Honors Classical Physics I

PHY141

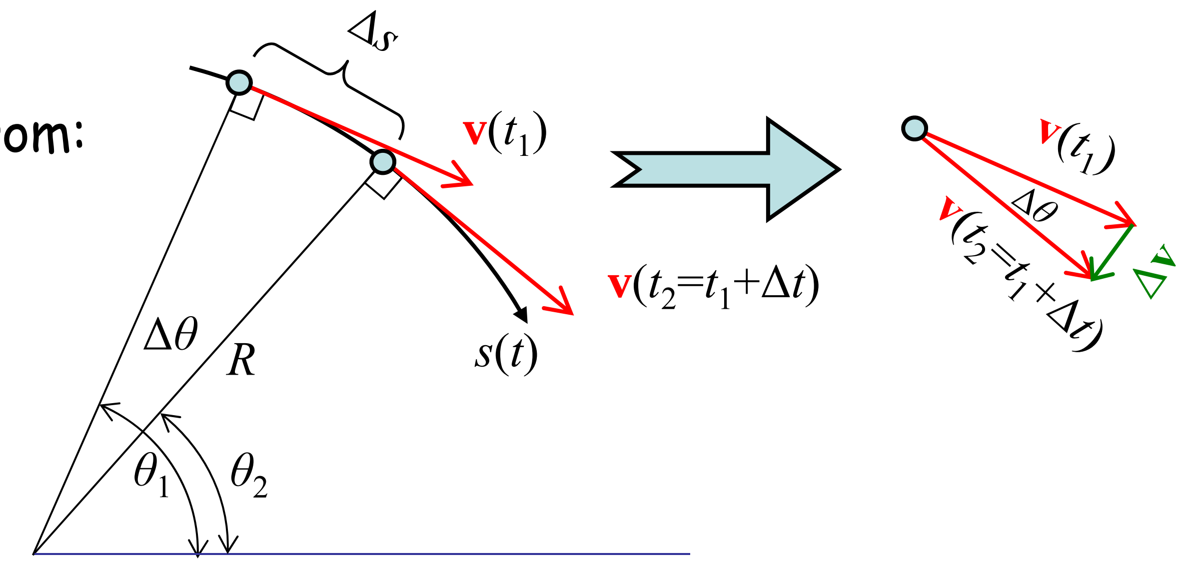
Lecture 8

Circular motion: dynamics

Forces in Circular Motion

- We saw before:

  $a_{rad} = a_c = v^2/R$
- This was derived from:



- From this:

$$\mathbf{a} \equiv \frac{d\mathbf{v}}{dt} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t} \neq 0$$

$$\text{Thus} \quad a = \lim_{\Delta t \to 0} \frac{|\Delta \mathbf{v}|}{\Delta t} = \lim_{\Delta t \to 0} \frac{v\Delta\theta}{\Delta t} = \lim_{\Delta t \to 0} \frac{v\,\Delta s/R}{\Delta t} = \frac{v}{R} \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{v}{R} v = \frac{v^2}{R} = \omega^2 R$$

- Direction of **a**: pointing towards center of rotation! (*centripetal*)
- If there is an $\mathbf{a}_c$, there must be a Net Force $\mathbf{F}_c$ causing it! (2nd Law)

.

