Cognitive-Agent-Based Modeling of a Financial Market

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Outline

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Introduction

- Agent-based modeling has become a widely accepted tool for studying the dynamics of financial markets;
- Santa Fe Institute Artificial Stock Market (SFI-ASM);
- We thought interesting to investigate the use of richer, more sophisticated agent types, like BDI agents;
- Idea: Let the agents evolve under the pressure of competition-driven selection.

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Market Simulator

- Asset (no dividend) + Money (no interest).
- Agents hold:
  - an inventory of the asset;
  - cash.
- Market Value of Inventory + Cash = NAV.
- Trading: buy/sell n contracts at limit price $\in [0, +\infty]$.
- Market matches orders at each period by executing a single-price auction (no fees).

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Price Formation

Single-Price Auction

bid	price	ask
	⋮	
Joe, 100	9.22	Lou, 25
	9.21	Mal, 75 · Rick, 10
Gus, 10 · Al, 50	9.20	Bob, 10 · Pat, 10 · Chuck, 20
Ed, 100	9.19	Dan, 200
Ted, 100 · Ike, 20	9.18	
	⋮	

Technical Indicators

#	description	depth
1	Price went up this period	2
2	Price went up one period ago	3
3	Price went up two periods ago	4
4	Price went up three periods ago	5
5	Price went up four periods ago	6
6	Price $>$ 5-period SMA	6
7	Price $>$ 10-period SMA	11
8	Price $>$ 20-period SMA	21
9	Price $>$ 5-period EMA	6
10	Price $>$ 10-period EMA	11
11	Price $>$ 20-period EMA	21
12	Price / 5-period SMA $>$ 20-period SMA	21
13	Price / 5-period SMA $>$ 50-period SMA	21
14	Price / 5-period EMA $>$ 20-period EMA	21
15	Price / 5-period EMA $>$ 50-period EMA	21
16	Price / 5-period SMA went up this period	7
17	Price / 5-p'd SMA went up one period ago	8
18	Price / 5-p'd SMA went up two periods ago	9
19	Price / 10-period EMA went up this period	12
20	Price / 10-p'd EMA went up one period ago	13
21	Price / 10-p'd EMA went up two periods ago	14
22	MACD	26

Fuzzy Propositions

Definition (Fuzzy Interpretation)

A fuzzy interpretation is an assignment of truth degrees in $[0, 1]$ to all atomic propositions (or atoms, for short) defined in the problem domain. Given a set of atoms $\mathcal{A}$, a fuzzy interpretation is a function

$$\mathcal{I} : \mathcal{A} \to [0, 1],$$

which assigns a truth degree $\mathcal{I}(p) \in [0, 1]$ to all atoms $p \in \mathcal{A}$.

The atomic propositions defined in our model are the following:

- the price is going up (u);
- the money balance is above the minimum threshold (m);
- the asset inventory is above the minumum threshold (a);
- buy (b);
- sell (s).

Therefore, $\mathcal{A} = \{u, m, a, b, s\}$.

Mental State

The state of an agent is completely described by a fourtuple $\mathcal{S} = \langle \mathcal{K}, \mathcal{B}, \mathcal{R}_J, \mathcal{J} \rangle$, where

- $\mathcal{K}$ is a fuzzy interpretation on $\mathcal{A}$ (knowledge);
- $\mathcal{B}$ is a fuzzy interpretation on $\mathcal{A}$ (beliefs);
- $\mathcal{R}_J$ is a set of desire-generation rules, such that, for each desire d, $\mathcal{R}_J$ contains at most one rule of the form $\delta \Rightarrow_D^+ d$;
- $\mathcal{J}$ is a fuzzy set of literals.

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Desire-Generation Rules

Definition (Desire-Generation Rule)

A desire-generation rule is an expression of the form
$R = \{\kappa_R, \beta_R, \psi_R \Rightarrow_D^+ d \mid \kappa_R, \beta_R, \psi_R \in \mathcal{L}, d \in \{a, \neg a\}, a \in \mathcal{A}\}$.

Example:

$$\begin{aligned} \top, u, \top &\Rightarrow_D^+ b, \\ \top, \neg u, \top &\Rightarrow_D^+ s. \end{aligned}$$

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Changes in the Mental State

Changes in the agent's knowledge are straightforward.

Definition (Belief Change Operator)

Upon receiving information β (a literal) from a source trusted to degree α, the new fuzzy set of beliefs $\mathcal{B}' = \mathcal{B} * \frac{\alpha}{\beta}$ is such that, for all $a \in \mathcal{A}$,

$$\mathcal{B}'(a) = \begin{cases} \mathcal{B}(a) \cdot (1-\alpha) + \alpha, & \text{if } \beta \models a; \\ \mathcal{B}(a) \cdot (1-\alpha), & \text{if } \beta \models \neg a; \\ \mathcal{B}(a), & \text{otherwise.} \end{cases} \quad (1)$$

Changes in $\mathcal{J}$ and $\mathcal{G}$ are computed by iteratively applying the rules in $\mathcal{R}_{\mathcal{J}}$ until a fixpoint is reached.

