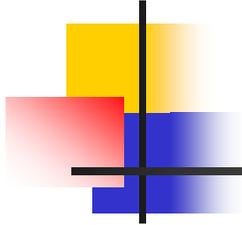
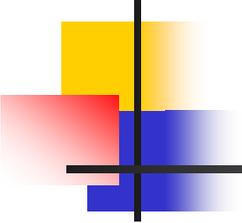


Chapter 14

Risk Analysis

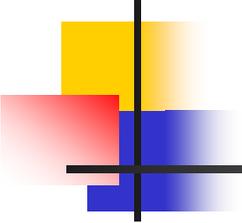




Frequency definition of probability

- Given a situation in which a number of possible outcomes might occur, the **probability of an outcome** is the proportion of times that it occurs if the situation exists repeatedly.
- $P(A) = \frac{r}{R}$
is the probability of r outcomes of A in R trials.
- In many cases **subjective probabilities** are very vague, but the best assessment a person can make.

Probability distribution and expected value



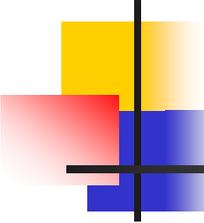
Profit	Probability
\$1,000,000	0.6
\$ - 600,000	0.4

Expected value of profit:

$$(\$1,000,000) 0.6 + (\$-600,000) 0.4 = \$360,000$$

π = Profit, P = Probability

$$\text{Expected profit} = E(\pi) = \sum_{i=1}^N \pi_i P_i$$



Constructing a decision tree

- **Decision fork:** a juncture representing a choice where the decision maker is in control of the outcome
- **Chance fork:** a juncture where “chance” (some call it “nature”) controls the outcome

Constructing a decision tree



Figure 15.1 Decision Tree, Jones Corporation

Tomco Oil Corporation

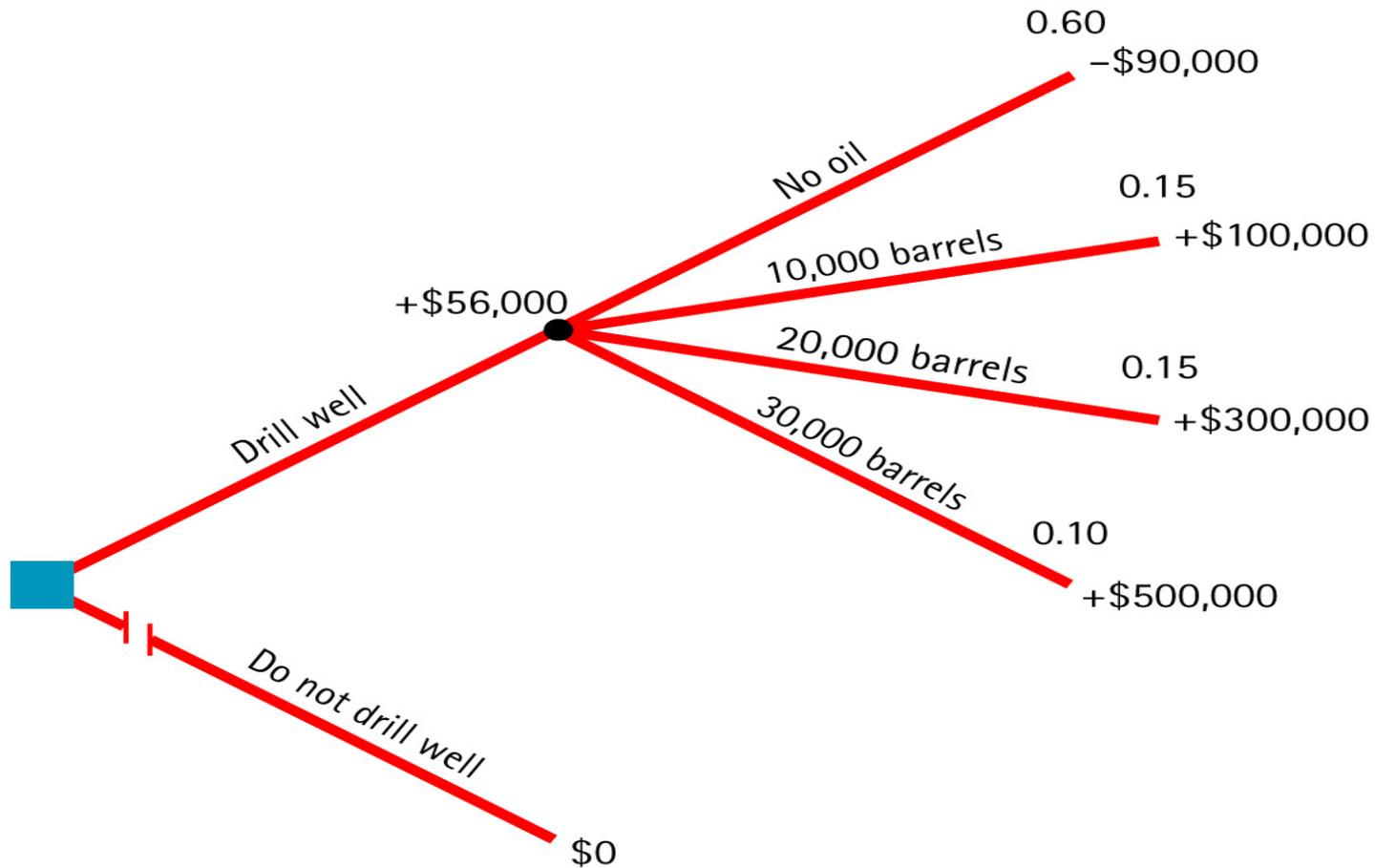
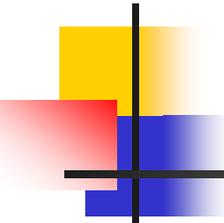
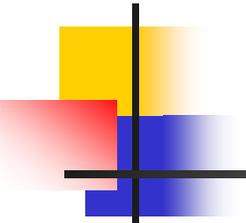