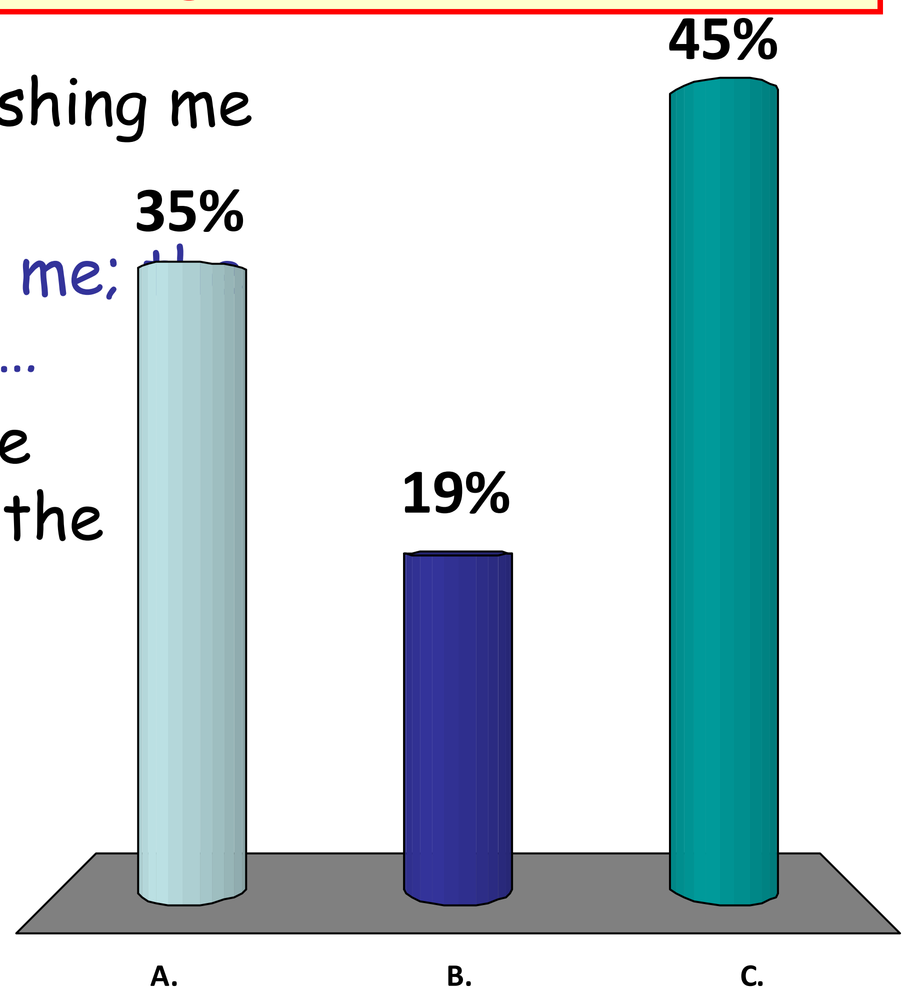
Choose the correct statement:

I'm a passenger in a car going at high, but constant speed through a curve…

A. The centrifugal force pushing me sideways…

B. There is no real force on me; the centrifugal force is fake…

C. The seatbelt and seat are accelerating me through the curve…



Free Body Diagram

- net force for ***uniform*** circular motion with speed v on a circle of radius R:

$$F_{Net} = ma_{rad} = ma_c = m\frac{v^2}{R}$$

(directed to the center of the circle!)

  - a simple way of "deriving" this is by use of units: $[a] = \mathrm{m/s^2}$ ➔ using v and R ➔ $a \propto [v^2]/[R] = \mathrm{m^2/s^2/m} = \mathrm{m/s^2}$

- in case of ***non-uniform*** circular motion, with instantaneous speed v:

$$\mathbf{F}_{Net} = m\mathbf{a} = m(\mathbf{a}_c + \mathbf{a}_{//})$$

$$\text{with:} \quad a_c = \frac{v^2}{R}$$

Example 1

- A car is going through a turn of radius R=40 m (about 130 ft). At a certain point in the turn the car has a speed of 36 mi/hr (i.e. v=36*1650m/3600 s =16.5 m/s), and is accelerating with 1.0 m/s^2 in the direction of motion: $a_{//} = 1.0 \text{ m/s}^2$.
  - Q1: calculate the **centripetal acceleration** $a_\perp$ at that time

$$a_c = \frac{v^2}{R} = \frac{(16.5\text{ m/s})^2}{40\text{ m}} = 6.8\text{ m/s}^2$$

