Time Value of Money

- Future value
- Present value
- Rates of return

If you deposit $100 in an account that pays 6% annual interest, what amount will you expect to have in the account at the end of the year.

Future value

$100	(starting value = present value (PV))
6	(interest = (0.06)(100) = 6)
$106	(ending value = future value (FV))

$$FV = PV + PV\ (\%\ change)$$

$$FV = PV\ (1 + \%\ change)$$

$$FV = PV\ (1 + i)$$

What if we leave the money for two years?

$ 100	(present value (PV))
6	(interest = 100*0.06 = 6)
$ 106	(future value year 1 (FV_1))
6.36	(interest = 106*0.06 = 6.36)
$112.36	(future value year 2 (FV_2))

How do we come up with a formula for multiple periods?

$106 = 100 (1 + 0.06)$

$FV_1 = PV (1 + i)$

$112.36 = 106 (1 + 0.06)$

$FV_2 = FV_1 (1 + i)$ (but from above)

$FV_2 = PV (1 + i) (1 + i)$

$FV_2 = PV (1 + i)^2$

Future value

In general, for any number of periods:

$FV_n = PV (1 + i)^n$

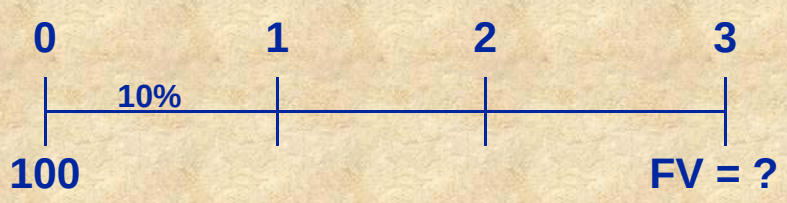
If interest is compounded during the year, we change the formula to:

$FV_n = PV (1 + i/m)^{n*m}$

Four Ways to Find FVs

- Solve the equation (using a regular calculator).
- Use tables.
- Use a financial calculator.
- Use a spreadsheet.

What's the FV of an initial $100 after 3 years if i = 10%?



Finding FVs is compounding.

Solve Equation

$FV_3 = PV(1 + i)^3$
$= \$100(1.10)^3 = \$100(1.331)$
$= \$133.10.$

On calculator:

1.10 $\boxed{y^x}$ 3 $\boxed{=}$ 1.331

100 $\boxed{X}$ 1.331 $\boxed{=}$ 133.10

Using tables

	2%	4%	6%	8%	10%
1	1.0200	1.0400	1.0600	1.0800	1.1000
2	1.0404	1.0816	1.1236	1.1664	1.2100
3	1.0612	1.1249	1.1910	1.2597	1.3310
4	1.0824	1.1699	1.2625	1.3605	1.4641
5	1.1041	1.2167	1.3382	1.4693	1.6105

FV_3 = PV (FVIF) = 100 (1.3310) = 133.10

Valuation using the calculator:

N = total number of periods

I = interest rate per period

PV = present value

FV = future value

BA II Plus

- **Initially:**
  - **2nd P/Y 1 ENTER CE/C**
  - **2nd FORMAT 4 ENTER CE/C**
- **To clear display:**
  - **CE/C**
- **To clear time value keys:**
  - **2nd CLRTVM**
- **Enter values by typing the number and then pressing the key**

HP10BII

- (2nd indicates the second function key)
- Initially:
  - 1 2nd P/YR
  - 2nd DISP 4
- To clear display:
  - C
- To clear time value keys:
  - 2nd C ALL
- Enter values by typing the number and then pressing the key

TI-83

- Older versions:
  - 2nd Finance
- Newer versions:
  - APPS Finance
- Next Choose: TVM_Solver
- Be sure P/Y and C/Y at bottom are set equal to 1

Financial Calculator Solution

INPUTS	3	10	-100		
	N	I/YR	PV	PMT	FV
OUTPUT					

Type numbers then hit key for BAII+ and HP10B Plus.
Type numbers beside corresponding item for TI-83 (use arrow keys to move)

Financial Calculator Solution

INPUTS	3	10	-100		
	N	I/YR	PV	PMT	FV
OUTPUT					133.10

To get answer:
BAII+: Press CPT and then key (FV)
HP10B Plus: Just press key (FV)
TI-83: Use arrow keys to put cursor next to item for which you are solving, then press Alpha Solve

Spreadsheet (Excel)

Formulas can be entered into spreadsheets to calculate the time value of money, or you can use available financial functions.

