AG Quantenoptik und Spektroskopie

Quantum information processing with trapped ions

Courtesy of Timo Koerber
Institut für Experimentalphysik
Universität Innsbruck

1. Basic experimental techniques
2. Two-particle entanglement
3. Multi-particle entanglement
4. Implementation of a CNOT gate
5. Teleportation
6. Outlook

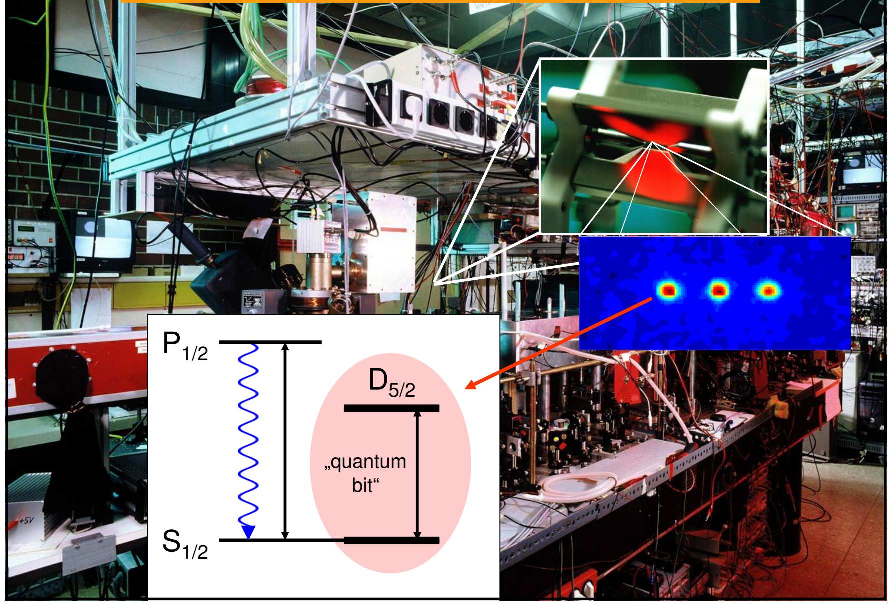
Lectures 16 - 17

The requirements for quantum information processing

D. P. DiVincenzo, Quant. Inf. Comp. **1** (Special), 1 (2001)

I. Scalable physical system, well characterized qubits
II. Ability to initialize the state of the qubits
III. Long relevant coherence times, much longer than gate operation time
IV. “Universal” set of quantum gates
V. Qubit-specific measurement capability

Experimental Setup

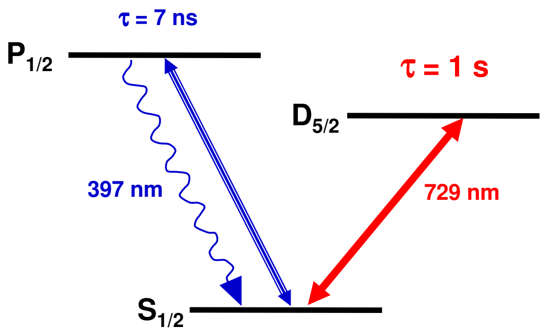


Important energy levels

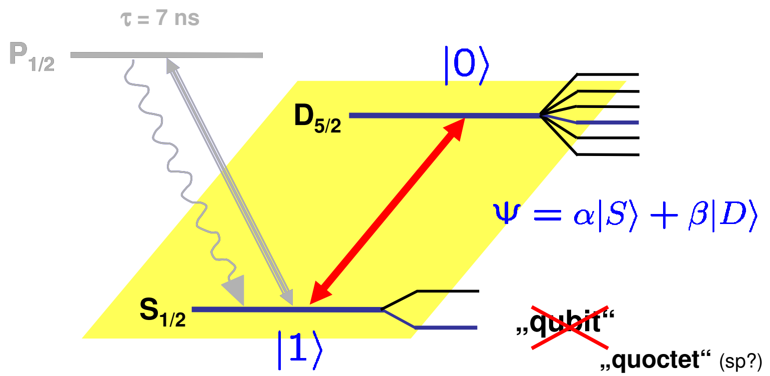
- The important energy levels are shown on the next slides; a fast transition is used to detect ion fluorescence and for Doppler cooling, while the narrow D5/2 quadrupole transition has a lifetime of 1 second and is used for coherent manipulation and represents out quantum bit. Of course a specific set of Zeeman states is used to actually implement our qubit. The presence of other sublevels give us additional possibilities for doing coherent operations.

Ca+: Important energy levels

$S_{1/2} - D_{5/2}$: quadrupole transition



Ca+: Important energy levels



Qubits with trapped ions

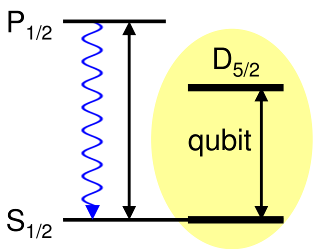
Encoding of quantum information requires long-lived atomic states:

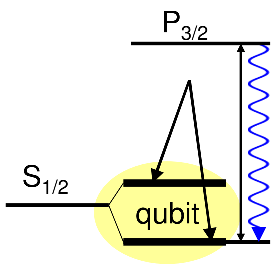
- optical transitions

  Ca^+, Sr^+, Ba^+, Ra^+, Yb^+, Hg^+ etc.

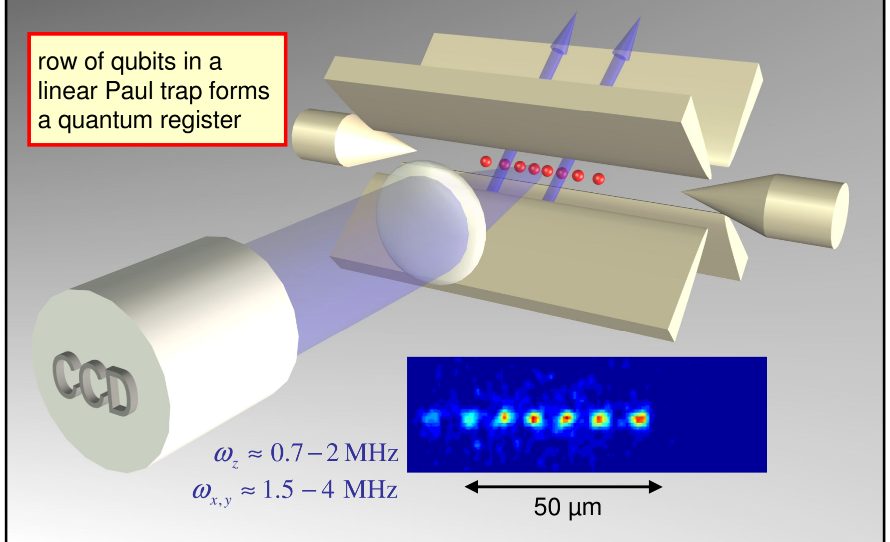
- microwave transitions

  $^{9}Be^+$, $^{25}Mg^+$, $^{43}Ca^+$, $^{87}Sr^+$, $^{137}Ba^+$, $^{111}Cd^+$, $^{171}Yb^+$

