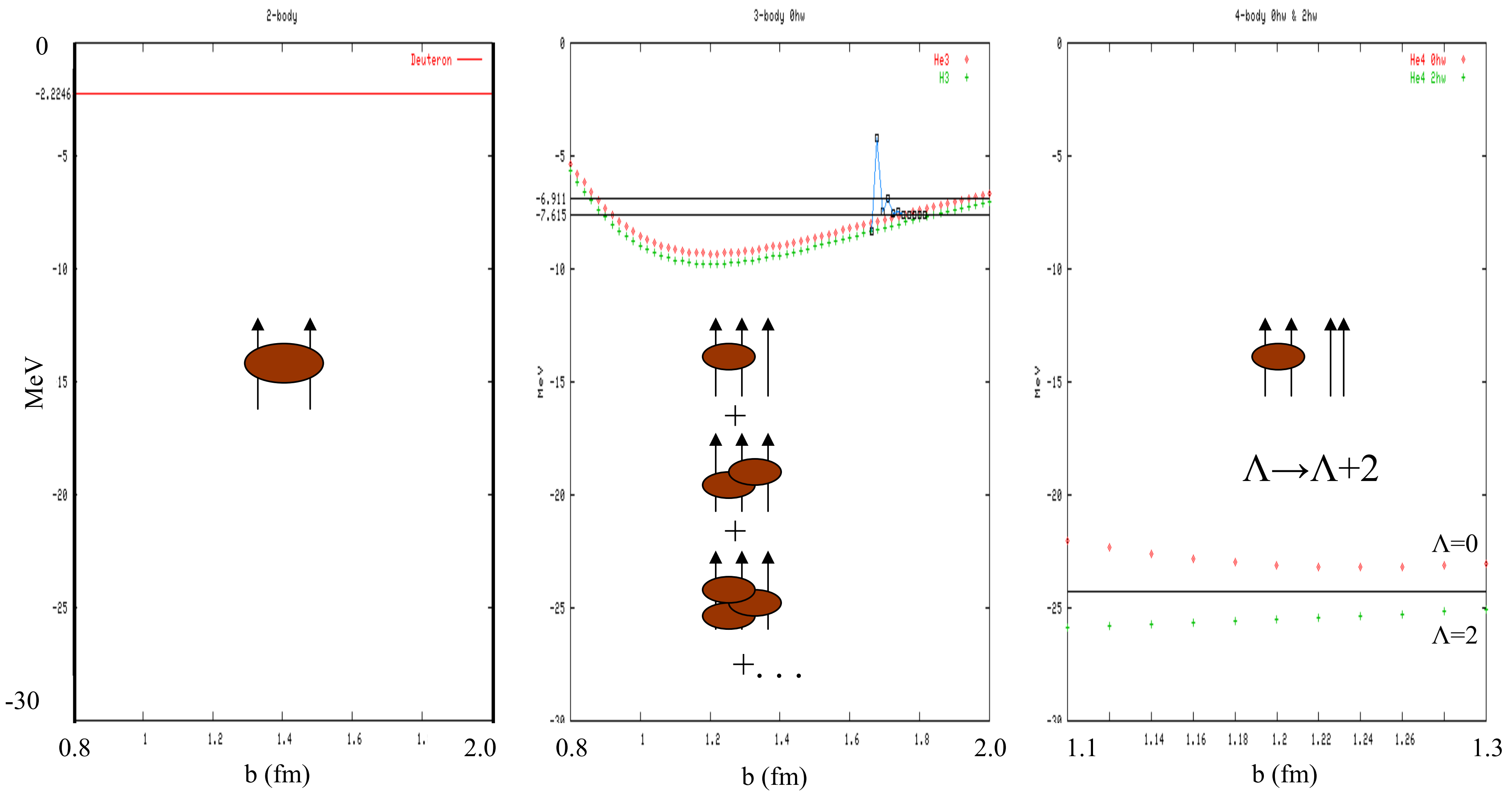
$\overline{G_0\, t_{12}(p`, p; E)\, G_0}$




