Enhancement of Crossing Elongated Structures in Images

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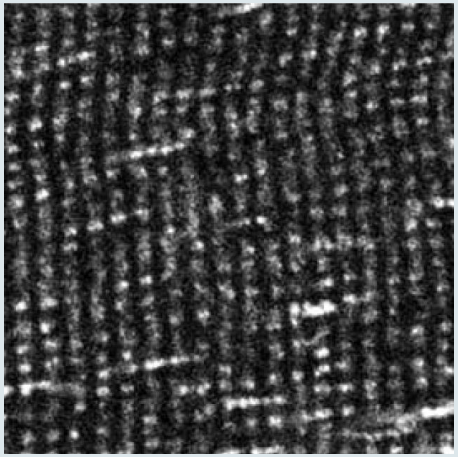
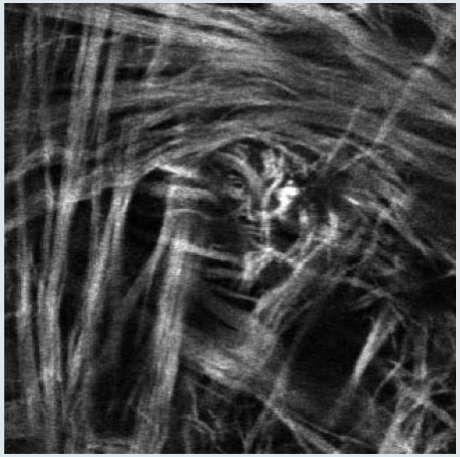
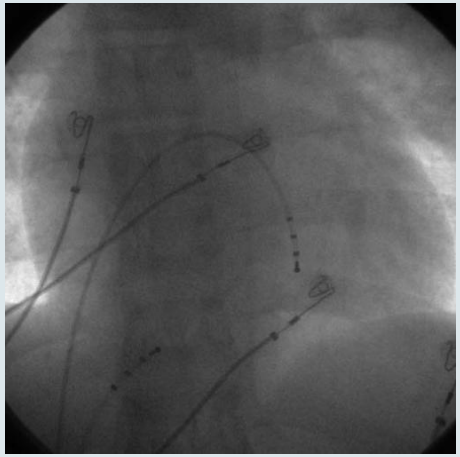
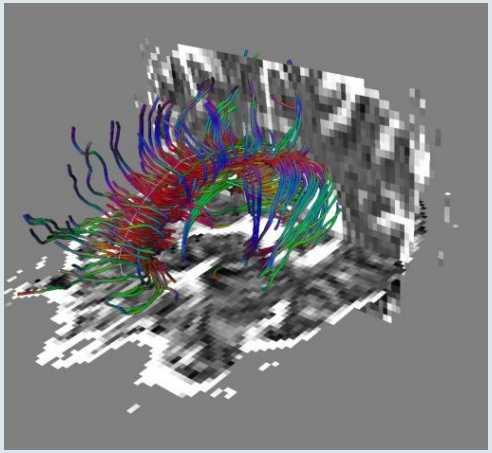
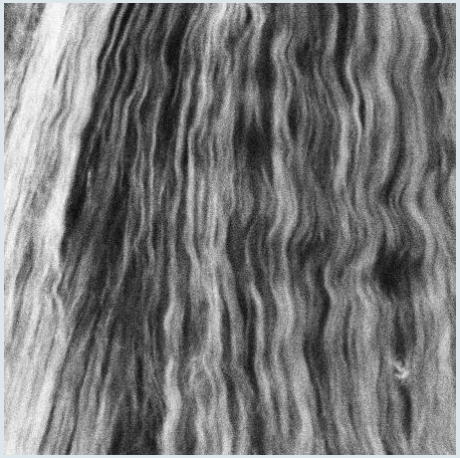
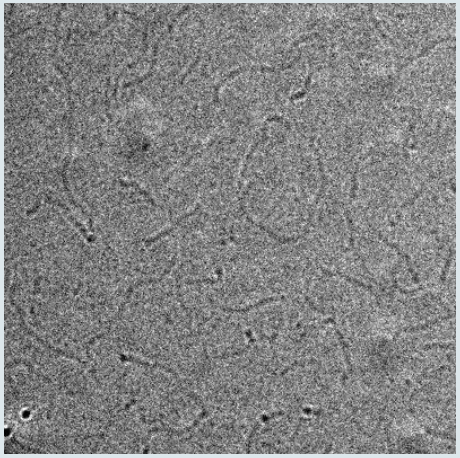
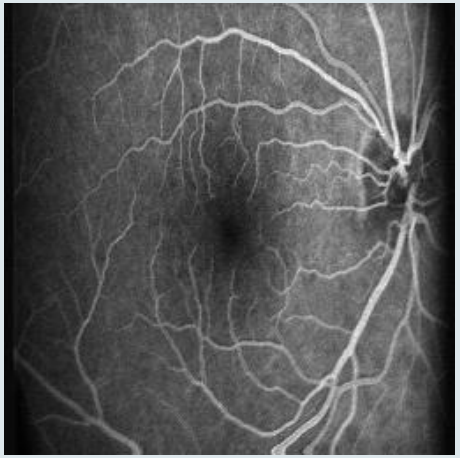
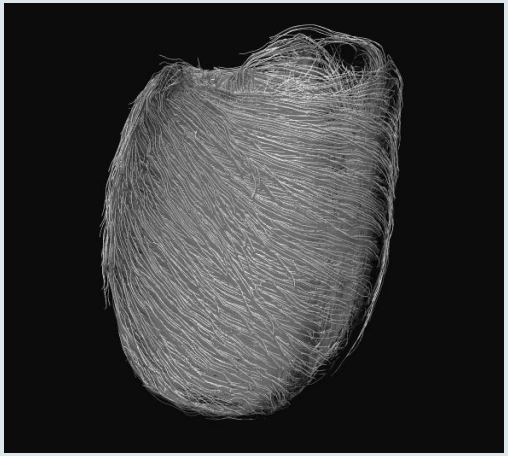
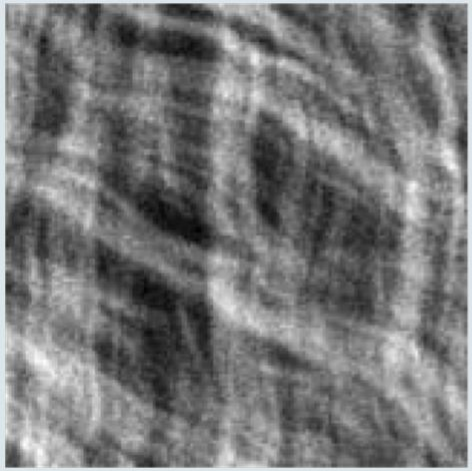
Evoluon, Eindhoven

Many elongated structures in medical images

Our goal

- Develop image processing methods for *enhancing elongated structures* in *noisy* images
- Appropriately handle *crossing* elongated structures



Original image

Result of old method

Result of new method

Image processing

- A 2D image is a function
  where the domain corresponds to spatial positions
  and the codomain corresponds to grey-values
- Note: a single grey-value *f*(*x,y*) *does not say anything* about elongated structures
- How to obtain information about orientation?
  → apply an oriented filter (i.e., a convolution)

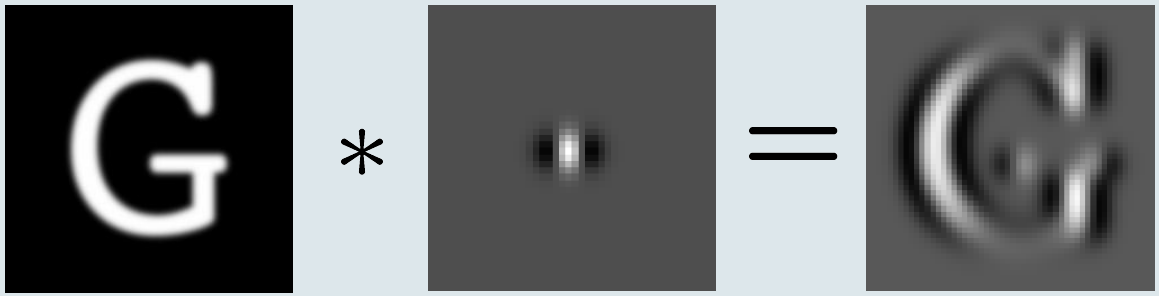
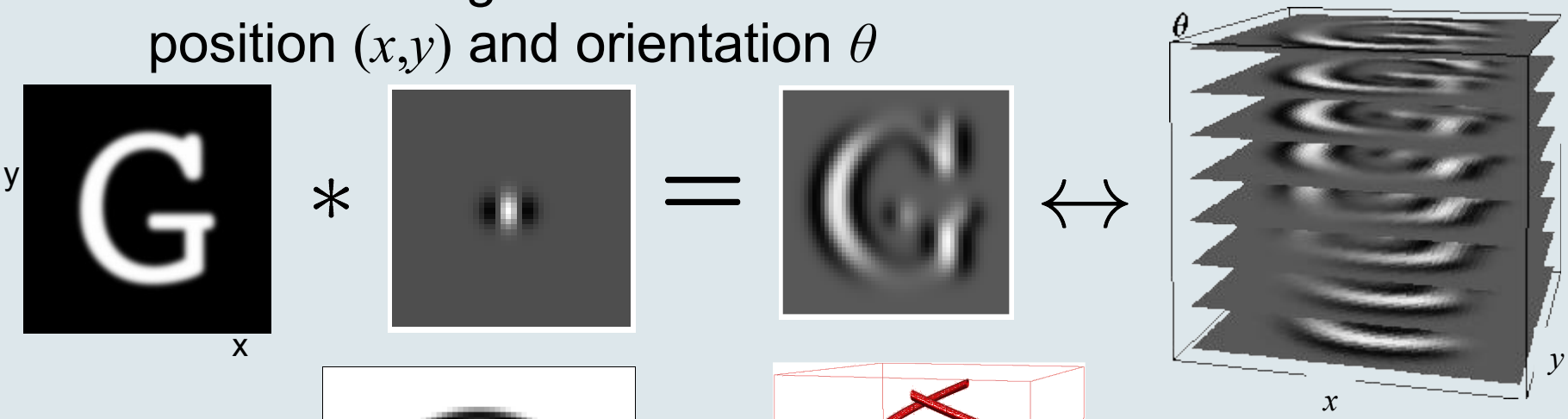


image filter kernel filtered image

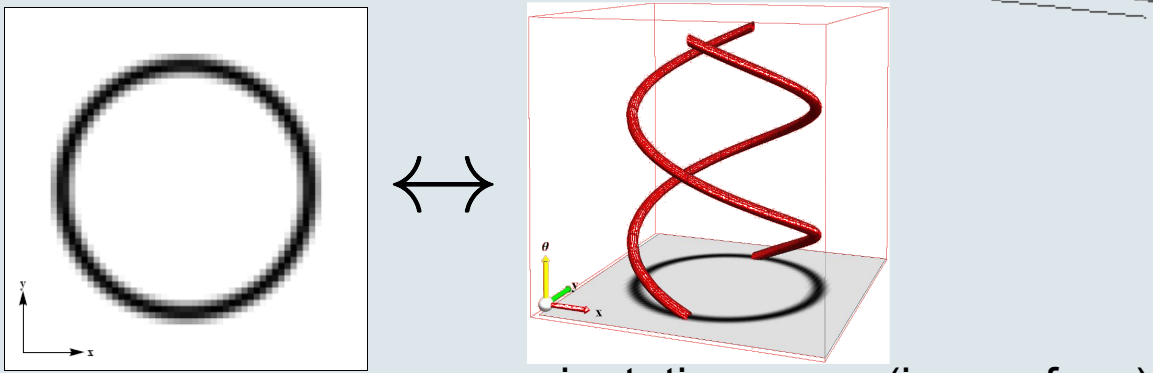
- No a priori knowledge on what orientations occur
  → so image should be filtered for *all orientations*