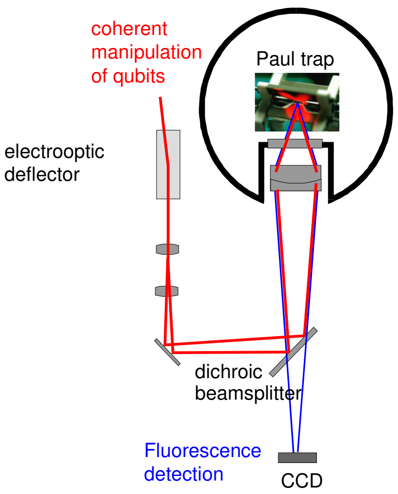


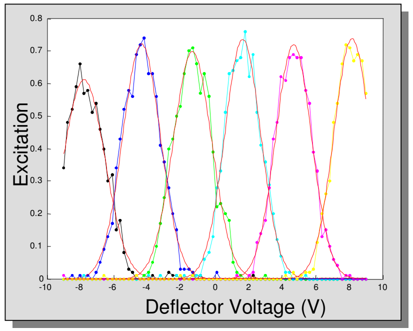


String of Ca+ ions in linear Paul trap



Addressing of individual ions





- inter ion distance: ~ 4 µm
- addressing waist: ~ 2.5 µm

< 0.1% intensity on neighbouring ions

Ion addressing

The ions can be addressed individually on the qubit transition with an EO deflector which can quickly move the focus of the 729 light from one ion to another, using the same optical path as the fluorescence detection via the CCD camera.

How well the addressing works is shown on the previous slide: The graph shows the excitation of the indiviual ions as the deflector is scanned across the crystal.

External degree of freedom: ion motion

Notes for next slides:

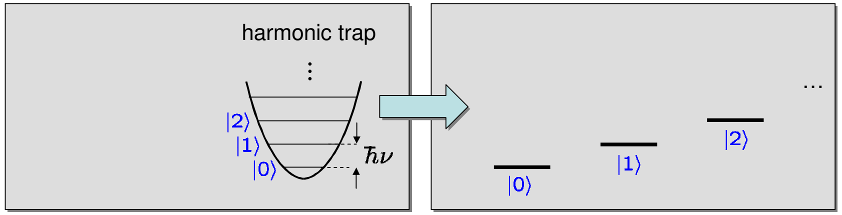
Now let's have a look at the qubit transition in the presence of the motional degrees of freedom. If we focus on just one motional mode , we just get a ladder of harmonic oscillator levels.

The joint (motion + electronic energy level) system shows a double ladder structure. With the narrow laser we can selectively excite the carrier transition, where the motional state remains unchanged...

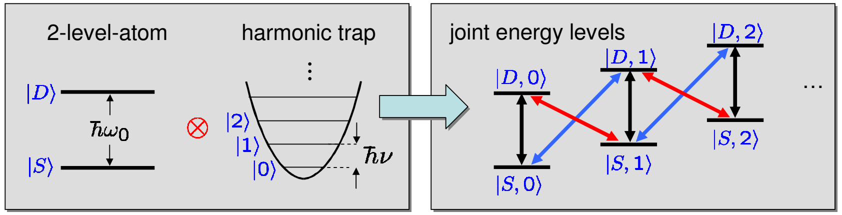
Or use the blue sideband and red sideband transitions, where we can change the motional state.

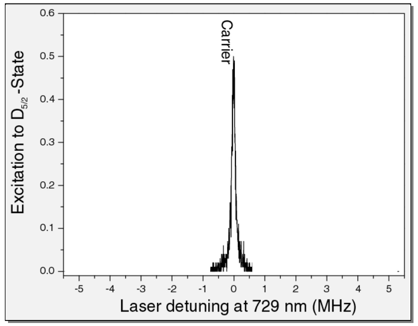
We can walk down the double ladder by exciting the red sideband and returning the ion dissipatively to the grounsstate. With this we can prepare the ions in the motional ground state with high probability, thereby initializing our quantum register.

External degree of freedom: ion motion

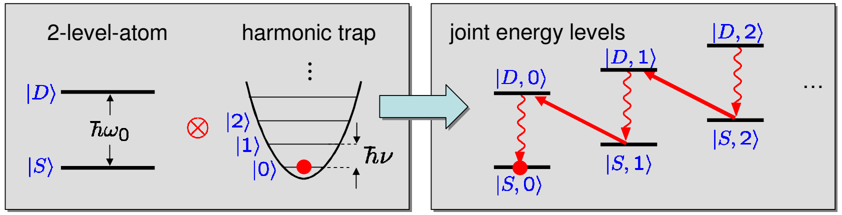


External degree of freedom: ion motion





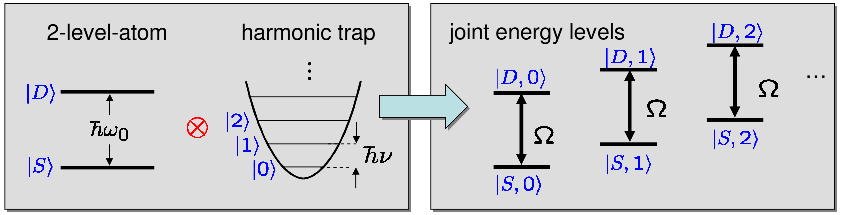
External degree of freedom: ion motion



Laser cooling to the motional ground state:

Cooling time: 5-10 ms
> 99% in motional ground state

Coherent manipulation



Interaction with a resonant laser beam :

$$H_I = \hbar\Omega \left(|D\rangle\langle S| e^{i\phi} + |S\rangle\langle D| e^{-i\phi} \right)$$

Ω : Rabi frequency

ϕ : phase of laser field

Laser beam switched on for duration τ :

$$U = e^{-i\frac{H}{\hbar}\tau}$$

θ : rotation angle $\quad \theta = 2\Omega\tau$

If we resonantly shine in light pulse at the carrier transition, the system evolves for a time tau with this Hamiltonian, where the coupling strength Omega depends on the sqroot of the intensity, and phi is the phase of the laser field with respect to the atomic polarization.

Coherent manipulation

Let's now begin to look at the coherent state manipulation. If we resonantly shine the light pulse at the carrier transition, the system evolves for a time τ with this Hamiltonian, where the coupling strength Ω depends on the square root of the intensity, and ϕ is the phase of the laser field with respect to the atomic polarization.

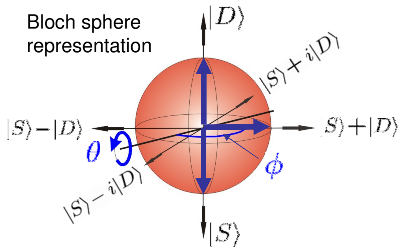
The effect of such a pulse is a rotation of the state vector on the Bloch sphere, where the poles represent the two states and the equator represents superposition states with different relative phases. The roation axis is determined by the laser frequency and phase. The important message is here that we can position the state vector anywhere on the Bloch sphere, which is a way of saying that we can create arbitrary superposition states.

The same game works for sideband pulses. With a $\pi/2$ pulse, for example, we entangle the internal and the motional state! Since the motional state is shared by all ions, we can use the motional state as a kind of bus to mediate entanglement between different qubits in the ion chain.

