

# History of chaos theory

“The idea that small causes can sometimes have large effects has been noted by historians and others since antiquity, and captured for example in "for want of a nail ... a kingdom was lost".” [Wolfram, *A New Kind of Science*, Ch. 7]



# History of chaos theory

This idea is in contrast to the idea of

## Continuity

where small changes in the input  
create small changes in the output.

This type of change would be called  
smooth, or predictable.



“During the 1800’s, ... Maxwell discussed how collisions between hard sphere molecules could lead to progressive amplification of small changes and yield microscopic randomness in gases... Poincaré found sensitive dependence on initial conditions in a particular case of the three-body problem, and later proposed that such phenomena could be common, say in meteorology.”



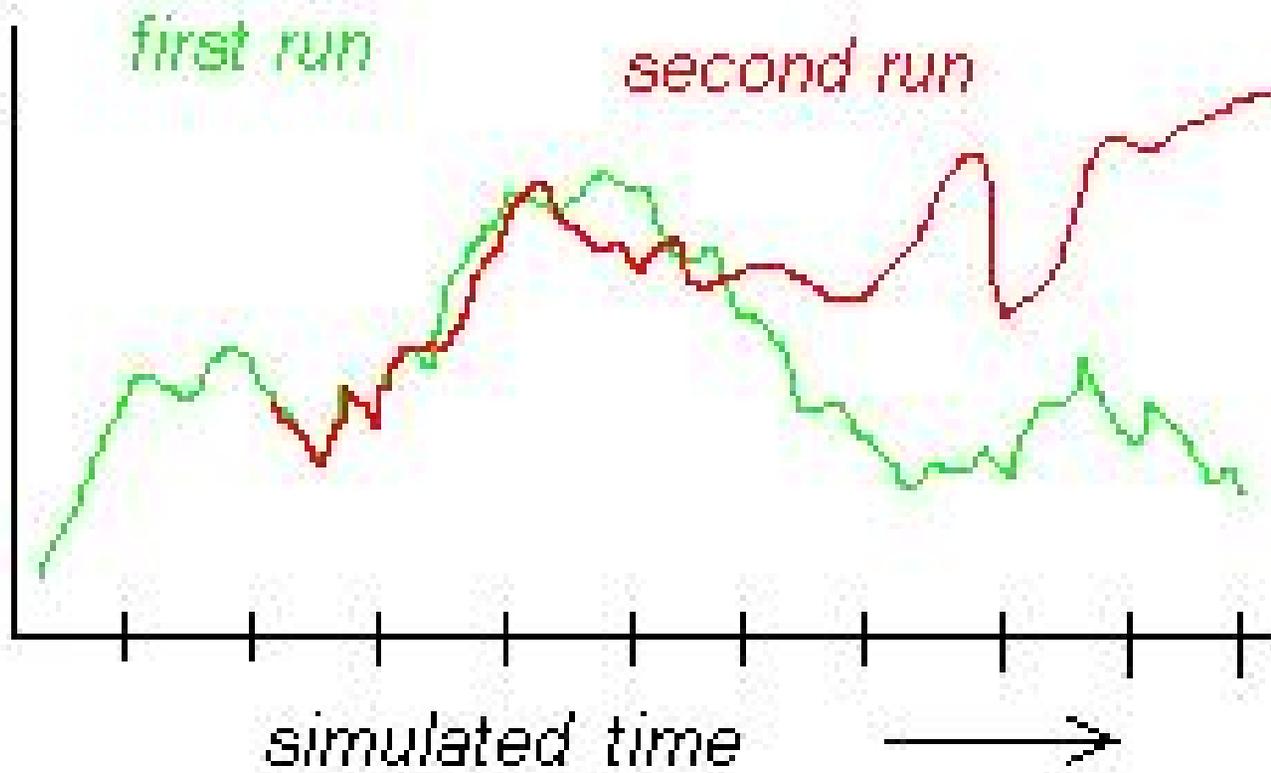
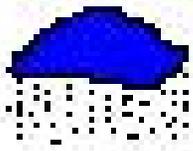
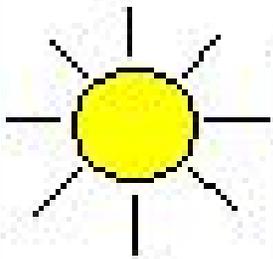
## Edward Lorenz

“In 1960 a meteorologist named Edward Lorenz was researching into the possibilities of long term weather prediction. He created a basic computer program using mathematical equations which could theoretically predict what the weather might be.”

[<http://people.bath.ac.uk/ma3mbc/Chaostheory.html>]

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‘Here are some defining features of chaotic systems:

- They are deterministic, something determines their behavior.
- They are very sensitive to their initial conditions. A slight change can lead to enormously different outcomes. (The Butterfly Effect).
- Although chaotic systems (appear to) behave randomly, they always have a sense of order and pattern. Truly random systems are NOT chaotic.”

[http://www.cmp.caltech.edu/~mcc/chaos\\_new/Lorenz.html](http://www.cmp.caltech.edu/~mcc/chaos_new/Lorenz.html)



Now let's turn to a model of chaotic behavior that you can play with and experience right now.

