Overview
Introduction
System Model
Independence Model
Performance Analysis
Matched Receiver
Channel Thresholding
Numerical Results
Summary

Error Correction and Opportunistic Scheduling Protocols in Random Wireless Networks

Amogh Rajanna

Advisor Mos Kaveh

Co-Advisor Itsik Bergel

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Thesis Overview

Performance analysis of HARQ, a joint error correction and retransmission protocol, and multiuser diversity scheduling in wireless adhoc networks (WANETs).

- **Rateless Codes in a Single hop WANET**:
  Studies the achievability of network ERD with short delays using rateless codes. The achievable rate density in the WANET is quantified and shown to have near ERD performance. The dynamic variations of the instantaneous rate in the WANET is characterized.

- **HARQ in a Cooperative Multihop WANET**:
  Studies the performance of HARQ protocol with opportunistic routing in a cooperative multihop WANET. A convex approximation to the transport capacity of the WANET is developed using the concept of decoding cells. The parameters of the HARQ protocol are optimized to operate the WANET at maximal transport capacity.

Thesis Overview

- **Multiuser Diversity in Downlink Channels**:
  Amogh Rajanna and Nihar Jindal

  Presents a cost-benefit analysis of multiuser diversity in downlink cellular channels emphasizing the cost of multiuser diversity in terms of the feedback resources bandwidth and power for acquiring user CQI at the base station. Characterizes the cost-benefit tradeoff for 2 types of CSI feedback methods, dedicated feedback and SNR dependent feedback, quantifying how many users should feedback CSI[1].

[1] A. Rajanna and N. Jindal *Multiuser diversity in downlink channels: When does the feedback cost outweigh the spectral efficiency benefit?* IEEE Tran Wire Comm, 11(1):408 - 418, January 2012.

Rateless Codes in a Single hop WANET[2]

- Introduction
- System Model
- Approximate Model
- Performance Analysis
- Improvements
- Numerical Results

[2] A. Rajanna, I. Bergel, and M. Kaveh. *Performance Analysis of Rateless Codes in an ALOHA WANET*, submitted IEEE Transactions on Wireless Communication, 2 Dec 2014.

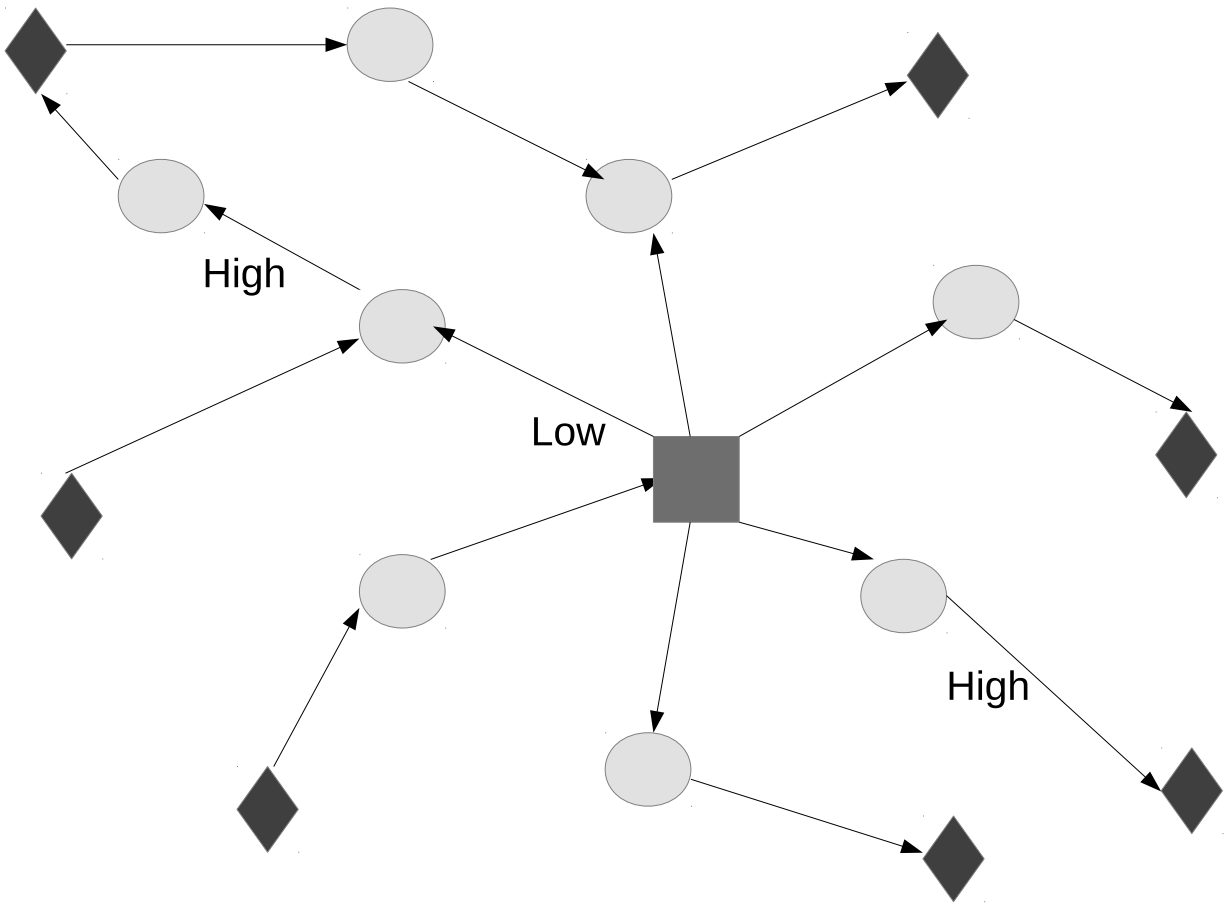


Figure: Wireless AdHoc Network (WANET)

Background

Rate Density - *number of information bits/sec communicated reliably per unit area of the network*

- Outage Rate Density (ORD): Rate subject to an outage constraint
- Ergodic Rate Density (ERD): Maximum achievable rate

The metric used depends on the type of network and type of traffic.

ORD

- Suited for previous generation wireless networks
- Information packets are successfully decoded or discarded at the 1st attempt.
  Even retransmissions are treated as fresh packets (ARQ).
- Best suited for delay constrained traffic - voice, video