Coherent excitation: Rabi oscillations

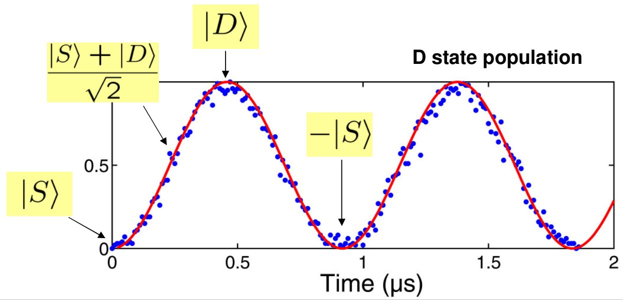
„Carrier“ pulses:

$|S\rangle \longleftrightarrow |D\rangle$



$\theta/2 = \Omega\tau$

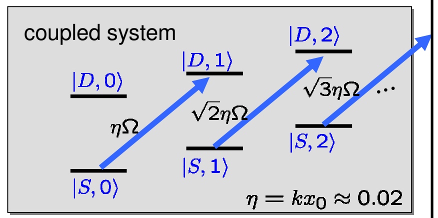
τ2π?



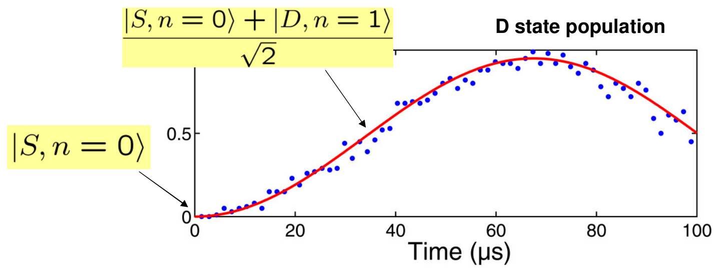
Coherent excitation on the sideband

„Blue sideband“ pulses:

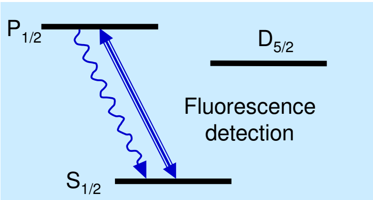
$|S\rangle|n\rangle \longleftrightarrow |D\rangle|n+1\rangle$



$\theta = \pi/2$: Entanglement between internal and motional state !

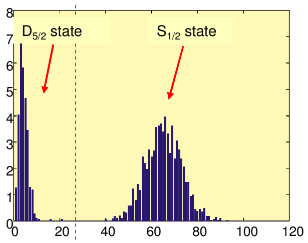


Experimental procedure



1. Initialization in a pure quantum state: laser cooling,optical pumping
2. Quantum state manipulation on $S_{1/2} – D_{5/2}$ qubit transition
3. Quantum state measurement by fluorescence detection

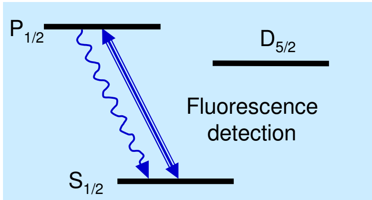
One ion : Fluorescence histogram



50 experiments / s

Repeat experiments 100-200 times

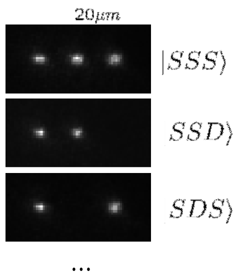
Experimental procedure



1. Initialization in a pure quantum state: Laser sideband cooling
2. Quantum state manipulation on $S_{1/2} – D_{5/2}$ transition
3. Quantum state measurement by fluorescence detection

Multiple ions:

Spatially resolved detection with CCD camera:

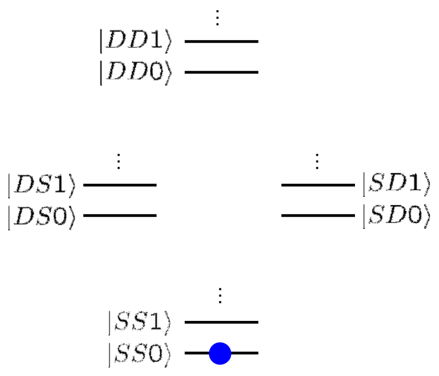


50 experiments / s

Repeat experiments 100-200 times

$$\frac{1}{\sqrt{2}}(|S\rangle|D\rangle + |D\rangle|S\rangle)$$

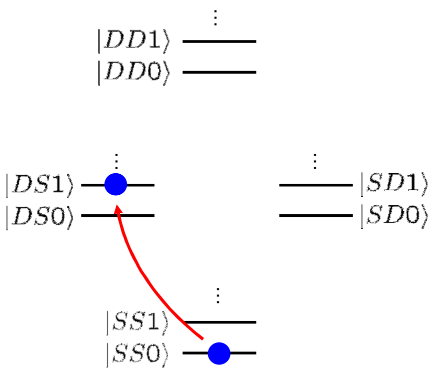
Creation of Bell state



Pulse sequence:

$|SS0\rangle$

Creation of Bell states

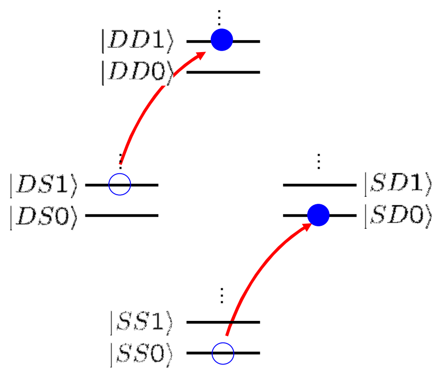


Pulse sequence:

Ion 1: π/2 , blue sideband

$$|SS0\rangle + |DS1\rangle$$

Creation of Bell states



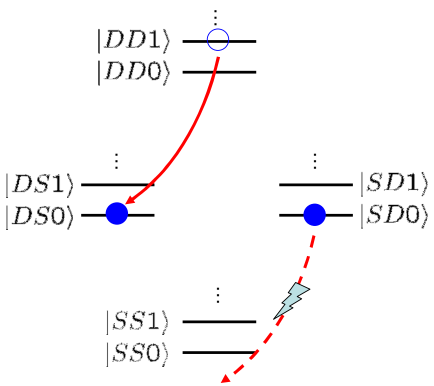
Pulse sequence:

Ion 1: π/2 , blue sideband

Ion 2: π , carrier

$$|SD0\rangle + |DD1\rangle$$

Creation of Bell states



Pulse sequence:

Ion 1: $\pi/2$, blue sideband

Ion 2: π , carrier

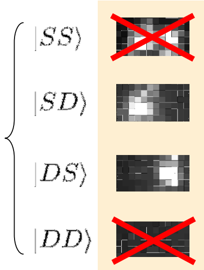
Ion 2: π , blue sideband

$$(|SD\rangle + |DS\rangle)|0\rangle$$

Analysis of Bell states

$$|SD\rangle + |DS\rangle$$

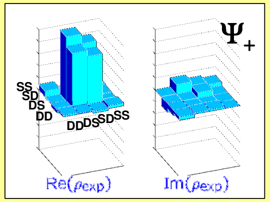
Fluorescence detection with CCD camera:



Coherent superposition or incoherent mixture ?

What is the relative phase of the superposition ?