Orientation Scores (OS)

- From 2D image to orientation score with position (x,y) and orientation θ



Example:

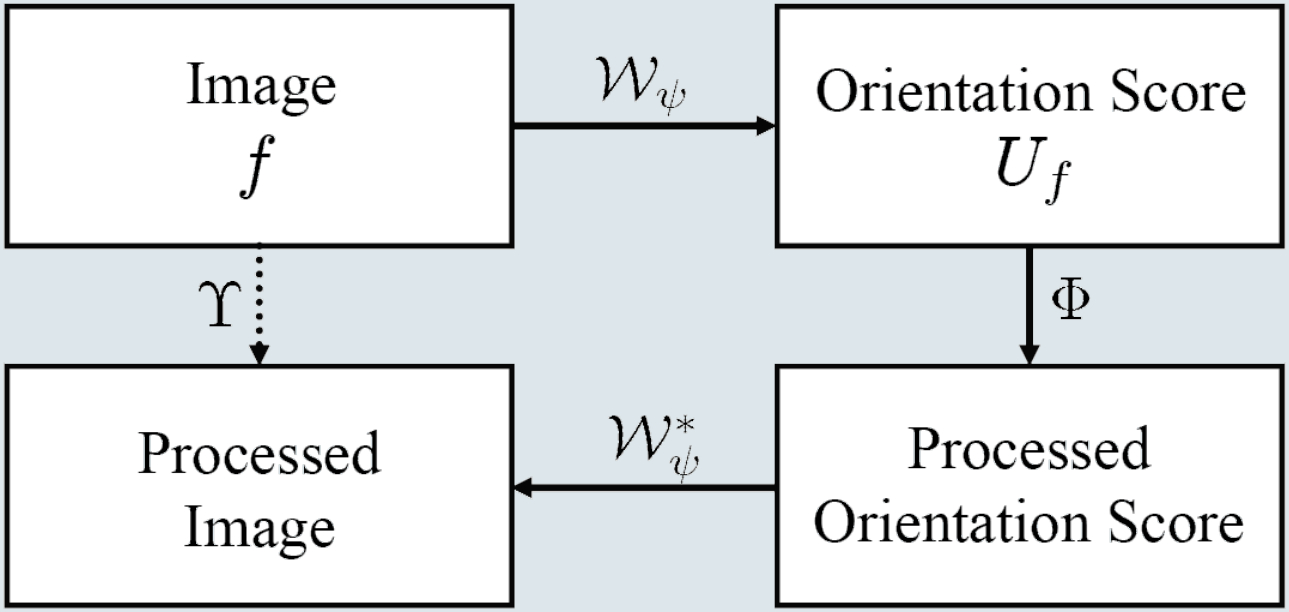


image

orientation score (iso-surface)

- Image can be reconstructed from OS (for specific choices of the kernel)

Our approach: image processing using OS

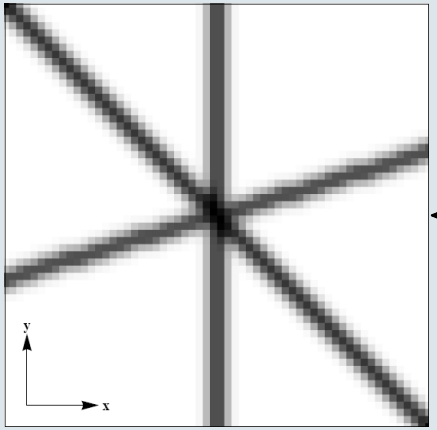


We consider orientation score as

where

Why processing via this “detour”?

Crossing lines are “torn apart”



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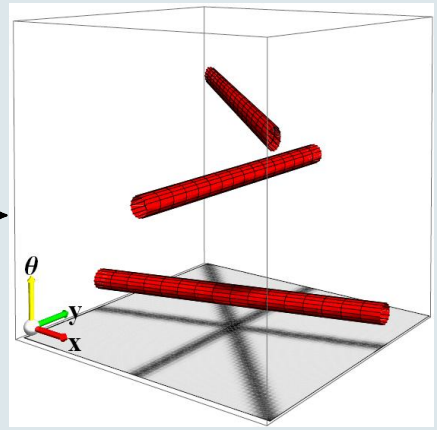


orientation score
(iso-surface)

→ makes it possible to handle singular points

(a) crossing (b) bifurcation (c) T-junction (d) end-point (e) discontinuity in slope

Outline

- **Invertible Orientation Scores**
- Operations in Orientation Scores (OS)
- Measuring Image Features in OS
- Coherence-Enhancing Diffusion in OS
- 3D Orientation Scores
- Conclusions

Invertible orientation score transformation

Image to orientation score

$$U_f(\mathbf{x},\theta) = \mathcal{W}_\psi[f](\mathbf{x},\theta) = \int_{\mathbb{R}^2} \overline{\psi(\mathbf{R}_\theta^{-1}(\mathbf{x}'-\mathbf{x}))} f(\mathbf{x}')d\mathbf{x}'$$

Oriented wavelet

Orientation score to image:

$$\tilde{f}(\mathbf{x}) = \mathcal{W}_\psi^*[U_f](\mathbf{x}) = \int_0^{2\pi}\int_{\mathbb{R}^2} \psi(\mathbf{R}_\theta^{-1}(\mathbf{x}'-\mathbf{x}))U_f(\mathbf{x}')d\mathbf{x}'$$

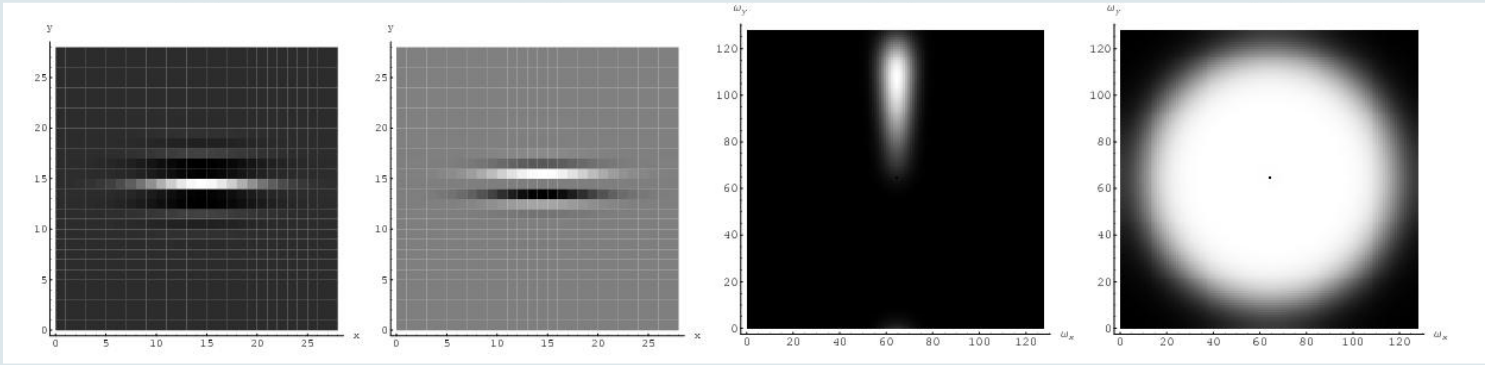
Stable reconstruction requires

$$0 < M_\psi(\boldsymbol{\omega}) = \int_0^{2\pi} |\mathcal{F}[\mathcal{R}_\theta[\psi]]|^2 d\theta < \infty$$

i.e. “fill up the entire Fourier spectrum”.

Invertible Orientation Score Transformation

Design considerations: reconstruction, directional, spatial localization, quadrature, discrete number of orientations



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Orientation score is a function on SE(2)

Properties of SE(2)

- Group element

$$g = (\mathbf{b}, \theta), \ \mathbf{b} \in \mathbb{R}^2 \text{ and } e^{i\theta} \in \mathbb{T}$$

translation rotation

- Group product

$$g_1 \, g_2 = (\mathbf{b}_1 + \mathbf{R}_{\theta_1} \mathbf{b}_2, \theta_1 + \theta_2)$$