  - Note that this is large, about 70% of g!
  - Q2: calculate the TOTAL acceleration.
    - Note: acceleration is a VECTOR, and thus I must specify either the *__components__* (in a well-defined axis system) or the *__magnitude and direction__* (with respect to a pre-defined axis)

$$\mathbf{a} = a_{//}\mathbf{i}_{//} + a_c\mathbf{j}_\perp = (1.0\,\mathbf{i}_{//} + 6.8\mathbf{j}_\perp)\text{m/s}^2, \qquad \text{or:}$$

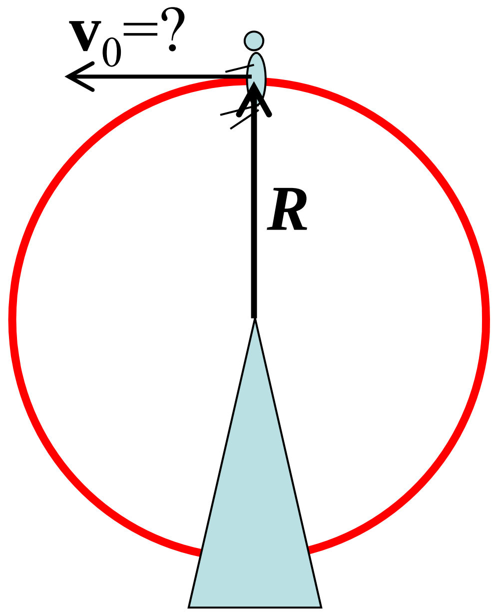
$$a = \sqrt{{a_{//}}^2 + {a_c}^2} = \sqrt{1.0^2 + 6.8^2} = 6.9\text{ m/s}^2 \text{ (magnitude)}$$

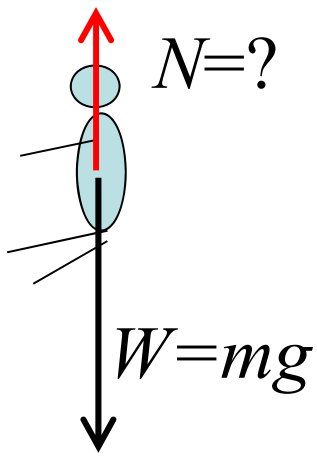
$$\varphi = \arctan\left(\frac{a_c}{a_{//}}\right) = \arctan\left(\frac{6.8}{1.0}\right) = 82^\circ \text{ w.r.t motion (direction)}$$

(inwards)

Example 2

- I'm sitting in a vertical Ferris Wheel of radius R. At the top of the ride I feel "weightless". Calculate the speed of the wheel…
- Sketch:





- Discuss:
  - "Weightless" means … I don't feel the seat pressing up onto my pants: **N**=0 !
  - NET FORCE here is therefore **W** !
  - Thus:

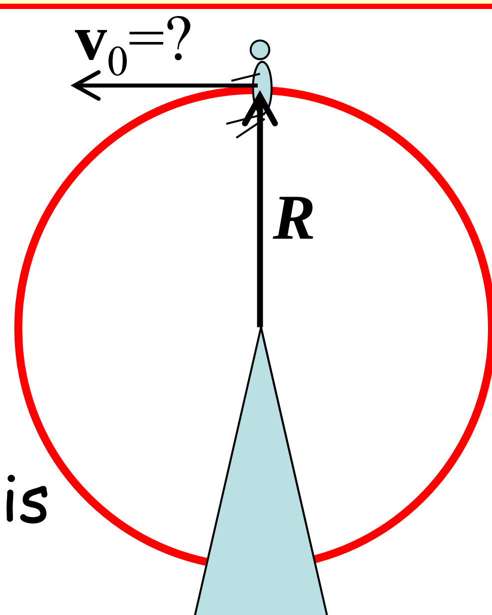
$$-W = mg = ma = m\,v_0^2/R \quad \Rightarrow a = v_0^2/R = g \quad \Rightarrow v_0 = \sqrt{gR}\ !$$

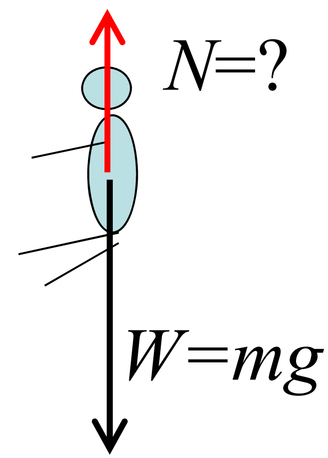
Assuming the Ferris wheel keeps going at the rate we just calculated, what happens at the bottom?