$\longrightarrow$ **Measurement of the density matrix:**



Reconstruction of a density matrix

Representation of ρ as a sum of orthogonal observables A_i :

$$\rho = \sum_i \lambda_i A_i \ \text{ with } \ Tr(A_i A_j) = \delta_{ij}$$

ρ is completely detemined by the expectation values $\langle A_i \rangle$:

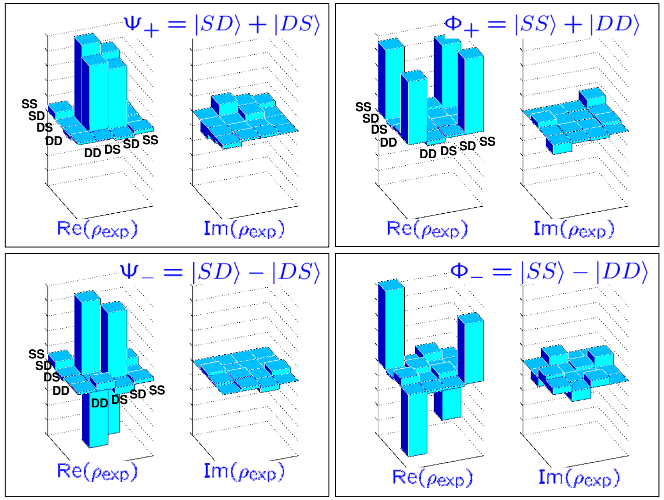
$$\langle A_j \rangle = Tr(\rho A_j) = \sum_i \lambda_i Tr(A_i A_j) = \lambda_j$$

Finally: maximum likelihood estimation (Hradil '97, Banaszek '99)

For a two-ion system : $A_i \in \{\sigma_i^{(1)} \otimes \sigma_j^{(2)}, \sigma_i \in \{I, \sigma_x, \sigma_y, \sigma_z\}\}$

$\longrightarrow$ Joint measurements of all spin components $\sigma_i^{(1)} \otimes \sigma_j^{(2)}$

Preparation and tomography of Bell states



Fidelity:

F = 0.91

Entanglement of formation:

E(ρ_{exp}) = 0.79

Violation of Bell inequality:

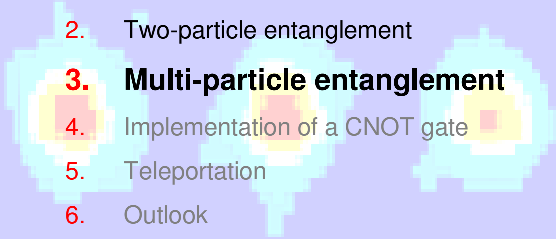
S(ρ_{exp}) = 2.52(6) > 2

C. Roos et al., Phys. Rev. Lett. **92**, 220402 (2004)

Different decoherence porperties

sensitive to:

	laser frequency	magnetic field	exc. state lifetime
$\|S\rangle + \|D\rangle$	⊗	⊗	⊗
$\|SD\rangle + \|DS\rangle$	○	○	⊗
$\|SS'\rangle + \|S'S\rangle$	○	○	○
$\|SS'\rangle + \|DD'\rangle$	⊗	○	⊗



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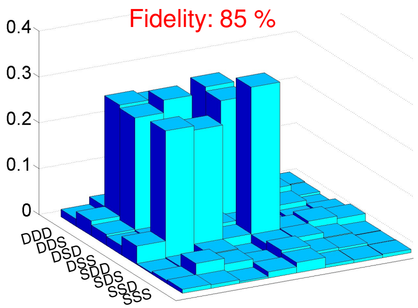
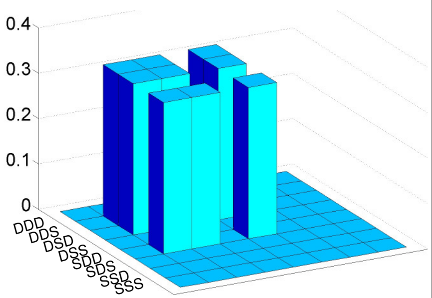
$$\frac{1}{\sqrt{3}}(|SDD\rangle + |DSD\rangle + |DDS\rangle)$$

Density matrix of W – state

$$|\Psi\rangle = \frac{1}{\sqrt{3}}\left(|SDD\rangle + |DSD\rangle + |DDS\rangle\right)$$



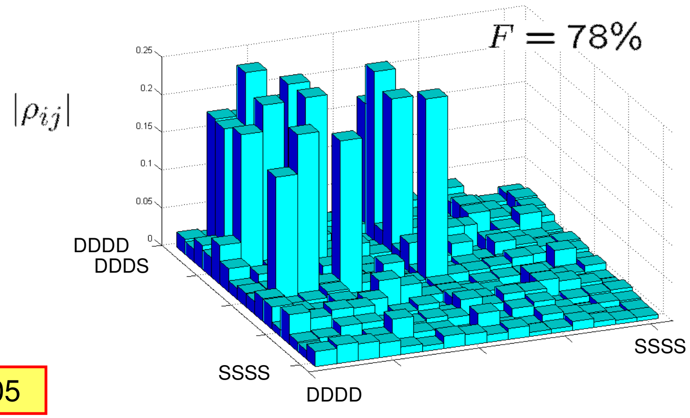
experimental result



theoretical expectation

Four-ion W-states

$$|\Psi_4\rangle = \frac{1}{2}\left(|SDDD\rangle + |DSDD\rangle + |DDSD\rangle + |DDDS\rangle\right)$$

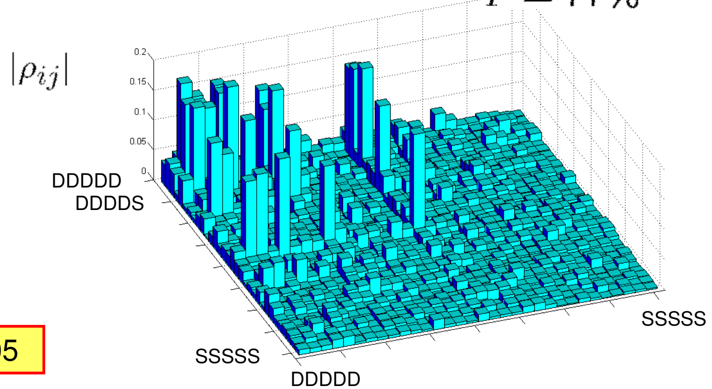


14.4.2005

Five-ion W-states

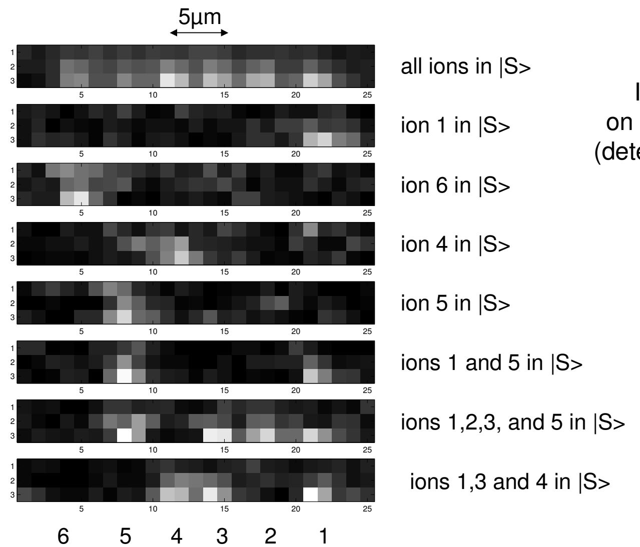
$$\Psi_5 = \frac{1}{\sqrt{5}}(|DDDDS\rangle + |DDDSD\rangle + |DDSDD\rangle + |DSDDD\rangle + |SDDDD\rangle)$$

