Supersolidity in electron-hole bilayers with a large density imbalance

Francesca Maria Marchetti

UAM
UNIVERSIDAD AUTONOMA
DE MADRID

In collaboration with

Meera Parish & Peter Littlewood
Cavendish Laboratory,
University of Cambridge, UK

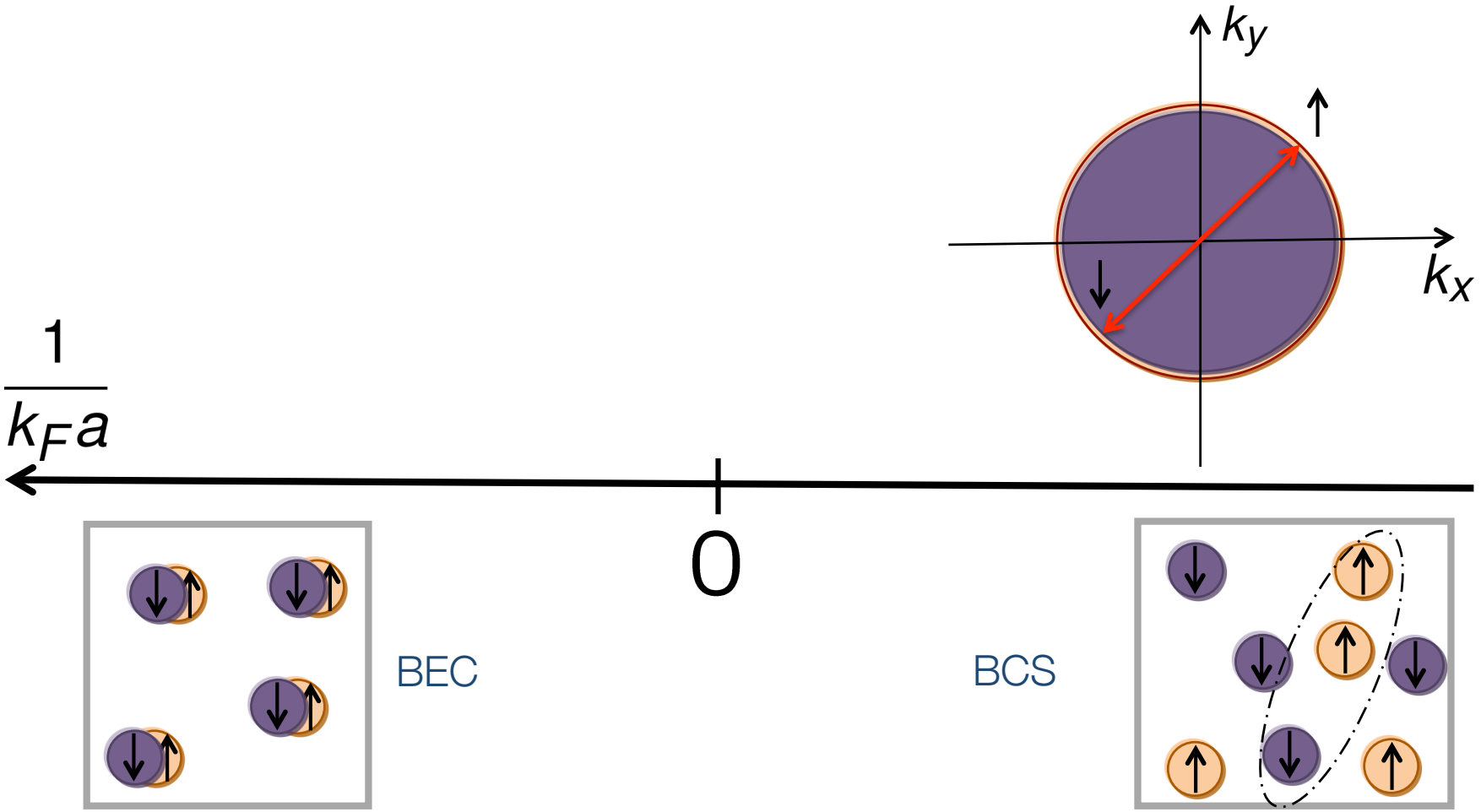
[1] arxiv > cond.mat > 1009.1420

Previous related work on imbalanced Fermi gases:

[2] Nature Physics **3**, 124 (2007)
[3] Phys. Rev. Lett. **98**, 160402 (2007)