Background

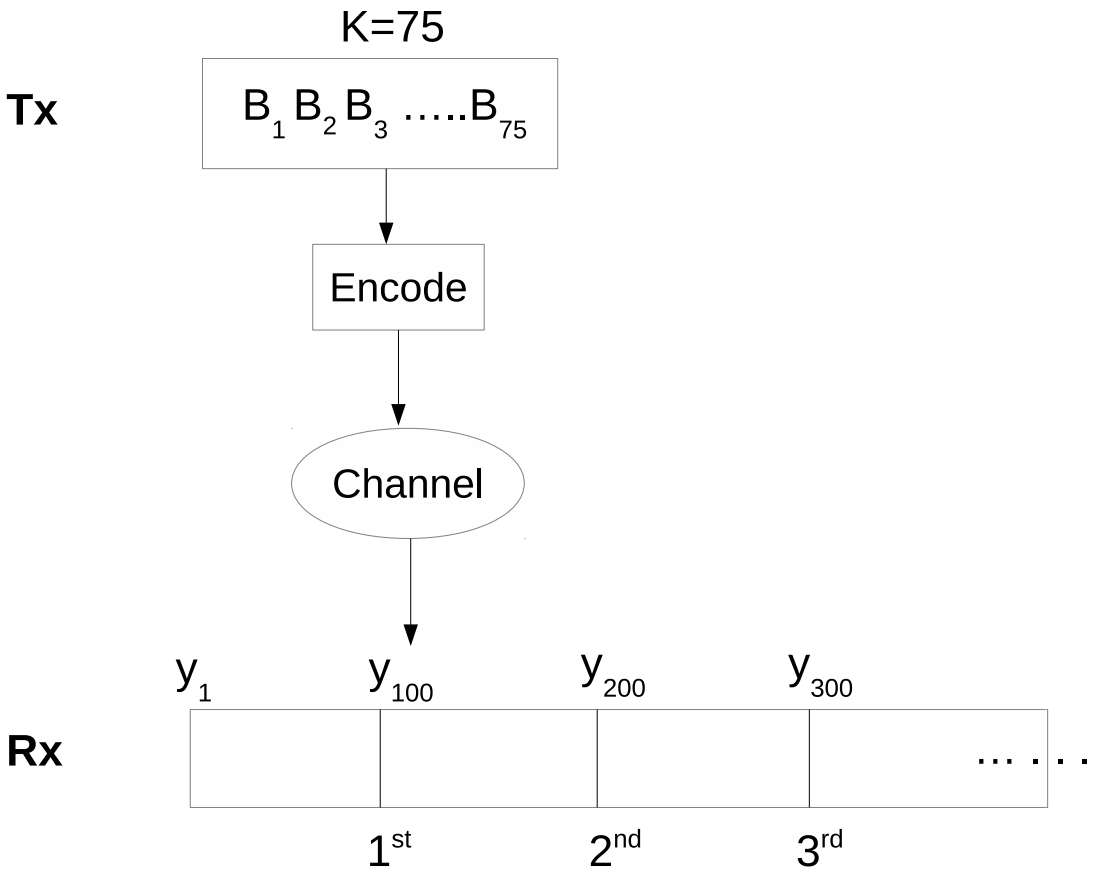
ERD

- Suited for wireless networks where links can achieve Shannon capacity.
- Achieving Shannon capacity in time varying wireless channels
  - Spreading the codeword of an information packet either in time or frequency before transmission.
  - Incremental Redundancy based HARQ.
    Punctured parity symbols of a codeword are transmitted on a block by block basis in response to receiver's ACK/NACK.
- Characterized by high instantaneous rate and substantial delay - Packet Data transmission

Motivation

- Sophisticated Error Correction Codes: Can achieve near (close to) ERD performance for delay constrained traffic.
- Fountain Codes -
  - Given an information bit vector, a Tx incrementally generates and sends the parity symbols.
  - At **periodic** intervals, the Rx attempts decoding using the received parity symbols upto that time.
- For example: $K = 75$
  - Tx - Encodes the bits by a Fountain Code and sends QPSK symbols
  - Rx - For every 100 channel uses, attempts to decode bits
  - Attempts continue until all $K = 75$ bits are decoded.

Fountain Codes



Motivation

- Rateless Codes have better performance than Fixed Rate codes.
  - Capacity achieving with shorter delays.
  - Robust to no CSIT transmission since it follows the channel closely.
- *Provide a setup to achieve near ERD performance in a WANET for delay constrained traffic using rateless codes.*

System Model

- We assume a Continuous Time ALOHA WANET
- The transmitter process is a space time homogeneous PPP Ψ with intensity λ_s / m^{-2}/sec.
- Notation $\Psi = \{X_i, T_i\}$ where $X_i \in \mathbb{R}^2$ is the location of node i and T_i is the packet transmission start time.
- Each node $X_i \in \Psi$ has a receive node at Y_i at a distance of d m. Assume $d = 1$.
- MAC state of node i at $t \in \mathbb{R}$ is given by

$$e_i(t) = 1\left(T_i \leq t \leq T_i + B_i\right) \tag{1}$$

B_i: packet transmission time

System Model

- Encoding - Decoding operations
  - Tx: Encodes K bits with a rateless code and sends Gaussian symbols incrementally over the channel.
  - Rx: For every L channel uses, attempts to decode a subset of K bits.
  - Process continues until Rx decodes K bits, and sends an ACK to Tx.
- Fading
  - Fading coherence time T_c is **large** relative to packet transmission time.
  - IID Block fading
    Independent fading coefficient from one packet transmission to next.

Packet Transmission Time

- The interference power at receiver i at time t is given by

$$I_i(t) = \sum_{k \neq i} \rho |h_{ki}|^2 |X_k - Y_i|^{-\alpha} e_k(t) \tag{2}$$

ρ : Transmit SNR
$|h_{ki}|^2$: Fading coefficient from transmitter k to receiver i
α : Path loss exponent
$e_k(t)$: MAC state of transmitter k

- The average interference upto time t where $t > T_i$ is

$$\hat{I}_i(t) = \frac{1}{t - T_i} \int_{T_i}^{t} I_i(\tau) \, \mathrm{d}\tau. \tag{3}$$

Packet Transmission Time

- The SINR at receiver i is

$$\hat{\mathrm{SINR}}_i(t) = \frac{\rho |h_{ii}|^2}{1 + \hat{I}_i(t)} \tag{4}$$

- Receiver i uses a nearest neighbor decoder.
  Not capable of estimating the instantaneous interference and noise power during the packet reception.

- Achievable rate of the nearest neighbor decoder at receiver i is

$$C_i(t) = \log_2\left(1 + \hat{\mathrm{SINR}}_i(t)\right) \tag{5}$$

  (5) is *Gaussian* receiver

Packet Transmission Time

- The time to decode K information bits for the Gaussian receiver

$$\hat{B}_i = \min\left\{t : \frac{K}{t} < \log_2\left(1 + \hat{\mathrm{SINR}}_i(t)\right)\right\} \tag{6}$$

- Each packet transmission of K bits is subject to a delay constraint of D.
  Without a delay constraint, Tx's with poor fading states cause unbounded packet times.

- Packet Transmission Time of receiver i is

$$B_i = \min\left(D, \hat{B}_i\right) = \begin{cases} D, & \hat{B}_i > D \\ \hat{B}_i, & \hat{B}_i \leq D \end{cases} \tag{7}$$

Exact Model

- (2)-(7): Referred to as the Exact Model
- $$\hat{B}_i = f_1\left(B_1, B_2 \cdots B_j \cdots\right) \tag{8}$$

  Distribution of B_i is dependent on $\{B_j\}_{j \neq i}$.
- Similar to (6)
  $$B_j = f_2\left(B_i\right) \tag{9}$$

  Each of $\{B_j\}_{j \neq i}$ is **again coupled** to B_i.
- With such an interdependence, distribution of B_i is intractable.

Independence Model

- WANET performance: Studying a reference Tx-Rx pair
- Tx 0 is the reference transmitter
  - Located at $X_0 = (d, 0)$.
  - Rx 0 is located at $Y_0 = (0, 0)$.
  - Tx 0 starts the packet transmission at $T_0 = 0$.
- $\bar{B}_1, \bar{B}_2, \bar{B}_3 \cdots$ represent the interferer packet transmission times.
- **Independence Approximation**:
  - $\bar{B}_1, \bar{B}_2, \bar{B}_3 \cdots$ are IID.
  - Statistically independent of $\tilde{B}_0$, the packet transmission time of Tx 0.
  - $\bar{B}_1, \bar{B}_2, \bar{B}_3 \cdots$ have the same distribution as $\tilde{B}_0$.

Independence Model

$$\tilde{e}_k(t) = 1\left(T_k \leq t \leq T_k + \bar{B}_k\right)$$

$$\tilde{I}_0(t) = \sum_{k \in \Psi} \rho |h_k|^2 |X_k|^{-\alpha} \tilde{e}_k(t)$$

$$\tilde{I}_a(t) = \frac{1}{t} \int_0^t \tilde{I}_0(\tau) \ \mathrm{d}\tau = \sum_{k \in \Psi} \rho |h_k|^2 |X_k|^{-\alpha} \eta_k(t) \tag{10}$$

$$\eta_k(t) = \frac{1}{t} \int_0^t \tilde{e}_k(\tau) \ \mathrm{d}\tau \tag{11}$$

$$\tilde{\mathrm{SINR}}_a(t) = \frac{\rho |h_0|^2}{1 + \tilde{I}_a(t)}$$

$$\tilde{C}_0(t) = \log_2\left(1 + \tilde{\mathrm{SINR}}_a(t)\right)$$

$$\tilde{B}_0 = \min\left\{t : K < t \ \tilde{C}_0(t)\right\}, \ \bar{B}_0 = \min\left(D, \tilde{B}_0\right) \tag{12}$$

Performance Analysis

Performance of the Gaussian receiver is quantified.