E=-20MeV  b=1.2 fm  $^1S_0$

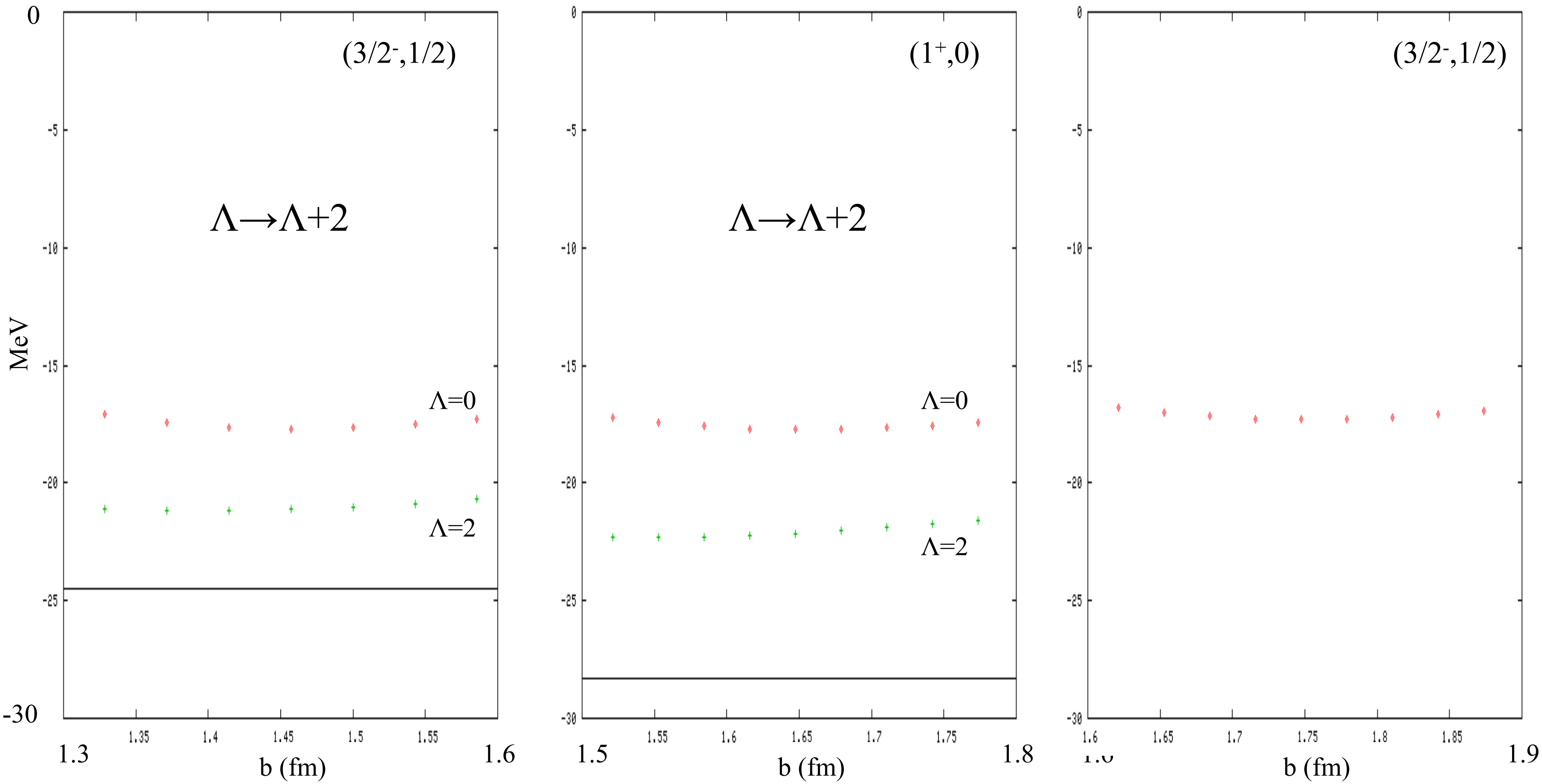
# Finally, some results. . .

