

From Concrete to Abstract in the Measurement of Length

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Two Rasch Measurement Experiments that measure length

- Results in logits can be compared to deterministic measurement of length in mm
- The unit of length could be reproduced in logits

Experiment 1

- Design
 - Each of 99 matches broken in two at random
 - The length of 99 headed match sticks was compared with the length of 33 no-head sticks used as reference lengths
- Scoring
 - 1: Headed match longer than reference length
 - 0: Headed match shorter than reference length
- Data
 - 99 rows of 33 0s and 1s
 - 33 columns: each reference length vs 99 headed sticks
- Joint Maximum Likelihood Estimation of length in logits

The 99 headed matches



04/06/2006

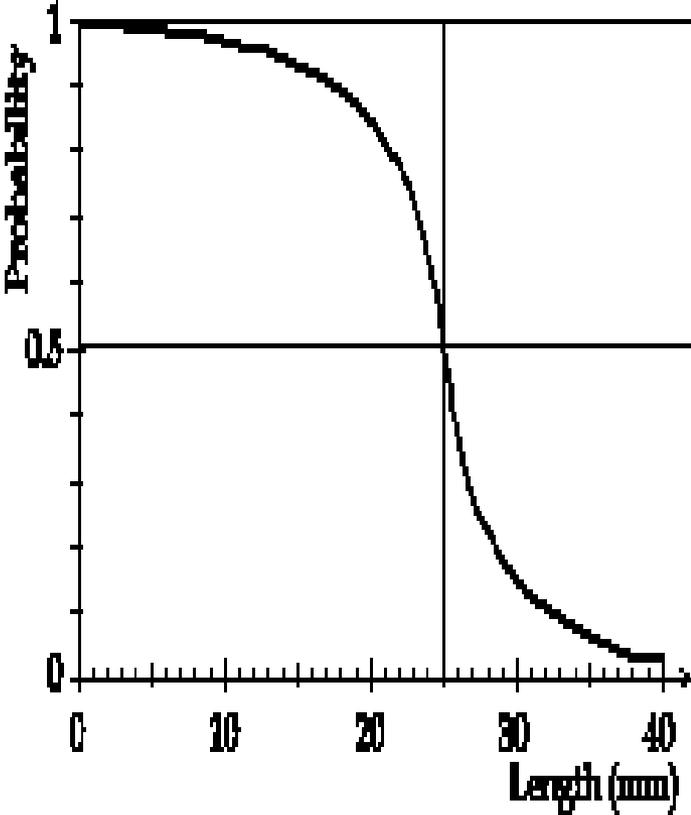
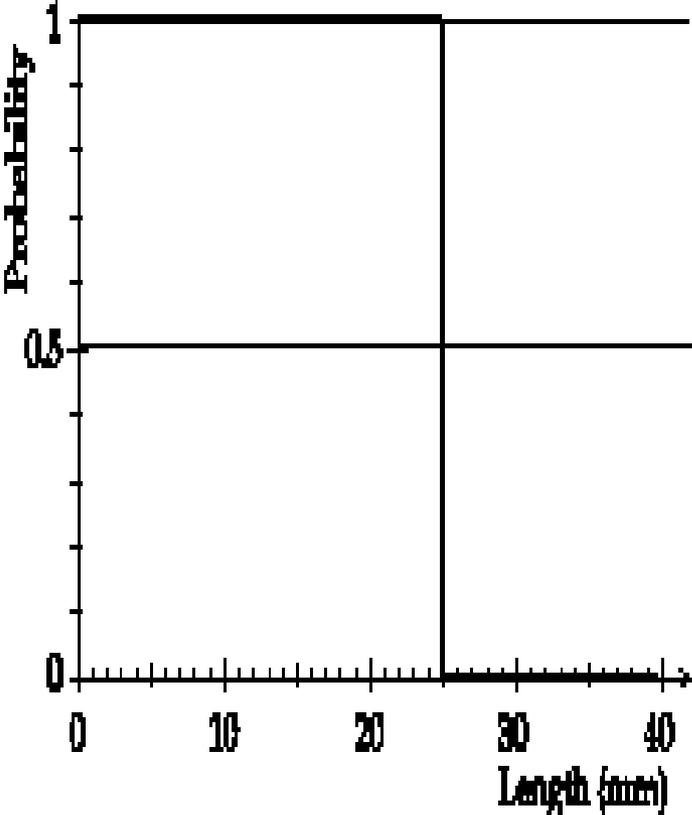
The 33 reference lengths