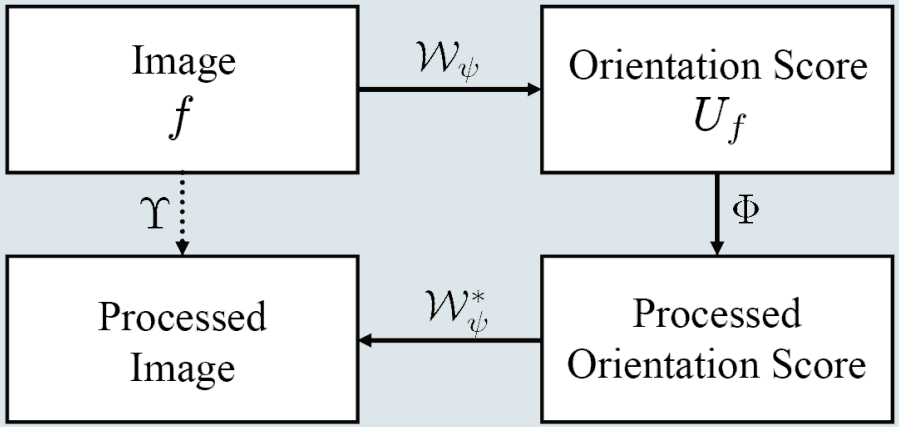
- Group inverse

Important representations

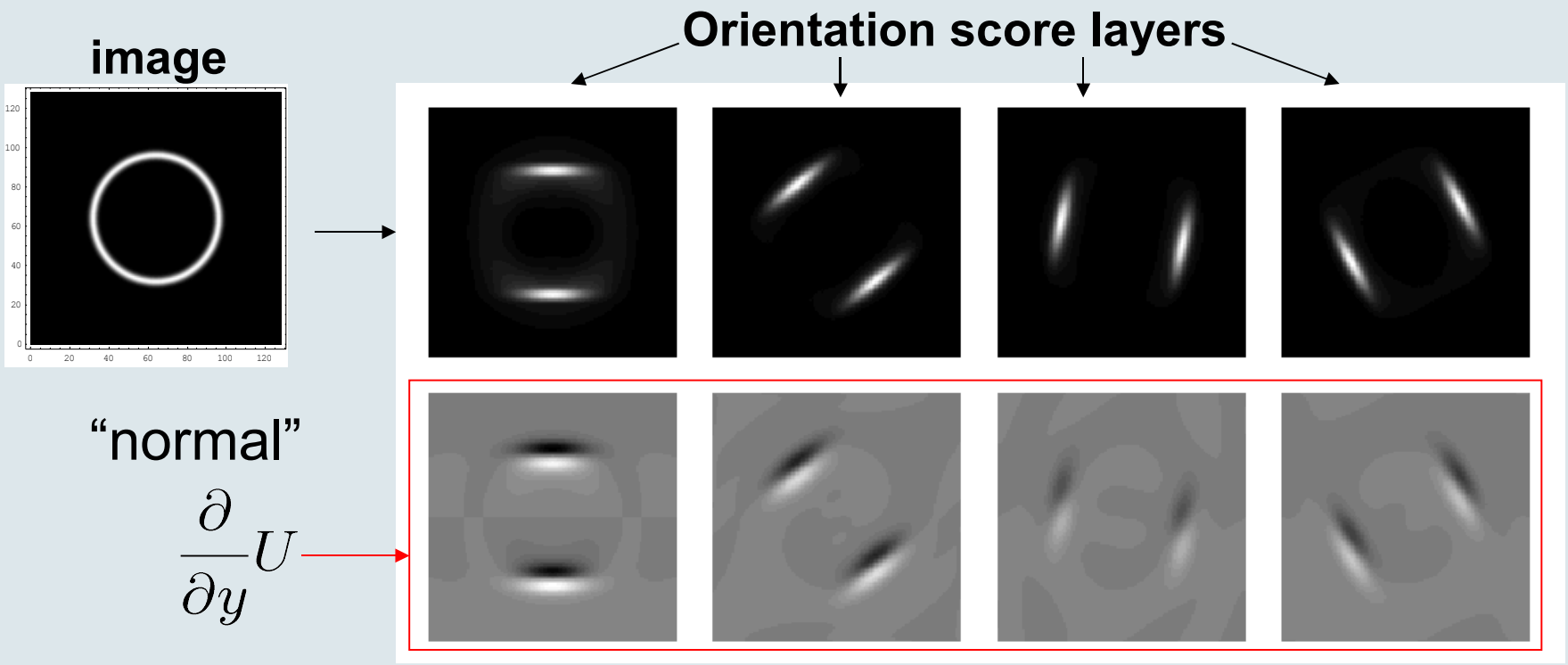
We need *left-invariant* operators

- The effective operator on the image Υ should be Euclidean invariant $\mathcal{U}_g(\Upsilon(f)) = \Upsilon(\mathcal{U}_g(f))$
- This implies that the operator Φ on the orientation score must be *left-invariant*, i.e.

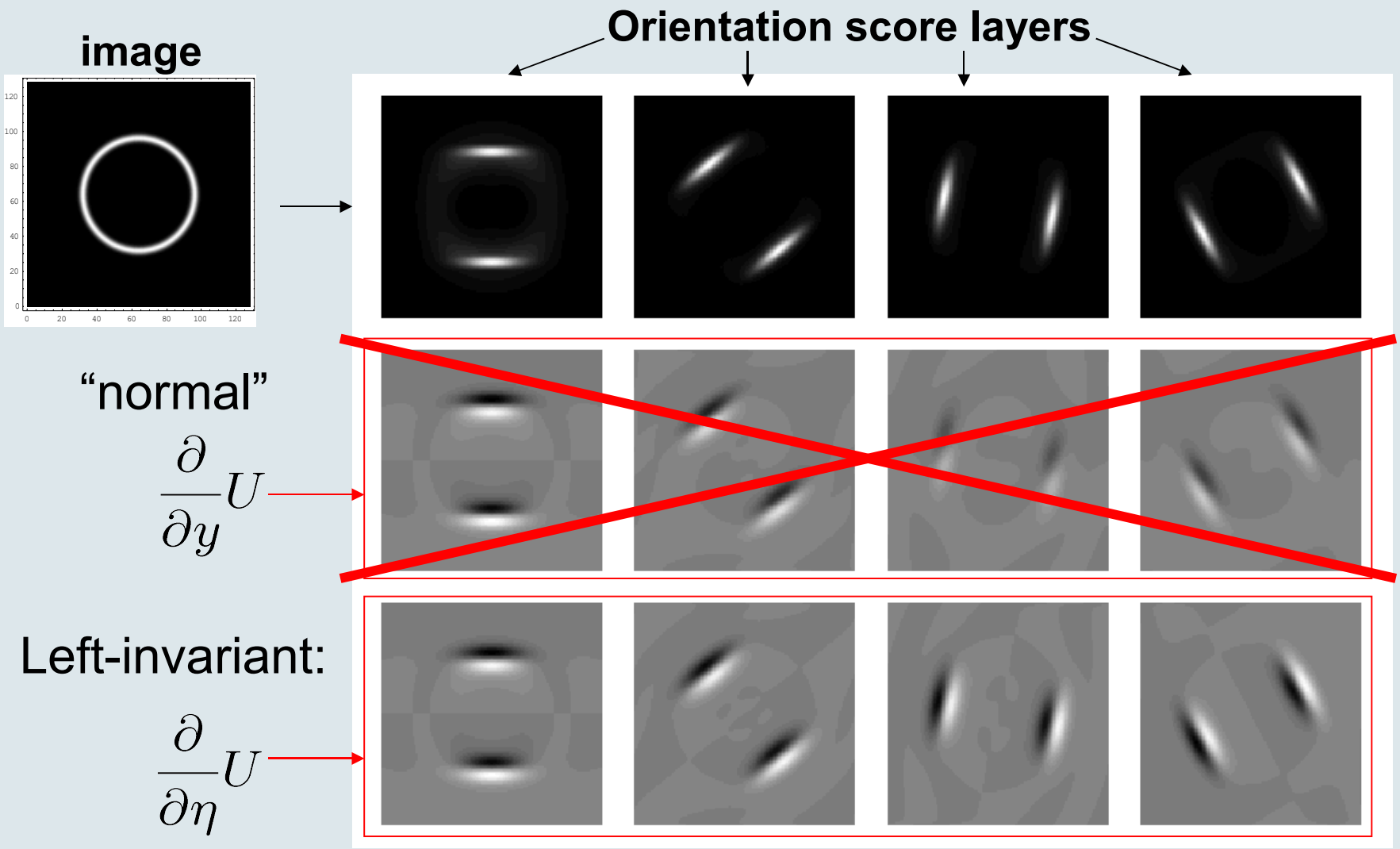
$$\mathcal{L}_g(\Phi(U)) = \Phi(\mathcal{L}_g(U))$$



Taking Derivatives of Orientation Scores



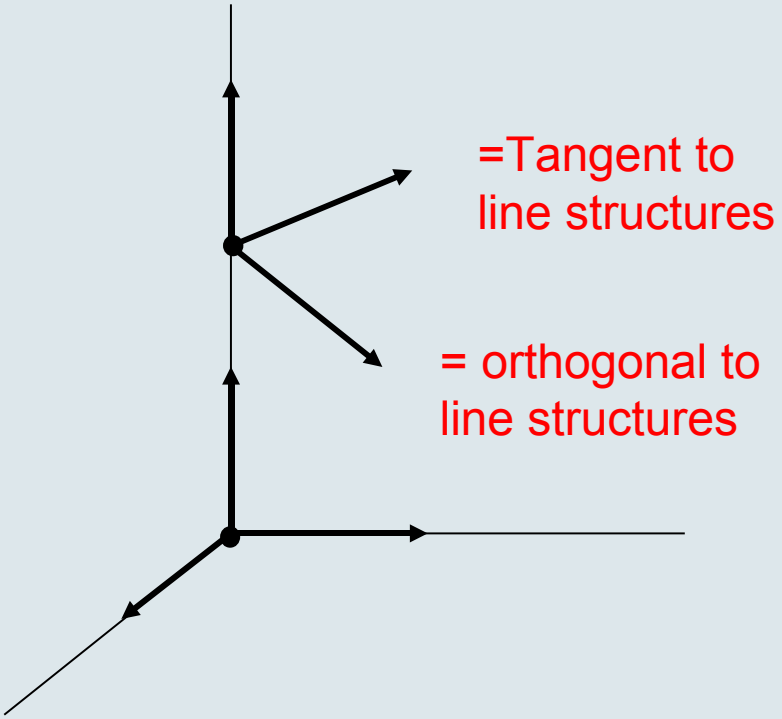
Taking Derivatives of Orientation Scores



Left-invariant Derivative Operators

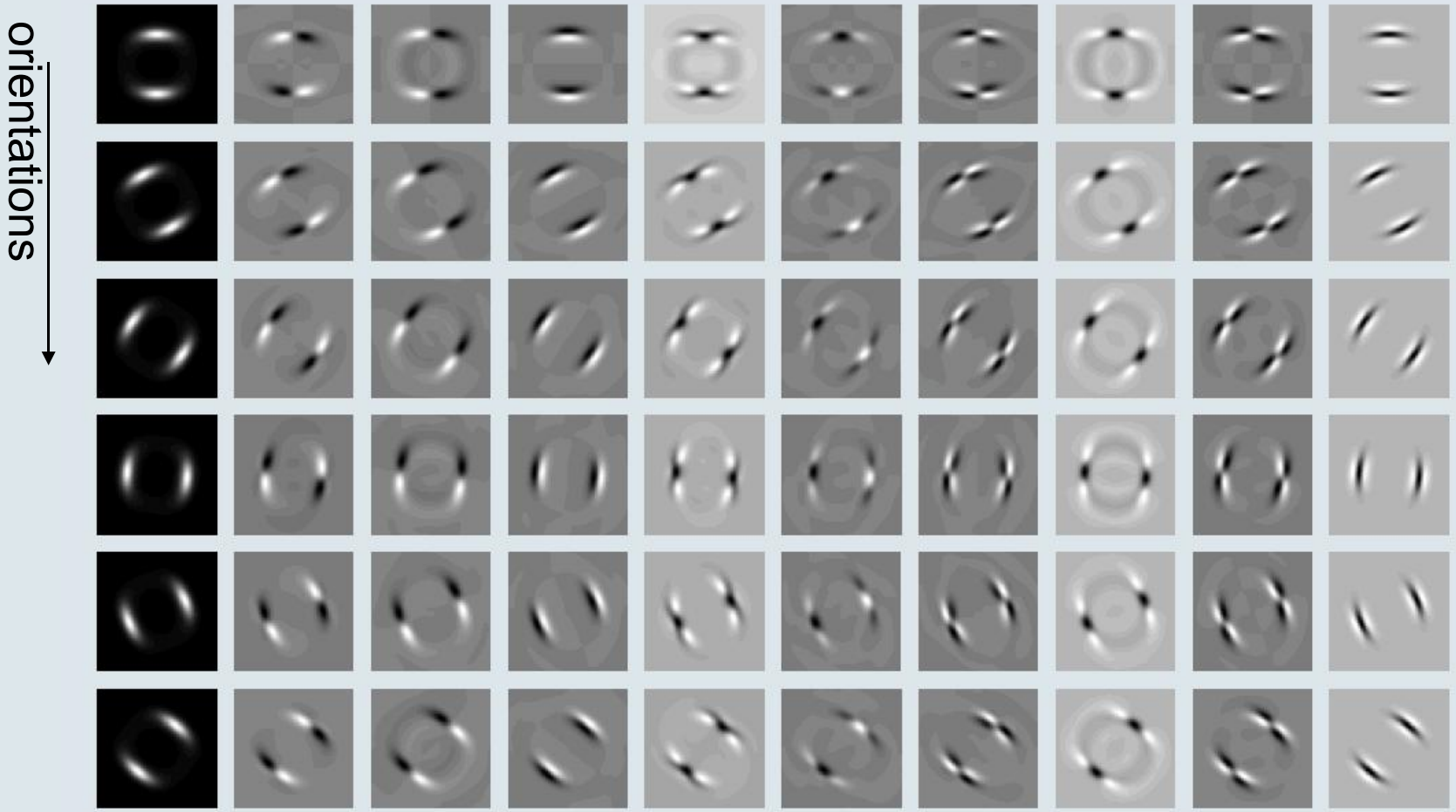
$$\partial_\xi = \cos\theta\,\partial_x + \sin\theta\,\partial_y$$
$$\partial_\eta = -\sin\theta\,\partial_x + \cos\theta\,\partial_y$$
$$\partial_\theta = \partial_\theta$$