$F = 77\%$



15.4.2005

Detection of six individual ions

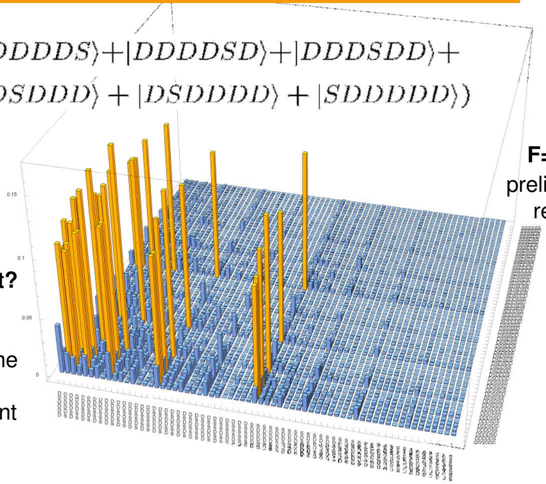


Ion detection
on a CCD camera
(detection time:4ms)

Six-ion W-state

$$\Psi_6 = \frac{1}{\sqrt{6}}(|DDDDDS\rangle + |DDDDSD\rangle + |DDDSDD\rangle + |DDSDDD\rangle + |DSDDDD\rangle + |SDDDDD\rangle)$$

$|\rho_{ij}|$



F=73%
preliminary
result

Is there 6-particle entanglement present?

- 6-particle W-state can be distilled from the state (O. Gühne)
- 6-particle entanglement present, unresolved issues with error bars

22.4.2005

729 settings, measurement time >30 min.

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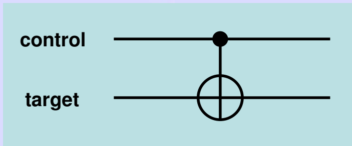


Cirac-Zoller two-ion controlled-NOT operation

VOLUME 74, NUMBER 20 PHYSICAL REVIEW LETTERS 15 MAY 1995

Quantum Computations with Cold Trapped Ions

J. I. Cirac and P. Zoller*

Institut für Theoretische Physik, Universität Innsbruck, Technikerstrasse 25, A-6020 Innsbruck, Austria

(Received 30 November 1994)

A quantum computer can be implemented with cold ions confined in a linear trap and interacting with laser beams. Quantum gates involving any pair, triplet, or subset of ions can be realized by coupling the ions through the collective quantized motion. In this system decoherence is negligible, and the measurement (readout of the quantum register) can be carried out with a high efficiency.

PACS numbers: 89.80.+h, 03.65.Bz, 12.20.Fv, 32.80.Pj

...allows the realization of a ***universal*** quantum computer !

$$|D\rangle|D\rangle \rightarrow |D\rangle|D\rangle$$

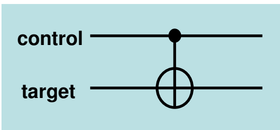
$$|D\rangle|S\rangle \rightarrow |D\rangle|S\rangle$$

$$|S\rangle|D\rangle \rightarrow |D\rangle|S\rangle$$

$$|S\rangle|S\rangle \rightarrow |S\rangle|D\rangle$$

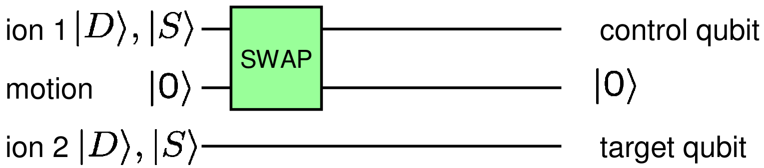
control target

other gate proposals include:
- Cirac & Zoller
- Mølmer & Sørensen, Milburn
- Jonathan & Plenio & Knight
- Geometric phases
- Leibfried & Wineland



Cirac-Zoller two-ion controlled-NOT operation

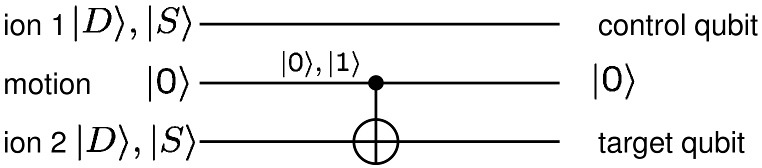
$|\varepsilon_1\rangle$ $|\varepsilon_2\rangle$



First we swap the quantum states of the control qubit and the motional qubit…

Cirac-Zoller two-ion controlled-NOT operation

$|\varepsilon_1\rangle$ $|\varepsilon_2\rangle$

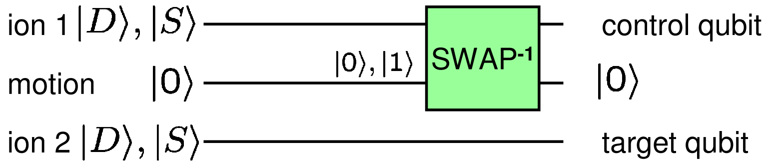


Next we perform a CNOT gate between the motional qubit and ion 2 and …

Cirac - Zoller two-ion controlled-NOT operation

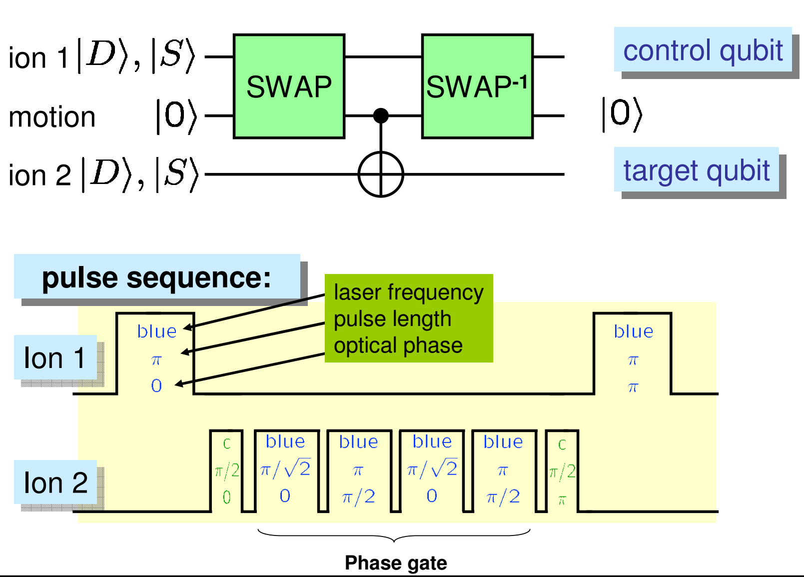
$|\varepsilon_1\rangle$ $|\varepsilon_2\rangle$

Finally we reverse the SWAP operation, we have used the motional qubit as a quantum messenger between the two ions.