Background: Two component Fermi gases

- Tunable interactions
- BEC-BCS crossover



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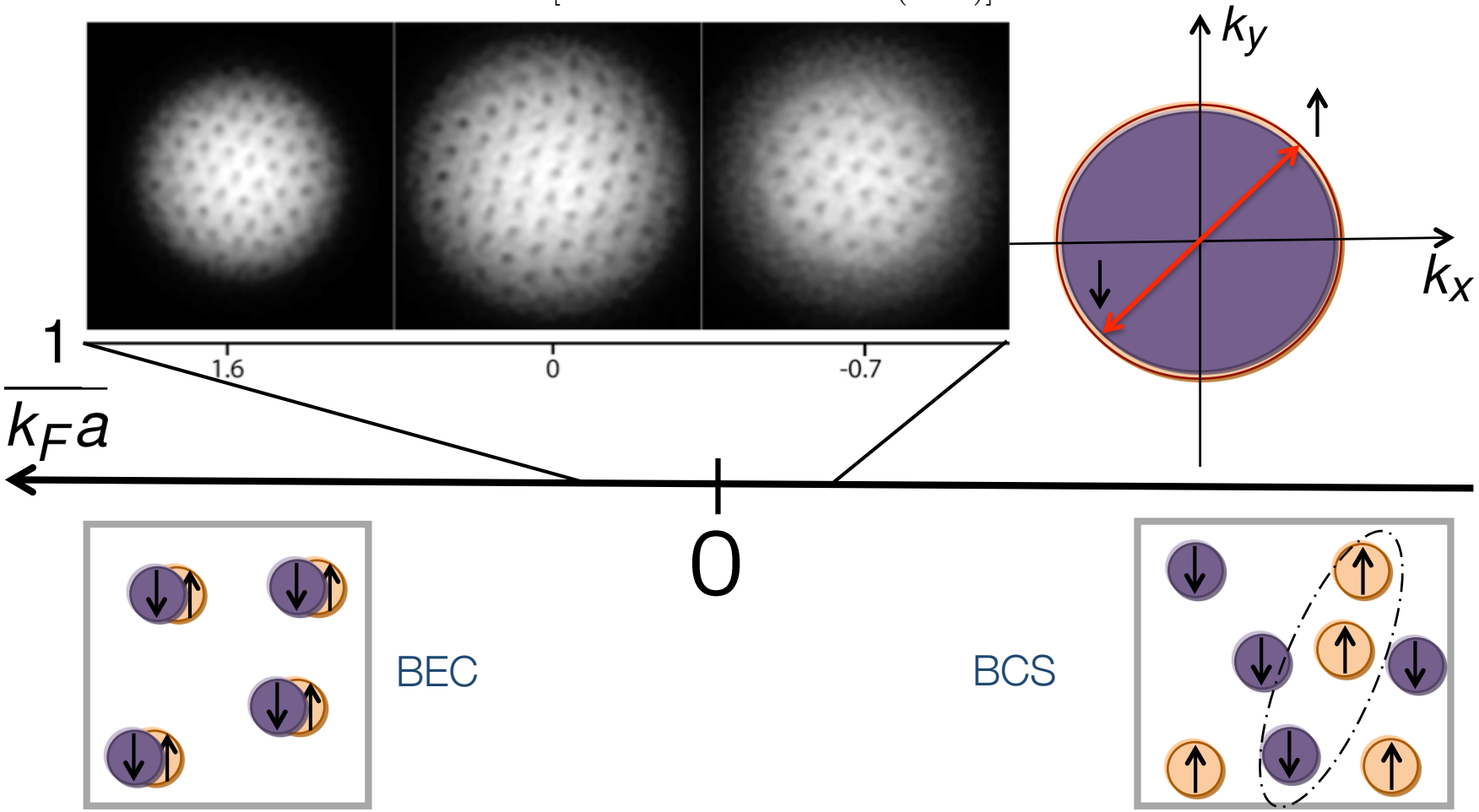
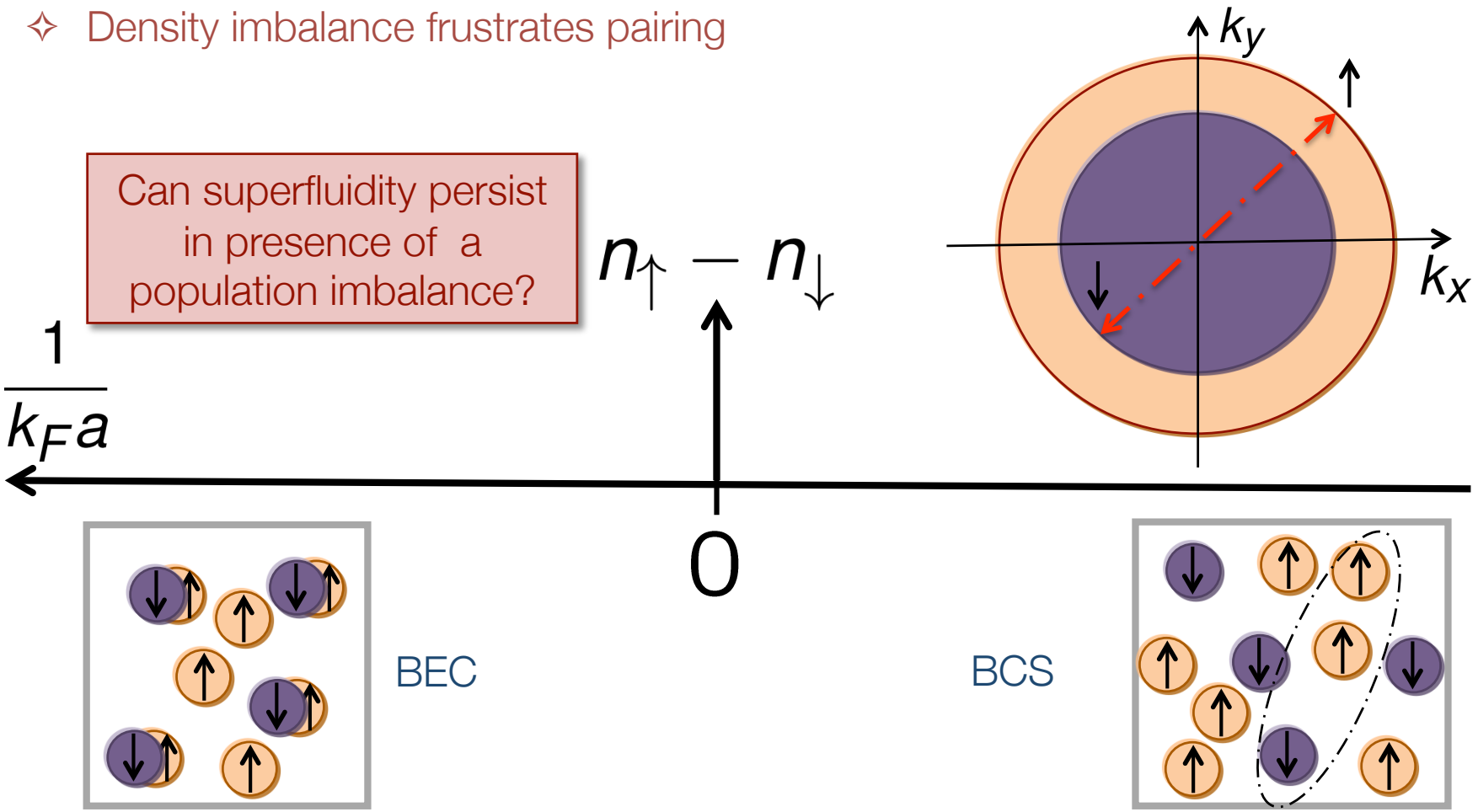
[Zwierlein et al. Nature (2005)]



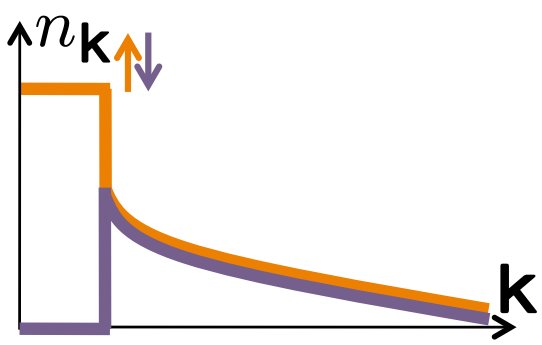
Background: Imbalanced Fermi gases

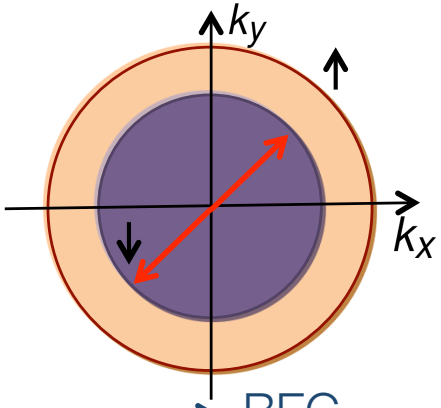
- Tunable interactions
- BEC-BCS crossover
- Density imbalance frustrates pairing

