

MC techniques

Suggested readings:

<http://pdg.lbl.gov/2005/reviews/monterpp.pdf>

- MCs assume a random number generator which generates uniform statistically independent values in the interval $[0,1)$
- **Inverse Transform Method**: if the probability density function is $f(x)$ with $-\infty < x < \infty$ its **cumulative distribution function** expressing the probability that $x \leq a$ is given by

$$F(a) = \int_{-\infty}^a f(x) dx$$

Notice this should be normalized to 1

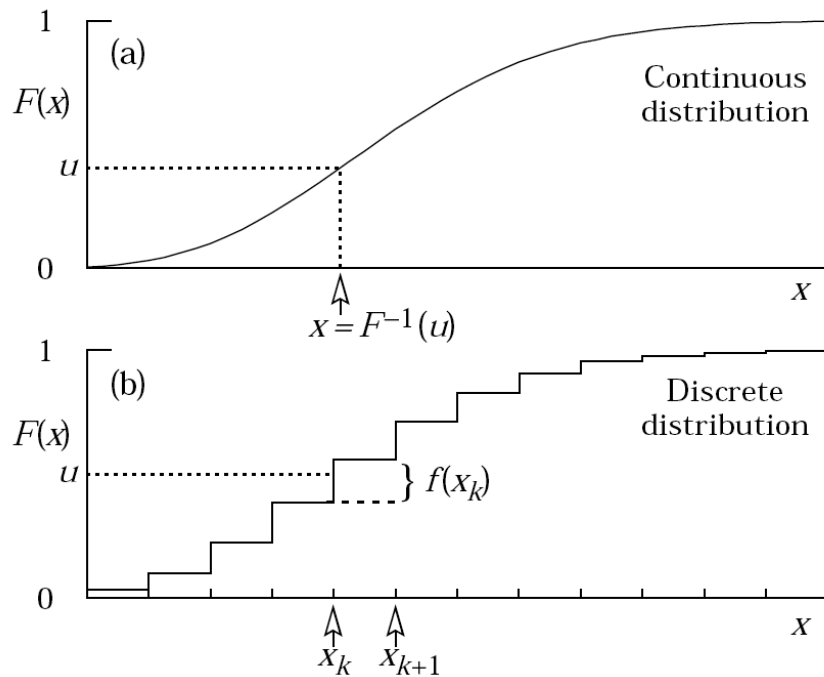
If a is chosen with pdf $f(a)$ then the integrated probability up to point a , $F(a)$ is itself a random variable which will occur with uniform probability in $[0,1]$. We can find a unique x chosen from the pdf f for a given u if we set

$$u = F(x)$$

Provided we can find an inverse function defined by

$$x = F^{-1}(u)$$

Inverse Transform Method



For a discrete function

$$F(x_{k-1}) < u \leq F(x_k) \equiv \text{Prob}(x \leq x_k) = \sum_{i=1}^k f(x_i)$$

Figure 33.1: Use of a random number u chosen from a uniform distribution $(0,1)$ to find a random number x from a distribution with cumulative distribution function $F(x)$.

Acceptance-rejection technique

When $F(x)$ is unknown or difficult to work out the inverse transform method cannot be applied. Let's suppose that we know enough $f(x)$ so that we can enclose it in a shape $h(x)$ (easily generated) that is $C > 1$ times $f(x)$. $h(x)$ can be a normalized sum of uniform distributions. To generate $f(x)$, first we generate a candidate x according to $h(x)$ and then calculate $f(x)$ and $Ch(x)$. We generate randomly u and test if $uCh(x) \leq f(x)$. If so we accept x , if not we reject x and retry.

The efficiency is the ratio of the areas $1/C$.