These are left-invariant derivatives , so



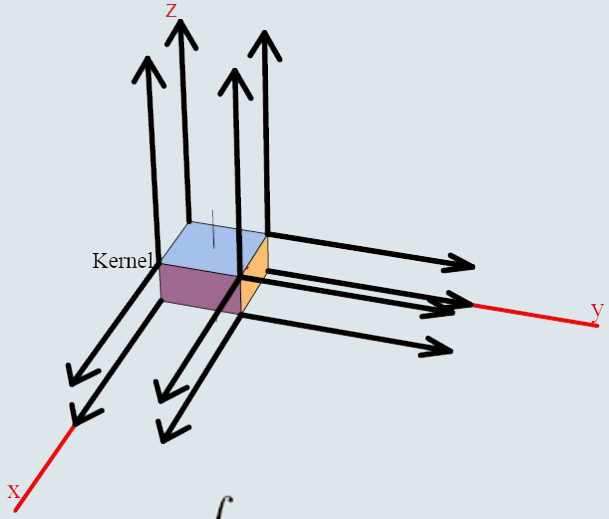
Not all left-invariant derivatives on SE(2) commute!

Example of a left-invariant Gaussian derivative jet

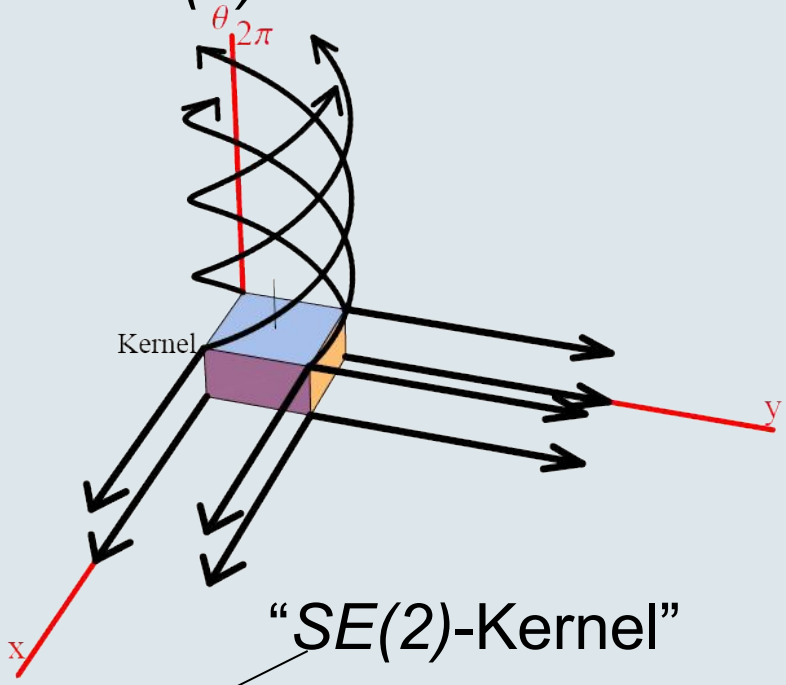


In general: all linear & left-invariant operators are *SE(2)-convolutions*

Normal 3D convolution – versus *SE(2)*-convolution on OS



$$[f * g](\mathbf{x}) = \int_{\mathbb{R}^n} f(\mathbf{x}-\mathbf{y})\, g(\mathbf{y})\, d\mathbf{y}$$



"*SE(2)*-Kernel"

$$[K *_G U_f](\mathbf{x},\theta) = \int_{\mathbb{R}^2}\int_0^{2\pi} K(R_{\theta'}^{-1}(\mathbf{x}-\mathbf{x}'),\theta-\theta')U_f(\mathbf{x}',\theta')\, d\theta' d\mathbf{x}'$$

Complexity of *Steerable* G-convolution $\mathcal{O}(MS^2 \log S) + \mathcal{O}(M^2S^2)$,
with M is # orientations, S is # pixels in x and y dimension.

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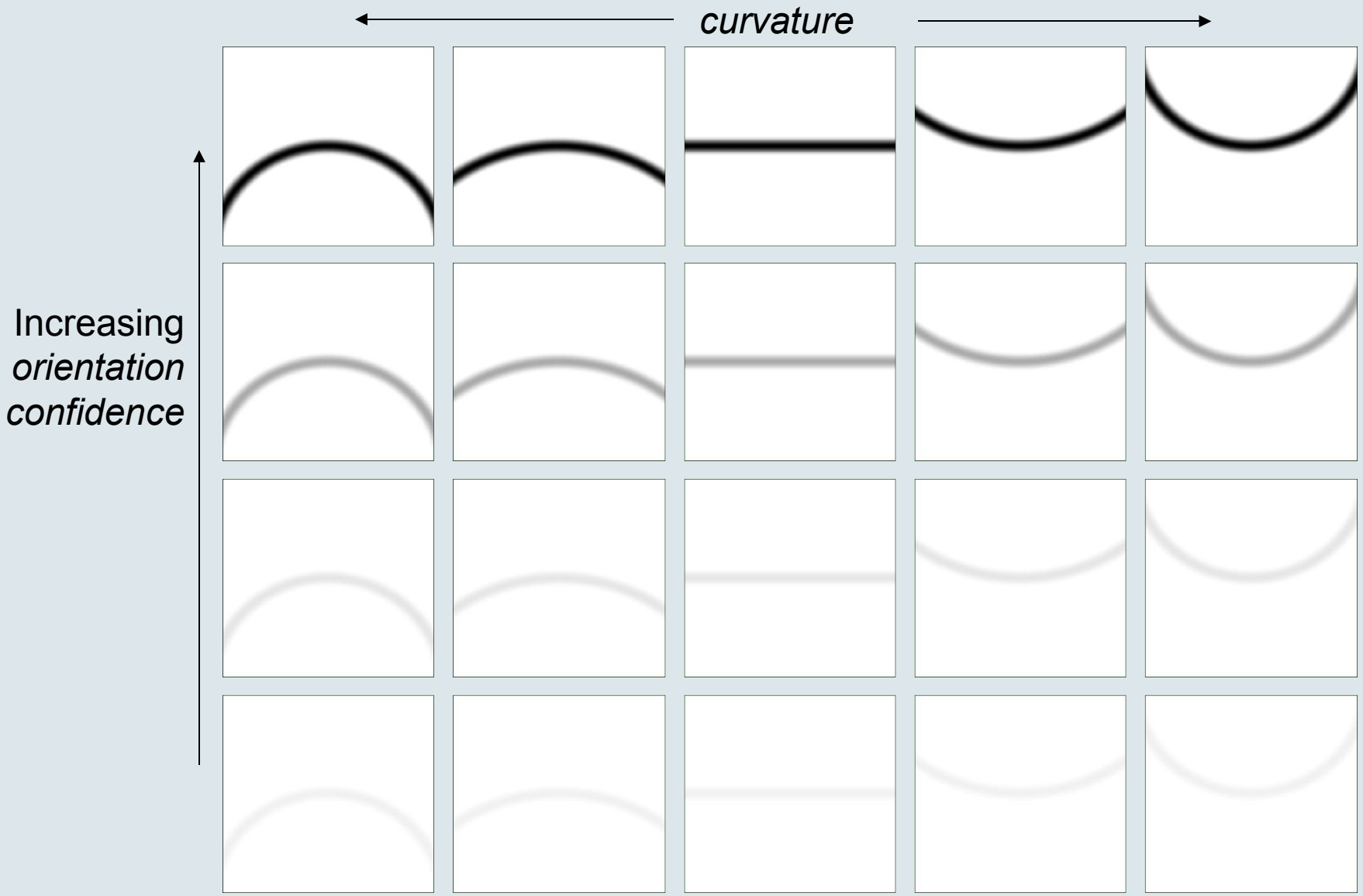
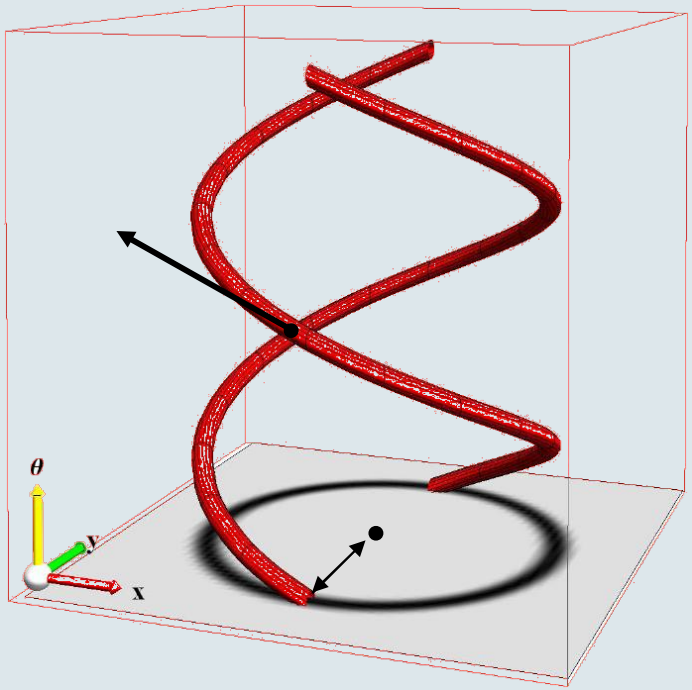
Measuring image features in orientation scores



Curvature estimation in OS

$\mathbb{R}^2$-curvature can be obtained from exp-curve ("tangent spiral") in OS.
→ What is curvature of the locally best fitting exponential curve?
→ Minimize the change of left-invariant gradient over the exp-curve



$$\min_{\mathbf{c}} \left\{ \left\| \frac{d}{ds} (\nabla U(\gamma_{\mathbf{c}}(s)))\Big|_{s=0} \right\|_\beta^2 \,\middle|\, \|\mathbf{c}\|_\beta = 1 \right\}$$

with $\nabla U = \left(\frac{\partial}{\partial_\theta}, \frac{\partial}{\partial_\xi}, \frac{\partial}{\partial_\eta} \right) U$ (components of a *co*vector)

Vector norm: $\|\mathbf{c}\|_\beta = (c_\theta)^2 + \beta^2 (c_\xi)^2 + \beta^2 (c_\eta)^2$

[1] [1/length] [length]

"Covector norm" $\left\|\hat{\mathbf{b}}\right\|_\beta = (b_\theta)^2 + \beta^{-2} (b_\xi)^2 + \beta^{-2} (b_\eta)^2$

Solving the Minimization Problem

To minimize $\min_{\mathbf{c}} \left\{ \left\| \frac{d}{ds} (\nabla U(\gamma_{\mathbf{c}}(s))) \Big|_{s=0} \right\|_{\beta}^2 \Big| \, \|\mathbf{c}\|_{\beta} = 1 \right\}$

Euler-Lagrange gives $\nabla_{\mathbf{c}} \|(\mathcal{H}U)\mathbf{c}\|_{\beta}^2 - \lambda(1 - \nabla_{\mathbf{c}} \|\mathbf{c}\|_{\beta}) = 0$

with $\mathcal{H}U = \nabla(\nabla U) = \begin{pmatrix} \partial_\theta^2 U & \partial_\xi \partial_\theta U & \partial_\eta \partial_\theta U \\ \partial_\theta \partial_\xi U & \partial_\xi^2 U & \partial_\eta \partial_\xi U \\ \partial_\theta \partial_\eta U & \partial_\xi \partial_\eta U & \partial_\eta^2 U \end{pmatrix}$ ← Note: non-symmetric!