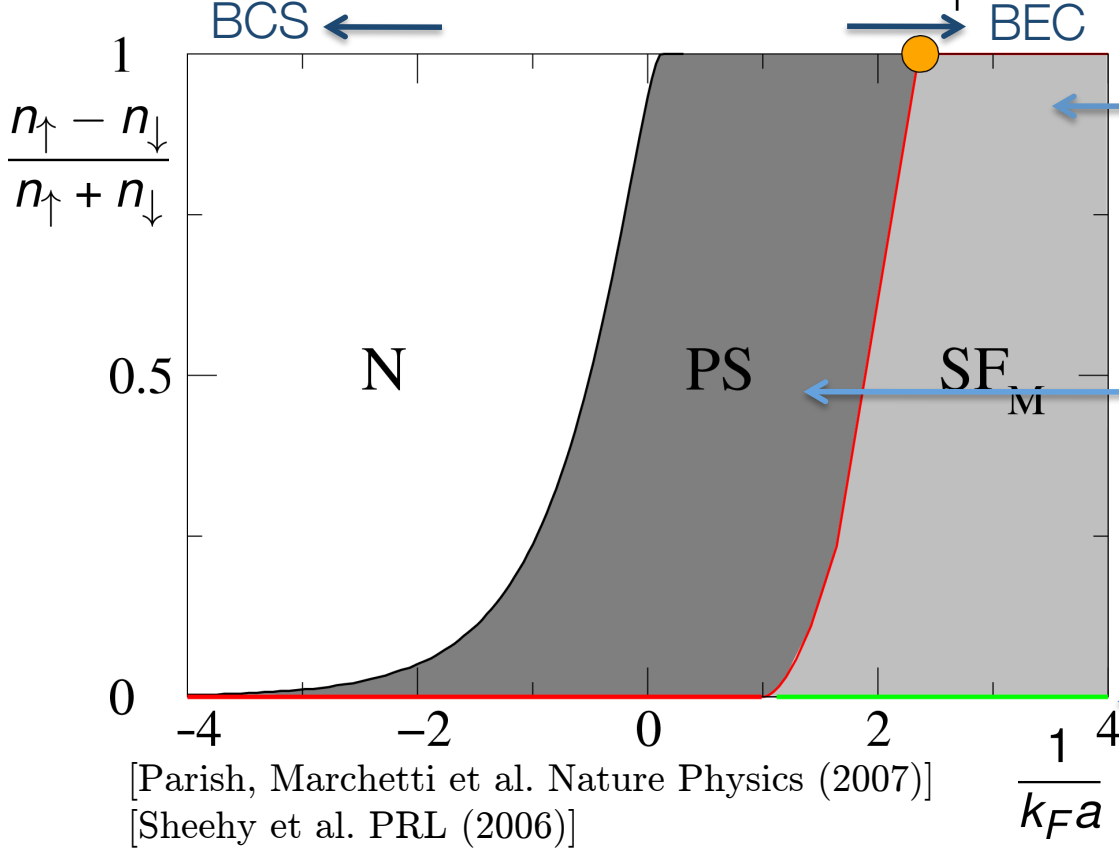
Can superfluidity persist in presence of a population imbalance?



Background: T=0 phase diagram



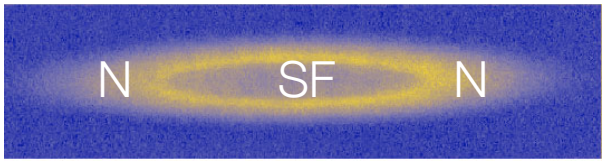




[Parish, Marchetti et al. Nature Physics (2007)]
[Sheehy et al. PRL (2006)]

Polarised SF or 1FS breached pairing (space homogeneous) → Bose-Fermi mixture

Macroscpic spatial phase separation between SF and N (inhomogeneous)

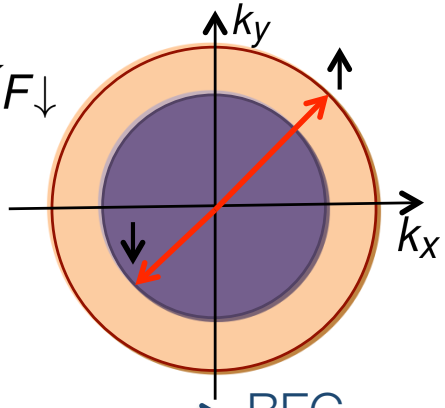


[Shin *et al.* PRL (2006)]

Fully paired SF phase

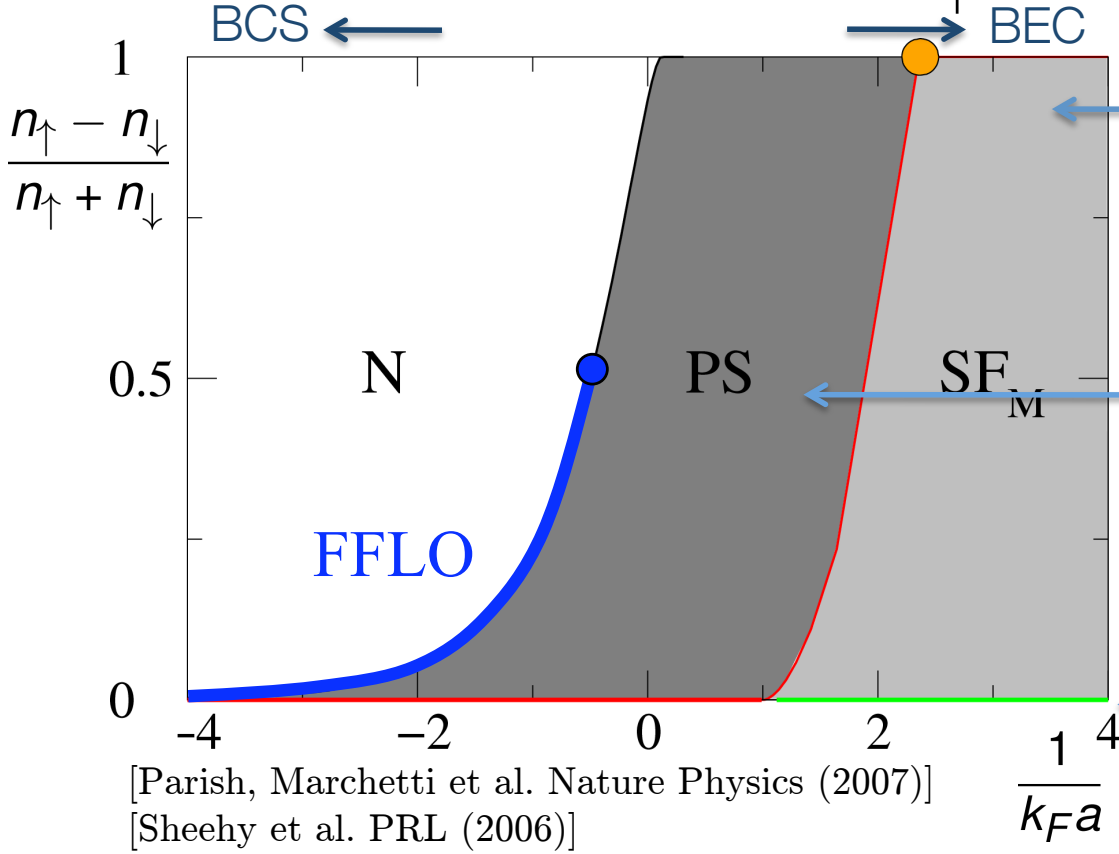
Background: T=0 phase diagram

$Q \sim k_{F\uparrow} - k_{F\downarrow}$



SUPERSOLID PHASE

FFLO phase $\Delta(\mathbf{r}) = \sum_{\mathbf{Q}} \Delta_{\mathbf{Q}} e^{i\mathbf{r}\cdot\mathbf{Q}}$