It is called the *logistic equation*. It tells how the population of a particular species varies with time.



# The Logistic Equation

Pierre Franois Verhulst (1804 - 1849, Brussels, Belgium) was a mathematician and a doctor in number theory from the University of Ghent in 1825. Verhulst published in 1838 a continuous logistic demographic model:

$$N'(t) = cN(t) \cdot \left(1 - \frac{N(t)}{K}\right)$$

where  $N(t)$  represents number of individuals at time  $t$ , 'c' the intrinsic growth rate and 'K' number of individuals in equilibrium.





P. F. VERHULST.

- For simplicity, we'll replace that equation with this Logistic Recurrence Relation or what we will call the logistic map.

$$y = 2x (1 - x)$$

- Let's see how it works.  $x$  stands for the percent of the possible population today.
- The “2” is the reproduction/starvation constant. We will experiment with various values other than 2 in a few minutes.
- The  $y$  is the percent for the next time period.

Copy this down.

- Since we will repeatedly apply this formula, we'll replace that equation with this one that uses  $x_n$  and  $x_{n+1}$ .

$$x_{n+1} = 2x_n(1 - x_n)$$

- Let's see how it works.  $x_n$  stands for the percent of the possible population on some day that we call the  $n^{\text{th}}$  day.

Copy this down too.



$$x_{n+1} = 2x_n(1 - x_n)$$

- The symbol  $x_{n+1}$  has a similar meaning, but it refers to the next day, the  $n + 1^{st}$  day. So, it stands for the percent of the possible population one day after the  $n^{th}$  day.
- For example,  $x_n = .20$  means that on the  $n^{th}$  day the population is at 20% of what the environment can support.



Let's start with first day population at 20%.

What does the formula say should happen the next day?

If  $x_1 = 0.2$ , (Day 1 has 20%), then

$$x_2 = 2(.2)(.8) = .32 = 32\%$$

$$x_3 = 2(.32)(.68) = .4352 = 44\%$$

$$x_4 = 2(.44)(.56) = .4928 = 49\%$$



What if we start higher than 50%?

Let's try starting Day 1 at 80%: Then  $x_1 = 0.8$ :

We'd get

$$x_2 = 2(.8)(.2) = .32 = 32\%$$

$$x_3 = 2(.32)(.68) = .4382 = 44\%$$

Now each of you can open an Excel spreadsheet on your own machine to play with this.



Move to Excel spread sheet from course web site:

Go to HANDOUTS;

Go down to *Chaos Demonstration with Logistic Equation*

SAVE to Desktop.

Shrink website

Double click and open “LogisticWeb.xls” file. Find the Excel spreadsheet and right click to download to your machine.



Inside Excel, go to TOOLS on the top menu bar, and  
Pull down to OPTIONS.

Open EDIT “file”

Uncheck the third button.

Click OK.

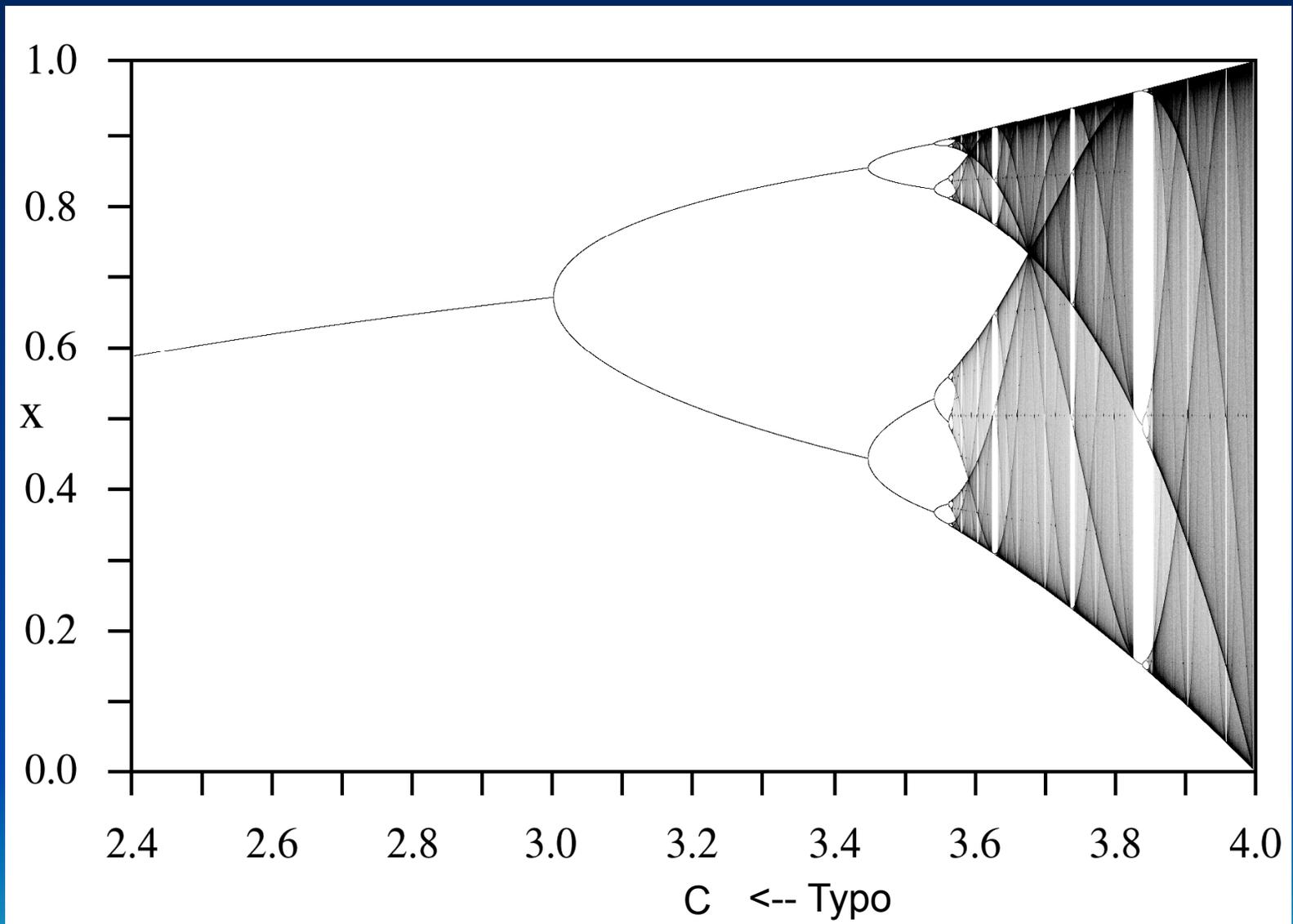
“Look For Seed=”