FV(rate,nper,pmt,pv,type)

- rate is the interest rate per period.
- nper is the number of periods.
- pmt is the payment amount per period.
- pv is the starting value.
- type indicates whether payments occur at the beginning or end of each period.

Spreadsheet (Excel)

- NOTE: pmt and type are for annuities. For lump sum problems set pmt equal to zero and ignore type.

Enter

=FV(0.1,3,0,-100)

$133.10

Answer

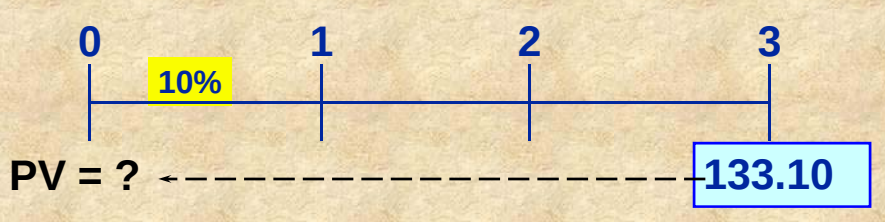
Present value

When we talk about present values of future cash flows, we use the same type of analysis.

All we do is rearrange the equation to solve for the present value.

What's the PV of $133.10 due in 3 years if i = 10%?

Finding PVs is <u>discounting</u>, and it's the reverse of compounding.



Solve $FV_n = PV(1 + i)^n$ for PV:

$$PV = \frac{FV_n}{(1+i)^n} = FV_n\left(\frac{1}{1+i}\right)^n.$$

$$PV = \$133.10\left(\frac{1}{1.10}\right)^3 = \$100(PVIF_{i,n})$$

$$= \$133.10(0.7513) = \$100$$

Financial Calculator Solution

INPUTS	3	10		0	133.10
	N	I/YR	PV	PMT	FV
OUTPUT			-100		

This means that if you put in $100 today earning 10% per year you would have $133.10 after 3 years.

We will deal with 3 different rates:

i_{Nom} = nominal, or quoted, or stated rate per year.

i_{Per} = periodic rate.

EAR = EFF% = effective annual rate

What is the FV of $100 after 3 years under 10% semiannual compounding?

	N	I/YR	PV	PMT	FV
INPUTS	3x2	10/2	-100		
OUTPUT					134.01

What is the PV of $500 received in 3 years under 10% semiannual compounding?

	N	I/YR	PV	PMT	FV
INPUTS	3x2	10/2			500
OUTPUT			373.11		

- **Effective Annual Rate** (EAR = EFF%):

The annual rate that causes PV to grow to the same FV as under multi-period compounding.

$$EFF = \left(1 + \frac{i_{Nom}}{m}\right)^m - 1$$

$$= \left(1 + \frac{0.10}{2}\right)^2 - 1.0$$

$$= (1.05)^2 - 1.0$$

$$= 0.1025 = 10.25\%.$$

Will the FV of a lump sum be larger or smaller if we compound more often, holding the stated I% constant? Why?

LARGER! If compounding is more frequent than once a year--for example, semiannually, quarterly, or daily--interest is earned on interest more often.

Will the PV of a lump sum be larger or smaller if we compound more often, holding the stated I% constant? Why?

SMALLER! If compounding is more frequent than once a year--for example, semiannually, quarterly, or daily--interest is earned on interest more often so you can start with a smaller amount and reach the same goal in the same amount of time.

On a financial calculator:

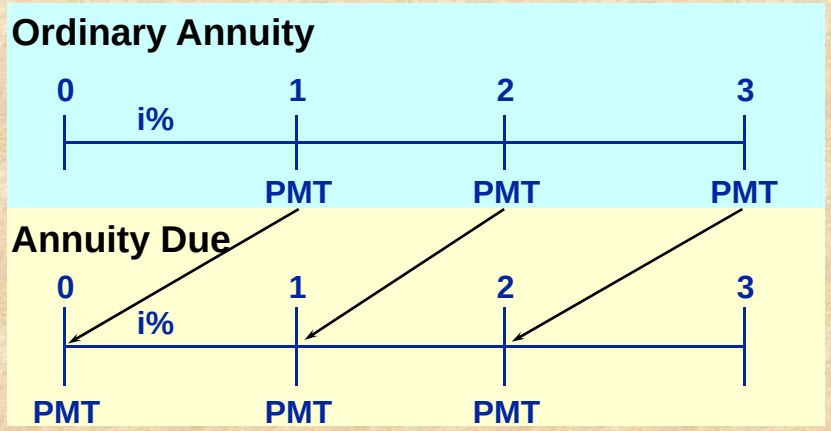
	N	I/YR	PV	PMT	FV
INPUTS	2	10/2	-100		
OUTPUT					110.25