Figure 15.2 Decision Tree, Tomco Oil Corporation



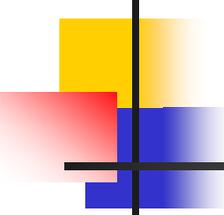
The expected value of perfect information

- How much resources should you spend in order to get perfect information (but you do not know in advance what that information is going to be)?
- Example: Jones Corp: (15.1) should the firm increase price or not
- In case of accurate information, the firm is able to make an correct decision



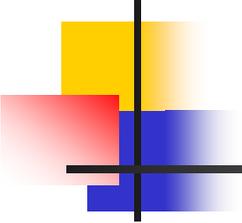
- Reason for uncertainty is advertising campaign

- If campaign would be successful, price increase would be chosen
- If unsuccessful, no price increase would be chosen
- But you do not know now which case will apply:
 - Therefore 50% chance that one of the cases will apply
 - $E(\pi) = 0.5(\$800000) + 0.5(\$200000) = \$500000$
- Value of perfect information is \$300000.
 - Because otherwise no price increase (\$200000) would have been chosen
- Firm should spend up to \$300000 to find out.



Simple decision rule

- Use expected value of a project
- How do people really decide?

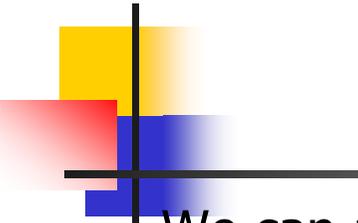


Expected utility

- **Expected utility:** Calculate utility of each outcome and then calculate expected value of these utilities.

$$EU(\pi) = \sum_{i=1}^N U(\pi_i)P_i \text{ with } \pi_i \text{ as profits and } P_i \text{ as probabilities}$$

- In general expected utility is different from expected profits:
 - Risk-averting, risk-neutral or risk-seeking individuals are possible.
 - Because the individual can assess the utility of a risky project different as the utility of a certain one



Should a decision maker maximize expected profit or expected utility?