This is our  $x_1$ . PLAY with this spreadsheet by  
trying various seeds and watch the column of  
values for successive days.



- Now return the SEED value to 0.5.  
Feel free to play with it later.
- Now we'll play around with the reproduction/  
starvation coefficient 'C'.
- Set  $C = 0.5, 0.8, 1, 1.1, 1.2, 1.3, \dots,$   
 $2.7, 2.8, \dots 3, 3.1, \dots$
- Now let's look at something called the  
**BIFURCATION DIAGRAM**

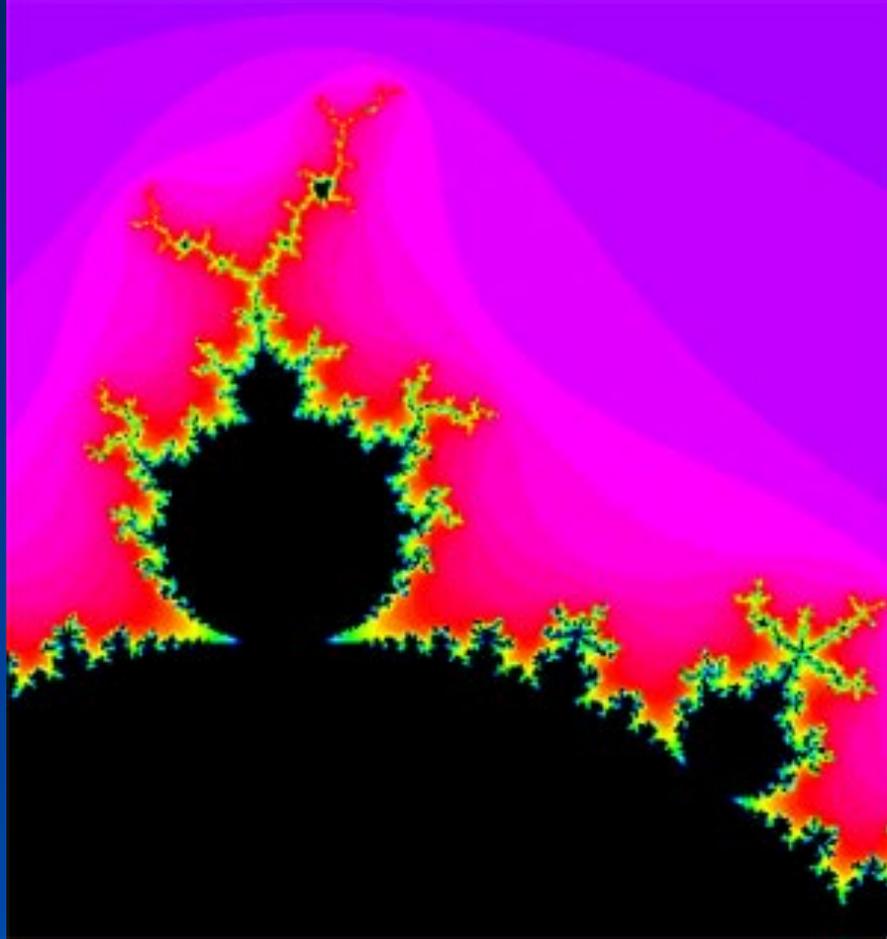




Another place you have seen chaotic behavior are the many fractal pictures that are quite popular.

Right now, Google “FRACTAL”  
and click on IMAGES.





The Mandelbrot Set



*THANKS SO MUCH*

Chaos: Making a New Science  
by James Gleick, 1987