→ $(\mathrm{M}_\beta \mathcal{H}U \mathrm{M}_\beta)^T (\mathrm{M}_\beta \mathcal{H}U \mathrm{M}_\beta) \, \tilde{\mathbf{c}}^* = \lambda \, \tilde{\mathbf{c}}^*$

with $\tilde{\mathbf{c}}^* = \mathrm{M}_\beta^{-1} \mathbf{c}^*$ and $\mathrm{M}_\beta = \mathrm{diag}\{1, 1/\beta, 1/\beta\}$

→ Eigensystem analysis on 3x3 matrix, pick right eigenvector $\mathbf{c} = (c_\theta, c_\xi, c_\eta)$ → $\kappa_{est} = \frac{c_\theta \mathrm{sign}(c_\xi)}{\sqrt{c_\xi^2 + c_\eta^2}}$

Orientation confidence can be found from the eigenvalues

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Inspiration from Image Diffusion Techniques

$$\begin{cases} \frac{\partial}{\partial t} u = \nabla \cdot \mathrm{D} \nabla u \\ u(\mathbf{x}; 0) = f(\mathbf{x}) \end{cases}$$

f = image

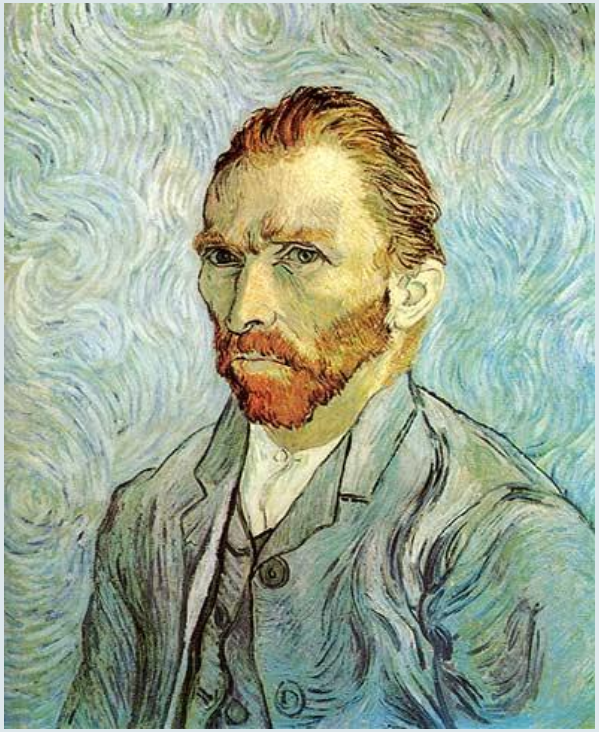
u = scale space of image

D = diffusion tensor

$$\nabla = \left(\frac{\partial}{\partial x} \quad \frac{\partial}{\partial y} \right)^T$$

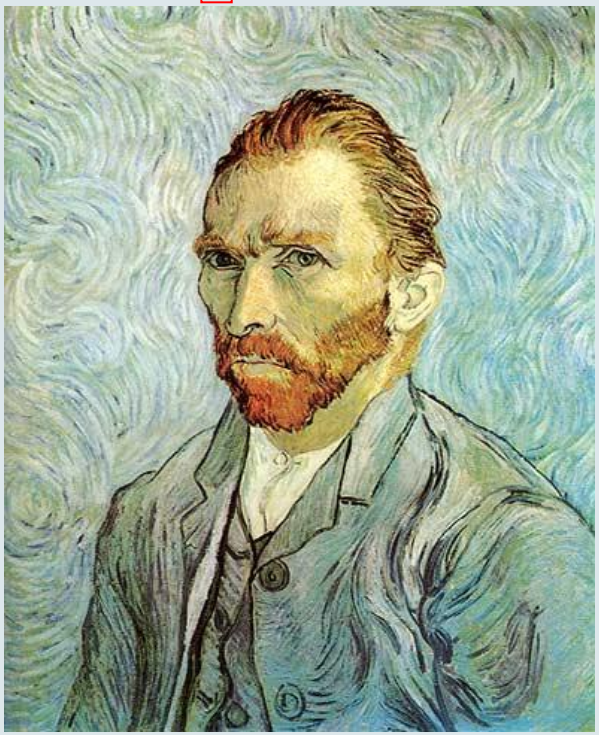
Linear diffusion

$\mathrm{D} = \mathrm{I}$



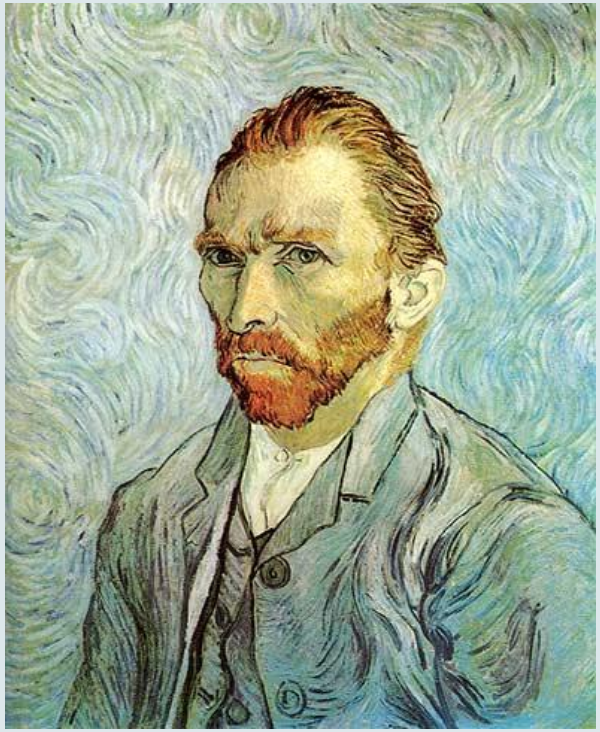
Perona&Malik

$\mathrm{D}(\mathbf{x}) = g(|\nabla u(\mathbf{x})|)\,\mathrm{I}$



Coherence-enhancing diff.

$\mathrm{D}(\mathbf{x}) = s(\mathbf{x})\,\mathbf{v}(\mathbf{x})\,\mathbf{v}^T(\mathbf{x}) + \alpha\,\mathrm{I}$



Diffusion Equation in Orientation Scores

$$\partial_t u = \begin{pmatrix} \partial_\theta & \partial_\xi & \partial_\eta \end{pmatrix} \begin{pmatrix} D'_{11} + D_{22}\kappa^2 & D_{22}\kappa & 0 \\ D_{22}\kappa & D_{22} & 0 \\ 0 & 0 & D_{33} \end{pmatrix} \begin{pmatrix} \partial_\theta \\ \partial_\xi \\ \partial_\eta \end{pmatrix} u$$

curvature

Evolving orientation score

Left-invariant derivatives

Diffusion in orientation

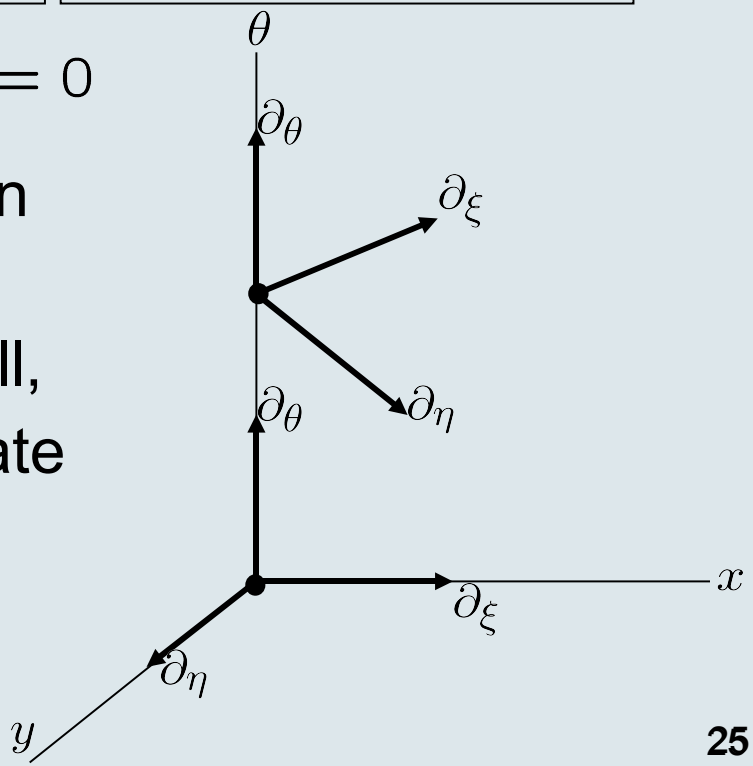
Diffusion tangent to oriented structures