- We can attach a utility to each outcome (say, profit or marks in final exam), that means we would talk about *utility of profit* instead of just profit.
- For marks in final exams, it seems natural to attach a utility – in order to compare it to other events, say the utility of going on vacation.
- For profits it seems at first sight odd to think of utility of profit (of income, or wealth)? We would rather like to talk about profits directly.
- This is ok, as far as profits are certain.
- In the case of uncertainty, using profits only as a decision criterion would mean:
 - the manager looks only at the expected profit
 - She does not care, if more/less risk is involved in the project, as far as expected profit is the same

Construct a utility function: Tomco Oil Corporation

- Utility function is not unique:
 - you can add a constant term
 - You can multiply by a constant factor

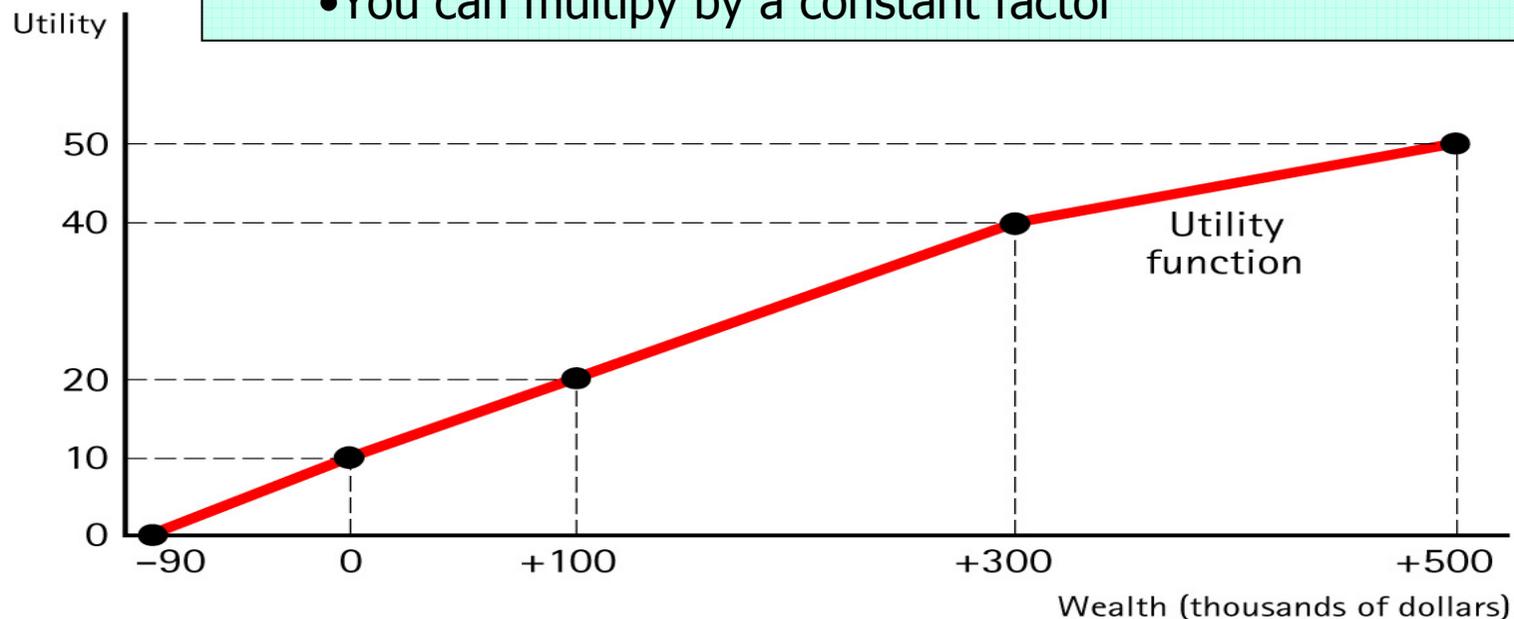
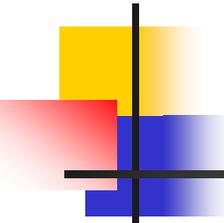
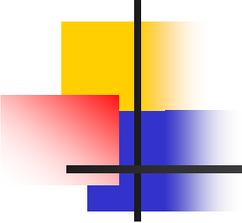


Figure 15.3 Utility Function



How do you get these points?