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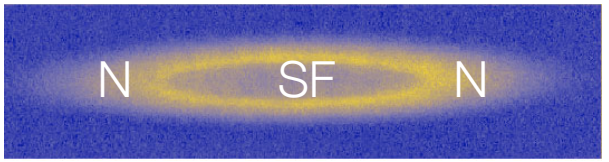

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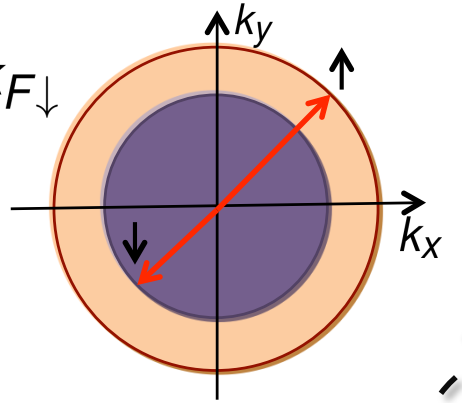
Fully paired SF phase

Background: FFLO unlikely in 3D Fermi gases

$$\mathbf{Q} \sim k_{F\uparrow} - k_{F\downarrow}$$



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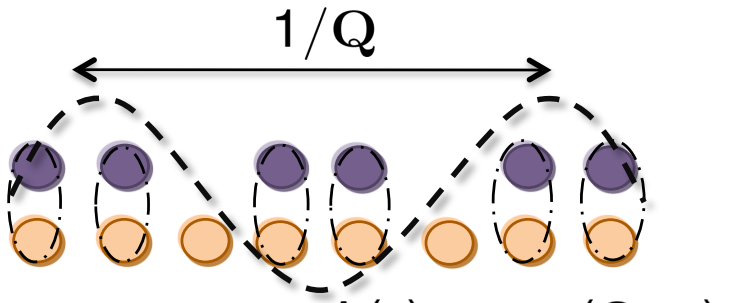
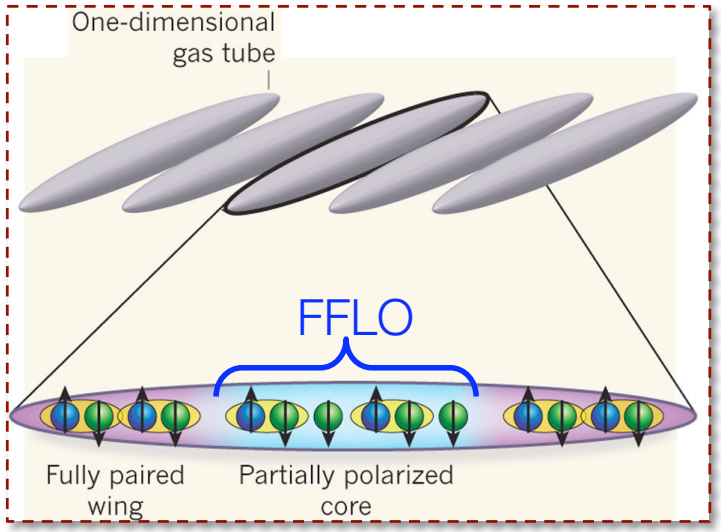


- Displacement of Fermi surface:
  - …allows the system to polarise
  - …allows Fermi surface nesting
- However
  - …it costs kinetic energy
  - …nesting is partial
- Phase separation dominates over FFLO
- Experiments (Rice & MIT): FFLO elusive



[Liao et al. Nature (2010)]

Motivation: Relevance to other systems

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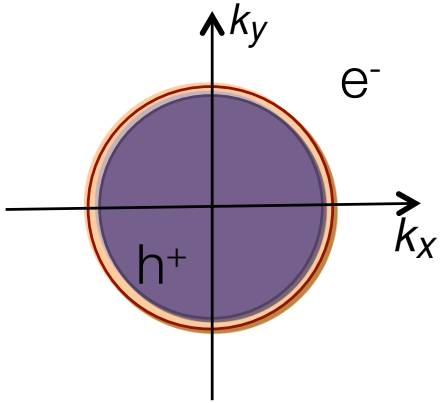