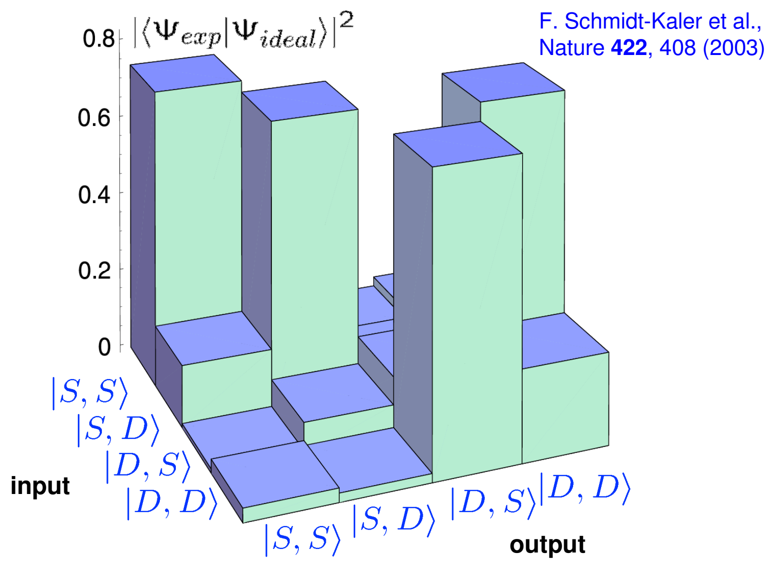


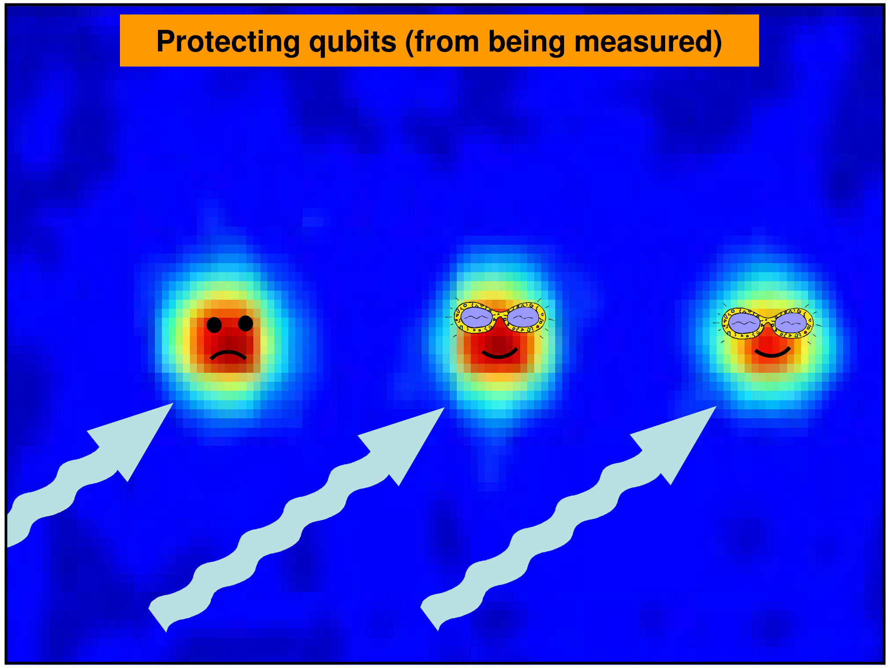
F. Schmidt-Kaler et al., Nature **422**, 408 (2003)

Cirac - Zoller two-ion controlled-NOT operation



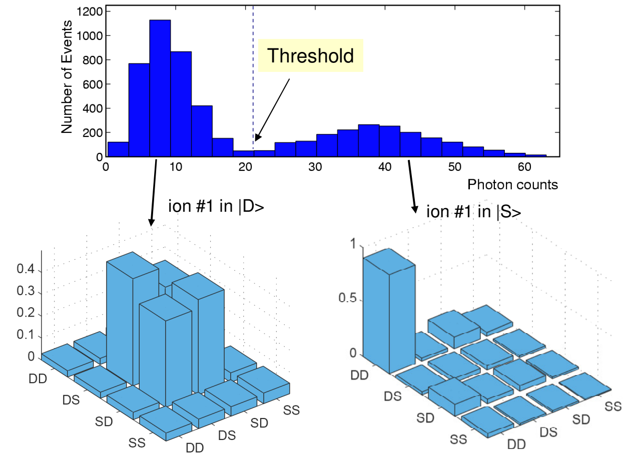
Experimental fidelity of Cirac-Zoller CNOT operation





Selective read-out of an atom in a W-state

$$|\Psi\rangle_W = \frac{1}{\sqrt{3}}\left(|D\underline{DS}\rangle + |D\underline{SD}\rangle + |S\underline{DD}\rangle\right)$$



Tomography **after** the measurement result is available!

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Quantum state teleportation

Phys. Rev. Lett. 70, 1895 (1993)

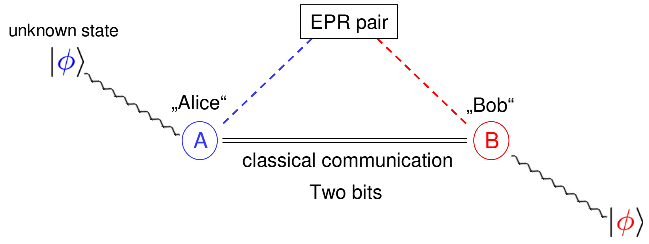
VOLUME 70 29 MARCH 1993 NUMBER 13

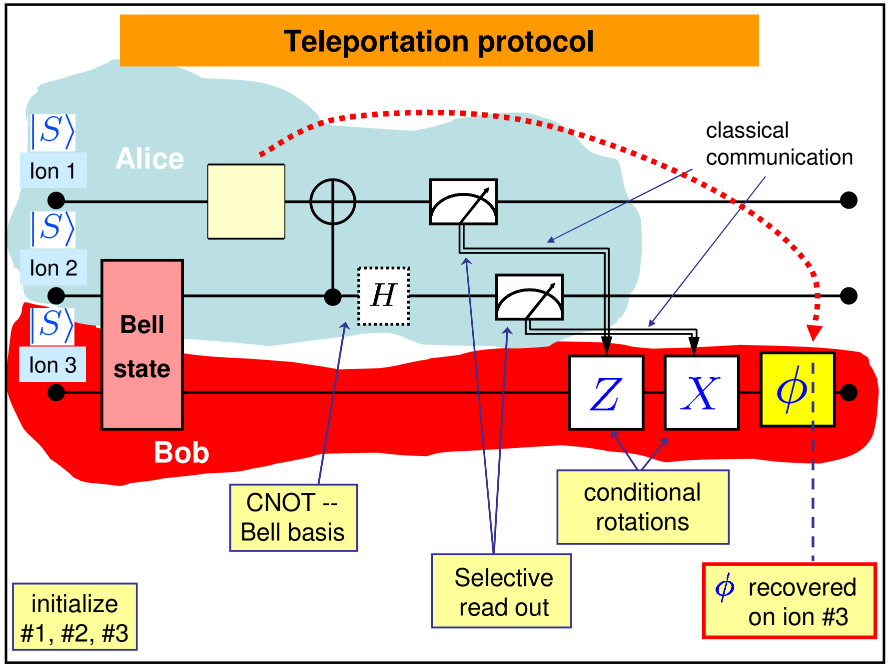
Teleporting an Unknown Quantum State via Dual Classical and Einstein-Podolsky-Rosen Channels