Diffusion orthogonal to oriented structures

Gaussian derivatives: $D22 = D33, \kappa = 0$

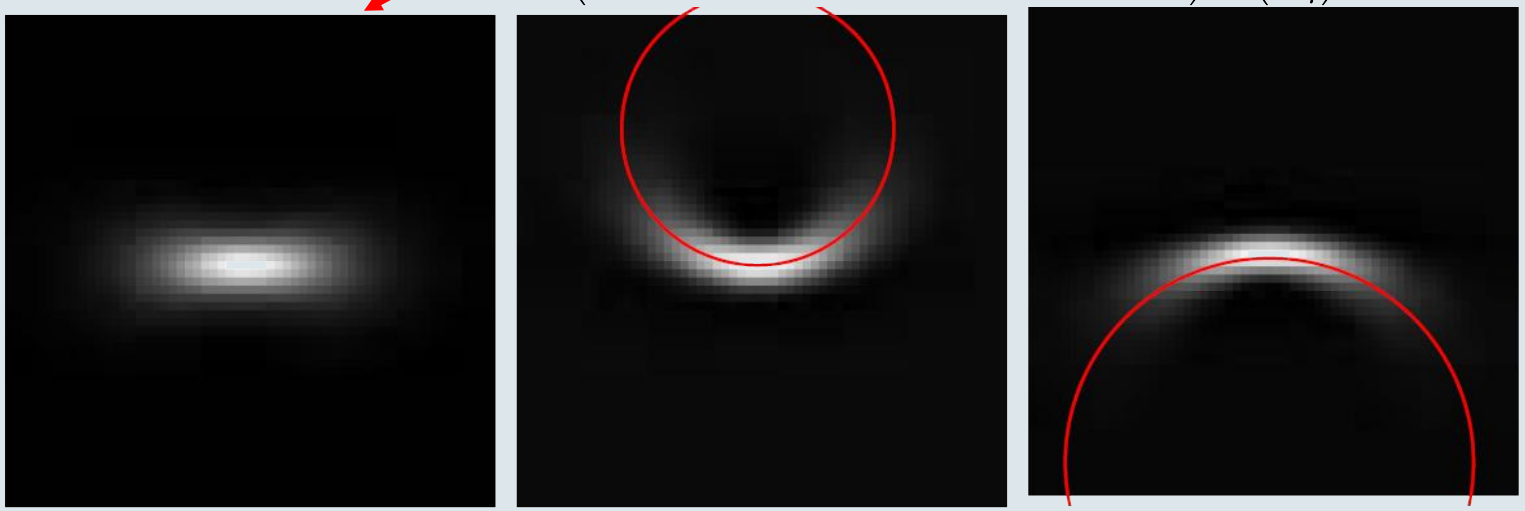
CED-OS: non-linear adaptive diffusion tensor coefficients

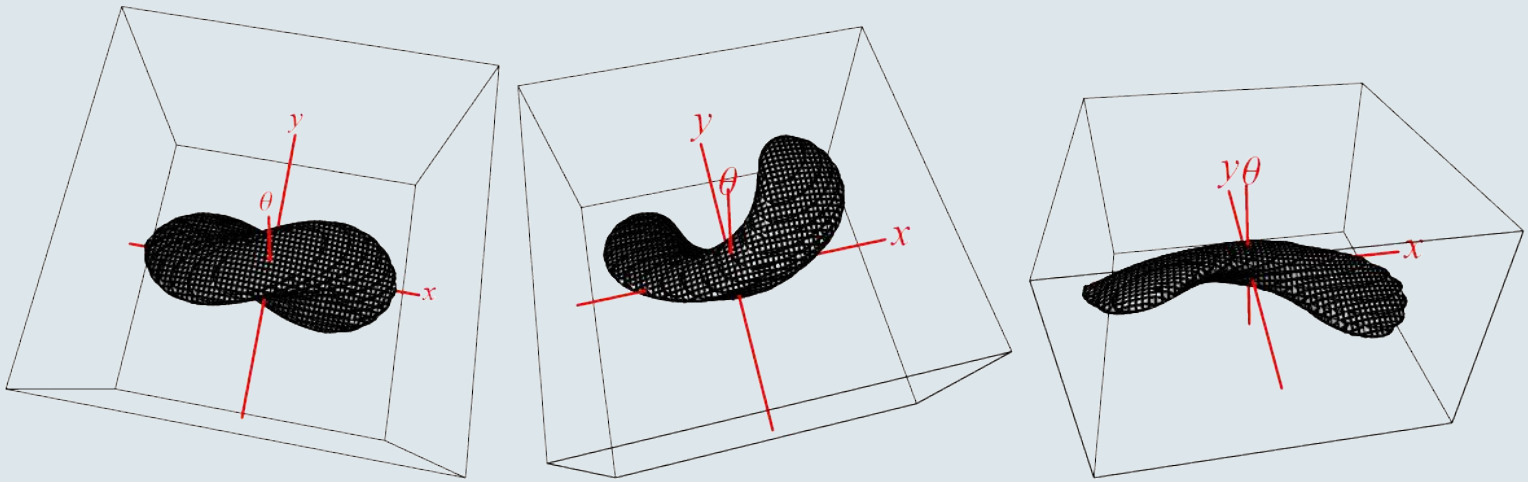
- Oriented regions: D'_{11} and D_{33} small, D_{22} large and κ according to estimate
- Non-oriented regions: D'_{11} large, $D_{22}=D_{33}$ large, $\kappa = 0$



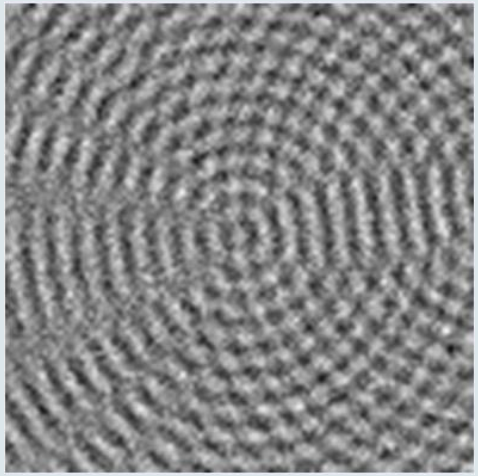
Example diffusion kernels

$$\partial_t u = \begin{pmatrix} \partial_\theta & \partial_\xi & \partial_\eta \end{pmatrix} \begin{pmatrix} D'_{11} + D_{22}\kappa^2 & D_{22}\kappa & 0 \\ D_{22}\kappa & D_{22} & 0 \\ 0 & 0 & D_{33} \end{pmatrix} \begin{pmatrix} \partial_\theta \\ \partial_\xi \\ \partial_\eta \end{pmatrix} u$$