- Start with any values $U(-90)=0$, $U(500)=50$
- Then ask the decision maker questions about indifference cases
 - Find value for 100
 - Do you prefer the certainty of a \$100 gain to a gamble of 500 with probability P and -90 with probability $(1-P)$?
 - Try several values of P until the respondent is indifferent
 - Suppose outcome is $P=0.4$
 - Then it follows
 - $U(100) = 0.4U(500) + 0.6U(-90)$
 - $\implies U(100) = 0.4(50) + 0.6(0) = 20$



Expected utility for Tomco Oil

$$E(U) = .6U(\$-90) + .15U(100) + .15U(300) + .10 U(500)$$

$$=.6 (0) + .15(20) + .15(40) + .10(50) = 14$$

$$\neq E(\pi) = .6(-90) + .15 (100) + .15(300) \\ + .10(500) = 56$$

Probability Distribution

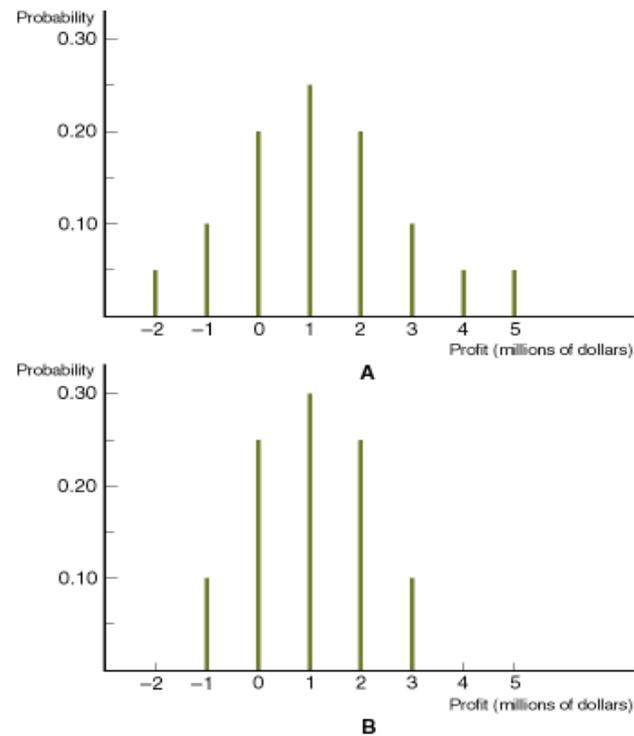
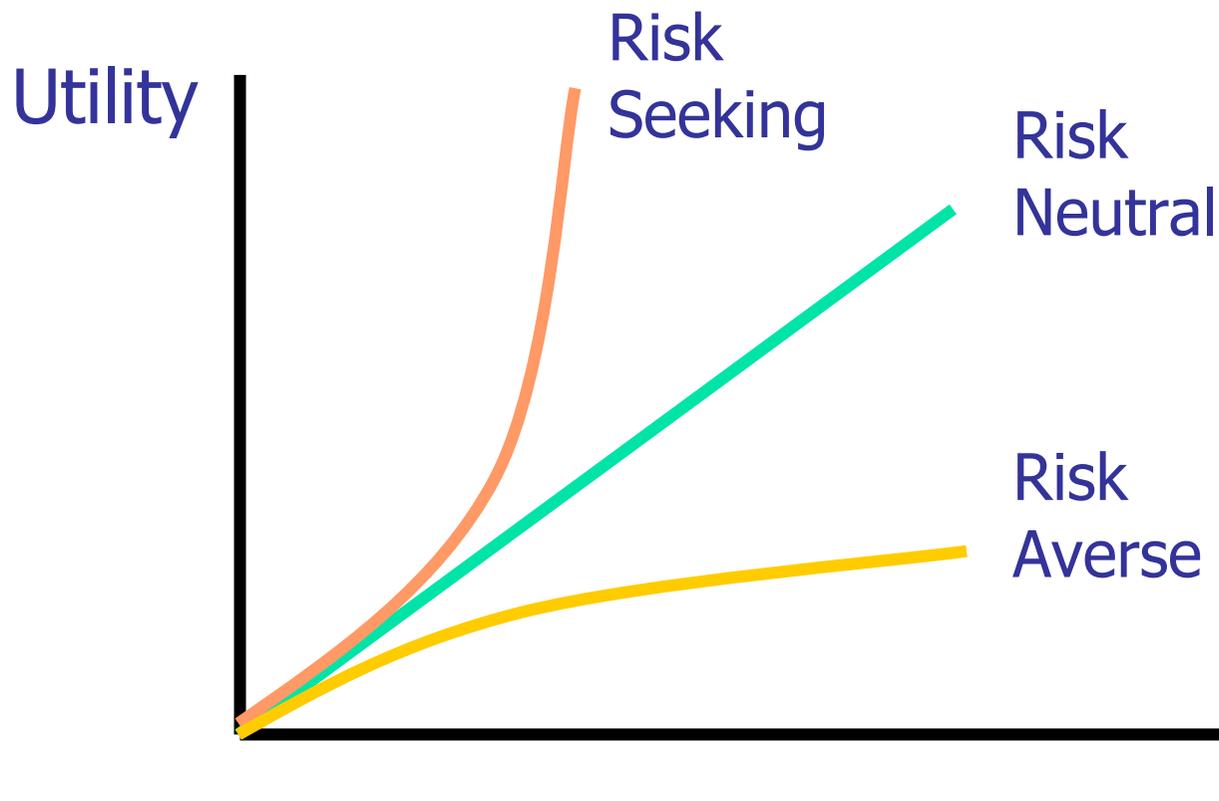
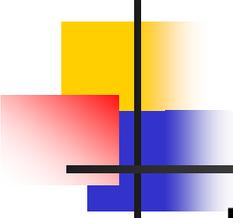