Experiment 2

- Design
 - 17 unevenly spaced units on ordinal ruler
 - At each hash mark: “Is the object at or above here?”
 - Range: 1.1 to 33.5 cm
- 42 physical objects of measurement
 - Various books, pictures, mirror, coffee mug, etc.
 - Range: 1 to 50 cm
- Scoring
 - 2: Object longer than reference length
 - 1: Object at reference length
 - 0: Object shorter than reference length
- Data
 - 42 rows: Each object by 17 reference lengths
 - 17 columns: Each reference length by 42 objects
- Modified pairwise comparison method

Guttman vs Rasch length probabilities



Same Design Elements Across Experiments

- Model (odds = B/D)

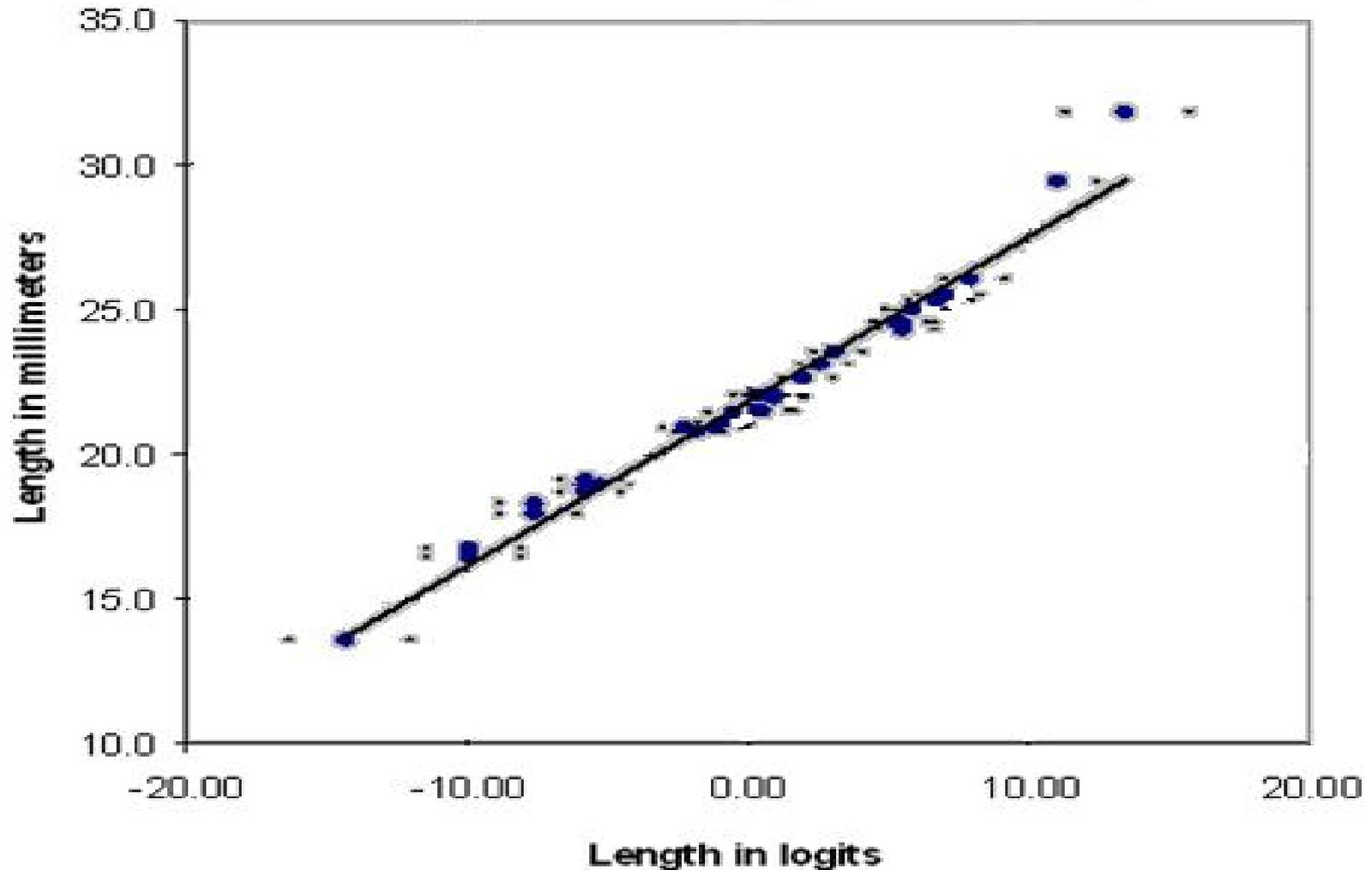
$$\Pr\{X_{ni} = 1\} = \frac{e^{\beta_n - \delta_i}}{1 + e^{\beta_n - \delta_i}}$$