Av18 S-shell



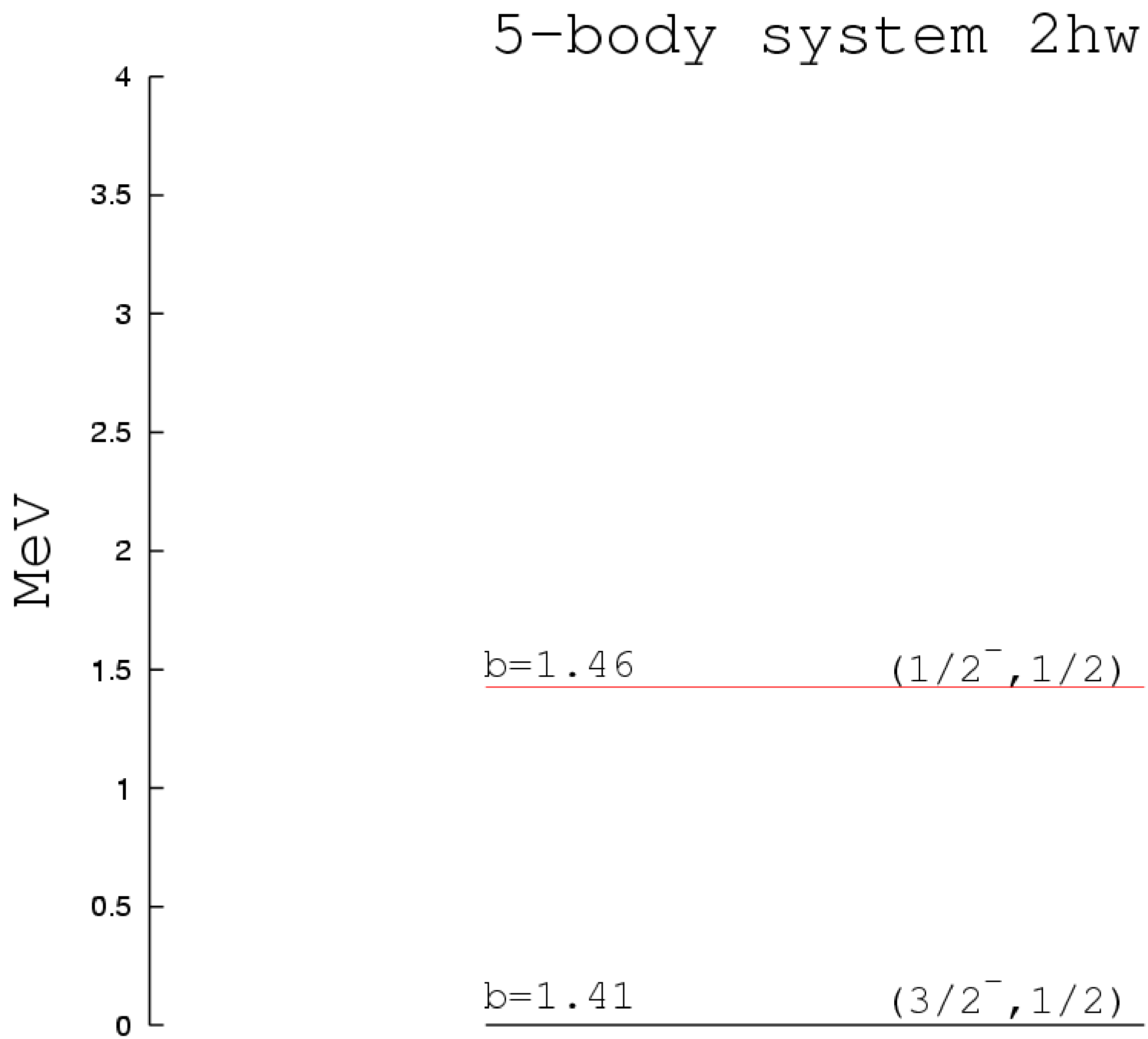
# Some P-Shell nuclei

5-body 6-body 7-body

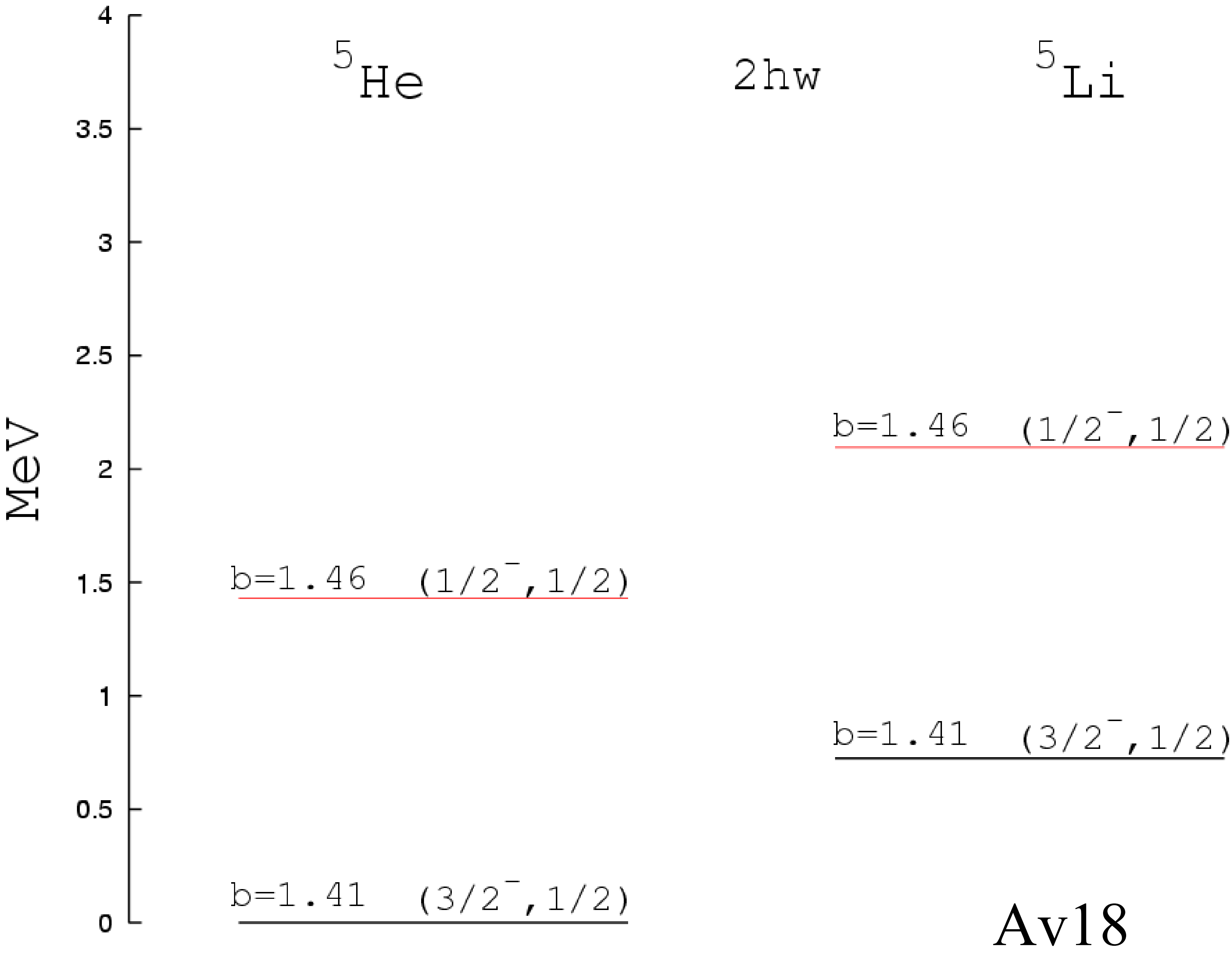



Av8’