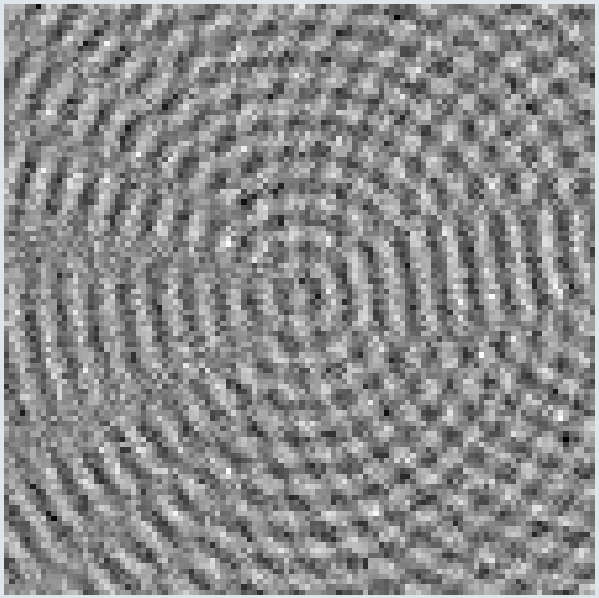




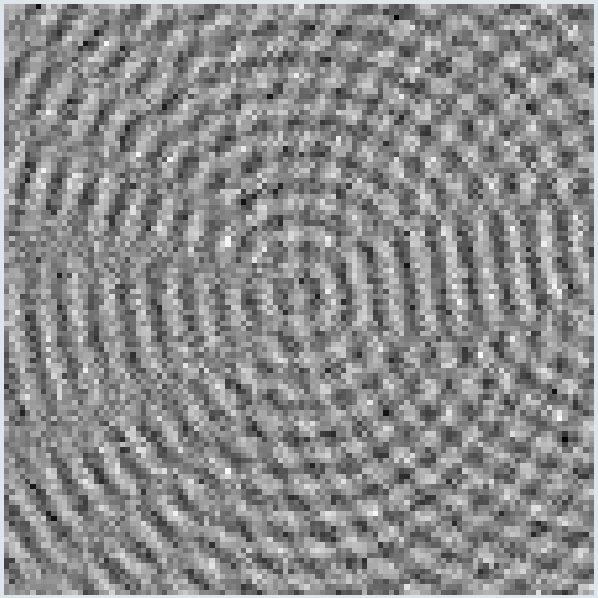
Example of CED-OS



Size: 128 x 128 x 64



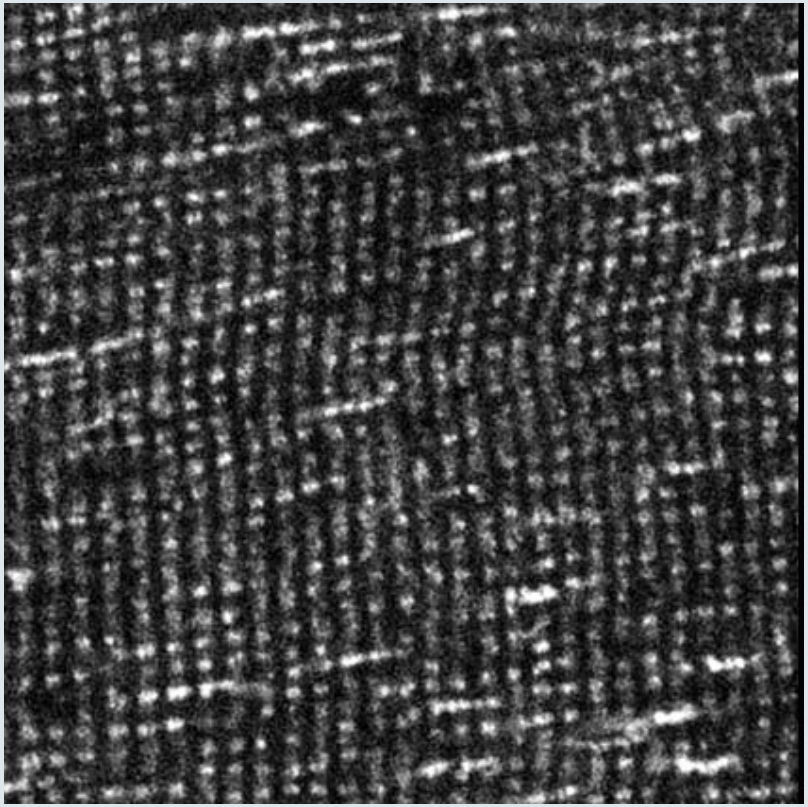
CED-OS



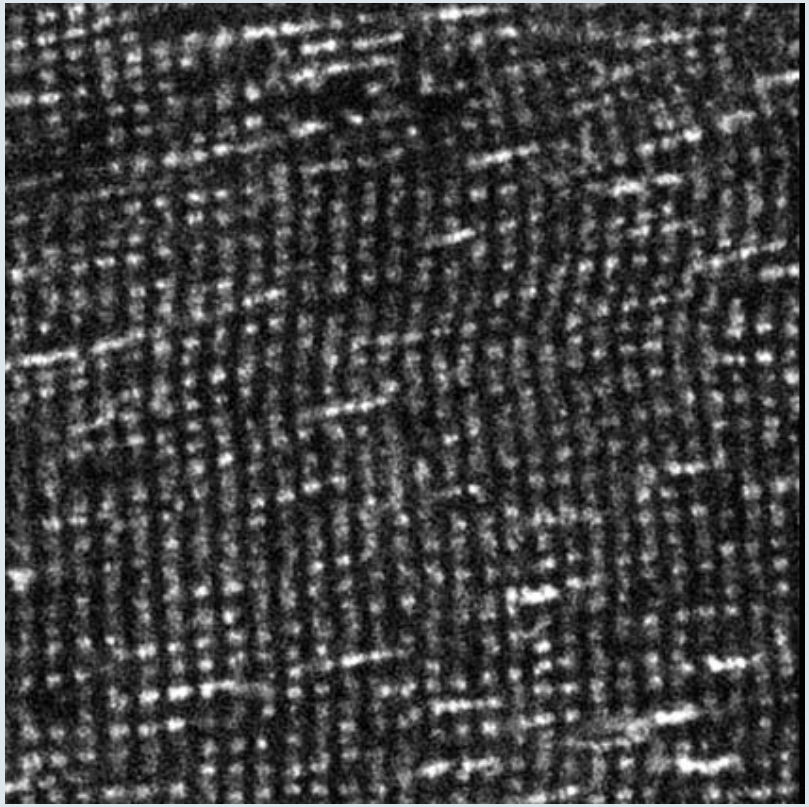
CED

Biomedical Example: Muscle Cell

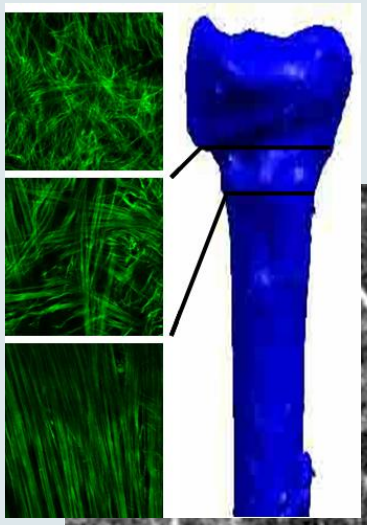
Original



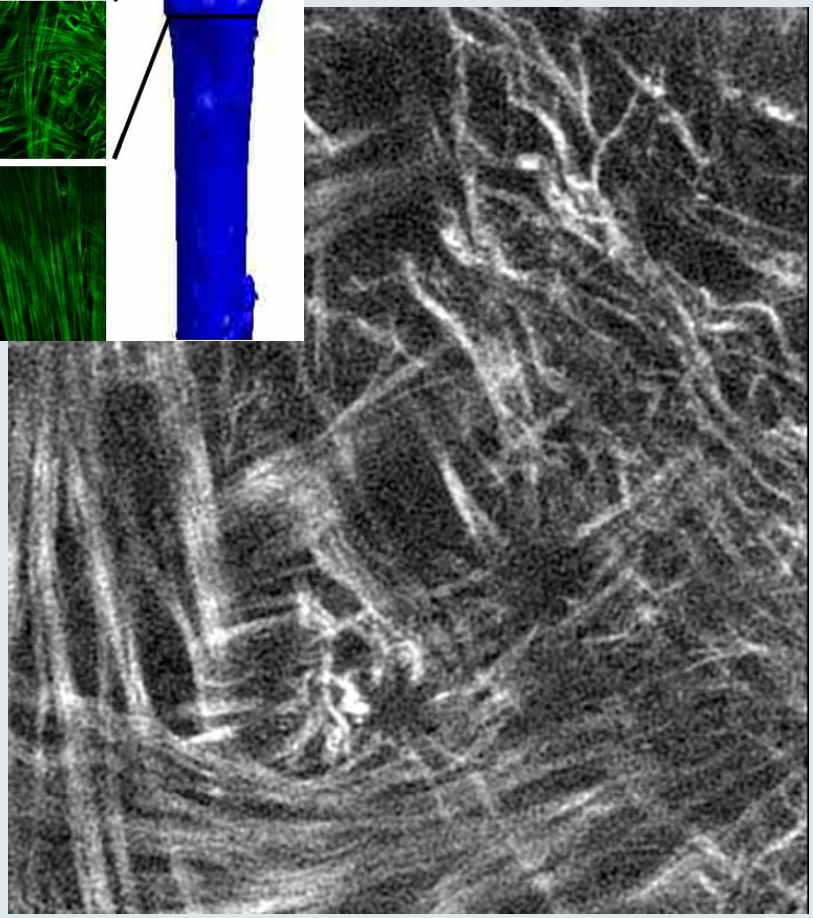
CED-OS



Biomedical Example: bone structure



Original



CED-OS

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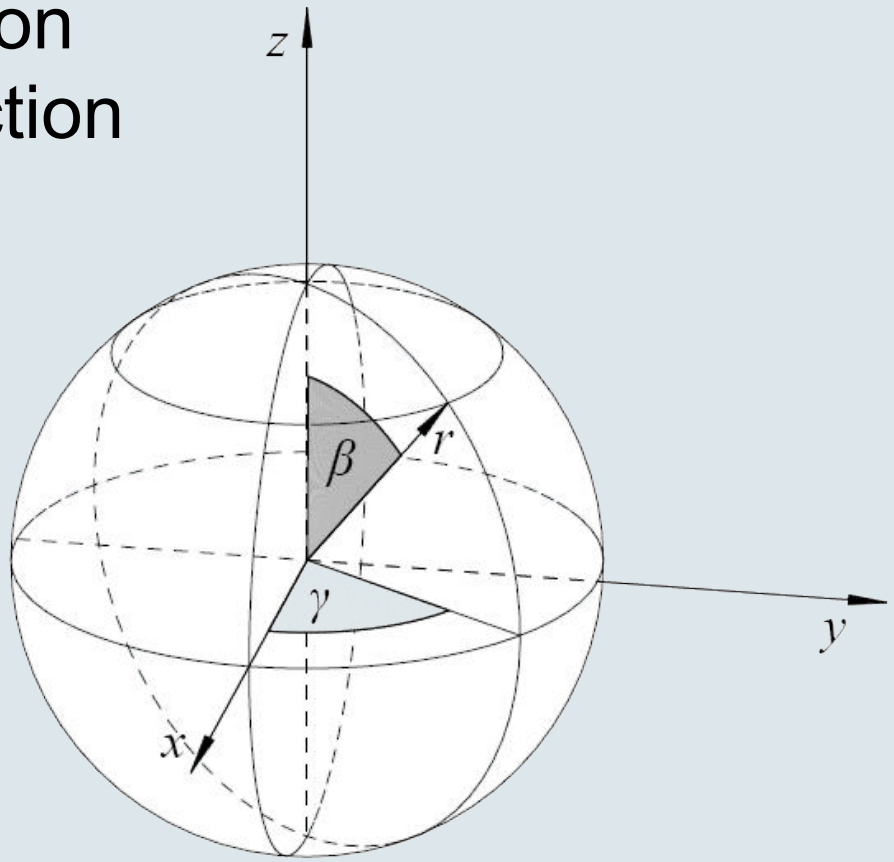
- Measuring Image Features in OS
- Coherence-Enhancing Diffusion in OS
- **3D Orientation Scores**
- Conclusions

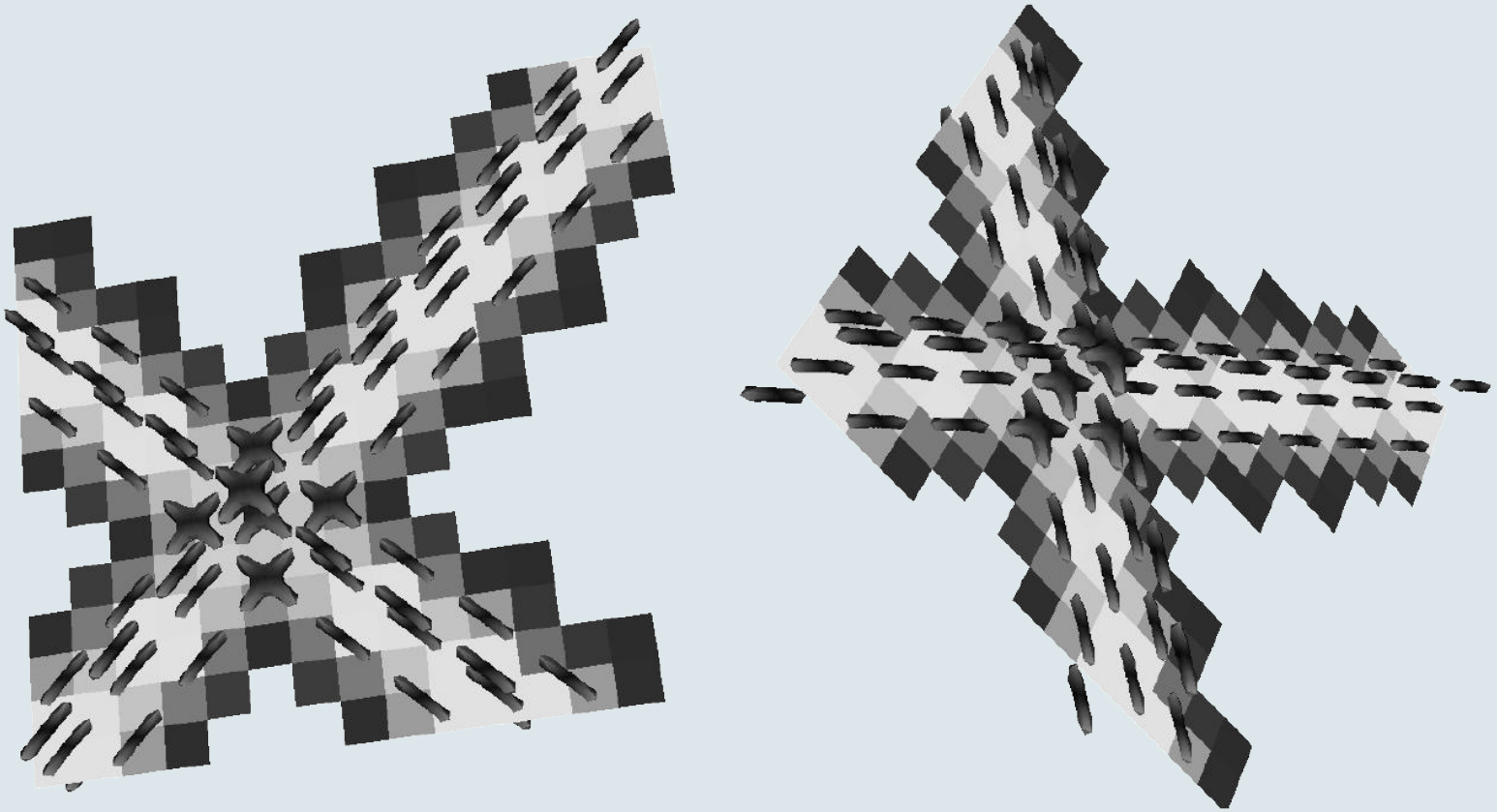
3D Orientation Scores

- Relevant for medical applications: high angular resolution diffusion imaging (HARDI).
- On each spatial position (x,y,z) we have a function on the sphere S^2.
- So a 3D orientation score is a *5D space*, parameterized (e.g.) by (x,y,z,β,γ).