- Joint Maximum Likelihood Estimation
- WINSTEPS software (Linacre, 2013)

Experiment 1 Results

Logits and millimeters

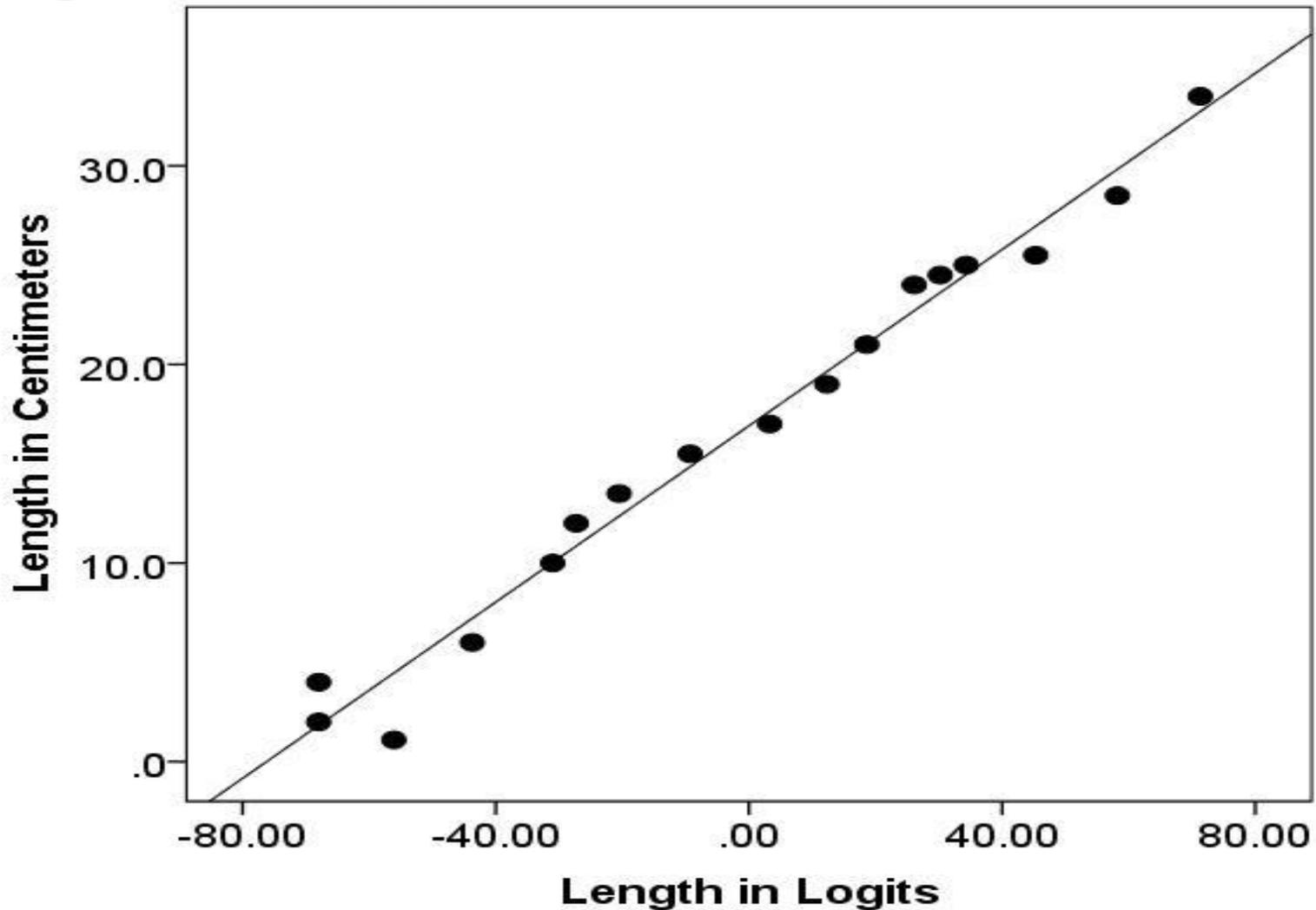
$$\text{Length in mm} = 0.52 \text{ logits} + 21.7$$