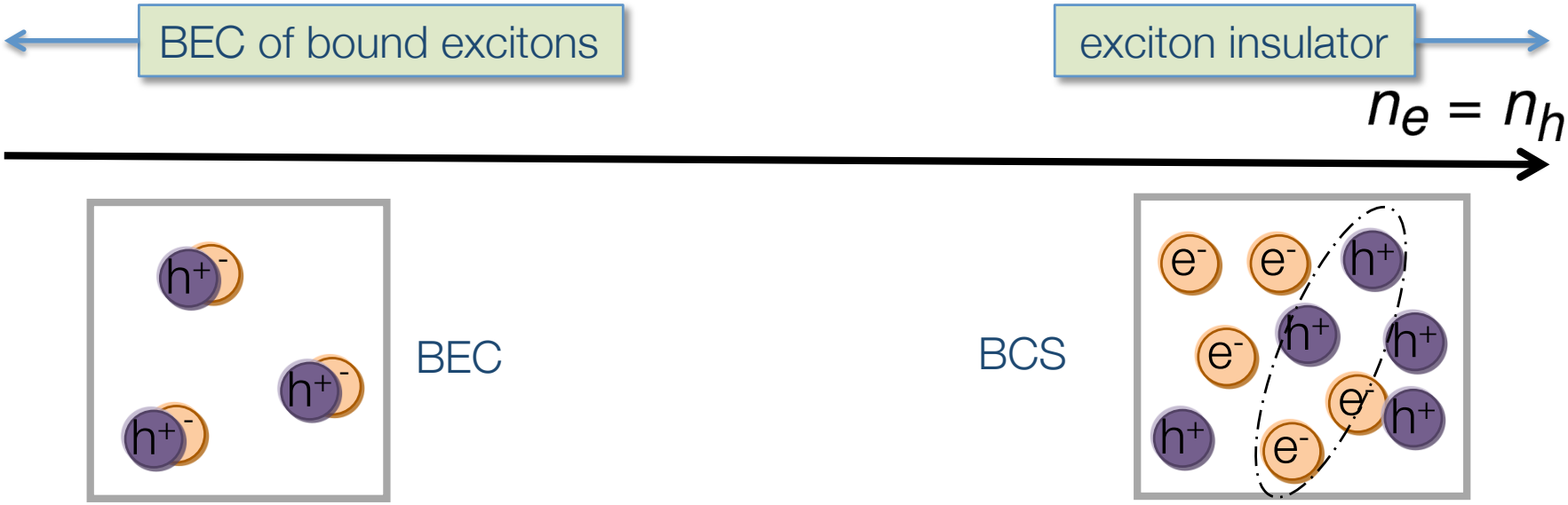

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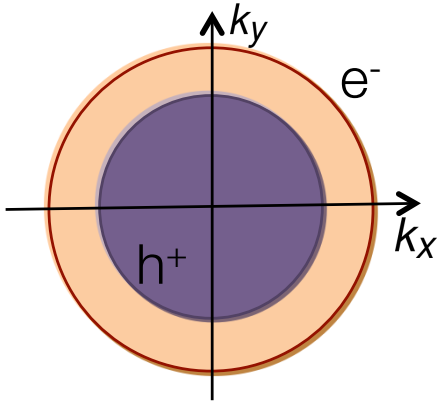
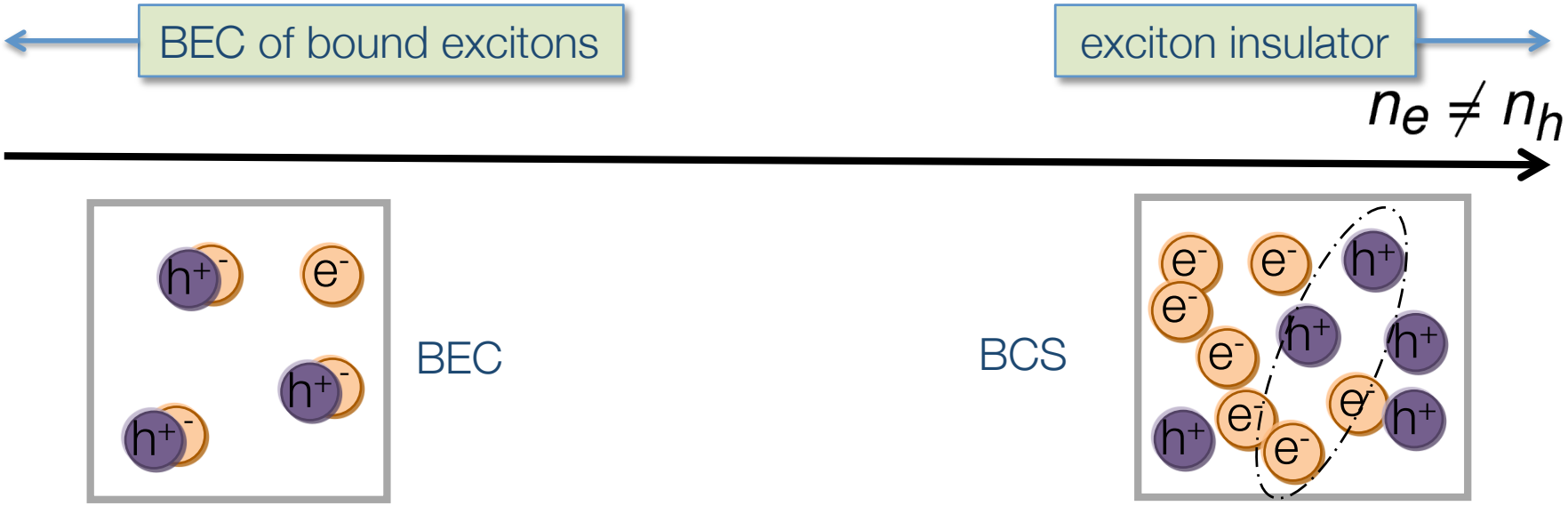
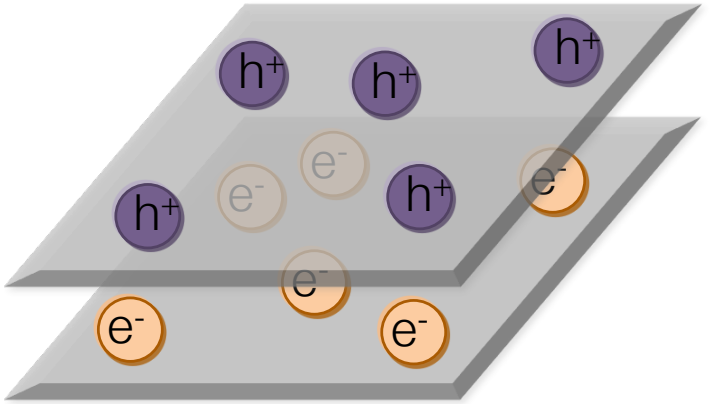


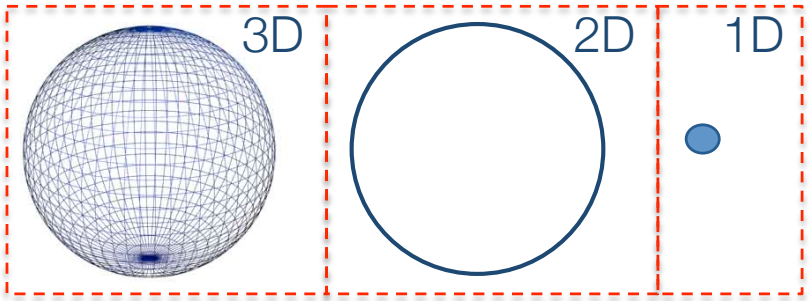


Electron-hole bilayers: FFLO more ‘likely’

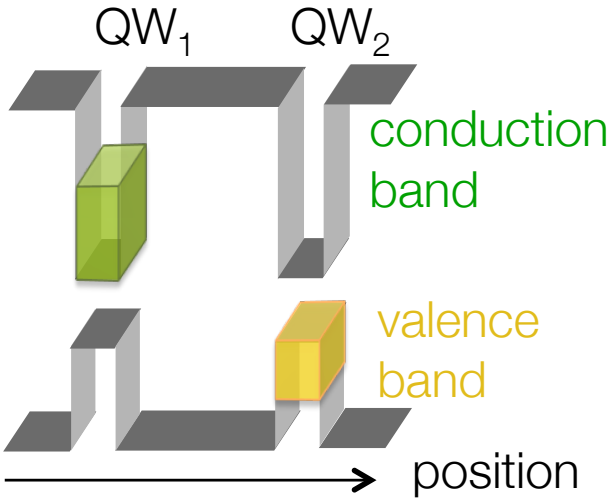
electron-hole bilayers:
better route to achieve FFLO!