FIGURE 14.5 Probability Distribution of Profit from Investment in New Plant

Attitudes toward risk

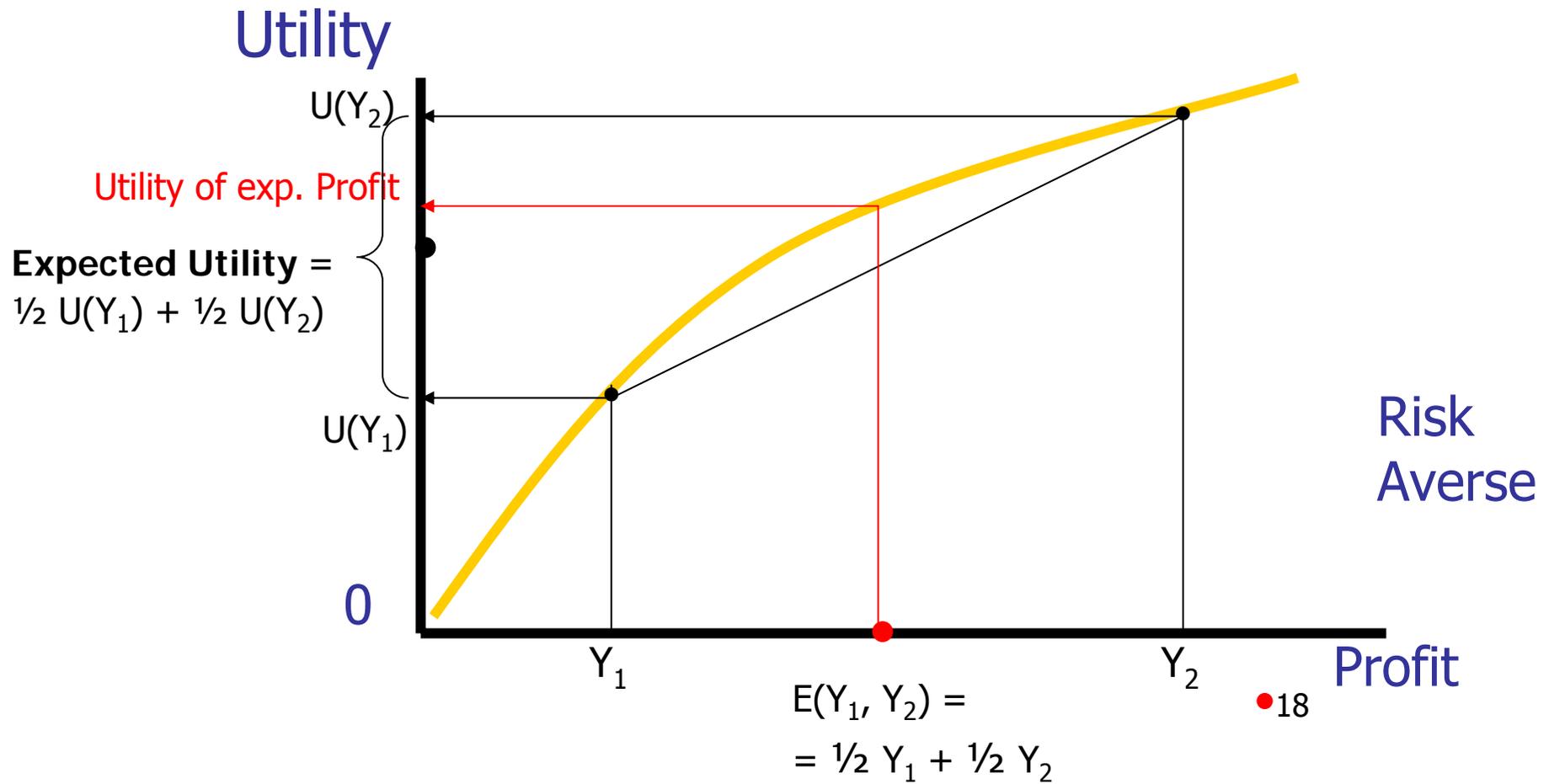


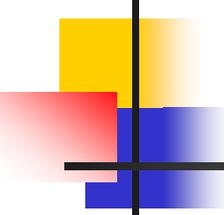


Attitudes towards risk

- Risk-averse: expected utility is lower than utility of expected profit: the individual fears a loss more than she values a potential gain
- Risk-neutral: the person looks only at expected value (profit), but does not care if the project is high- or low-risk.
- What attitude towards risk do most people have? (maybe you wanna differentiate between long-term investment and, say, Lotto)
- What attitude towards risk should a manager of a big (publicly traded) company have?
- What's the effect of the managers' risk attitude?

Example: A risk averse person gets Y_1 or Y_2 with probability of $\frac{1}{2}$
Expected Utility < Utility of expected value