- CCDF of the packet transmission time $\tilde{B}$.
- Achievable Rate Density in the WANET.

CCDF of $\tilde{B}$:

$$\mathrm{P}\left(\tilde{B} > b\right) = \mathrm{P}\left(\int_0^b 1\left(\frac{K}{u} < \tilde{C}(u)\right) \mathrm{d}u < 1\right) \tag{13}$$

Complex to analyse.

Novel and tractable upper bound on the CCDF of $\tilde{B}$.

CCDF Upper Bound

Theorem

An upper bound on the CCDF of the packet transmission time of the Gaussian receiver, $\tilde{B}$ in (12), is given by

$$\mathrm{P}\left(\tilde{B} > b\right) \leq 1 - \exp\left(-\frac{2^{K/b}-1}{\rho} - \pi\lambda_s\left(2^{K/b}-1\right)^{\delta}\right.$$

$$\left.\Gamma\left(1-\delta\right)\Gamma\left(1+\delta\right)\ \frac{\hat{g}\left(\bar{B}\right)}{b^{\delta}}\right) \tag{14}$$

$$\triangleq \mathrm{P}_u \tag{15}$$

$$\hat{g}\left(\bar{B}\right) = b^{\delta}\ \mu^{\delta}\max\left(b,\mu\right)^{1-\delta} + \frac{1-\delta}{1+\delta}\min\left(b,\mu\right)^{1+\delta} \tag{16}$$

$$\mu :- \int_0^D \mathrm{P}_u\ \mathrm{d}b - \mu = 0 \tag{17}$$

$\delta = 2/\alpha$

CCDF Upper Bound

Implicit Nature:

- $\mu = \mathbb{E}[\bar{B}]$ cannot be determined in advance.
- Guess μ, evaluate (14) and reevaluate $\mathbb{E}[\bar{B}]$.
- The tuning procedure in (17) is quite efficient.

Approximation:
Replacing (16) with

$$\hat{g}\left(\bar{B}\right) = \mu\ b^{\delta}\left(1 + \frac{1-\delta}{1+\delta}\frac{b}{D}\right) \tag{18}$$

results in a very useful approximation to the CCDF.

Proof

- $$\mathrm{P}\left(\tilde{B} > b\right) \leq \mathrm{P}\left(K \geq b \log_2\left(1 + \mathrm{SI\tilde{N}R}_a(b)\right)\right) \tag{19}$$
- Let $v = 2^{K/b} - 1$

$$\mathrm{P}\left(\mathrm{SI\tilde{N}R}_a(b) \leq v\right) = 1 - \exp\left(-\frac{v}{\rho}\right) \mathcal{L}_{\tilde{I}_a(b)}\left(\frac{v}{\rho}\right) \tag{20}$$

- $$\mathcal{L}_{\tilde{I}_a(b)}(s) = \exp\left(-\pi\lambda\mathbb{E}\left[\int_0^\infty \left(1 - e^{-s\rho r^{-\alpha}|h|^2\eta(b)}\right) \mathrm{d}r^2\right]\right)$$
- $$\mathrm{P}\left(\tilde{B} > b\right) \leq 1 - \exp\left(-\frac{2^{K/b} - 1}{\rho} - \pi\lambda_s\left(2^{K/b} - 1\right)^\delta \Gamma(1-\delta)\Gamma(1+\delta)\ 2N_T\ \mathbb{E}\left[\eta(b)^\delta\right]\right) \tag{21}$$

Proof

- $$\eta(b) = \frac{1}{b}\int_0^b 1\left(T \le \tau \le T + \bar{B}\right)\ \mathrm{d}\tau \quad T \sim \mathrm{Unif}\left[-N_T, N_T\right]$$
- $$\mathbb{E}\left[\eta(b)^\delta\right] = \frac{1}{2N_T b^\delta}\left[\left(b\int_0^b \bar{b}^\delta + \frac{1-\delta}{1+\delta}\int_0^b \bar{b}^{1+\delta} + b^\delta\int_b^D \bar{b} + \frac{1-\delta}{1+\delta}\, b^{1+\delta}\int_b^D\right)\ \mathrm{d}F_{\bar{b}}\right] \tag{22}$$
- Consider $\delta = 1$

$$\mathbb{E}\left[\eta(b)\right] = \mathbb{E}\left[\bar{B}\right]/2N_T \tag{23}$$

- Let $\mu = \mathbb{E}\left[\bar{B}\right]$ and apply tuning procedure

$$\int_0^D CCDF\ \mathrm{d}b - \mu = 0$$

Proof

- Upper bound on $\mathbb{E}\left[\eta(b)^{\delta}\right]$

$$\mathbb{E}\left[\eta(b)^{\delta}\right] = \frac{1}{2N_T b^{\delta}}\,\mathbb{E}\Bigg[b\;\bar{B}^{\delta}\;1\left(\bar{B}<b\right) + b^{\delta}\;\bar{B}\;1\left(\bar{B}>b\right) + \frac{1-\delta}{1+\delta}\left(\bar{B}^{1+\delta}\;1\left(\bar{B}<b\right) + b^{1+\delta}\;1\left(\bar{B}>b\right)\right)\Bigg]$$
$$\triangleq \frac{1}{2N_T b^{\delta}}\,\mathbb{E}\left[g\left(\bar{B}\right)\right] \tag{24}$$

$g\left(\bar{B}\right)$ in (24) is concave in $\bar{B}$.

- Jensen's inequality for concave functions yields (16).

$$\mathbb{E}\left[g\left(\bar{B}\right)\right] \leq g\left(\mathbb{E}\left[\bar{B}\right]\right) \tag{25}$$

- Linear lower bound on $g\left(\bar{B}\right)$ yields (18).

Achievable Rate Density

Rate Density:
In a time space volume of area A and duration τ, *the sum of bits that were generated and received successfully divided by the volume*

$$\mathrm{RD} = \lim_{A,\tau\to\infty} \frac{1}{\tau A} \sum_{i\in\mathcal{M}} q_i K_i \tag{26}$$

$q_i \in \{0,1\}$ indicates the packet reception success or failure.

The rate density for our WANET system model is

$$\mathrm{RD} = \lambda_s \; K\,(1-\epsilon) \tag{27}$$
$$\epsilon = \mathrm{P}\left(\tilde{B} > D\right)$$

Achievable Rate Density

Ergodic Rate Density:
A slotted ALOHA WANET using time (frequency) diversity or IR-HARQ

$$\mathrm{ERD}\left(\lambda\right) = \lambda\ \mathbb{E}\left[\log_2\left(1+\mathrm{SINR}\right)\right] \tag{28}$$

$$\mathrm{SINR} = \frac{\rho|h_0|^2}{1+\sum_{j\in\Phi}\rho|h_j|^2|X_j|^{-\alpha}},$$

$\lambda\ /m^2$- Transmit density

To compare RD and ERD, transmit density in our system model is

$$\lambda = \lambda_s\ \mathbb{E}[\tilde{B}] \tag{29}$$

Compare RD to ERD

The deviation of RD from ERD can be accounted for by 2 factors

- Lack of fading and interference diversity in the channel.
- Lack of instantaneous interference power estimates at receiver.

Asymptotic Limit:

$$\frac{\mathrm{RD}}{\mathrm{ERD}} \xrightarrow{\lambda \to 0} \frac{\ln 2 \; K \; \exp\left(-\frac{2^{K/D}}{\rho}\right)}{E_1\left(\frac{1}{\rho}\right)\left(D - e^{\frac{1}{\rho}} \int_0^D \exp\left(-\frac{2^{K/b}}{\rho}\right) \mathrm{d}b\right)} \tag{30}$$

$E_1(x) = \int_1^\infty \frac{e^{-xt}}{t} \mathrm{d}t$- exponential integral function.