1. Enhanced Fermi surface nesting



2. No phase separation on macro-scales



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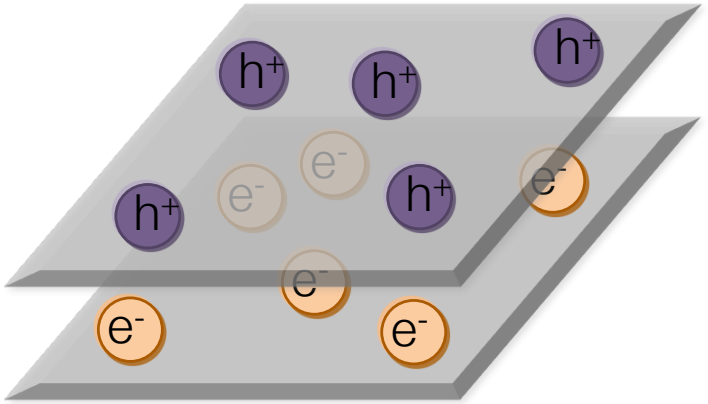

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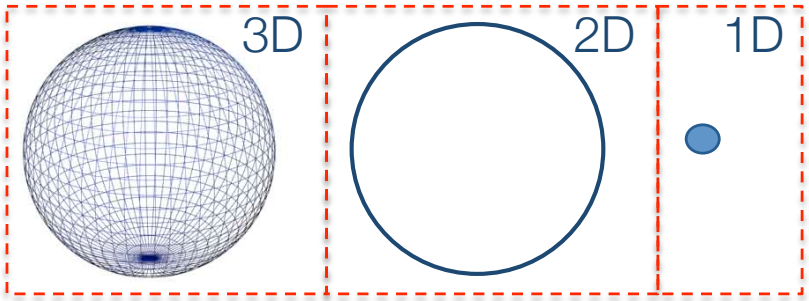

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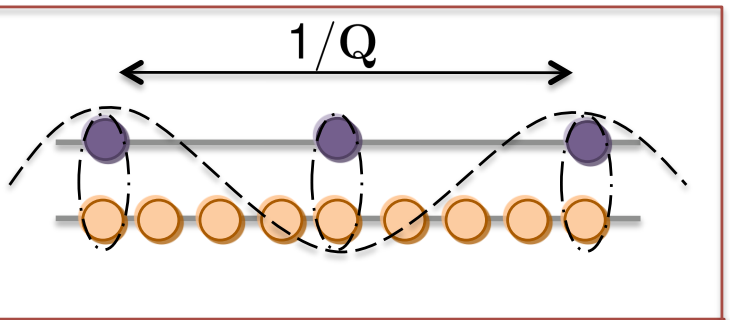
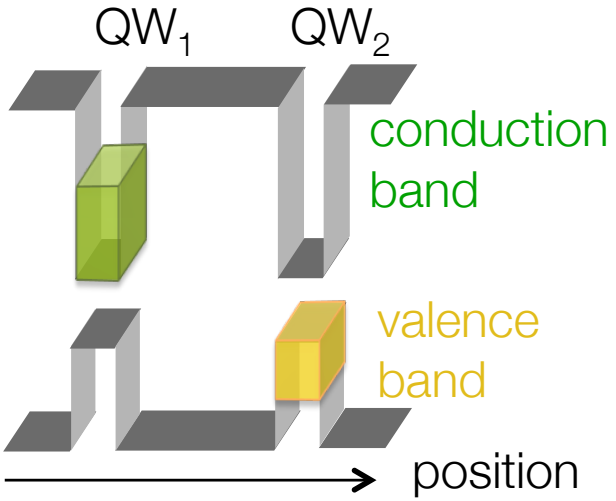
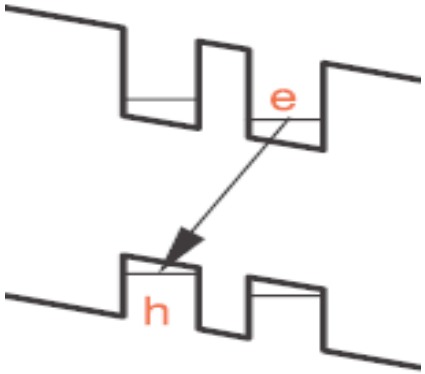


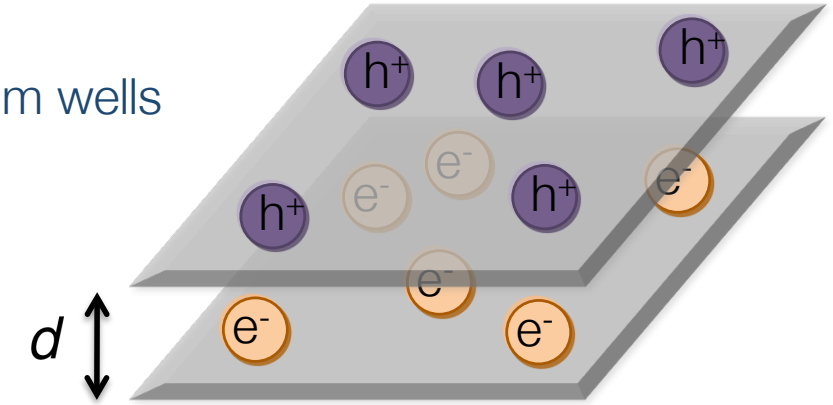
Novel ‘bosonic’ limit of FFLO



Electron-hole bilayers: Experimental realisations

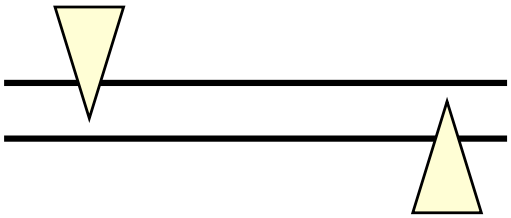
- Optically pumped coupled quantum wells



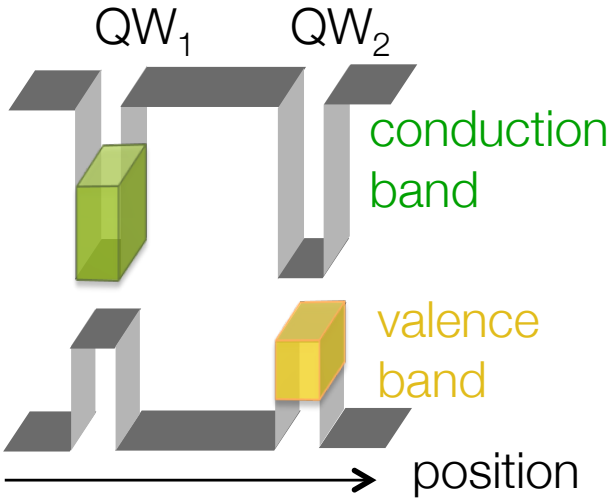


[Butov et al. Nature (2002)]
[Snoke et al. Nature (2002)]
[…and many more…]