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Genome

#	Symbol	Range	Meaning
0	S	$0, \ldots, 7$	One of eight pre-defined initial mental states
1	α_1	$[0, 1]$	Trust in technical indicator #1
$\vdots$	$\vdots$	$\vdots$	$\vdots$
22	α_{22}	$[0, 1]$	Trust in technical indicator #22
23	g_{b1}	$[0, 1]$	Fraction of money the agent is willing to invest at each period
24	g_{b2}	$[1, 25]$	Minimum threshold for asset
25	g_{b3}	$[0, 1]$	Incentive to buy if asset below threshold
26	g_{b4}	$[-2, 0]$	Parameter for price concession
26	g_{b5}	$[-2, 0]$	Parameter for price concession adaptation
28	g_{a1}	$[0, 1]$	Fraction of asset the agent is willing to divest at each period
29	g_{a2}	$[90, 115]$	Maximum threshold for asset
30	g_{a3}	$[0, 1]$	Incentive to sell if asset above threshold
31	g_{a4}	$[-2, 0]$	Parameter for price concession
32	g_{a5}	$[-2, 0]$	Parameter for price concession adaptation

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Action

Determining the Agent's Bid

1. the total value of the bid, V_{bid} is determined as

$$V_{\mathrm{bid}} = \begin{cases} M(g_{b1} + (1 - g_{b1})g_{b3}\mathcal{G}(b)), & A < g_{b2}, \\ Mg_{b1}, & A \geq g_{b2}; \end{cases}$$

2. a price concession c_t is determined as

$$c_t = \begin{cases} 2\mathcal{G}(b) + g_{b4}, & \text{first time,} \\ c_{t-1} + (1 - c_{t-1})(4\mathcal{G}(b) + g_{b5}), & \text{if previously unfilled;} \end{cases}$$

3. finally, the bid price and quantity are set as

$$\begin{aligned} p_{\mathrm{bid}} &= p_{t-1}c_t, \\ q_{\mathrm{bid}} &= V_{\mathrm{bid}}/p_{\mathrm{bid}}. \end{aligned}$$

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Determining the Agent's Offer

1. the quantity of the ask, q_{ask} is determined as

$$q_{\text{ask}} = \begin{cases} A(g_{a1} + (1 - g_{a1})g_{a3}\mathcal{G}(s)), & A < g_{a2}, \\ Ag_{a1}, & A \geq g_{a2}; \end{cases}$$

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Overview

Purpose

An evolutionary algorithm is used by the simulator to make the agents participating in the market evolve according to their trading proficiency.

Application

A generation is performed at regular intervals, whose length, i, measured in trading periods, is a parameter of the simulation.

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Fitness and Selection

Fitness

Fitness = NAV.

Redistribution

At each generation, the 30% of the individuals having the lowest NAV dies. Their money and asset inventory are distributed to the surviving agents, proportionately to their NAV.

Selection

Surviving agents are selected for reproduction with a probability proportional to their NAV. Selection is elitist: the agent with the highest NAV is always selected at least once.

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Representation, Initialization, and Reproduction

The parameters that make up the genome of an agent, are encoded as an array of 33 fixed-point numbers.

The genomes of the agents in the initial population are generated at random. However, every agent is constructed so that it takes only 11 of the 22 available technical indicators into account.

Recombination is performed by uniform crossover, whereas mutation applies a Gaussian perturbation (with mean 0 and standard deviation 0.01) to every gene of an agent's genome.

The new agent thus generated inherits 25% of the properties of both parents.

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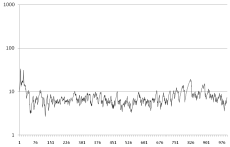
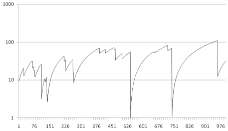
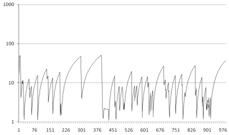
Charts



$i = 1$



$i = 10$



$i = 100$

The distribution of log-returns of the time series of asset prices is leptokurtic:

- for $i = 1$, excess kurtosis is 44;
- for $i = 10$, excess kurtosis is 1.28;
- for $i = 100$, excess kurtosis is 7.44.

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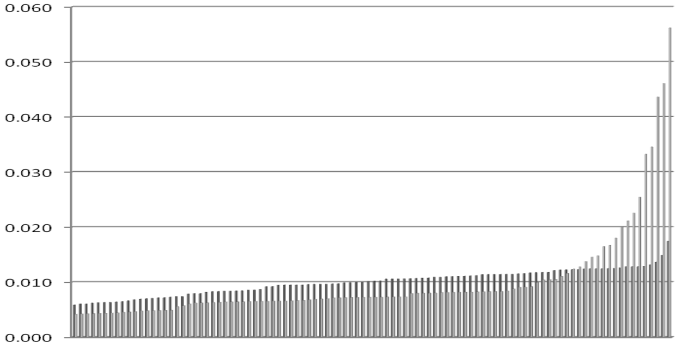
Distribution of Wealth



Distribution of wealth in two simulation scenarios:

- $i = 10$ (dark bars);
- $i = 100$ (light bars).

Conclusion

- The simulated market exhibits the key stylized facts of real financial markets: volatility clustering, leptokurtosis of log-returns, bubbles and crashes, and mean-reverting time series.
- First simulated market using cognitive agents.
- Future Work:
  - Introduce interest on money and dividend-paying securities.
  - Let evolution figure out the set of desire-generation rules without any constraints.

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