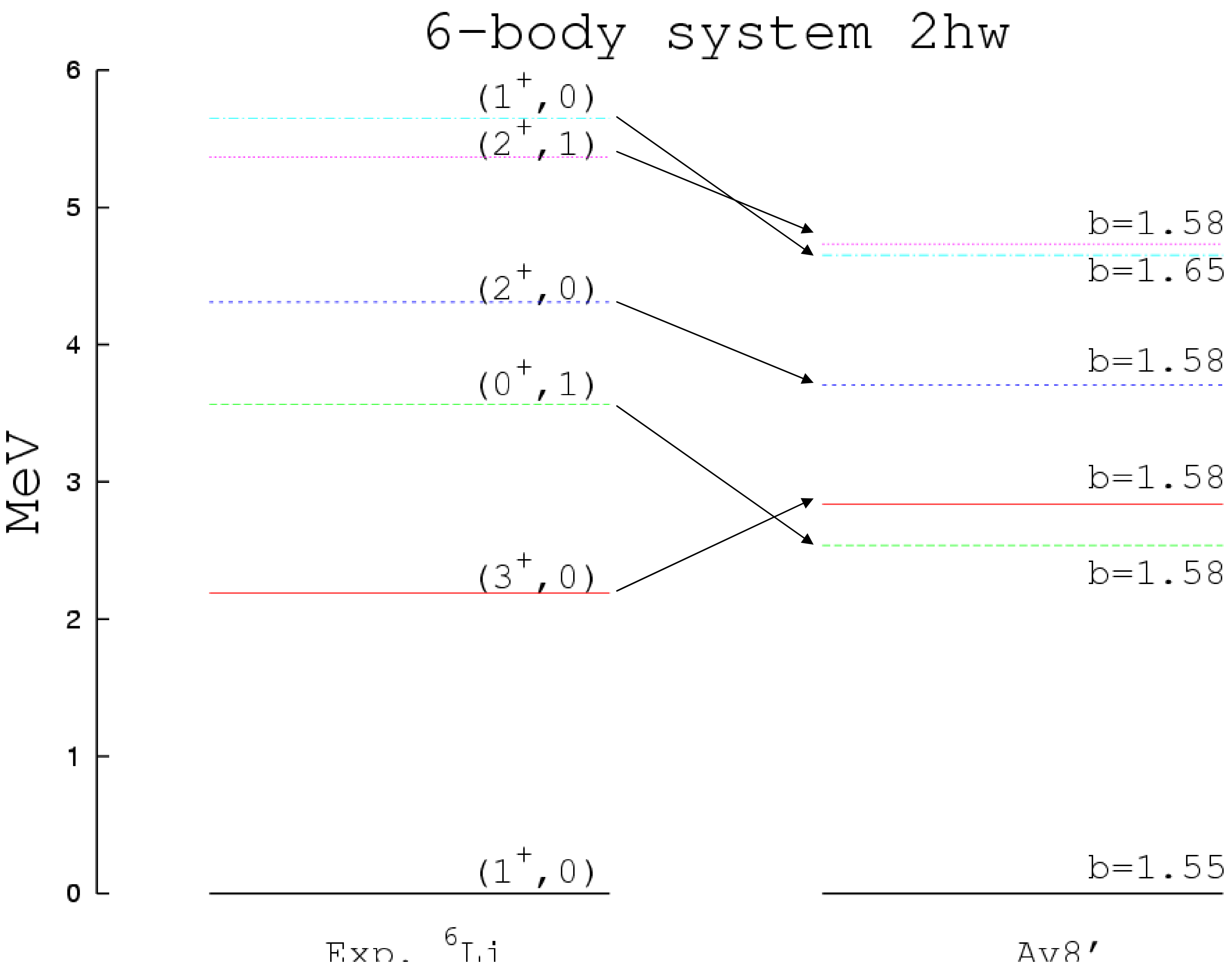
# What about excited states?