- Individually contacted doped layers



[Croxall et al. PRL (2008)]
[Seamons et al. PRL (2009)]

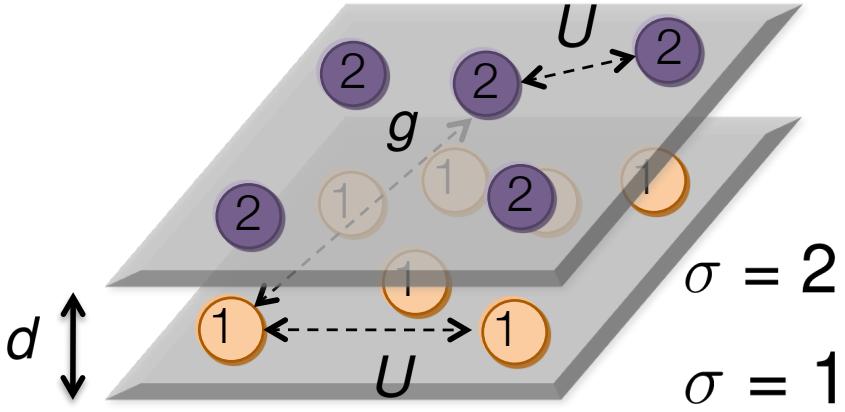


⇒ Equilibrium phase diagram

Electron-hole bilayers: Hamiltonian

$$\alpha = \frac{m_2}{m_1} \mapsto \frac{1}{\alpha} \quad (e^- \mapsto h^+)$$

$$\epsilon_{\mathbf{k}\sigma}$$



$$H = \sum \frac{k^2}{2m_\sigma} c_\sigma^\dagger c_\sigma + \frac{1}{\Omega} \sum g_q c_1^\dagger c_2^\dagger c_2 c_1 + \frac{1}{2\Omega} \sum U_q c_\sigma^\dagger c_\sigma^\dagger c_\sigma c_\sigma$$

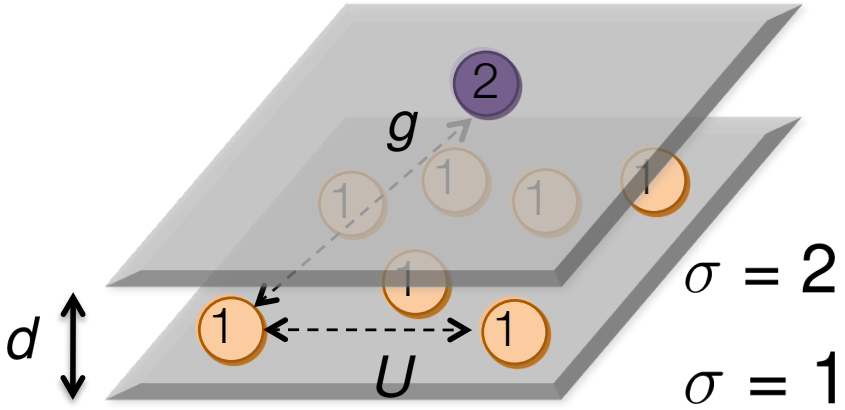
$U_q = \frac{2\pi e^2}{\varepsilon q}$ bare intra-layer Coulomb interaction

$g_q = -U_q e^{-qd}$ bare inter-layer

⇒ No spin (spin polarised)

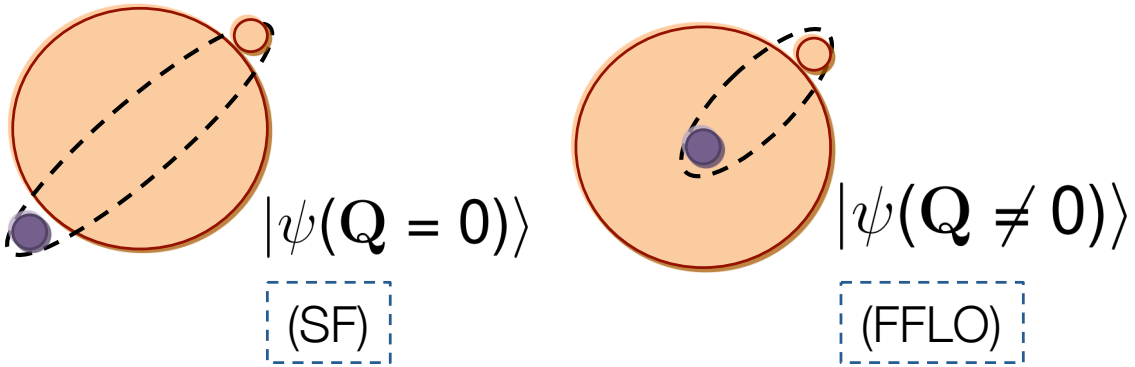
Limit of large imbalance: Variational ground state

✧ single particle in the 2nd layer + Fermi see in the 1st layer



relative k ↔ CoM Q

$$|\Psi(\mathbf{Q})\rangle = \sum_{\mathbf{k}>k_F} \varphi_{\mathbf{kQ}}\, c^{\dagger}_{\mathbf{Q}-\mathbf{k},2} c^{\dagger}_{\mathbf{k},1} |FS\rangle$$



✧ Good description of both low and high density limit

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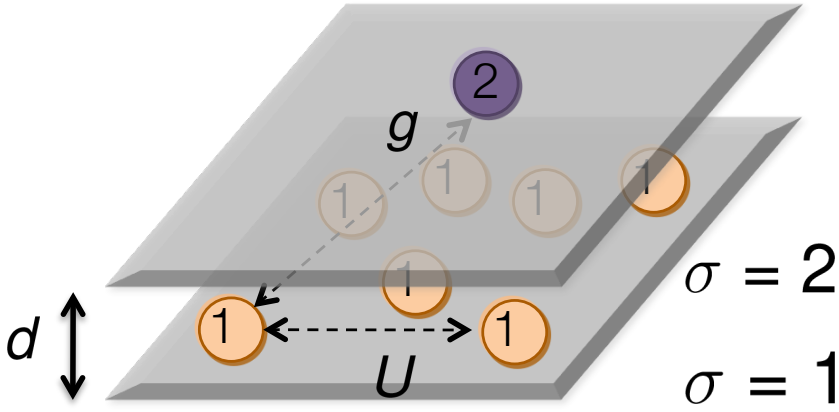

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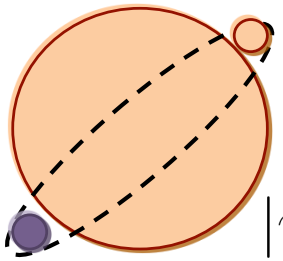
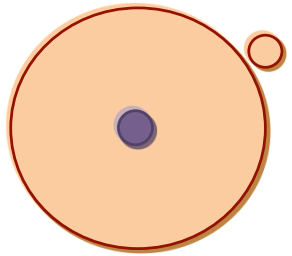


