Extending Similarity Measures of Interval Type-2 Fuzzy Sets to General Type-2 Fuzzy Sets

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Similarity Measures

One of the most common tools of fuzzy logic is similarity measures. A similarity measure between fuzzy sets indicates the degree to which the fuzzy sets are similar as a value between 0 and 1.

Although some methods have been developed for interval type-2 fuzzy sets, fewer methods exist for general type-2 fuzzy sets.

This presentation presents a general method of extending similarity measures on interval type-2 fuzzy sets to similarity measures on general type-2 fuzzy sets.

Similarity Properties

Four properties of similarity measures for fuzzy sets that are commonly used in the literature are:
Reflexivity: $s(\tilde{A}, \tilde{B}) = 1 \iff \tilde{A} = \tilde{B}$
Symmetry: $s(\tilde{A}, \tilde{B}) = s(\tilde{B}, \tilde{A})$
Transitivity: If $\tilde{A} \leq \tilde{B} \leq \tilde{C}$, then $s(\tilde{A}, \tilde{B}) \geq s(\tilde{A}, \tilde{C})$
Overlapping: If $\tilde{A} \cap \tilde{B} \neq \emptyset$, then $s(\tilde{A}, \tilde{B}) > 0$; otherwise, $s(\tilde{A}, \tilde{B}) = 0$
Note that it is not necessary for a similarity measures to have all of these properties as the application of the measure may not depend on all of them.

Existing Interval Type-2 Similarity Measures

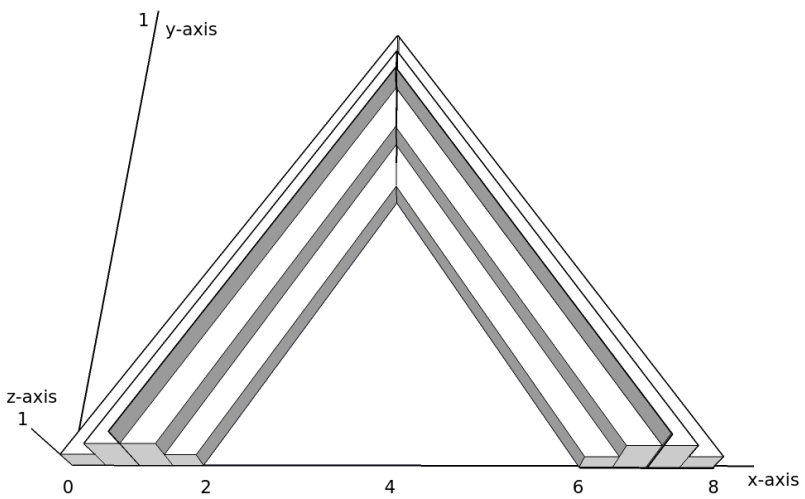
The following four existing interval type-2 similarity were reviewed and tested against general type-2 fuzzy sets.

- Zeng & Li
- Jaccard
- Gorzałczany
- Bustince

Additional similarity measures exist in the literature, but for conciseness only these four are currently considered.

zSlices-based General Type-2 Fuzzy Sets

A general type-2 fuzzy set can be represented by slicing the third dimension (z) at level z_i to create a zSlices-based general type-2 fuzzy set.
The resulting set will consist of zSlices which are interval type-2 fuzzy sets with a secondary membership grade of z_i.