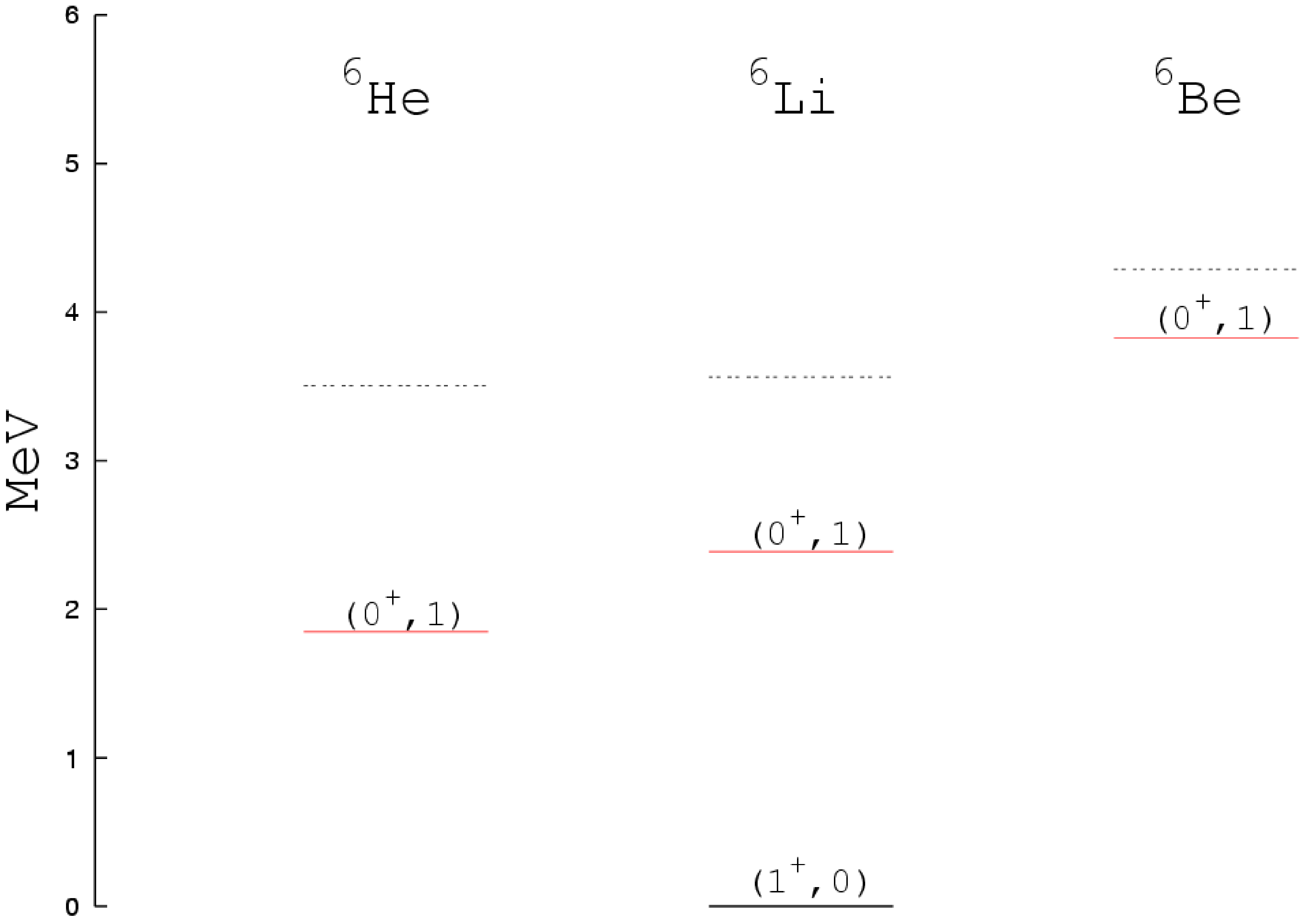



Av8’