EAR = 110.25 – 100 = 10.25%

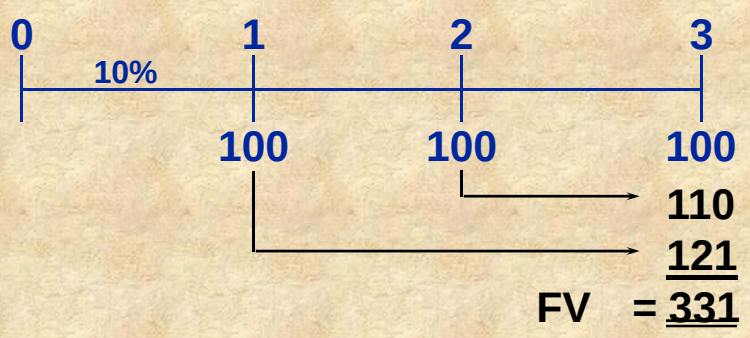
Any PV would grow to same FV at 10.25% annually or 10% semiannually.

The EAR is used to compare returns on investments with different compounding.

Annuities are sets of equal payments received at equal time intervals.



What's the FV of a 3-year ordinary annuity of $100 at 10%?



Annuity Valuation using the calculator:

N = total number of payments

I = interest rate per payment period

PV = present value

FV = future value

PMT = payment each period

END MODE for ordinary annuity

BEGIN MODE for annuity due

Begin/End Mode

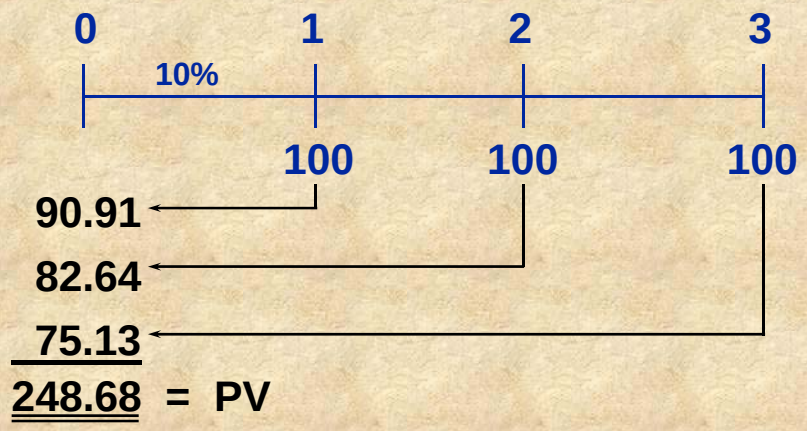
- BAII Plus:
  - 2nd BGN 2nd Set CE/C
- HP 10B Plus
  - 2nd BEG/END
- TI-83
  - At bottom of time value screen:
  - Use arrows keys to highlight Begin or End and then press ENTER key

Financial Calculator Solution

INPUTS	3	10		-100	
	N	I/YR	PV	PMT	FV
OUTPUT					331.00

You should be in “end” mode when you calculate the answer.

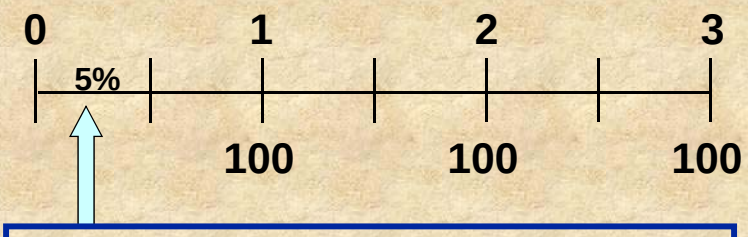
What's the PV of this ordinary annuity?