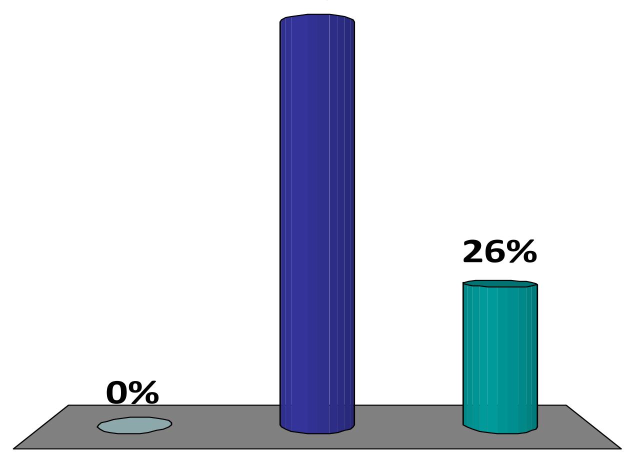
A. I feel weightless again …

B. I feel twice as heavy because the force on my pants is twice my weight …

C. I feel normal, because the acceleration at the bottom is simply g …



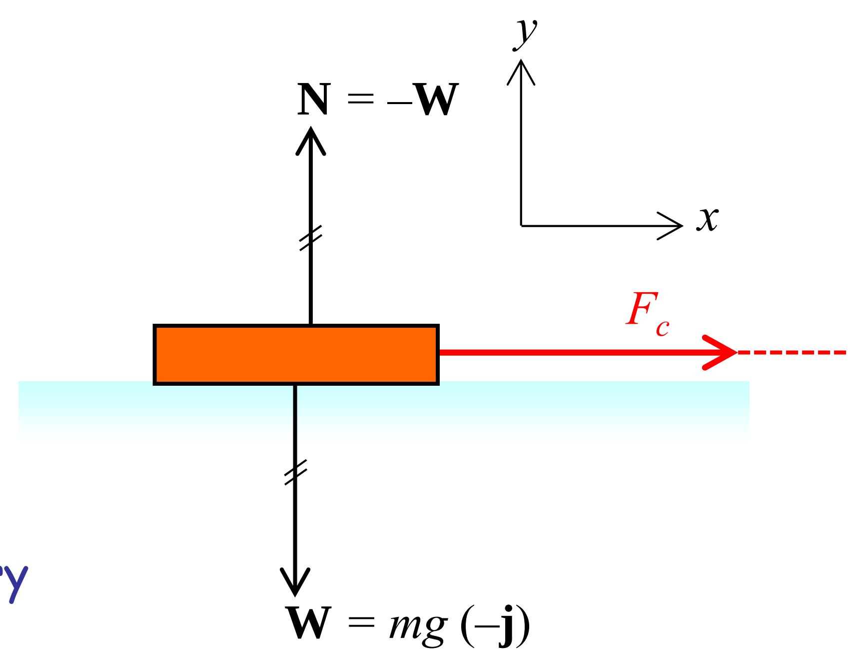




Example: hockey puck on a string

Consider a hockey puck on a sling without friction on a horizontal sheet of ice. The puck is tied with a massless string to a central post, and is sliding with a constant speed v along a circular path of radius R

- Free body diagram:



- $a_y = 0$
  ➔ $\mathbf{W}+\mathbf{N}=0$
  ➔ $N = mg$

- $\mathbf{F}_{Net} = \mathbf{F}_c = m\mathbf{a}_c = mv^2/R\ \mathbf{I}$
  - where v is the (constant) speed of the puck in its circular trajectory of radius R

- From this we can build all kinds of problems…

Conical Pendulum …

- see book, study yourself…
- quite similar to the next example: car on banked curve…

Car on a Banked Curve

For a car of mass m going through a (circular) curve in the road, a net force is required to keep them on a circular trajectory.