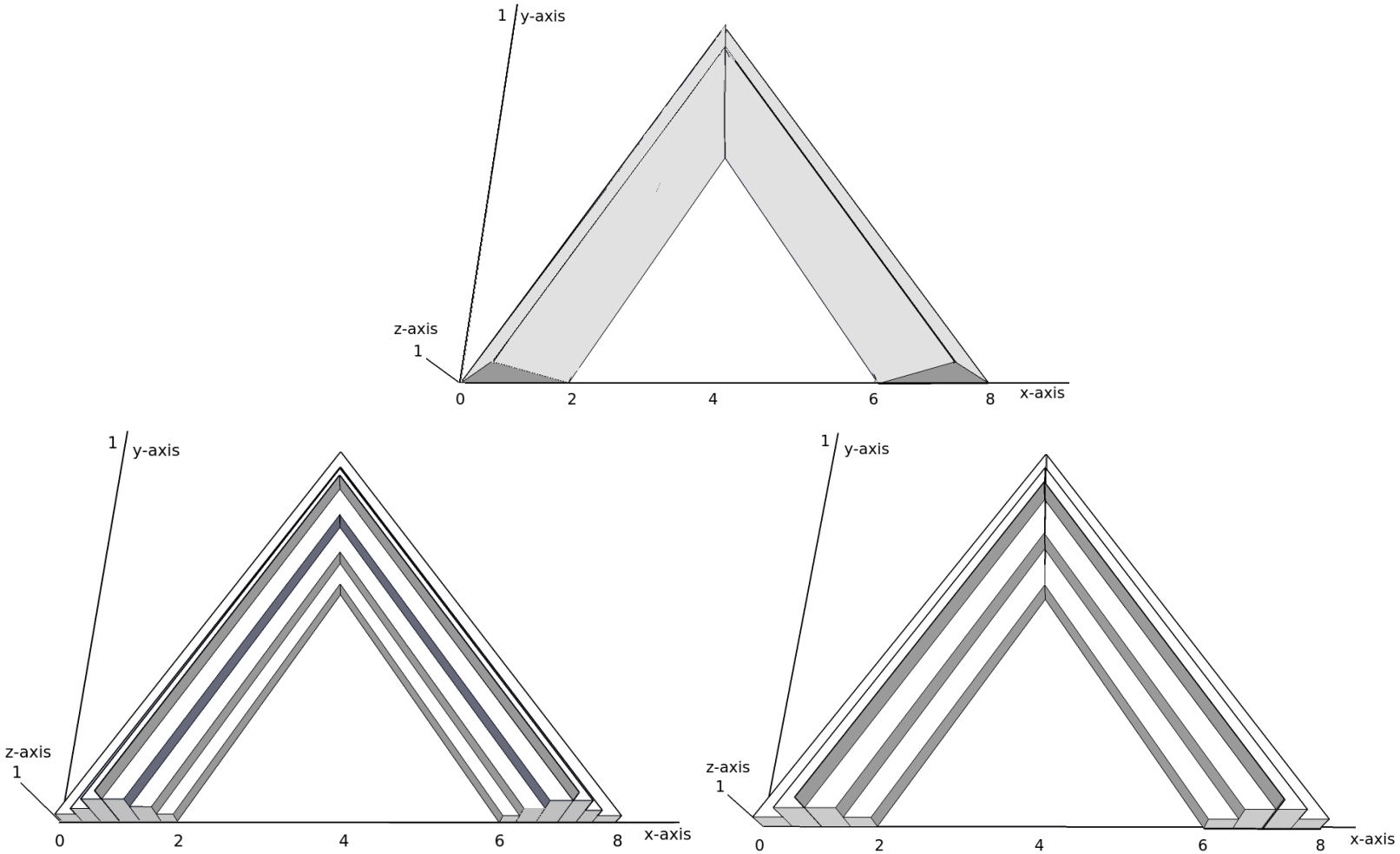
Similarity Measures for General Type-2 Fuzzy Sets

By using zSlices-based general type-2 fuzzy sets, a measure of similarity on interval type-2 fuzzy sets can be applied to each zSlice, and the results for each zSlice can be combined as follows:

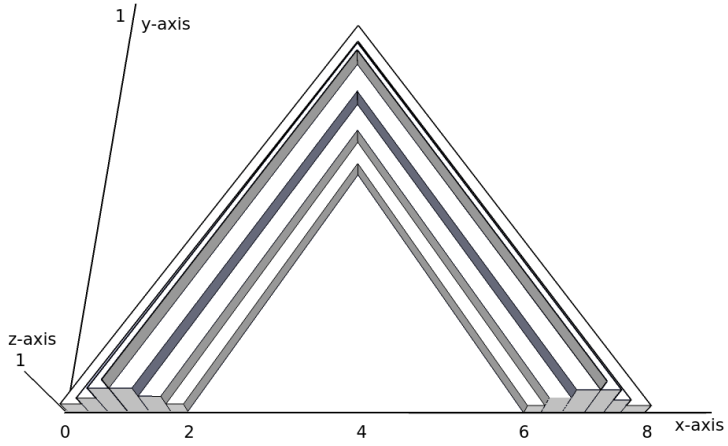
$$S_{ZS}(\tilde{A}, \tilde{B}) = \frac{\sum_{i \in L} z_i S_\lambda(\tilde{A}_{z_i}, \tilde{B}_{z_i})}{\sum_{i \in L} z_i}$$