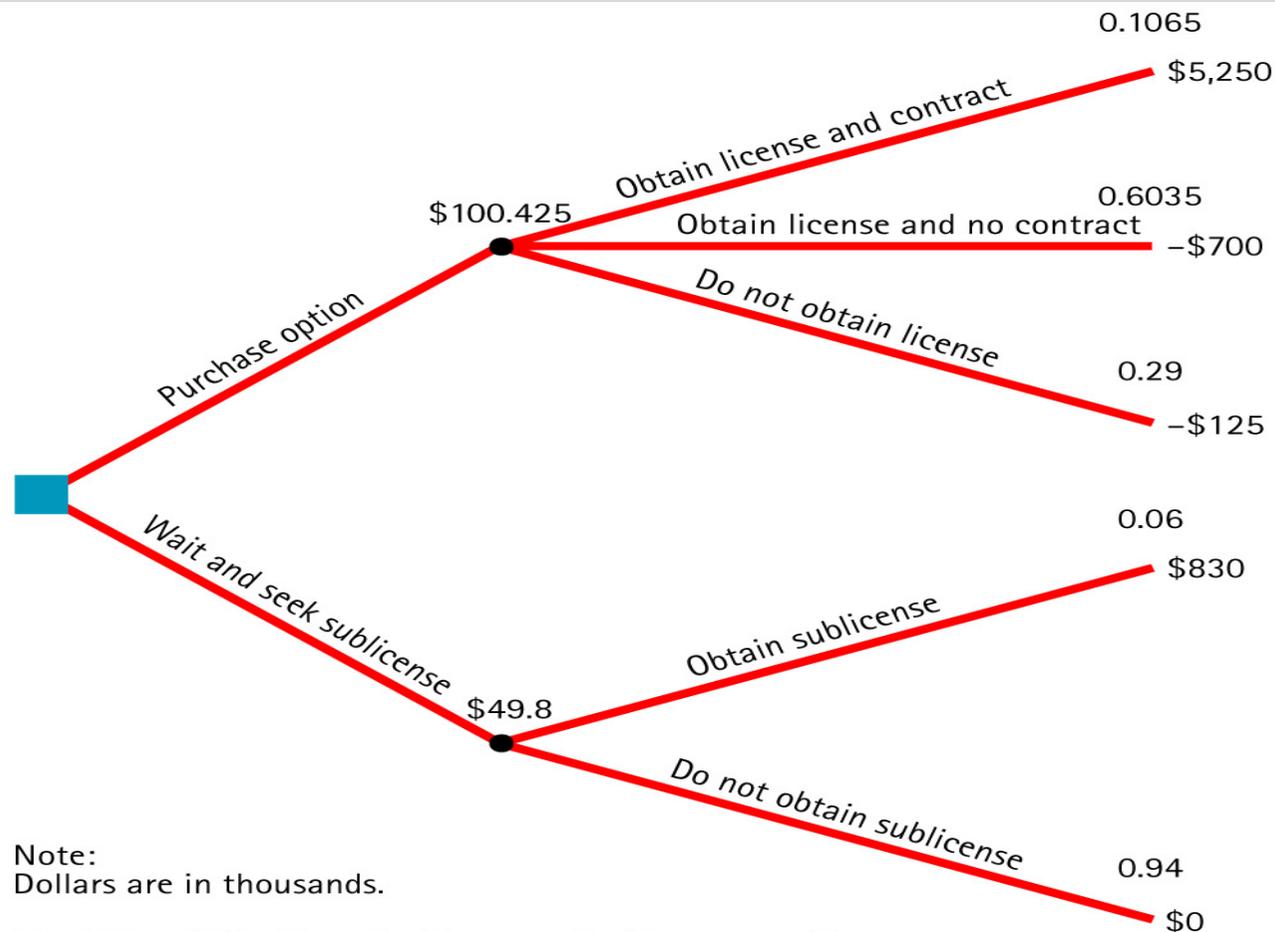




Expected utility

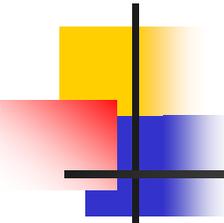
- We measure utility of wealth or net worth
- But not utility of a „change in income“
- Therefore for small changes, say Lotto, change in net worth is small and
 - Utility function rather linear
 - Approximation of risk neutrality not bad

Manager decided to wait and seek: Why?



Note:
Dollars are in thousands.

AMD 15.2 Deciding Whether to Buy an Option on a New Flight-Safety System



Adjusting the valuation model for risk

- Certainty equivalent approach:
- What fixed monetary value has the same U as the EU of an uncertain project (with known probabilities)?
- Construct indifference curves
- Risk premium: risk-averse individual accepts lower expected profit (return) if the risk is lower.
 - Maximum amount the individual would be willing to pay to an insurance company in order to be insured against risk

