A New Evolutionary Algorithm for Multi-objective Optimization Problems

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- Multi-objective Optimization Problems (MOP)
  - Definition

$$
\begin{aligned}
\text{Minimiz/Maximize} \quad & f_m(X), m = 1, 2, \ldots, M; \\
\text{Subject to} \quad & g_j(X) \geq 0, j = 1, 2, \ldots, J; \\
& h_k(X) = 0, k = 1, 2, \ldots, K; \\
& \alpha_i \leqslant x_i \leqslant \beta_i, i = 1, 2, \ldots, n. \\
& X = (x_1, x_2, \ldots, x_n)^T
\end{aligned}
$$

  - NP hard

Two Basic Considerations

- Elitism
  - means that elite individuals, which are not worse than any other solutions in the current population, should not be eliminated during evolutionary process.
  - Proved to improve the performance significantly in SPEA, NSGA-II, etc.
- No sharing factor (to retain diversity)
  - Drawbacks of sharing function approach: 1 problem dependent; 2 not easy to set fitness sharing factor properly
  - Diversity by Crowding Model

Two heuristics introduced from SOO

- Multi-parent Non-convex Crossover
  - *Definition*

$$V = \{X \mid X = \Sigma_{i=1}^{m} a_i x_i\}$$

where

$$\Sigma_{i=1}^{m} a_i = 1 \; and \; -0.5 \le a_i \le 1.5, i = 1, 2, ..., m.$$

  - *Effect :* When iteration times increase ∞, *V* could cover the whole search space.
  - *Advantage:* given a fixed size population, the more individuals get involved in crossover, the greater the probability of convergence to the optimum.

Two heuristics introduced from SOO (cont'd)

- Swarm Hill Climbing
  - *Definition:* It is realized by setting the stopping criterion of evolutionary process to be when the fitness difference between the best solution and the worst solution is small enough.
  - *Effect:* More chances are given for individuals that are trapped at the local peaks, to escape. All individuals climb the hills in parallel.

The Flow of the New MOEA

- *Step1* A population $P(0)$ is generated at random and $t = 0$ is set;
- *Step2* Select at random m individuals from $P(t)$ and create a single offspring y by multi-parent crossover;
- *Step3* Collect all individuals Dy from $P(t)$ that are dominated by y and calculate y'*s* rank and crowding distance value at the same time;
- *Step4* If $Dy <> \Phi$, delete one of the members of Dy at random and append y in $P(t)$ and go to Step5. Otherwise, append y and delete the lowest rank and the least widely spread solution.

The Flow of the New MOEA (cont'd)

- *Step5* If the ranks of all individuals are 0 and the difference between the biggest crowding distance value and the smallest crowding distance value is less than some small number ε, or the maximum evolution generation is reached, then stop and declare $P(t + 1)$ as the set of obtained non-dominated solutions; otherwise set $t = t + 1$ and go to Step 2.
- *Pareto Rank*: an individual's Pareto Rank is the number of individuals that dominate it
- *Crowding distance*:

$$d_{I_j^m} = d_{I_j^m} + \frac{f_m^{(I_{j+1}^m)} - f_m^{(I_{j-1}^m)}}{f_m^{max} - f_m^{min}}.$$