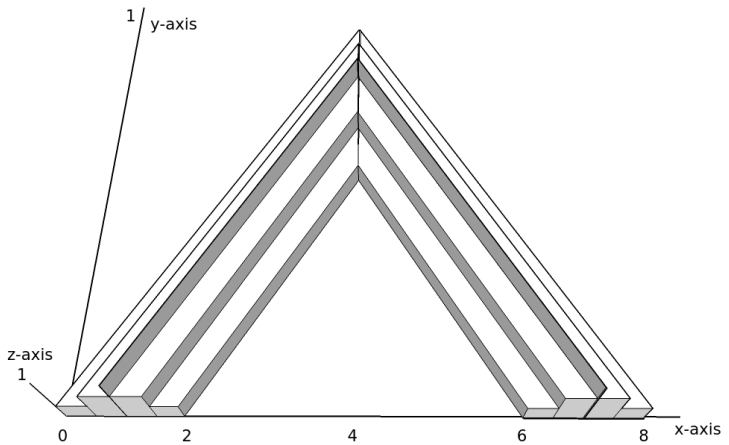
where $S_\lambda(\tilde{A}_{z_i}, \tilde{B}_{z_i})$ is any similarity measure for interval type-2 fuzzy sets. Sets $\tilde{A}_{z_i}$ and $\tilde{B}_{z_i}$ are zSlices from sets $\tilde{A}$ and $\tilde{B}$ at zLevel z_i, and L is the set of zLevels used by $\tilde{A}$ and $\tilde{B}$. For example, if $\tilde{A}$ and $\tilde{B}$ have three zLevels where $z_1 = 0.33$, $z_2 = 0.66$ and $z_3 = 1$ then $L = \{0.33, 0.66, 1\}$

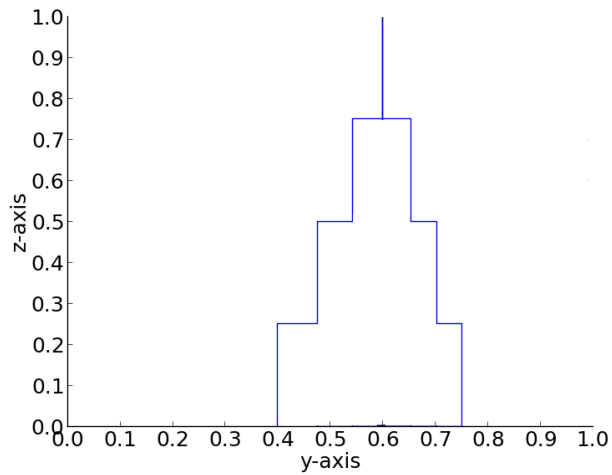
zSlices-based General Type-2 Fuzzy Sets with different numbers of zLevels