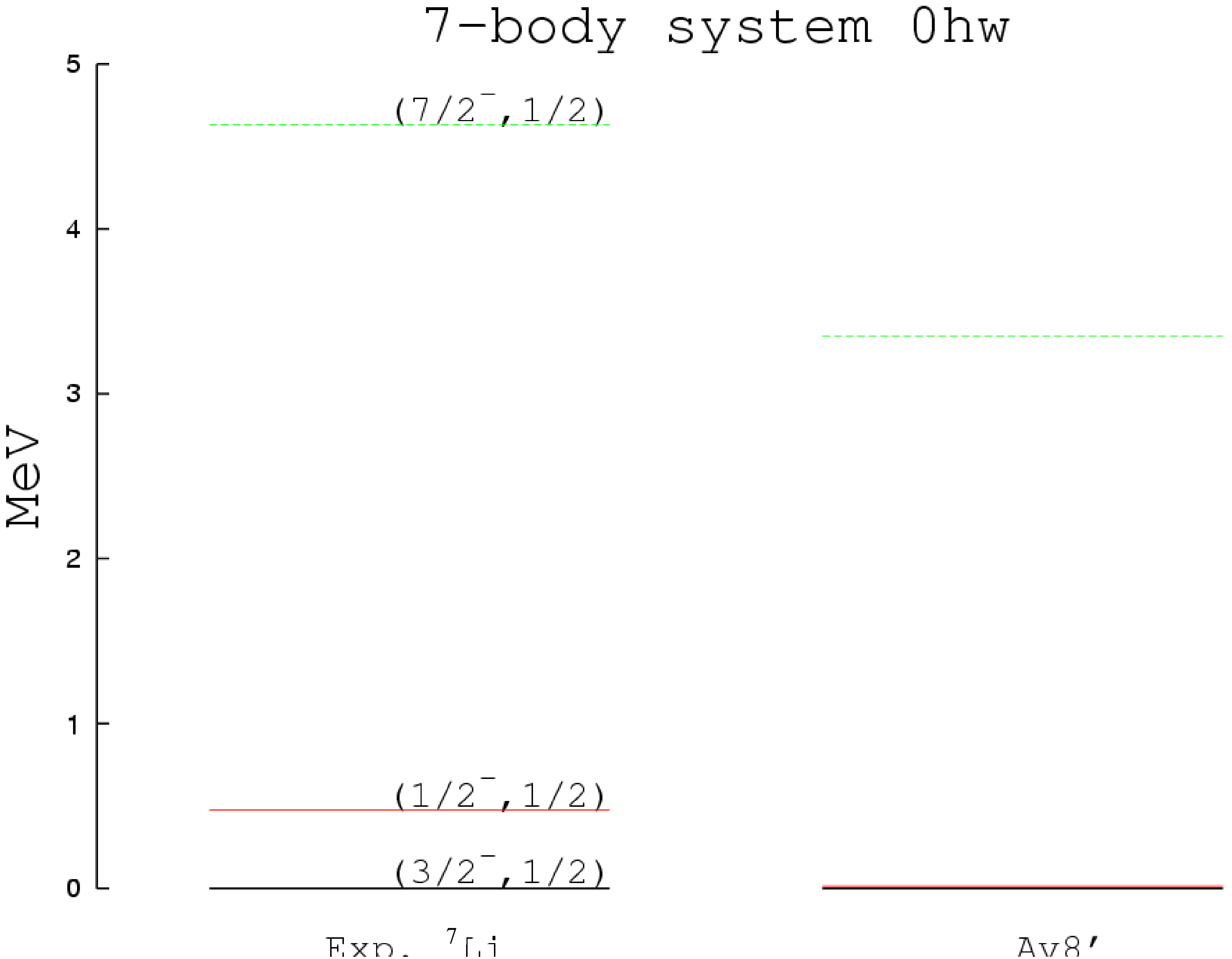


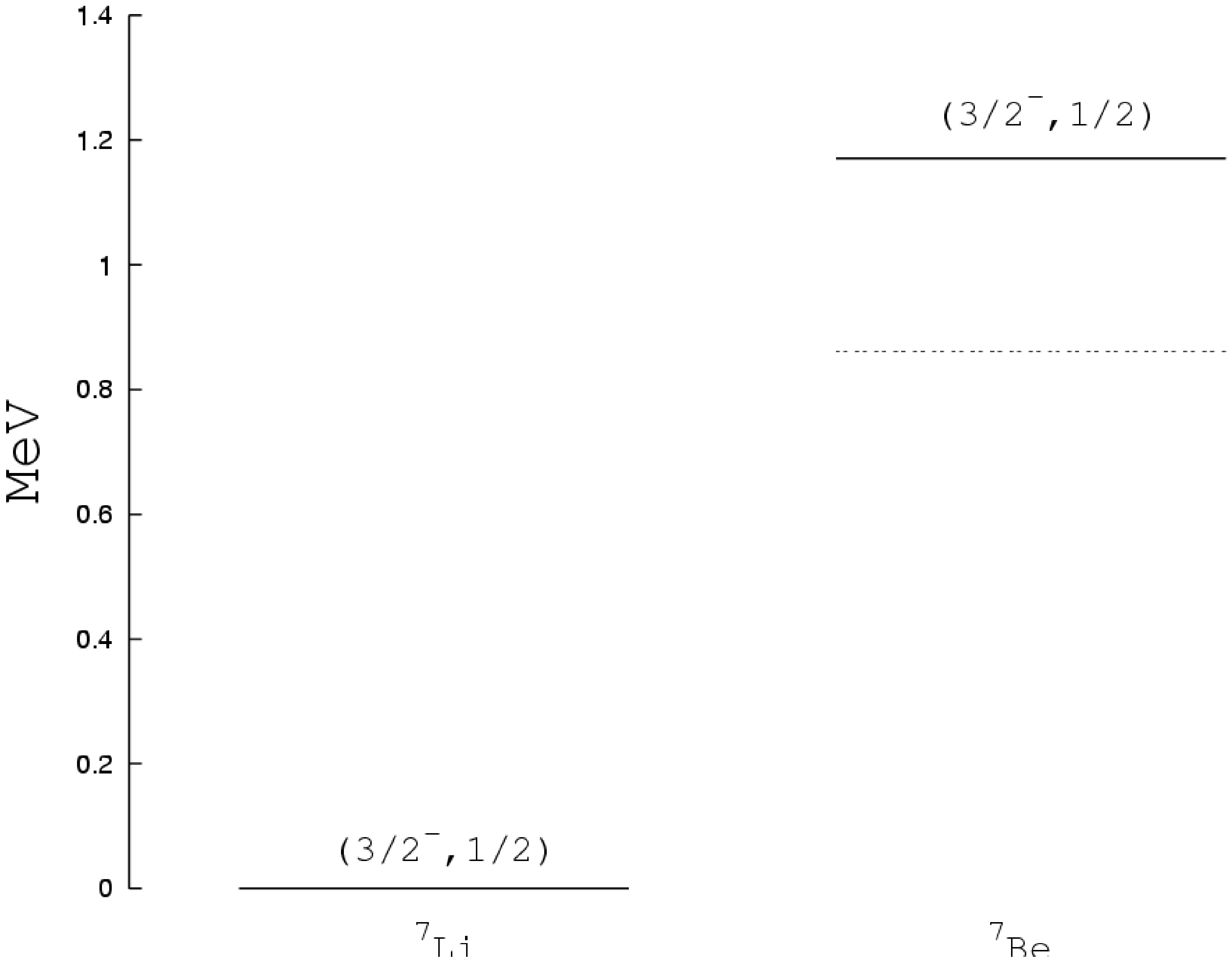
MeV
4
3.5
3
2.5
2
1.5
1
0.5
0
$^{5}$He
2hw
$^{5}$Li
b=1.46 (1/2$^{-}$,1/2)
b=1.46 (1/2$^{-}$,1/2)
b=1.41 (3/2$^{-}$,1/2)
b=1.41 (3/2$^{-}$,1/2)
Av18