Manager's Indifference Curve

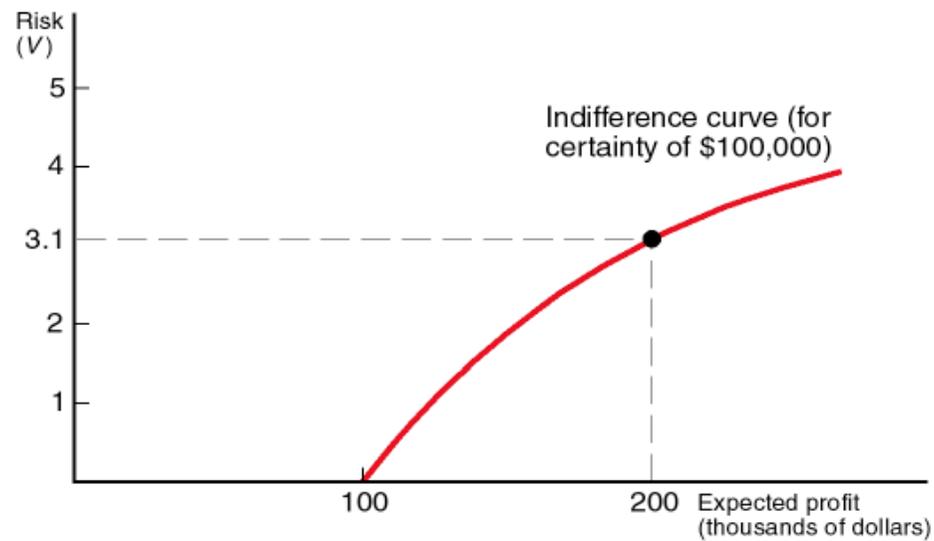


FIGURE 14.6 Manager's Indifference Curve between Expected Profit and Risk

Risk Premium

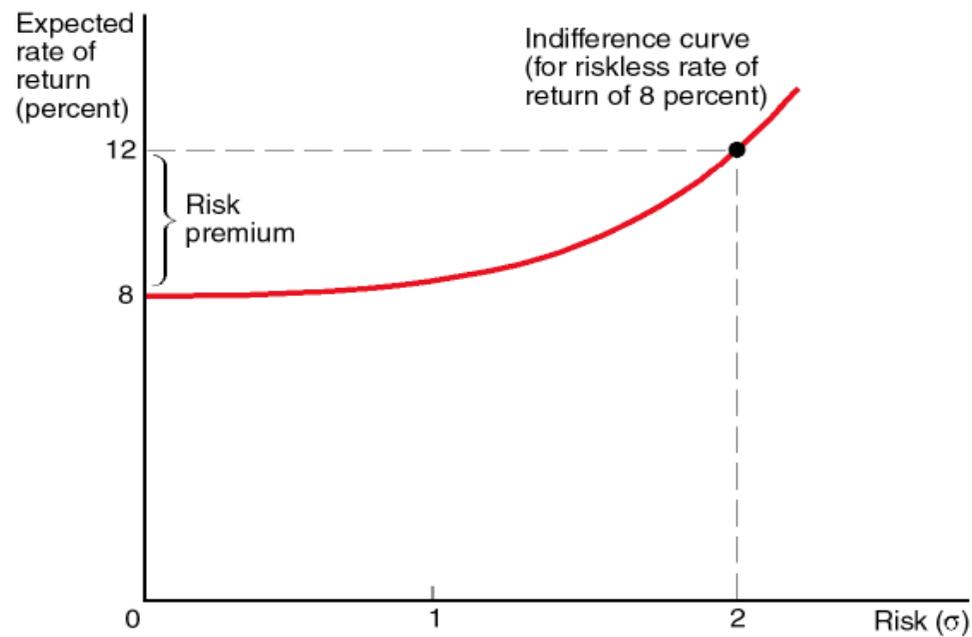
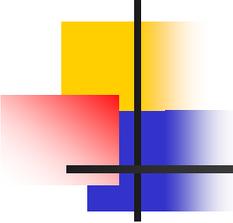


FIGURE 14.7 Manager's Indifference Curve between Expected Rate of Return and Risk

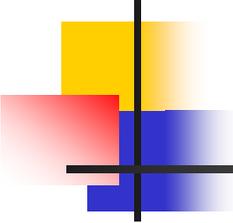
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Problem 6

The CEO of a publishing company says she is indifferent between the certainty of receiving \$7,500 and a gamble where there is 0.5 chance of receiving \$5,000 and a 0.5 chance of receiving \$10,000. Also, she says she is indifferent between the certainty of receiving \$10,000 and a gamble where there is a 0.5 chance of receiving \$7,500 and a 0.5 chance of receiving \$12,500.

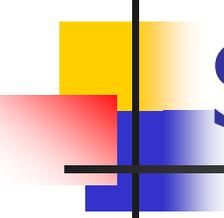
- a) Draw (on a piece of graph paper) four points on the utility function of this publishing executive.
- b) Does she seem to be a risk averter, a risk lover, or risk neutral?



Problem 10

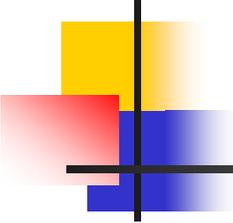