For $K = 75, D = 500$ & $\rho = 15$ dB, $\frac{\mathrm{RD}}{\mathrm{ERD}} \xrightarrow{\lambda \to 0} 0.7$

Improvements

Two approaches to improve the rate density

- **Matched Receiver**:
  Receiver which tracks *instantaneous* interference and noise power $\Rightarrow$ increases successful packet transmissions in the WANET and thus rate density

- **Channel Thresholding**:
  Tx with poor fading will likely transmit for a long duration and cause more interference and outage in the WANET. Allowing only the Tx with fading above a certain threshold to transmit, interference from poor Tx's can be reduced.

Matched Receiver

- The achievable rate upto time t at receiver 0 is

$$\tilde{C}(t) = \frac{1}{t}\int_0^t \log_2\left(1 + \mathrm{SI\tilde{N}R}\left(\tau\right)\right) \ \mathrm{d}\tau \tag{31}$$

- Packet transmission time $\tilde{B}$ is

$$\tilde{B} = \min\left\{t : \frac{K}{t} < \tilde{C}(t)\right\} \tag{32}$$

- CCDF of $\tilde{B}$ is

$$\mathrm{P}\left(\tilde{B} > b\right) = \mathrm{P}\left(\frac{K}{b} \geq \tilde{C}(b)\right) \tag{33}$$

Distribution of $\tilde{C}(b)$ is hard!!!

Matched Receiver

A novel lower bound for the achievable rate $\tilde{C}(b)$.

$\mathcal{E}$: Strongest interference at receiver 0 in the interval $(0, b) < \gamma_{th}$

$$\tilde{C}(b) = \mathrm{P}\left(\mathcal{E}\right) \tilde{C}(b)\big|\mathcal{E} + \mathrm{P}\left(\mathcal{E}_c\right) \tilde{C}(b)\big|\mathcal{E}_c$$

$$\tilde{C}(b) \geq \mathrm{P}\left(\mathcal{E}\right) \tilde{C}(b)\big|\mathcal{E}$$

$$\tilde{C}(b) \geq \mathrm{P}\left(\mathcal{E}\right) \ \log_2\left(1 + \tilde{\mathrm{SINR}}_a\left(b\right)\big|\mathcal{E}\right) \tag{34}$$

$$\tilde{\mathrm{SINR}}_a\left(b\right) = \frac{\rho|h|^2}{1 + \frac{1}{b}\int_0^b \tilde{I}(\tau) \ \mathrm{d}\tau} \tag{35}$$

(34)- Jensen's inequality for convex functions to $\log_2\left(1 + \tilde{\mathrm{SINR}}\left(b\right)\right)$.

CCDF Upper Bound

Theorem

For any $\gamma_{th} > 0$, an upper bound on the CCDF of the packet transmission time of the matched receiver, $\tilde{B}$ in (32), is

$$\mathrm{P}\left(\tilde{B} > b\right) \leq 1 - \exp\left(-\frac{2^{K'/b} - 1}{\rho} - \pi\lambda_s \left(2^{K'/b} - 1\right)^{\delta} G\left(\gamma_{th}\right) \Gamma\left(1+\delta\right) \frac{\hat{g}\left(\bar{B}\right)}{b^{\delta}}\right) \triangleq \mathrm{P}_u \tag{36}$$

$$K' = K \exp\left(\pi\lambda_s \left(b+\mu\right) \Gamma\left(1+\delta\right) \left(\frac{\rho}{\gamma_{th}}\right)^{\delta}\right) \tag{37}$$

$$G\left(\gamma_{th}\right) = \int_{\frac{\gamma_{th}}{\rho}\left(2^{K'/b}-1\right)}^{0} \left(1 - e^{-t}\right) \, \mathrm{d}t^{-\delta} \tag{38}$$

CCDF Upper Bound

- The function $\hat{g}\left(\bar{B}\right)$ is defined in Theorem 1.
- Similiar to Theorem 1, the upper bound is implicit in nature and solved by the tuning procedure

$$\int_0^D \mathrm{P}_u \ \mathrm{d}b - \mu = 0 \tag{39}$$

- The best CCDF bound (approximation) is obtained by optimizing γ_{th} to yield the minimal expected packet time,

$$\gamma_{th}^* = \arg\min_{\gamma_{th}} \ \mu\left(\gamma_{th}\right) \tag{40}$$

- For $\gamma_{th} = \infty$ the CCDF upper bound in Theorem 2 reduces to the bound presented in Theorem 1.

Channel Thresholding

- In WANET system model, each transmit receive pair employs
  - A Gaussian receiver for reception and
  - A form of power control known as channel thresholding for transmission.
- The transmit SNR from transmitter i to the WANET is

$$\rho_i = \begin{cases} \rho, & |h_{ii}|^2 \geq \beta \\ 0, & |h_{ii}|^2 < \beta \end{cases} \tag{41}$$

- The rate density and transmit density are

$$\mathrm{RD} = \lambda_s p \; K \left(1 - \epsilon\right) \tag{42}$$

$$\lambda = \lambda_s p \; \mathbb{E}[\tilde{B}] \tag{43}$$

$$p = \mathrm{P}\left(|h|^2 \geq \beta\right)$$

Numerical Results

WANET simulation is implemented in 2 stages

- **Exact Model**:
  - The WANET is run for a total duration of 50000 channel uses.
  - Samples from the steady state region of the network operating time.
- **Independence Model**:
  - WANET performance is measured by simulating only Tx 0.
  - Network is run for a total duration of $2D$ channel uses around $T_0 = 0$.
  - Interferer packet durations are obtained based on the empirical distribution from the exact model simulation.
  - Network is simulated for 10000 iterations.