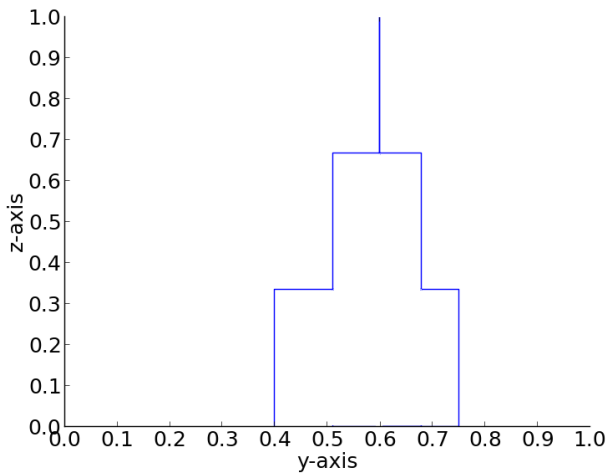


zSlices-based General Type-2 Fuzzy Sets with different numbers of zLevels









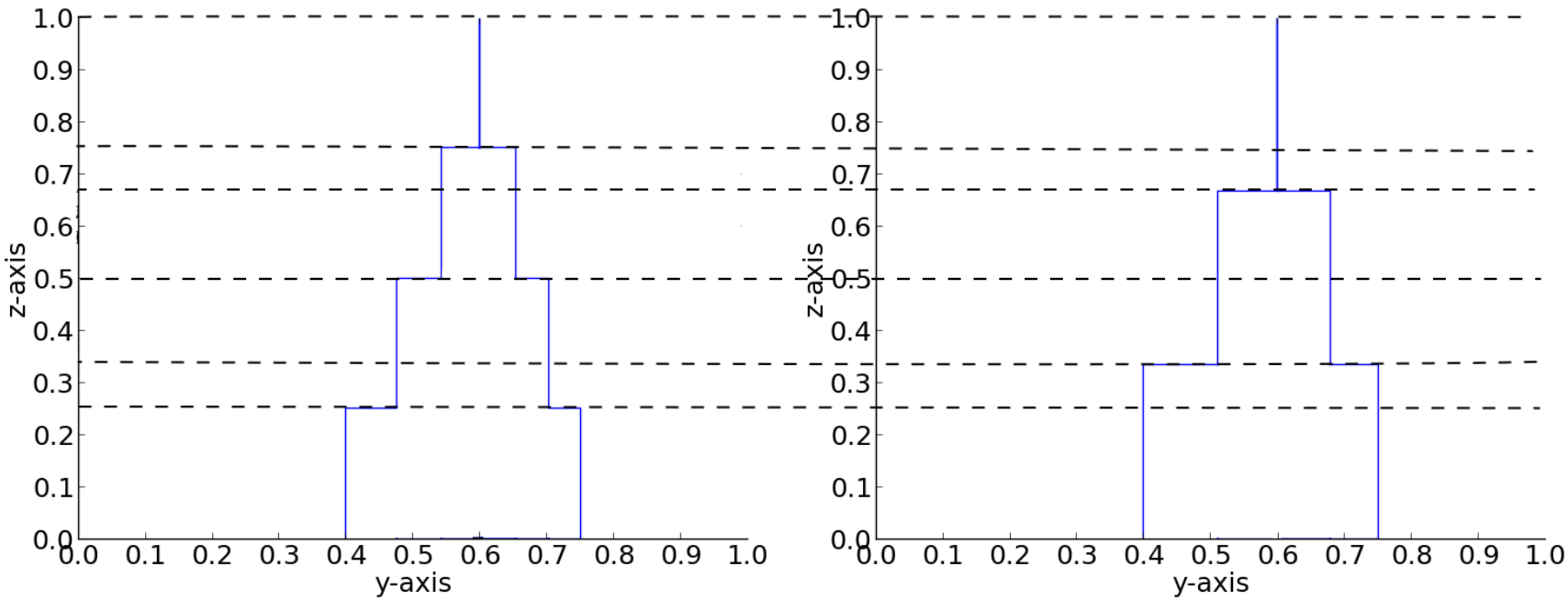
zSlices-based General Type-2 Fuzzy Sets with different numbers of zLevels

The similarity of sets with different numbers of zLevels is calculated using the union of their zLevels as follows:

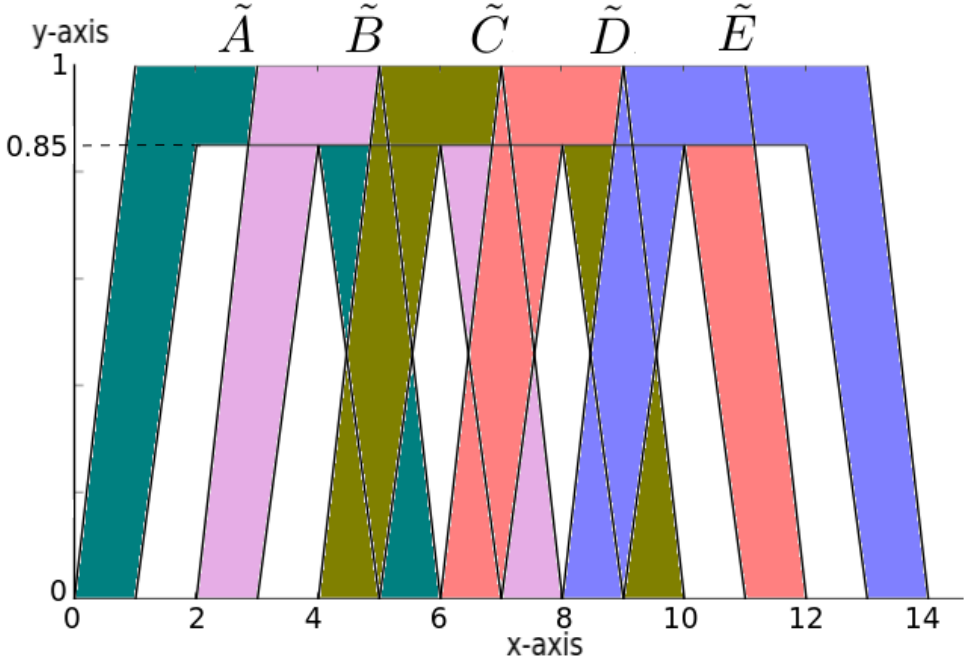
$$L = \bigcup_{m=1}^{M} z_m \cup \bigcup_{n=1}^{N} z_n$$