- This centripetal (=NET) force consists of two forces:
  - the horizontal component of the friction force of the road on the car, plus
  - the horizontal inward-pointing component of the normal force $N\sin\theta$ if the curve is banked…
  - Note, that the Normal force is a *reaction* force
  - Until the car starts slipping sideways, the friction force is the force of static friction between the tires and the road: $F_f = \mu_s N$

Car on a Banked Curve

- Free-Body diagram:
- Q: what is the ideal speed for this banked curve? (i.e. when friction is not needed…)

$\mathbf{F}_{\text{net}} = m\mathbf{a}$ is 2 equations:

Vertical:

$$ma_y = 0 = N\cos\theta - W \mp F_f \sin\theta = N\cos\theta \mp \mu_s N \sin\theta - mg$$

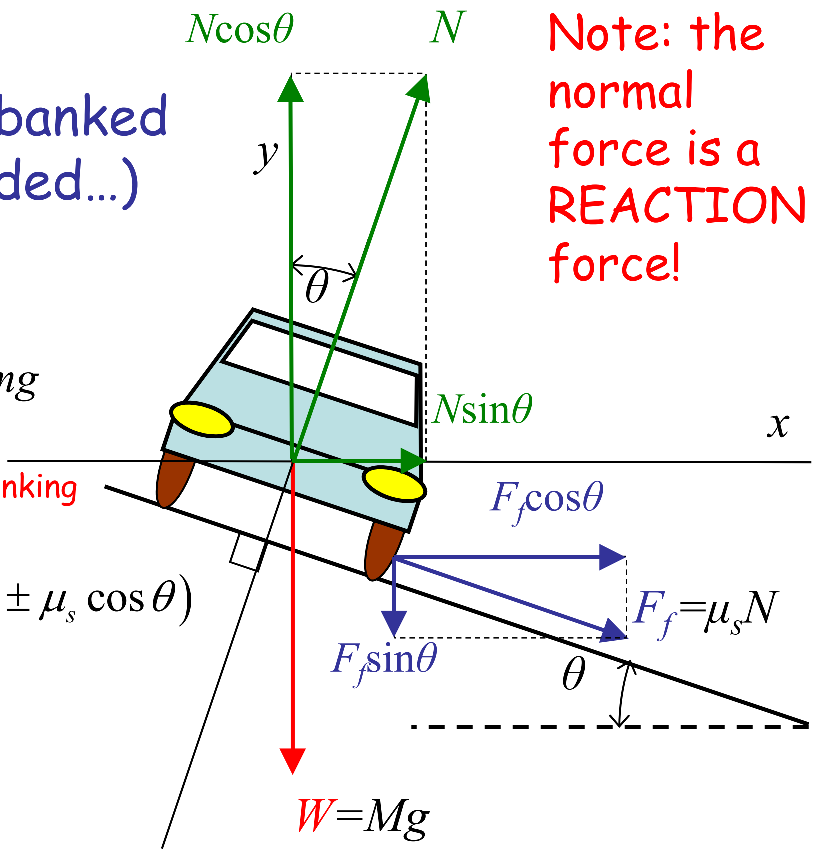
$$\Rightarrow N = \frac{mg}{\cos\theta \mp \mu_s \sin\theta}$$

top sign when $\mathbf{F}_f$ is as indicated;
the bottom sign when $\mathbf{F}_f$ is *up* the banking

Horizontal:

$$ma_x = ma_{rad} = m\frac{v^2}{R} = F_{Net,x} = N\sin\theta \pm F_f\cos\theta = N\left(\sin\theta \pm \mu_s \cos\theta\right)$$

$$\Rightarrow v^2 = gR\frac{\sin\theta \pm \mu_s\cos\theta}{\cos\theta \mp \mu_s \sin\theta} = gR\frac{\tan\theta \pm \mu_s}{1 \mp \mu_s \tan\theta}$$