Computational Results

- Test function 1

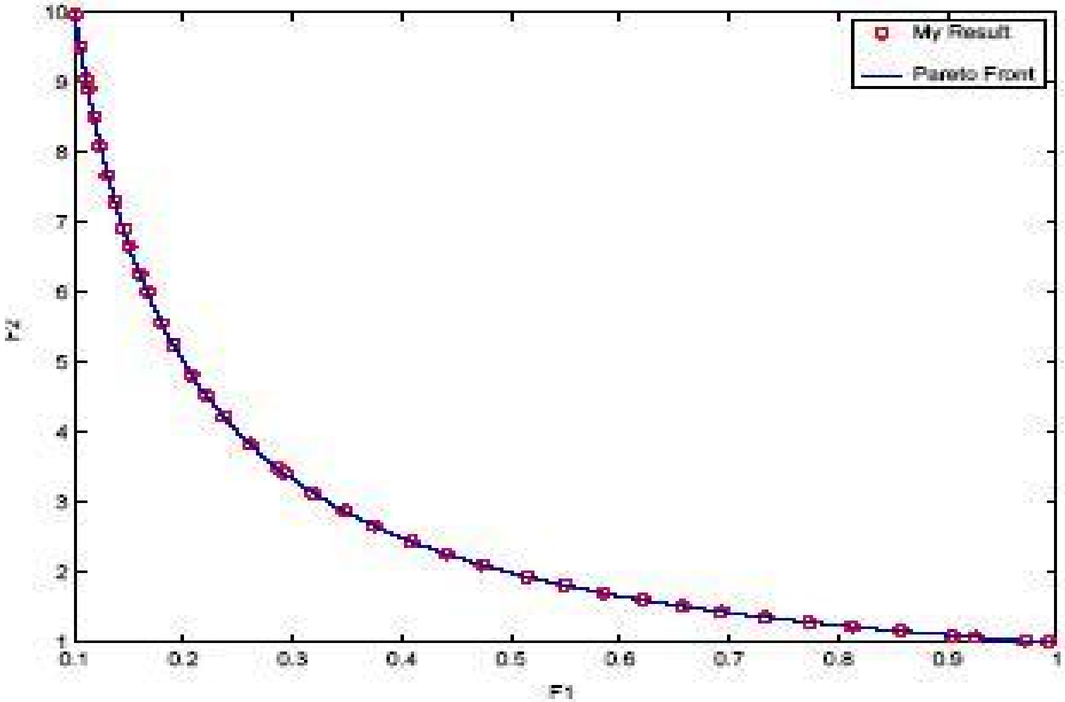

Figure 1. Test Function 1

Computational Results (cont'd)

- Test function 2

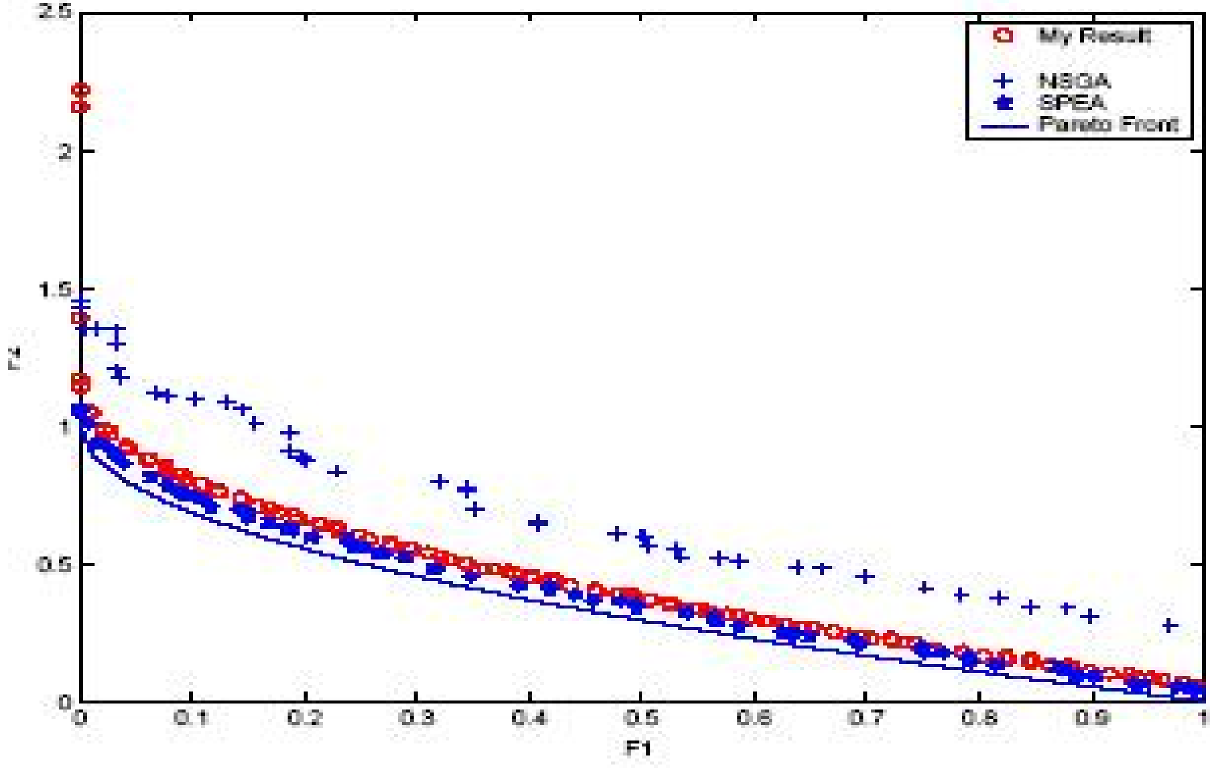

Figure 2. Test Function 2

Computational Results (cont'd)