Rate Density

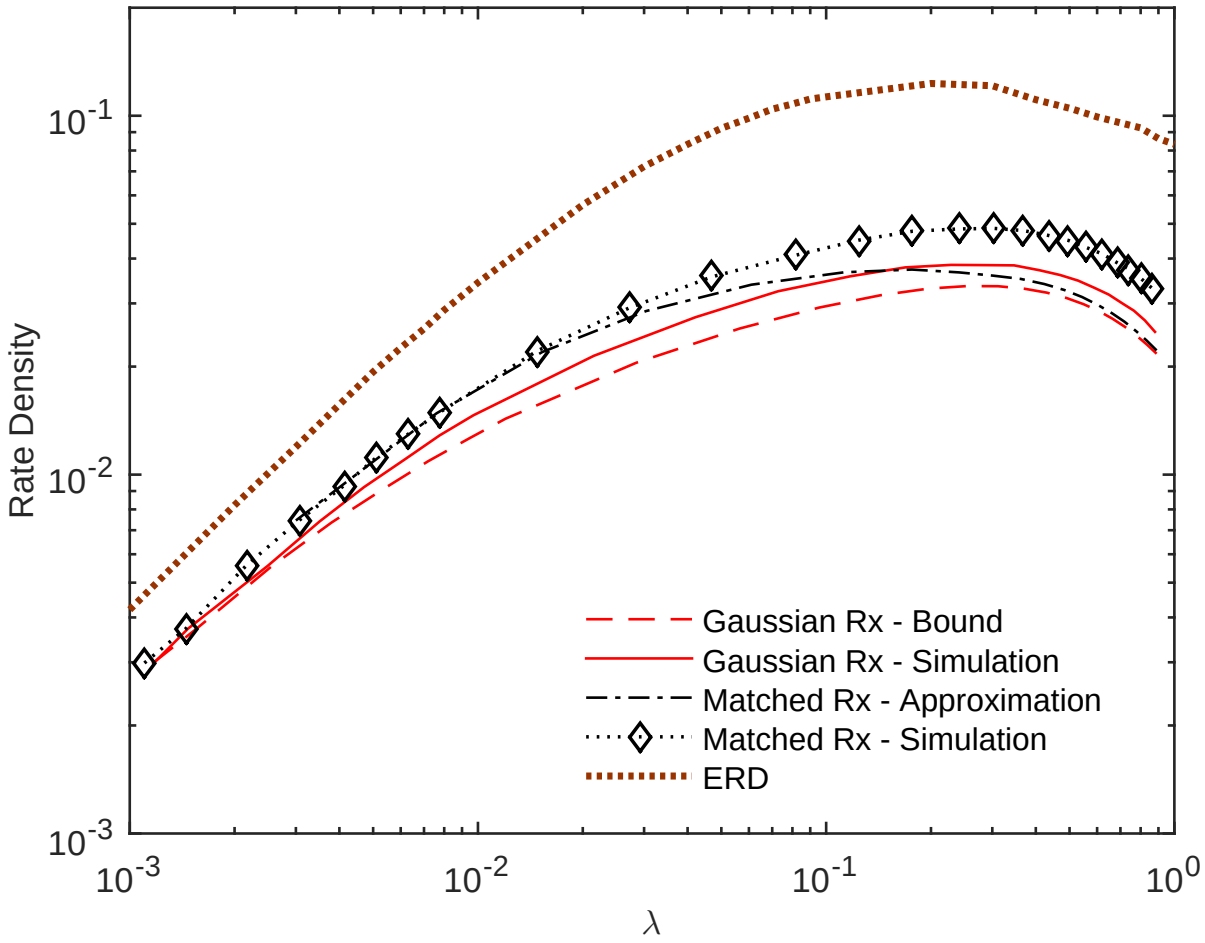


Figure: $\alpha = 3$ and $D = 500$

Channel Thresholding

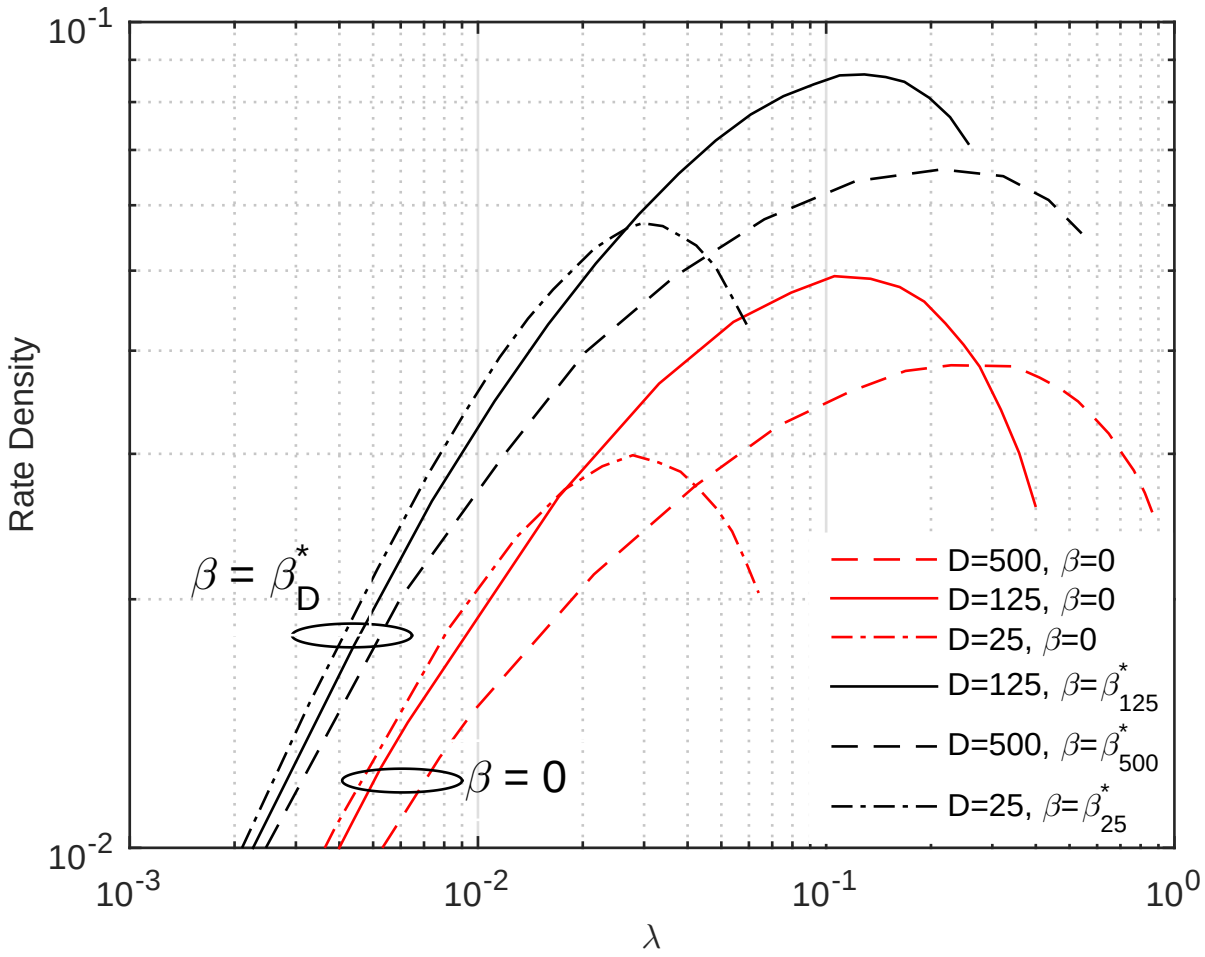


Figure: Gaussian Recevier- $D = \{500, 125, 25\}$

CCDF of $\tilde{B}$

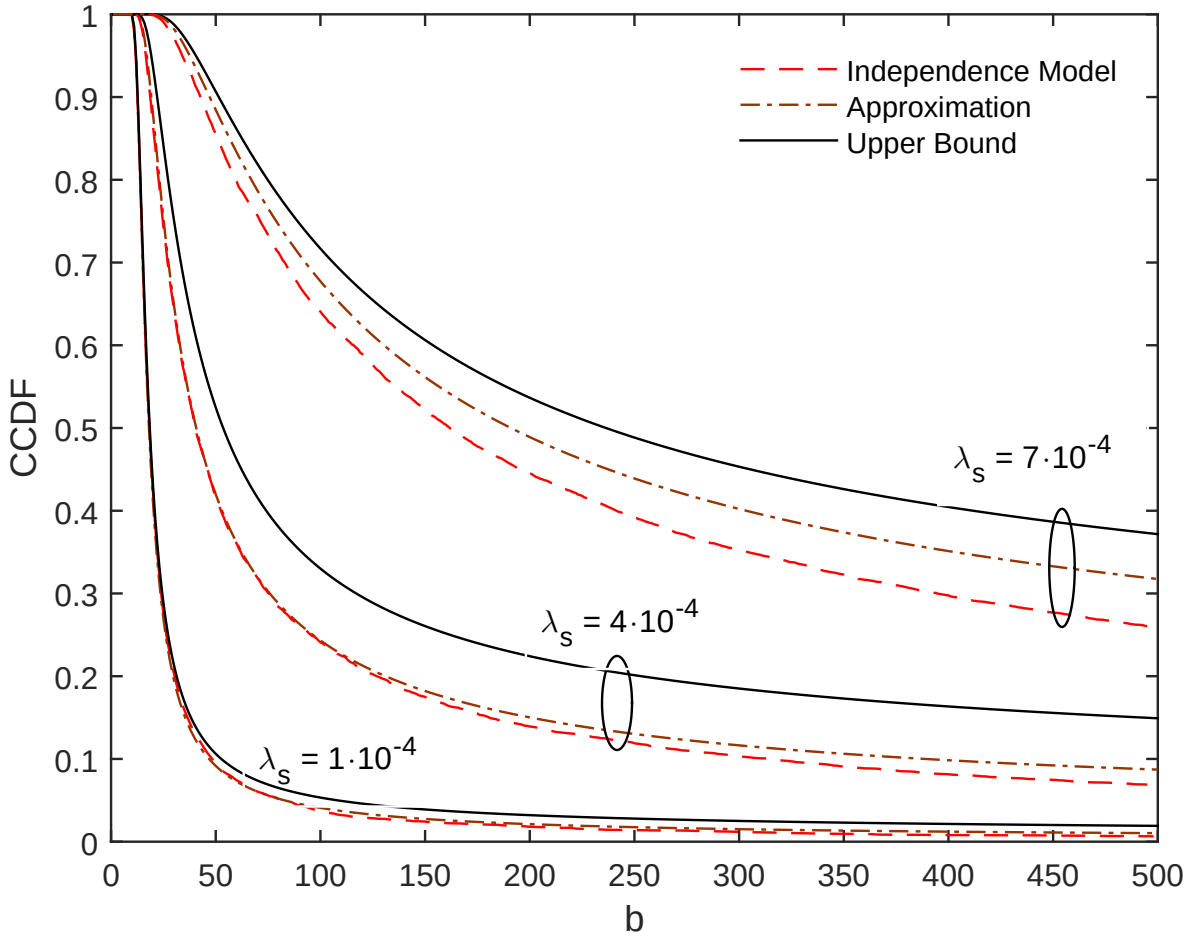


Figure: Gaussian Recevier- $\lambda_s = \{1, 4, 7\} \cdot 10^{-4}$

Plot of $\mathbb{E}\left[\bar{B}\right]$

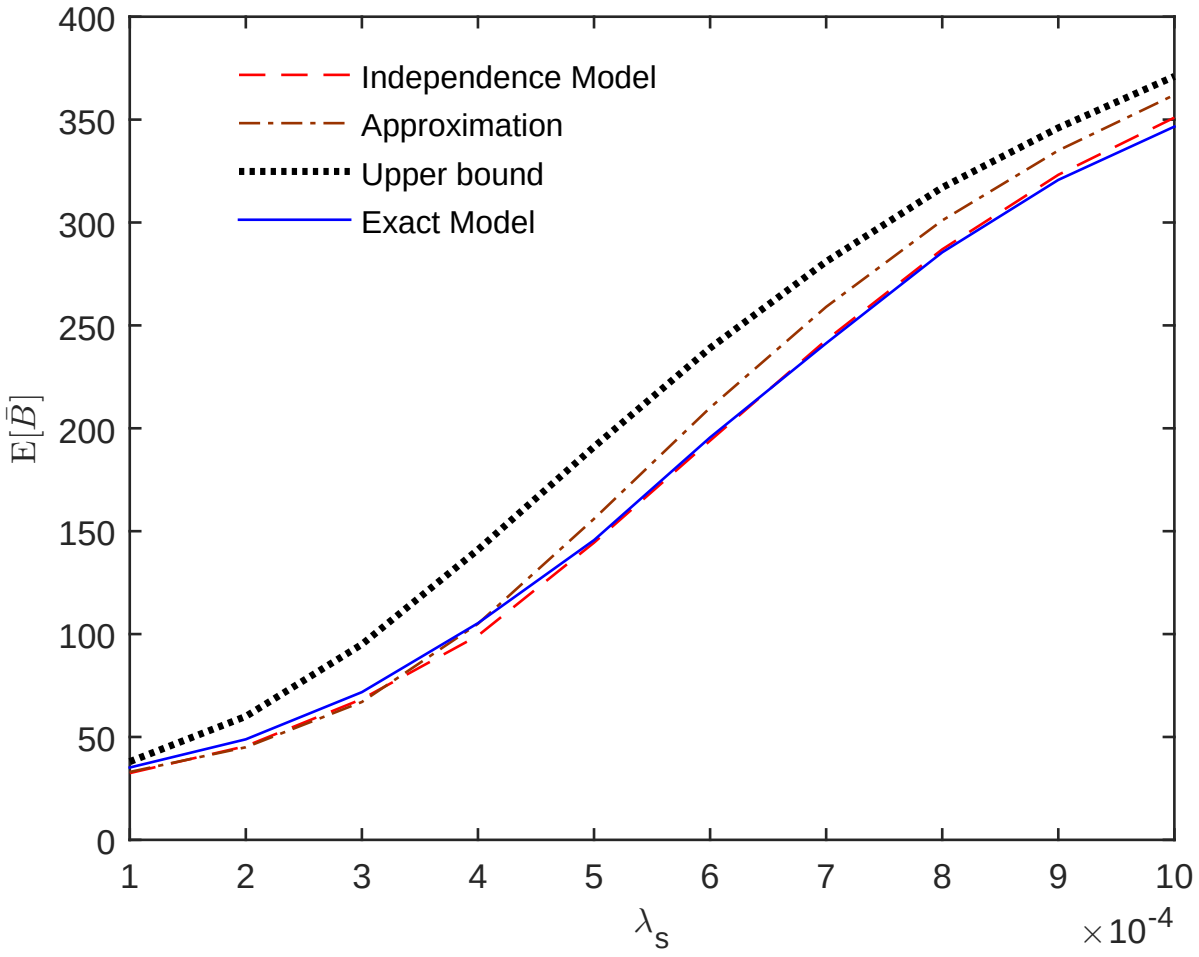


Figure: Gaussian Recevier $\alpha = 3$ and $D = 500$

Plot of SD $[\bar{B}]$

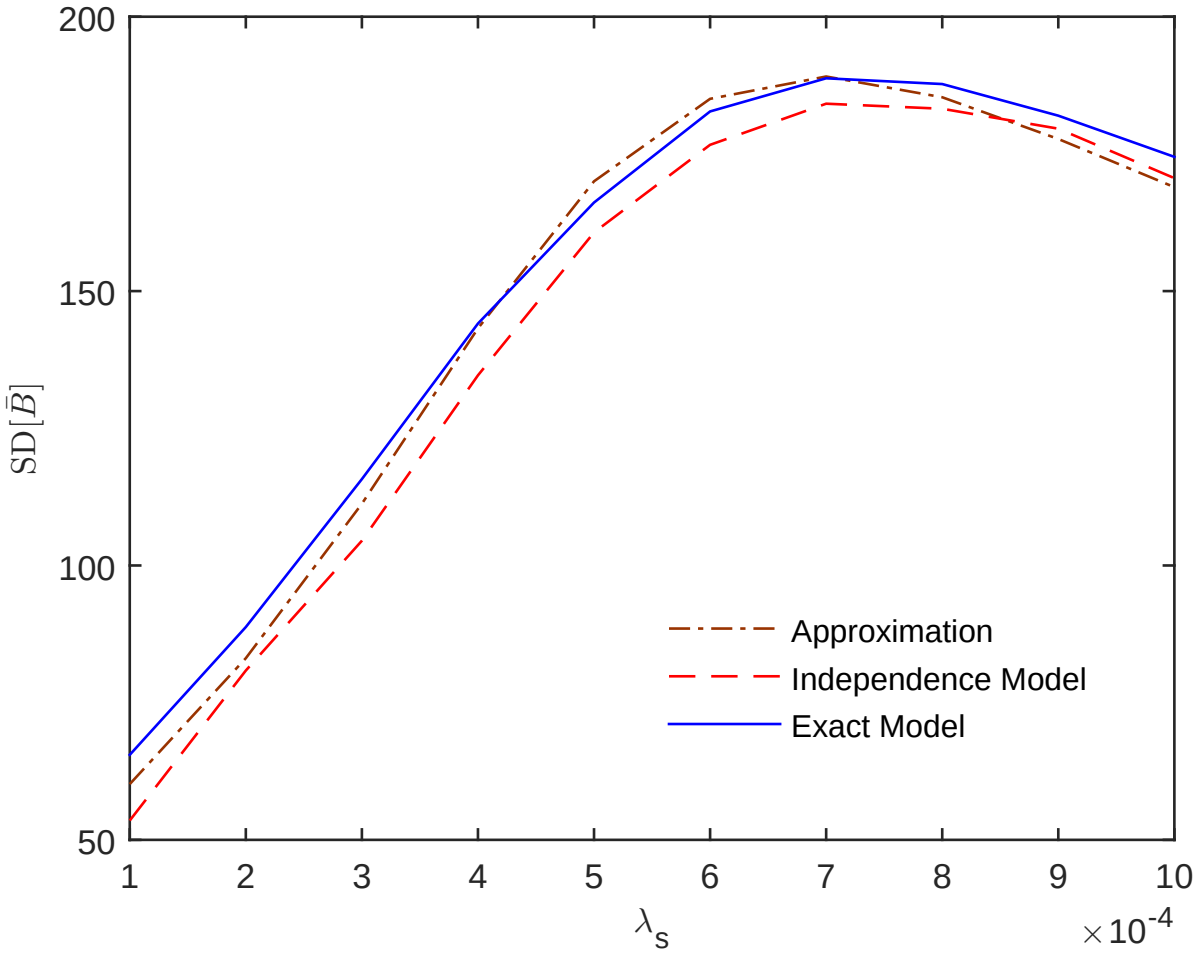


Figure: Gaussian Recevier- $\alpha = 3$ and $D = 500$

Summary

- Studies the achievability of network ERD in a WANET employing fountain codes in the physical layer.
- Presented a rateless coding setup in which a packet is transmitted within a single coherence time and it was shown that the achievable rate density can be upto 70% of the ERD.