Where M and N are the number of zLevels used by each respective fuzzy set. In this example, $M = 4$ and $N = 3$, so $L = \{0.25, 0.5, 0.75, 1.0\} \cup \{0.33, 0.66, 1.0\} = \{0.25, 0.33, 0.5, 0.66, 0.75, 1.0\}$.

zSlices-based General Type-2 Fuzzy Sets with different numbers of zLevels

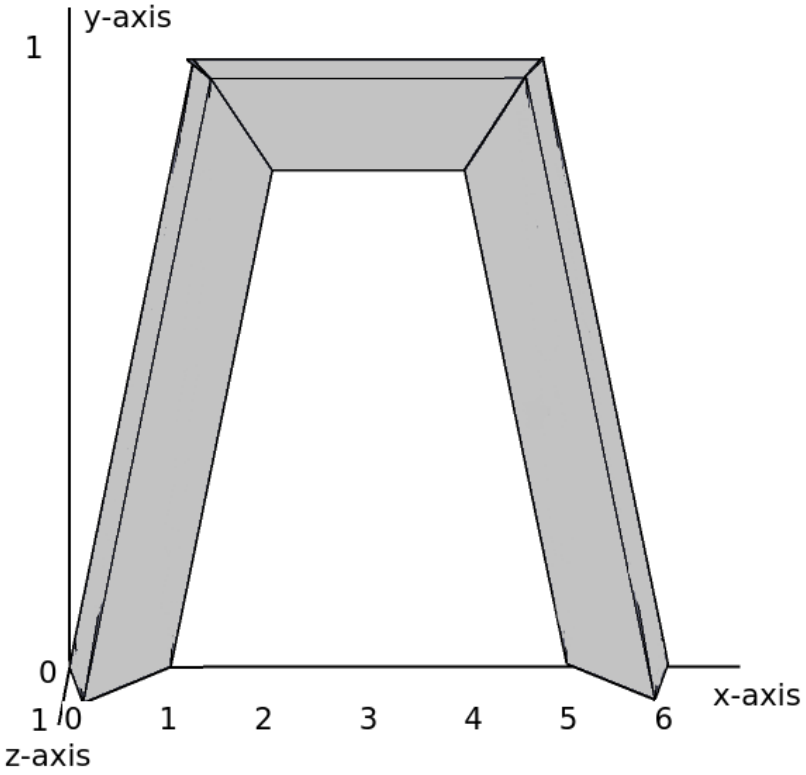


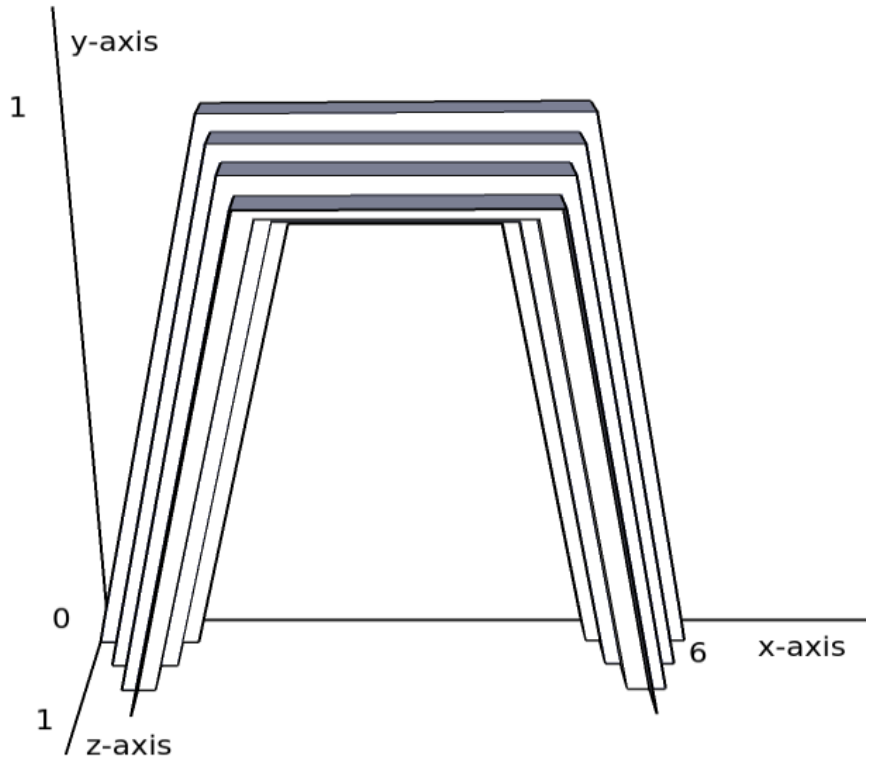
Reviewing Interval Type-2 Similarity Measures