- Test function 3

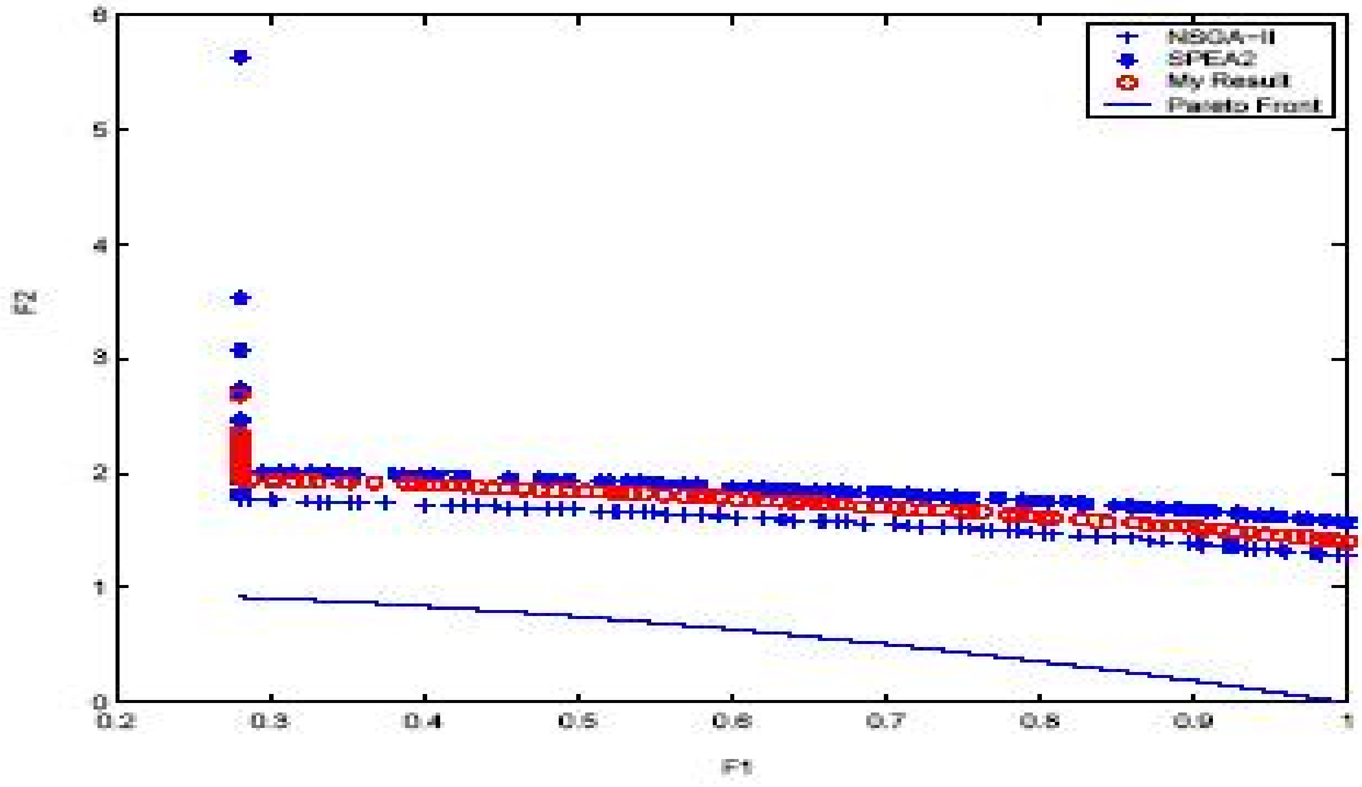

Figure 3. Test Function 3

Conclusion

- Performance is good and comparable with the results in recent research

- Unique advantage: find well distributed solutions.