- Dynamic variations of the instantaneous rate are quantified by deriving an upper bound to the CCDF of the packet transmission time.
- Studies a Gaussian receiver with low complexity and shows that its rate density is quite close to that of an ideal matched receiver.
- Shows that power control in the form of channel thresholding can lead to a 72% gain relative to constant power transmission.

HARQ in cooperative multihop WANET[3]

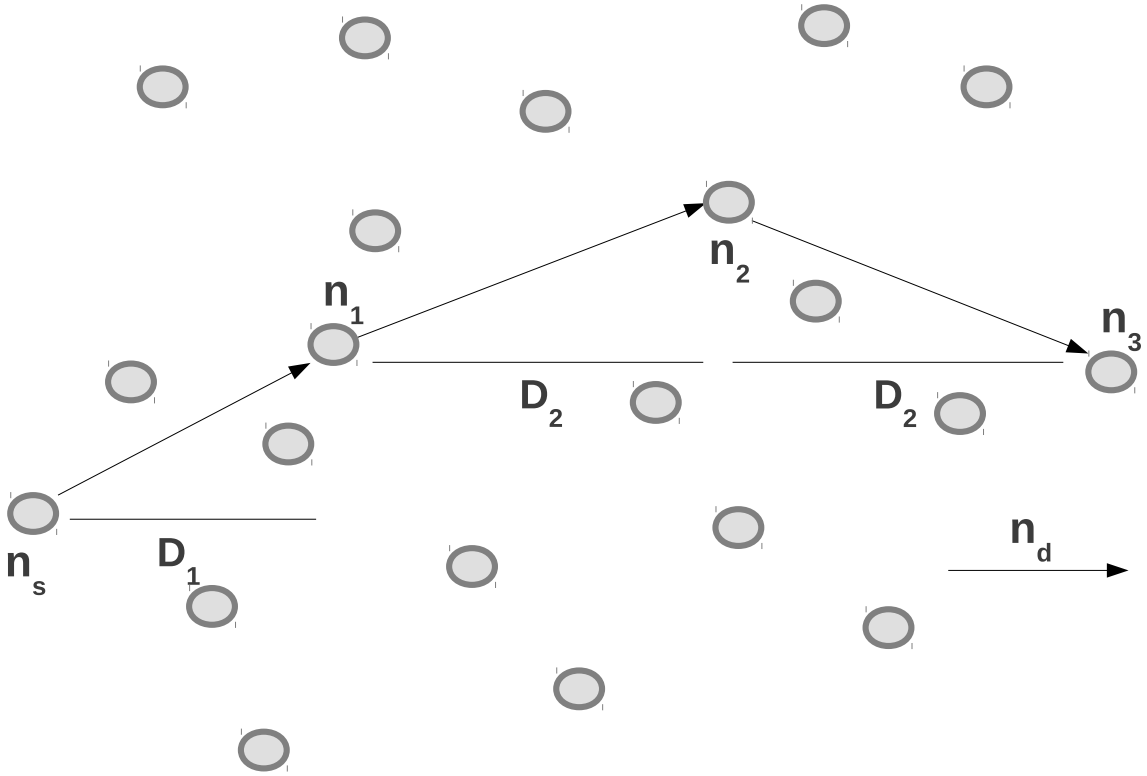


Figure: Source node n_s, destination node n_d and relays $\{n_1, n_2, n_3 \cdots\}$

[3] A. Rajanna and M. Kaveh. *Analysis of Cooperative HARQ with Poisson Interference and Opportunistic Routing.* submitted IEEE Transactions on Wireless Communication, July 2013.

Cooperative HARQ

- Nodes- 2-D homogeneous Poisson point process (PPP) $\Phi = \{X_i\}$ of intensity $\lambda\ m^{-2}$
- Source node n_s is at the origin and the destination n_d is along the X-axis at an asymptotic distance.
- MAC layer uses the slotted ALOHA protocol. In every slot,