Roy Lamb has an option on a particular piece of land, and must decide whether to drill on the land before the expiration of the option or give up his rights. If he drills, he believes that the cost will be \$200,000. If he finds oil, he expects to receive \$1 million; if he does not find oil, he expects to receive nothing.

a) Can you tell whether he should drill on the basis of the available information? Why or why not?



Solution Problem 10

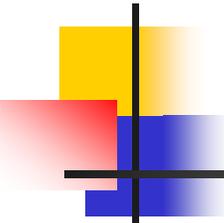
a) No, there are no probabilities given.



Problem 10 – Part 2

Mr. Lamb believes that the probability of finding oil if he drills on this piece of land is $\frac{1}{4}$, and the probability of not finding oil if he drills here is $\frac{3}{4}$.

- b) Can you tell whether he should drill on the basis of the available information. Why or why not?
- c) Suppose Mr. Lamb can be demonstrated to be a risk lover. Should he drill? Why?
- d) Suppose Mr. Lamb is risk neutral. Should he drill or not. Why?



Solution Problem 10 – Part 2

- b) $1/4(800) - 3/4(200) = 50 > 0$, so a person who is risk neutral would drill. However, if very risk averse, the person would not want to drill.
- c) Yes, since the project has both a positive expected value and contains risk, Mr. Lamb will be doubly pleased.
- d) Yes, Mr. Lamb cares only about expected value, which is positive for this project.