6-body system 2hw
MeV
6
5
4
3
2
1
0
(1+,0)
(2+,1)
(2+,0)
(0+,1)
(3+,0)
(1+,0)
b=1.58
b=1.65
b=1.58
b=1.58
b=1.58
b=1.55
Exp. 6Li
Av8′



6
5
4
3
2
1
0
MeV
$^6$He
$^6$Li
$^6$Be
$(0^+,1)$
$(0^+,1)$
$(1^+,0)$
$(0^+,1)$


7-body system 0hw
MeV
5
4
3
2
1
0
(7/2⁻,1/2)
(1/2⁻,1/2)
(3/2⁻,1/2)
Exp. ⁷Li
Av8′



1.4
1.2
1
0.8
0.6
0.4
0.2
0
MeV
(3/2⁻,1/2)
(3/2⁻,1/2)
7Li
7Be

# BH as a cluster expansion

Ansatz: Hierarchy in number of particles interacting in Q space

$$\begin{aligned} H_{eff}(E) =& H + H\frac{1}{E - QH}QH \\ =& H^2_{eff}(E) + H^3_{eff}(E) + \cdots \\ \sim& H^2_{eff}(E) + H^3_{eff}(E) + H^4_{eff}(E) \end{aligned}$$

# Why should cluster expansion work?

- Shell model interactions, though only 2-body in nature and phenomenological, work really well
- Short-ranged clustering is qualitatively unlikely: Pauli exclusion principle excludes s-wave interactions with A>4
- NCSM have shown systematic improvements by including 3eff on top of 2eff
- GFMC calculations show alpha clustering in $^{8}$Be
- Analogy with EFT?

# What do these interactions look like?

$$
\begin{aligned}
H_{eff}(E) &= H + H\frac{1}{E - QH}QH \\
&= H^2_{eff}(E) + H^3_{eff}(E) + \cdots \\
&\sim H^2_{eff}(E) + H^3_{eff}(E) + H^4_{eff}(E)
\end{aligned}
$$

$$\downarrow$$

$$H_{12} + H_{12}\frac{1}{E - QH_{12}}QH_{12}$$

Multi-valued

$$H_{12} = T_{12} + V_{12}$$

Higher A-body cluster interactions are a little bit trickier, but follow similar pattern. . .

# Why is this the way to go?

- Only need to solve up to the A=4 body system
- However, need to solve for multiple Λ to account for spectator dependence (i.e. multi-valued)
- Need to solve for wide range of energies E

# Conclusion

- Can solve effective interaction ‘exactly’ for s-shell nuclei non-perturbatively by using Faddeev techniques
- Can solve perturbatively for 2- and 3-body system
- LO BH calculation suggest 4-body system is perturbative as well
- LO BH calculation on p-shell nuclei not so impressive, even though interaction is multi-valued and includes cluster recoil
- For 0hw and 2hw calculations, LO BH gives spectra in the right ballpark, but convergence is still an issue
- BH as a cluster expansion is feasible, and may be the way to go—just need to do it and see