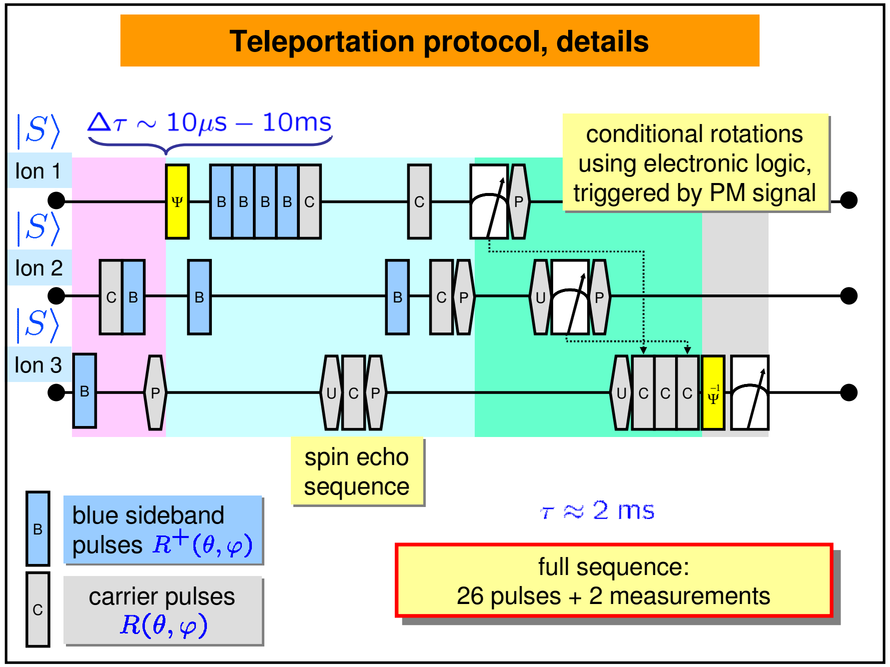
Charles H. Bennett,[1] Gilles Brassard,[2] Claude Crépeau,[2],[3] Richard Jozsa,[2] Asher Peres,[4] and William K. Wootters[5]

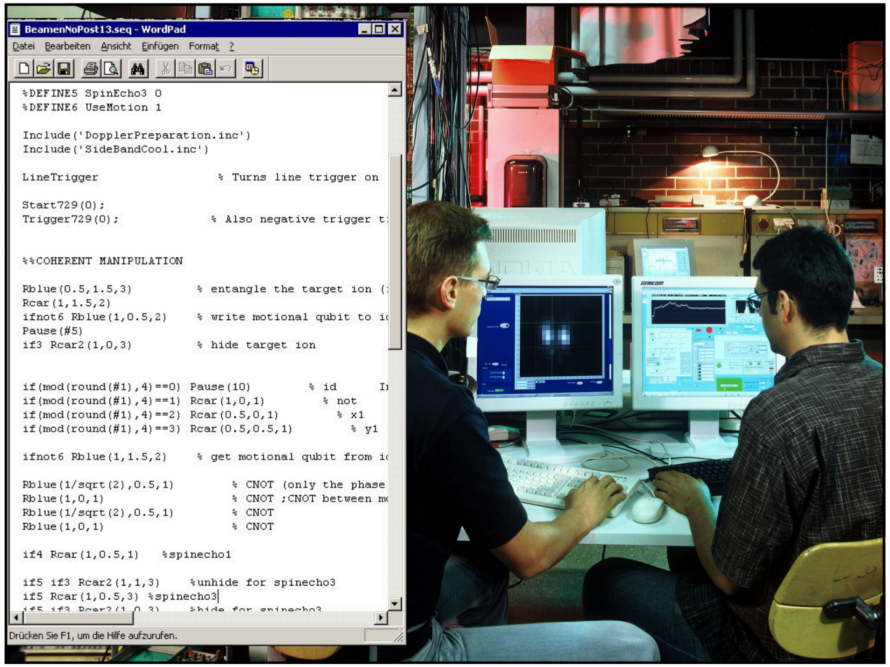
Is it possible to transfer an unknown quantum state $|\phi\rangle = \alpha|D\rangle + \beta|S\rangle$ from „Alice" to „Bob" by classical communication ?

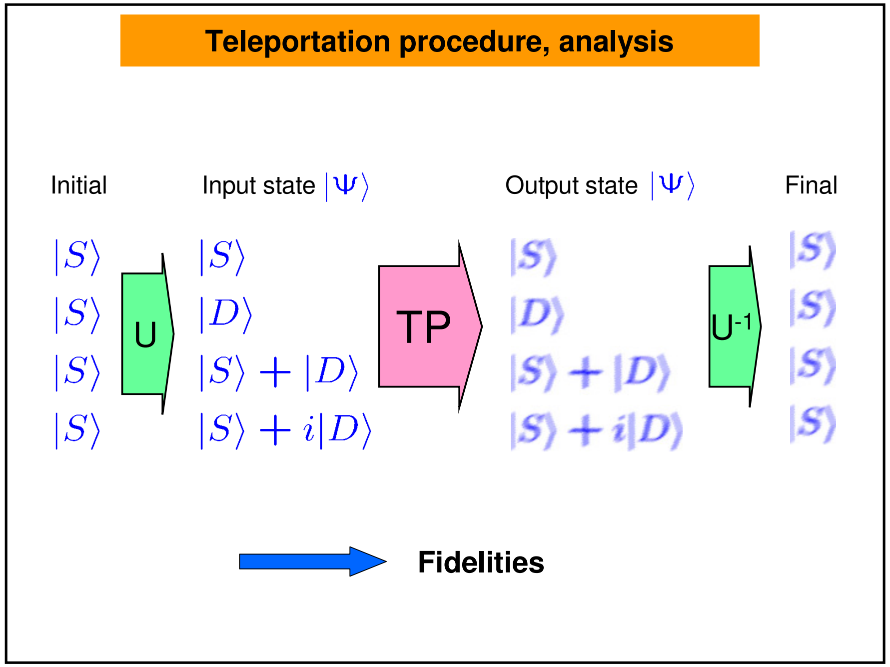
Yes, if Alice and Bob share a pair of entangled particles !



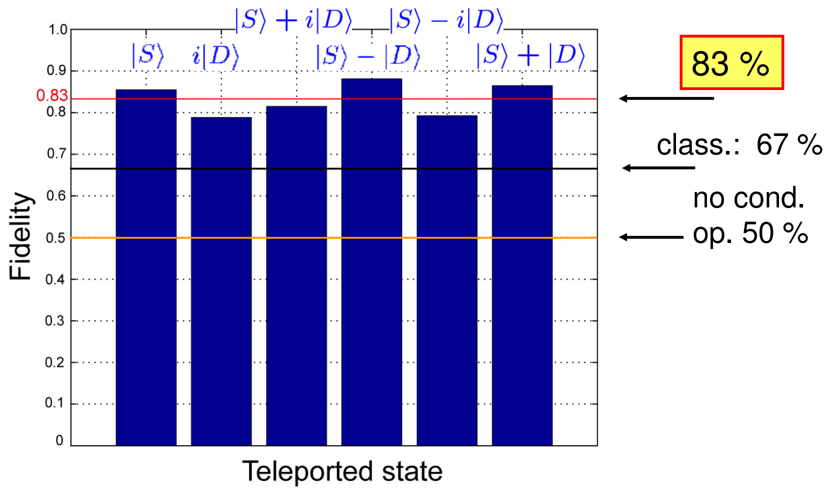

Teleportation protocol
classical communication
Alice
Ion 1
Ion 2
Ion 3
Bell state
H
Z
X
Bob
CNOT -- Bell basis
conditional rotations
Selective read out
initialize #1, #2, #3
recovered on ion #3


Teleportation protocol, details
Ion 1
Ion 2
Ion 3
conditional rotations using electronic logic, triggered by PM signal
spin echo sequence
blue sideband pulses
carrier pulses
τ ≈ 2 ms
full sequence: 26 pulses + 2 measurements