Experiment 2 Results

Logits and centimeters

$$\text{Length in cm} = 0.22 \text{ logits} + 18$$



Discussion

- The two experiments reproduce length units from ordinal observations. They are analogous to other work making the same kind of point concerning weight, distance, and density.
 - Choi, S. E. (1997). Rasch invents "Ounces." *Rasch Measurement Transactions*, 11(2), 557.
 - Moulton, M. (1993). Probabilistic mapping. *Rasch Measurement Transactions*, 7(1), 268.
 - Pelton, T., & Bunderson, V. (2003). The recovery of the density scale using a stochastic quasi-realization of additive conjoint measurement. *Journal of Applied Measurement*, 4(3), 269-281.

Discussion

- “We do not in real life have perfectly spherical balls moving on perfectly smooth horizontal planes—the trick lay in the fact that it occurred to Galileo to imagine these.”
- Galileo and others "were discussing not real bodies as we actually observe them in the real world but geometrical bodies moving in a world without resistance and without gravity..."
 - Butterfield, H. (1957). *The origins of modern science (revised edition)*. New York: The Free Press.

Discussion

- “A 'mathematical pendulum' is defined as 'a heavy point, swinging frictionless on a weightless string in vacuum'. A contraption like that was never seen; thus as a model for the motion of a real pendulum it is 'unrealistic'. Notwithstanding, it works quite well for a short time interval...”

- Rasch, G. (1973/2011). All statistical models are wrong! Comments from the Conference on Foundational Questions in Statistical Inference, Aarhus, Denmark, May 7-12, 1973. *Rasch Measurement Transactions*, 24(4), 1309.

Discussion

- “All models are wrong, but some are useful.”
 - Box, G. E. P. (1979). Robustness in the strategy of scientific model building. In R. L. Launer & G. N. Wilkinson (Eds.), *Robustness in statistics (pp. 201-235)*. New York: Academic Press, Inc.
- “*Models should not be true, but it is important that they are applicable.*”
 - Rasch, G. (1960). *Probabilistic models for some intelligence and attainment tests (Reprint, with Foreword and Afterword by B. D. Wright, Chicago: University of Chicago Press, 1980)*. Copenhagen, Denmark: Danmarks Paedagogiske Institut.

Conclusions

- The demonstrated capacity to reproduce mm and cm lengths from ordinal observations suggests untapped potential for stable and metrologically traceable units of measurement in psychology and the social sciences.
- Success in devising and applying such units will require thorough qualitative elaboration of the constructs measured.