3D Orientation Scores

Example visualization of a 3D orientation score



3D Orientation scores and *α-right-invariance*

- 3D Orientation score is a function on
  rather than 3D Euclidean motion group
- The unit sphere S^2 is isomorphic to the left cosets *SO(3)/SO(2)* given by

- So: we consider 3D OS as an α*-right-invariant* function
  which should fulfill
- Operators on 3D OS should *preserve* α*-right-invariance*

Operations on 3D Orientation Scores

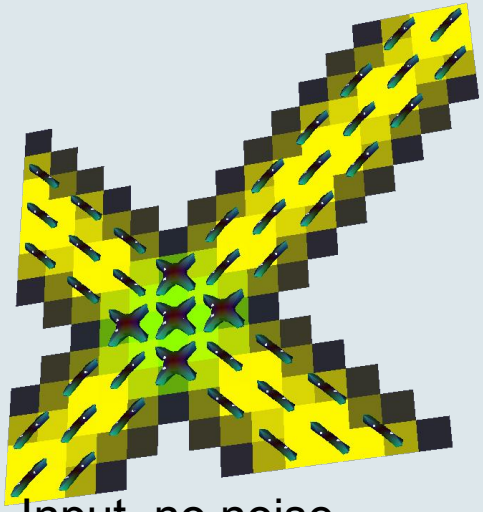
Similar to the 3 left-invariant derivatives in 2D, we now have 6 left-inv. Derivatives

Left-invariant diffusion on SE(3) is given by

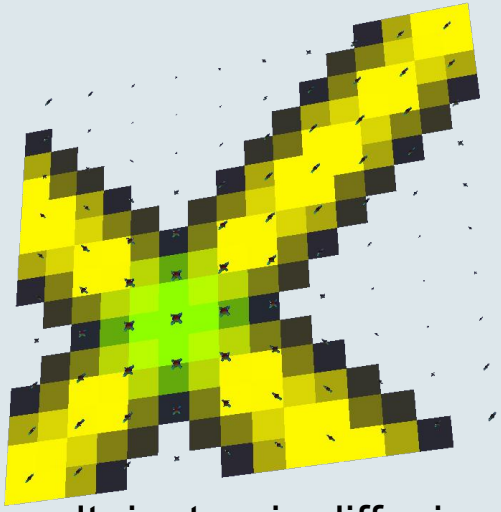
Constant diffusion: only

Adaptive diffusion: fine as long as the adaptivity is calculated in left-invariant way.

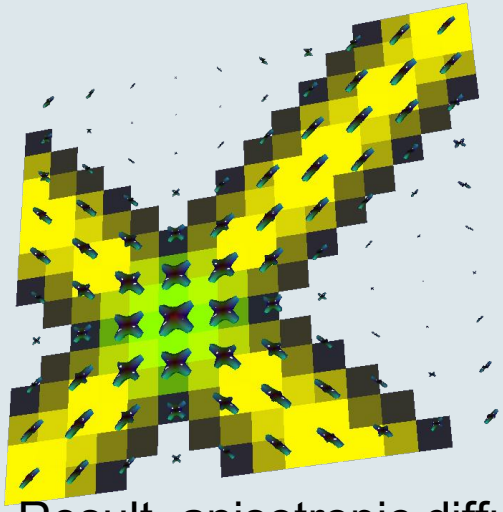
Results of linear diffusion on 3D OS



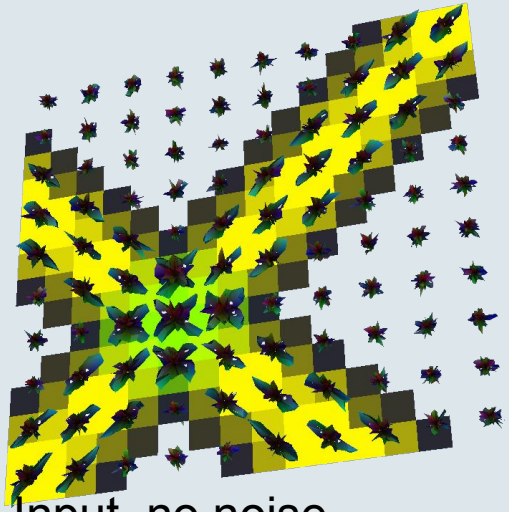
Input, no noise



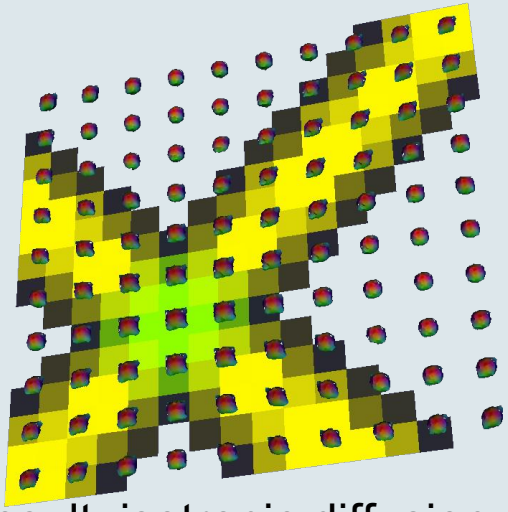
Result, isotropic diffusion



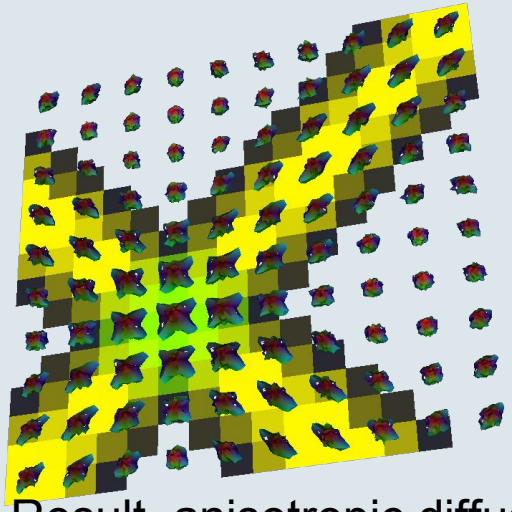
Result, anisotropic diffusion



Input, no noise

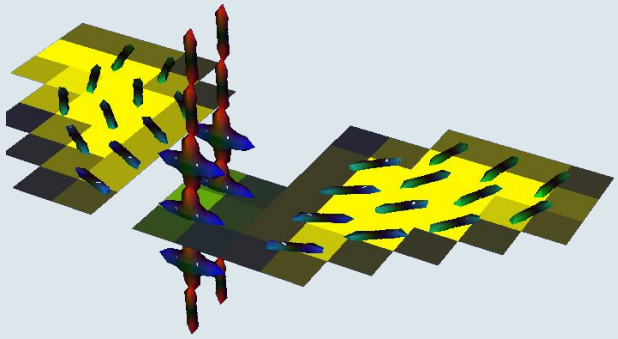


Result, isotropic diffusion

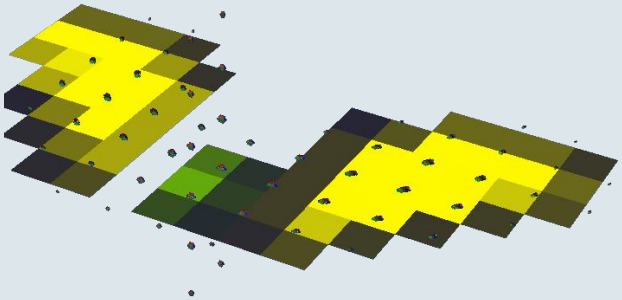


Result, anisotropic diffusion

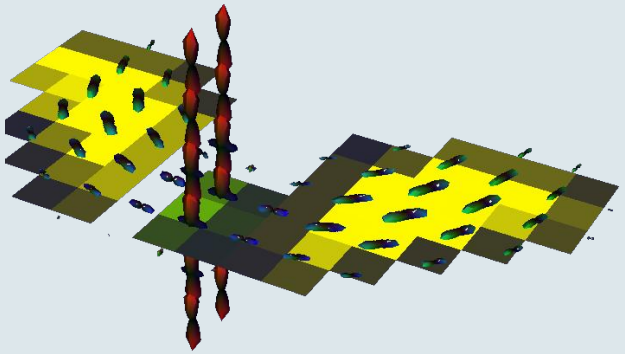
Results of linear diffusion on 3D OS



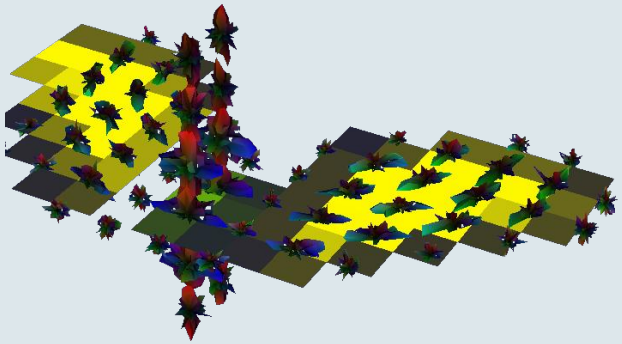
Input, no noise



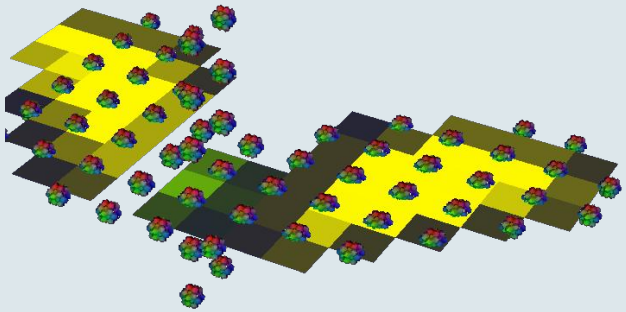
Result, isotropic diffusion



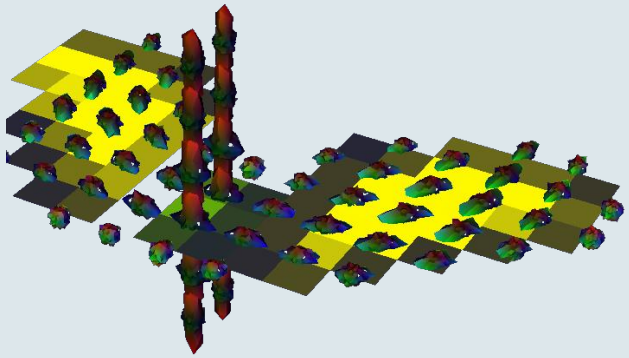
Result, anisotropic diffusion



Input, noisy



Result, isotropic diffusion



Result, anisotropic diffusion

Outline

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- Operations in Orientation Scores (OS)
- Measuring Image Features in OS
- Coherence-Enhancing Diffusion in OS
- 3D Orientation Scores
- **Conclusions**

Conclusions

- We developed a framework for image processing / analysis via orientation scores of 2D and 3D images
  Important notion: An orientation score is a function on *SE(n)* → use group theory.
- Developed nonlinear diffusion on orientation scores to enhance crossing line structures in 2D images
- All methods also apply to 3D orientation scores

Future work:

- Detection of elongated structures via OS
- Further develop 3D orientation scores, apply to HARDI