BeamenNoPost13.seq - WordPad
Datei Bearbeiten Ansicht Einfügen Format ?
%DEFINE5 SpinEcho3 0
%DEFINE6 UseMotion 1
Include('DopplerPreparation.inc')
Include('SideBandCool.inc')
LineTrigger % Turns line trigger on
Start729(0);
Trigger729(0); % Also negative trigger
%%COHERENT MANIPULATION
Rblue(0.5,1.5,3) % entangle the target ion
Rcar(1,1.5,2)
ifnot6 Rblue(1,0.5,2) % write motional qubit to
Pause(#5)
if3 Rcar2(1,0,3) % hide target ion
if(mod(round(#1),4)==0) Pause(10) % id
if(mod(round(#1),4)==1) Rcar(1,0,1) % not
if(mod(round(#1),4)==2) Rcar(0.5,0,1) % x1
if(mod(round(#1),4)==3) Rcar(0.5,0.5,1) % y1
ifnot6 Rblue(1,1.5,2) % get motional qubit from
Rblue(1/sqrt(2),0.5,1) % CNOT (only the phase
Rblue(1,0,1) % CNOT ;CNOT between
Rblue(1/sqrt(2),0.5,1) % CNOT
Rblue(1,0,1) % CNOT
if4 Rcar(1,0.5,1) %spinecho1
if5 if3 Rcar2(1,1,3) %unhide for spinecho3
if5 Rcar(1,0.5,3) %spinecho3
Drücken Sie F1, um die Hilfe aufzurufen.

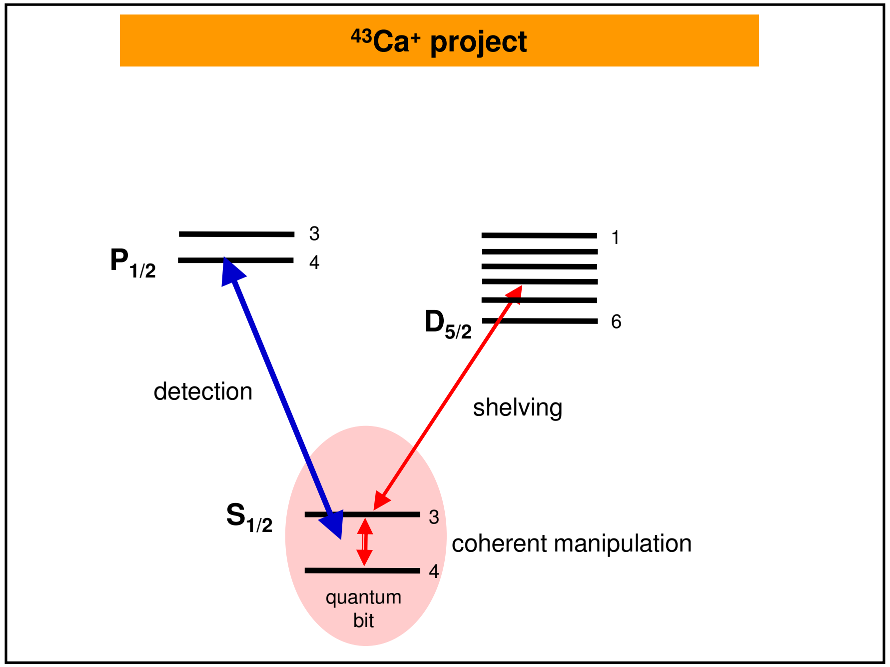

Teleportation procedure, analysis
Initial
Input state $|\Psi\rangle$
Output state $|\Psi\rangle$
Final
$|S\rangle$
$|S\rangle$
$|S\rangle$
$|S\rangle$
U
$|S\rangle$
$|D\rangle$
$|S\rangle + |D\rangle$
$|S\rangle + i|D\rangle$
TP
$|S\rangle$
$|D\rangle$
$|S\rangle + |D\rangle$
$|S\rangle + i|D\rangle$
U^{-1}
$|S\rangle$
$|S\rangle$
$|S\rangle$
$|S\rangle$
Fidelities

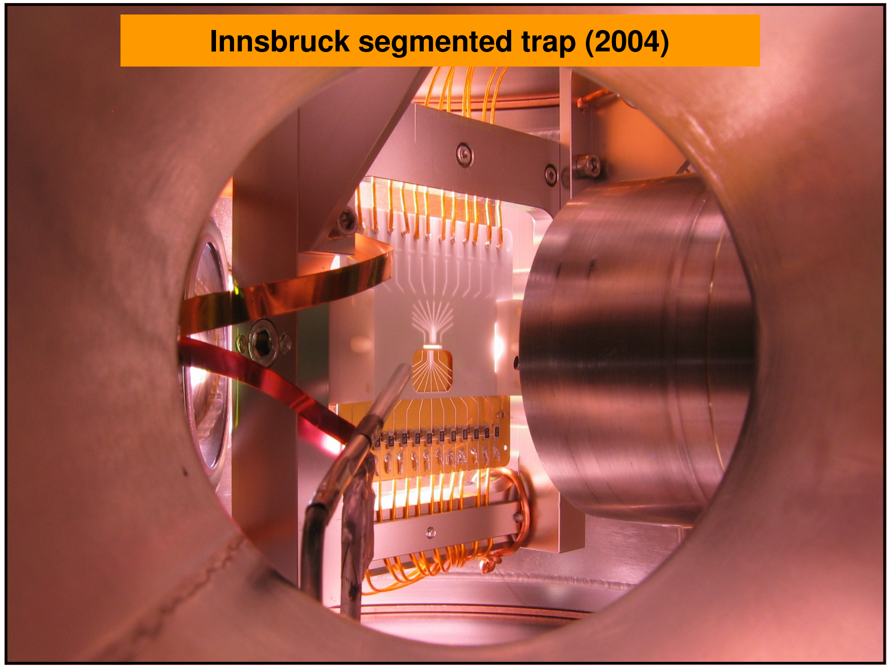
Quantum teleportation with atoms: results



1. Basic experimental techniques
2. Two-particle entanglement
3. Multi-particle entanglement
4. Implementation of a CNOT gate
5. Teleportation
6. **Outlook**

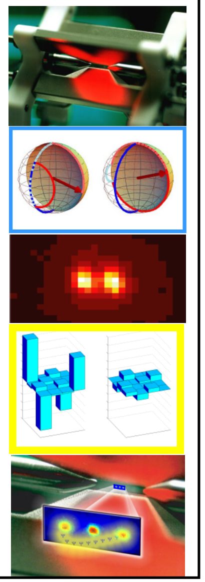
- optimization of Cirac-Zoller gate
  achieve 3 - 5 CNOT gate operations
- error correction protocols with three and five qubits
- qubit manipulation in DFS
- implementation with $^{43}Ca^{+}$
- test of segmented traps


43Ca+ project
P1/2
3
4
D5/2
1
6
detection
shelving
S1/2
3
4
coherent manipulation
quantum
bit


Innsbruck segmented trap (2004)

Summary

- Basic experimental techniques
- Bell, W and GHZ states – up to 6 ions
- State and process tomography
- C-not gate
- Deterministic teleportation with atoms
- Future: Ca43-qubit, segmented traps



The Innsbruck ion trap group

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References :

„*Realization of a controlled-NOT quantum gate*“, F. Schmidt-Kaler et al., Nature. **422**, 408 (2003)

„*Bell states of atoms with ultralong lifetimes*“, C. Roos et al., Phys. Rev. Lett. **92**, 220402 (2004)

„*Control and measurement of three-qubit entangled states*“, C. Roos et al., Science **304**, 1478 (2004)

„*Determistic quantum teleportation with atoms*“, M. Riebe et al., Nature **429**, 734 (2004)