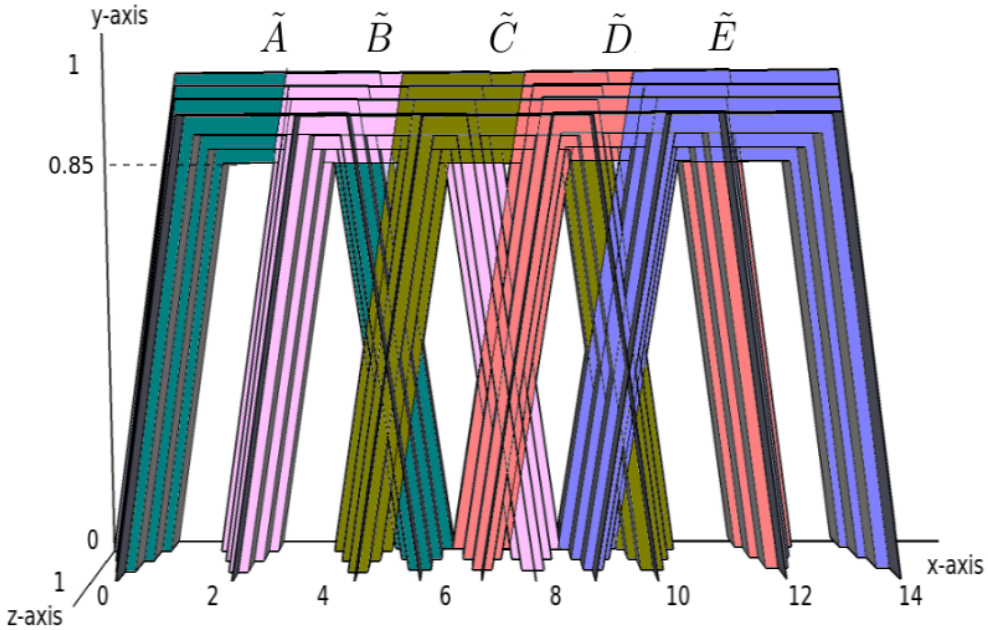
	$S(\tilde{A}, \tilde{A})$	$S(\tilde{A}, \tilde{B})$	$S(\tilde{A}, \tilde{C})$	$S(\tilde{A}, \tilde{D})$	$S(\tilde{A}, \tilde{E})$
Zeng & Li	1.0	0.538	0.345	0.371	0.461
Jaccard	1.0	0.342	0.071	0.0	0.0
Gorzałczany	(1.0, 1.0)	(1.0, 1.0)	(0.0, 1.0)	(0.0, 0.0)	(0.0, 0.0)
Bustince	(1.0, 1.0)	(0.0, 0.15)	(0.0, 0.15)	(0.0, 0.15)	(0.0, 0.15)

Converting Interval Type-2 Fuzzy Sets into General Type-2 Fuzzy Sets





Interval Type-2 Similarity Measures on zSlices-based General Type-2 Fuzzy Sets



	$S(\tilde{A}, \tilde{A})$	$S(\tilde{A}, \tilde{B})$	$S(\tilde{A}, \tilde{C})$	$S(\tilde{A}, \tilde{D})$	$S(\tilde{A}, \tilde{E})$
Zeng & Li	1.0	0.496	0.267	0.345	0.443
Jaccard	1.0	0.335	0.041	0.0	0.0
Gorzałczany	(1.0, 1.0)	(1.0, 1.0)	(0.33, 0.66)	(0.0, 0.0)	(0.0, 0.0)
Bustince	(1.0, 1.0)	(0.05, 0.1)	(0.05, 0.1)	(0.05, 0.1)	(0.05, 0.1)

Summary

We have introduced a general method of extending existing similarity measures on interval type-2 fuzzy sets to similarity measures on general type-2 fuzzy sets through the use of the zSlices based general type-2 fuzzy set representation.

The extension preserves all the common initial properties for similarity measures of the interval type-2 case, namely transitivity, symmetry, transitivity and overlapping.

Questions?