INPUTS	3	10		100	
	N	I/YR	PV	PMT	FV
OUTPUT			-248.69		

Again, make sure you are in "end" mode when you calculate the answer.

What's the value at the end of Year 3 of the following CF stream if the quoted interest rate is 10%, compounded semiannually?



10% compounded semiannually is 5% each ½ year.

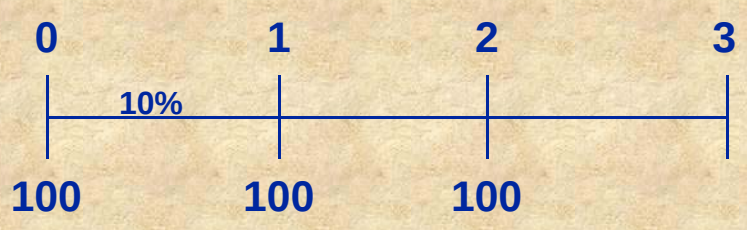
a. The cash flow stream is an annual annuity. First find EAR.

$$EAR = \left(1 + \frac{0.10}{2}\right)^2 - 1 = 10.25\%.$$

b. Calculate FV using EAR as interest rate.

	N	I/YR	PV	PMT	FV
INPUTS	3	10.25		-100	
OUTPUT					331.80

Find the FV and PV if the annuity were an annuity due with annual compounding of interest.

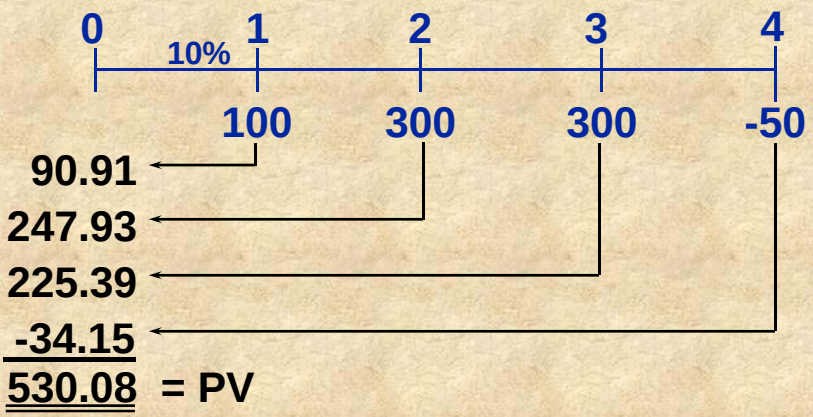


Switch from "End" to "Begin."

	N	I/YR	PV	PMT	FV
INPUTS	3	10		100	
OUTPUT					$364.10

	N	I/YR	PV	PMT	FV
INPUTS	3	10		100	
OUTPUT			-273.55		

What is the PV of this uneven cash flow stream?



- **Input in "CFLO" register:**

  CF_0 = 0

  CF_1 = 100

  CF_2 = 300

  CF_3 = 300

  CF_4 = -50

- **Enter I = 10, then press NPV button to get NPV = $530.09. (Here NPV = PV)**
- **TI-83:**
- **npv(10,0,{100,300,300,-50}) Enter**

Uneven cashflows

- Suppose you are offered an investment that pays $10,000 per year the first 8 years, $20,000 per year the next 12 years, and $30,000 per year the following 15 years. If the appropriate discount rate is 9%, what is the present value of the investment?

- Input in "CFLO" register:

  CF_0 = 0

  CF_1 = 10000 Frequency = 8

  CF_2 = 20000 Frequency = 12

  CF_3 = 30000 Frequency =15

- Enter I = 9, then press NPV button to get NPV = $170,371
- TI-83:
- npv(10,0,{10000,20000,30000},{8,12,15}) Enter

Uneven cashflows

- Suppose you are offered an investment that pays $10,000 per year the first 8 years, $20,000 per year the next 12 years, and $30,000 per year the following 15 years. If you invest all of the cashflows at an annual interest rate of 9%, what will be the future value of the cashflows at the end of the 35 years?

Future value of uneven cashflows

- We first calculate the PV of the uneven CFs, and then calculate the FV.
- From previous problem we have:
  - PV = 170,371

INPUTS	35	9	-170371		
	N	I/YR	PV	PMT	FV
OUTPUT					3,477,948.13