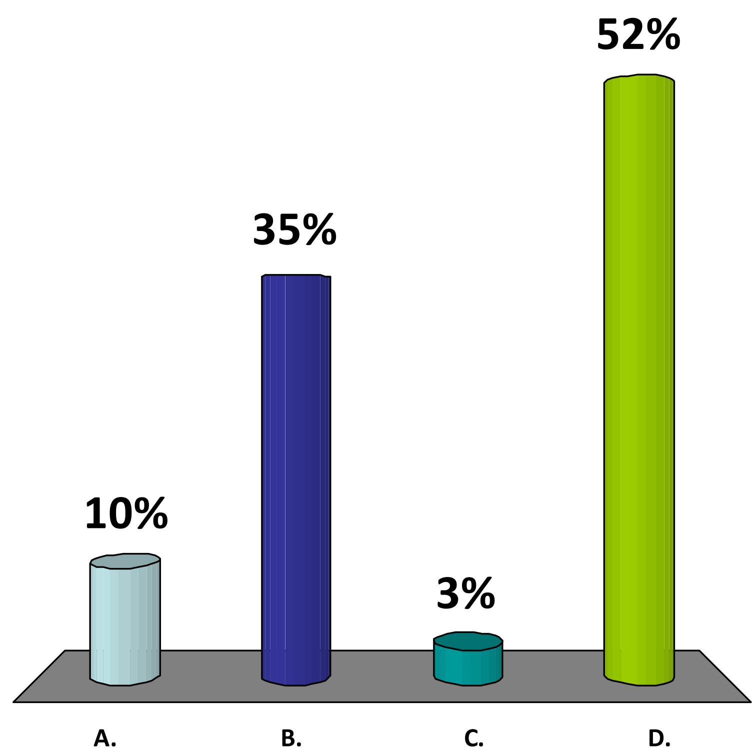


Note: the normal force is a REACTION force!

- Thus, for μ_s=0, $v=\sqrt{(gR\tan\theta)}$; check!
- For θ=0, we ***need*** friction: $v_{max}=\sqrt{(\mu_s gR)}$; check!
- Else: $v_{min} < v < v_{max}$ with: $v_{min}^2 = gR\frac{\tan\theta - \mu_s}{1+\mu_s\tan\theta}$, $v_{max}^2 = gR\frac{\tan\theta + \mu_s}{1-\mu_s\tan\theta}$
- e.g: for μ_s=0.70, θ=10º, R=30 m we find: v_{max}= 17 m/s (37 mi/hr) and v_{min} = 0 m/s

Choose the correct answer:
The force that accelerates the car is:

A. There is no force because the car goes through the turn at constant speed!
B. The force of static friction on tires…
C. The force of gravity…
D. Gravity and friction both, depending on the speed…



Example

A block of mass slides along a horizontal surface lubricated with a thick oil which provides a drag force proportional to the square root of velocity: $F = bv^{1/2}$

- Q: It starts with v_0; how long & far does the block slide?

$$F_{net} = -b\sqrt{v} = ma = m\frac{dv}{dt} \underset{\text{to find time}}{\Rightarrow} -\frac{b}{m}\int_0^t dt = \int_{v_0}^0 \frac{dv}{\sqrt{v}}$$

$$\Rightarrow -\frac{b}{m}t = \left[2\sqrt{v}\right]_{v_0}^0 = -2\sqrt{v_0}$$

$$F_{net} = -b\sqrt{v} = ma = m\frac{dv}{dt} = mv\frac{dv}{dx} \underset{\text{to find distance}}{\Rightarrow} -\frac{b}{m}\int_0^D dx = \int_{v_0}^0 \sqrt{v}dv$$

$$\Rightarrow -\frac{b}{m}D = \left[\frac{2}{3}v^{\frac{3}{2}}\right]_{v_0}^0 = -\frac{2}{3}v_0^{\frac{3}{2}}$$