$$\text{a node} \begin{cases} Tx, & \text{w.p} \quad p \\ Rx, & \text{w.p} \quad 1-p \end{cases}$$

- Source node transmits 1st block of the codeword at rate R. Relays selected for forwarding the packets transmit the subsequent blocks at the same code rate R.

Cooperative HARQ

- Transport capacity of the network C is

$$C = R\ \lambda p\ \mathbb{E}\Big[\mathrm{D}_2\left(\Phi, R, p\right)\Big] \tag{44}$$

- The progress D_2

$$\mathrm{D}_2\left(\Phi, R, p\right) = \mathrm{F}_2\left(\Phi, R, p\right) - \mathrm{D}_1\left(\Phi, R, p\right) \tag{45}$$

$$\mathrm{F}_2 = \max_{i \in \Phi^r} \left[\mathbf{1}\left(I_{i,0} + I_{i,Y_1} \geq R | \Phi^t, Y_1\right)\ |X_i| \cos\left(\theta\left(X_i\right)\right)\right]$$

$$\mathrm{D}_1 = \max_{i \in \Phi^r} \left[\mathbf{1}\left(I_{i,0} \geq R | \Phi^t\right)\ |X_i| \cos\left(\theta\left(X_i\right)\right)\right]$$

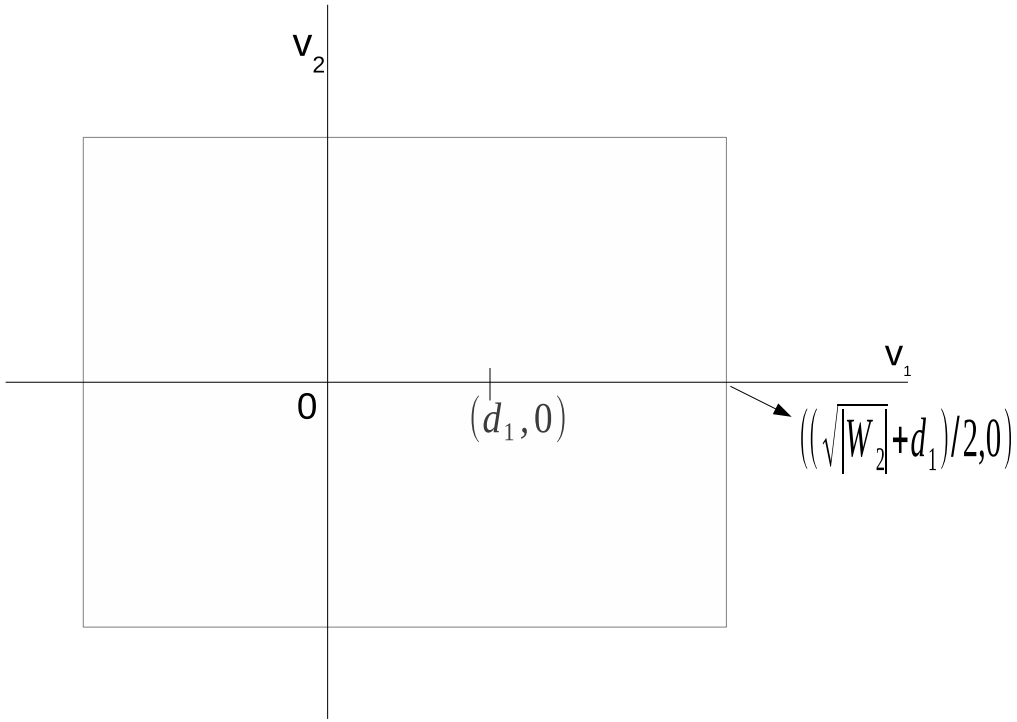
- **Optimization**:

$$\max_{R,p}\ \Big[R\ \lambda p\ \mathbb{E}\left[\mathrm{D}_2\right]\Big]$$

Cooperative HARQ

Expected Progress Approximation: Decoding Cell

$$\Sigma_2 = \left\{ v \in \mathbb{R}^2 : \; I_{v,0} + I_{v,\eta_1} \geq R \right\}$$
$$I_{v,0} = \log_2 \left(1 + \mathrm{SIR}_{v,0}\right), I_{v,\eta_1} = \log_2 \left(1 + \mathrm{SIR}_{v,\eta_1}\right)$$
$$\eta_1 = (d_1, 0) \tag{46}$$



Cooperative HARQ

- Expected progress approximation

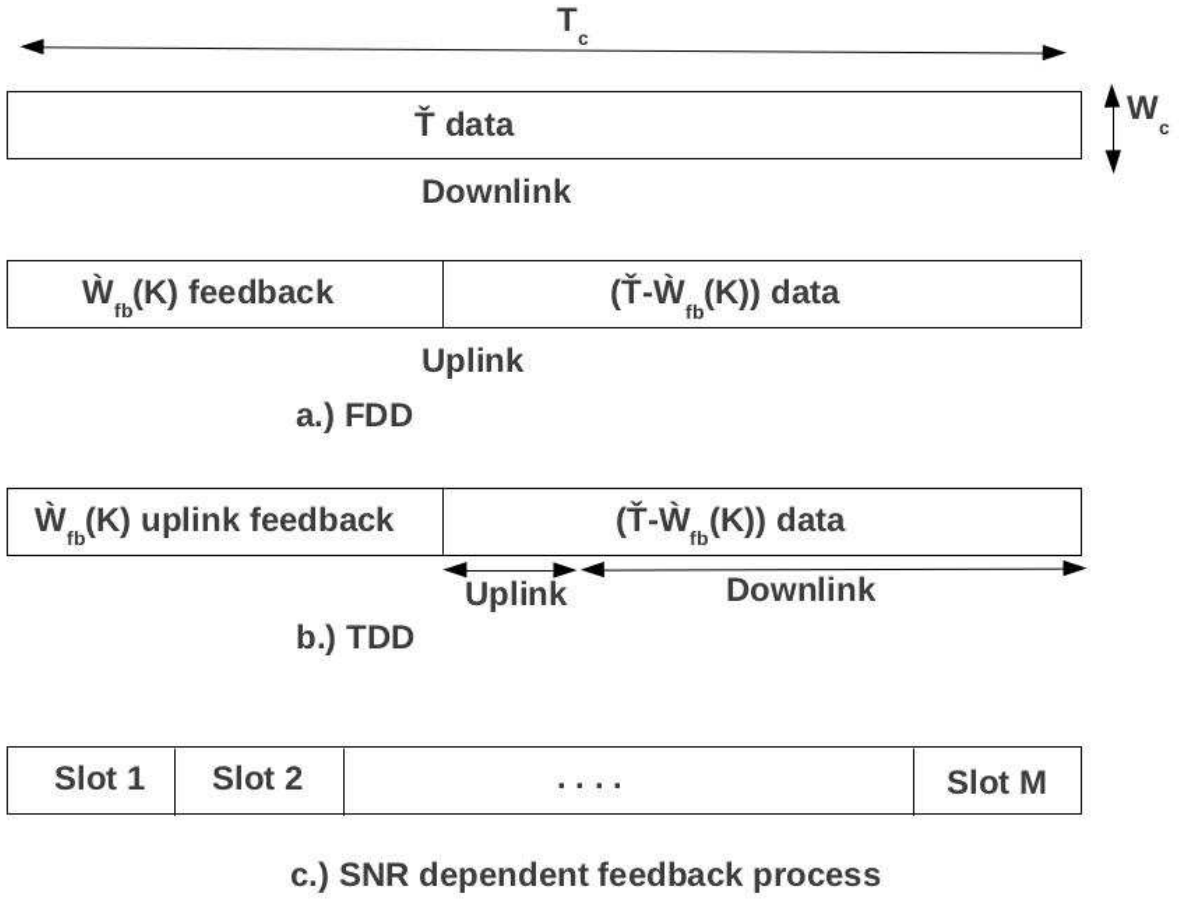
$$\tilde{d}_2\left(R,p\right) = \frac{\sqrt{|W_2|}+d_1}{2}\left(1-\frac{1-e^{-c_1}}{c_1}\right)-d_1 \qquad (47)$$

- Concave objective

$$\max_{R,p}\left[R\ \lambda p\ \tilde{d}_2\right]$$

- Similar extension to $M-$round HARQ based on decoding cells.
- **Repetition Time Diversity**: SINR combining treated in a similar manner.

Multiuser Diversity in Downlink Channels



Multiuser diversity

- Cost: Feedback bandwidth to acquire users CQI
- Benefit: Downlink spectral efficiency increases

Multiuser Diversity

- Benefit of multiuser diversity

$$\log_2\left(1+P\left(\log K-\log\log\log K\right)\right)+o(1)\leq \mathbb{E}\left[\log_2(1+P\gamma_{df})\right]\leq \log_2\left(1+P\left(0.58+\log K\right)\right) \tag{48}$$

- TDD system

Dedicated Feedback:

$$\max_K\left(1-\frac{K}{T\log_2\left(1+P\right)}\right)\mathbb{E}\left[\log_2\left(1+P\gamma_{df}\right)\right] \tag{49}$$

SNR dependent Feedback:

$$\max_{K,M}\left[1-\left(\frac{L_{fb}+\log_2 K}{\log_2(1+P)}\right)\frac{M}{\check{T}}\right]P\left(X\geq 1\right)\ \mathbb{E}[\log_2(1+P\gamma_{sdf})] \tag{50}$$

Multiuser Diversity

- FDD System

$$\max_{K} \left[\lambda_R R_D(K) + R_U(K)\right] \tag{51}$$

- MIMO V-BLAST

$$\max_{K} \left(1 - \frac{KL_{fb}}{\check{T}\log_2(1+P)}\right) \mathbb{E}\left[\max_{1\le i\le K} \log_2 \left|\mathbf{I} + \frac{P}{N}\mathbf{H}_i\mathbf{H}_i^{\dagger}\right|\right]$$

- Approximation

$$\log_2 \left|\mathbf{I} + \frac{P}{N}\mathbf{H}_i\mathbf{H}_i^{\dagger}\right| \sim \mathcal{N}(\mu, \sigma^2),$$

$$\mu = N\log_2\left(\frac{P}{e}\right), \ \sigma^2 = (\log_2 e)^2 \left[\log N + 1.58\right]$$

$$\max_{K} \left(1 - \frac{K}{T\log_2(1+P)}\right) \left[\mu + \sigma\sqrt{2\log K}\right]$$