$|\psi(\mathbf{Q}=0)\rangle$

(SF)

Interpolate by screening the interactions (within RPA) = dress with density fluctuations, including particle-hole excitations



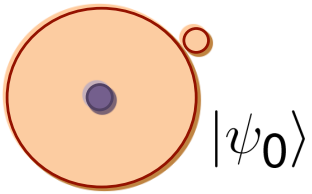
$|\psi_0\rangle$

(N)

✧ Good description of both low and high density limit

Phase diagram

1. Minimise $\langle \Psi(\mathbf{Q}) | H | \Psi(\mathbf{Q}) \rangle$
2. Eigenvalue equation
3. Unbinding transition $\langle \Psi_0 | H | \Psi_0 \rangle$

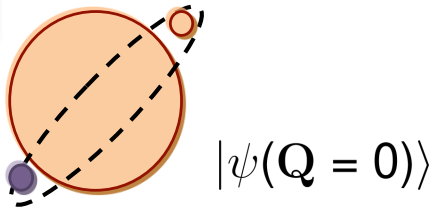


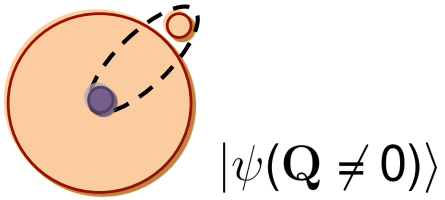
✧ Read also as mean-field (linearised) gap equation

⇒ Current theories
⇒… some neglect screening
⇒… some neglect finite Q FFLO (next talk)

[Yamashita et al. J Phys Soc Japan (2010)]
[Pieri et al. PRB (2007)]
[Subasi et al. PRB (2010)]

✧ SF & FFLO





✧ Parameters:

exciton Bohr radius $a_0 = \dfrac{\varepsilon}{me^2}$

⇒ mass ratio $\alpha = \dfrac{m_2}{m_1}$
⇒ interlayer distance d/a_0
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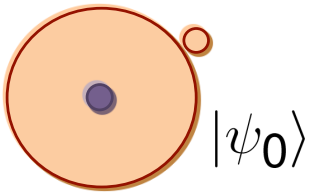

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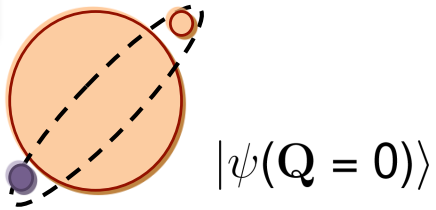

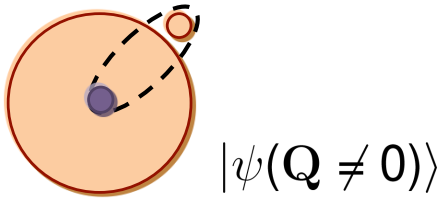

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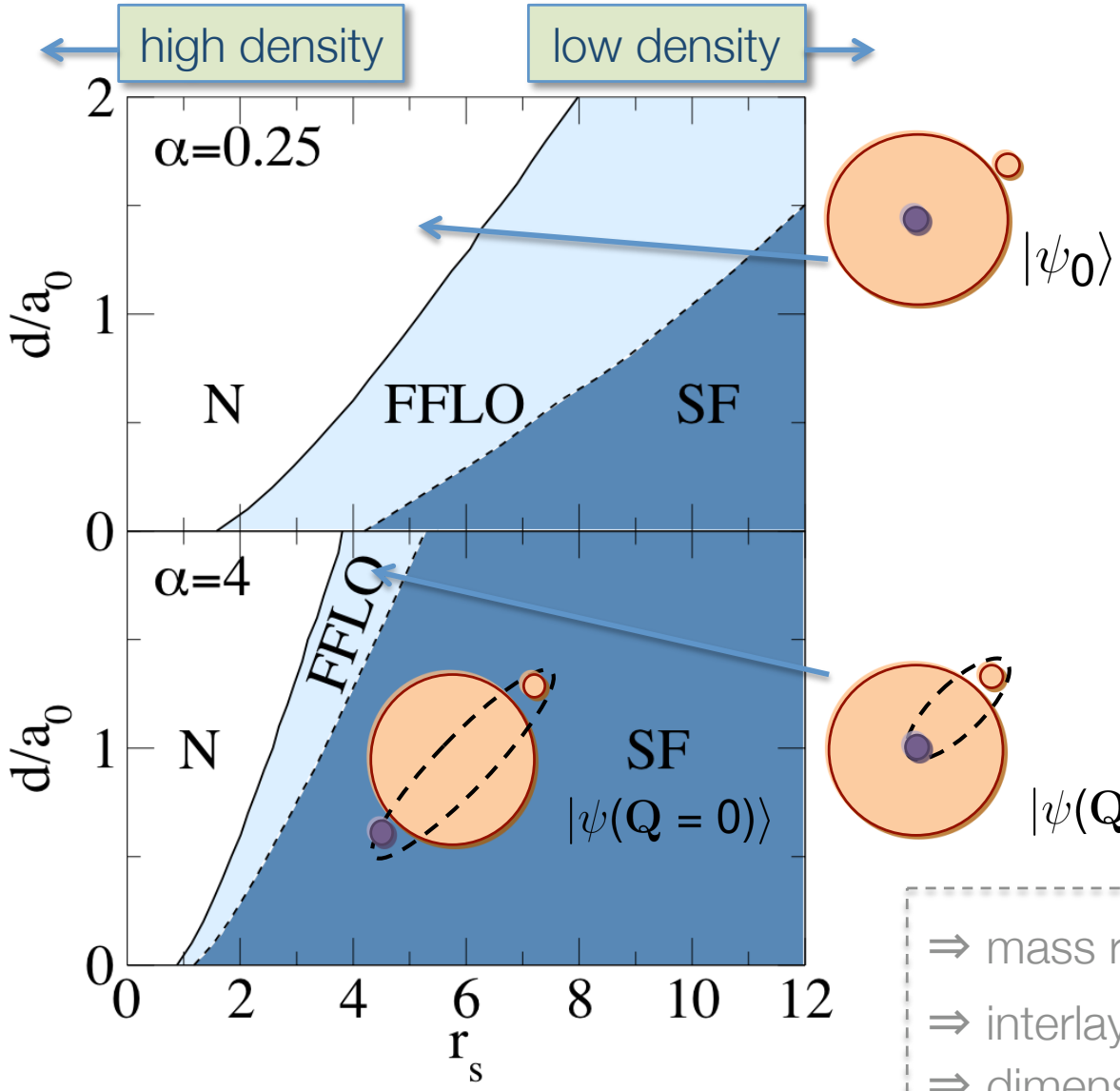
Phase diagram of fully imbalanced EH bilayer

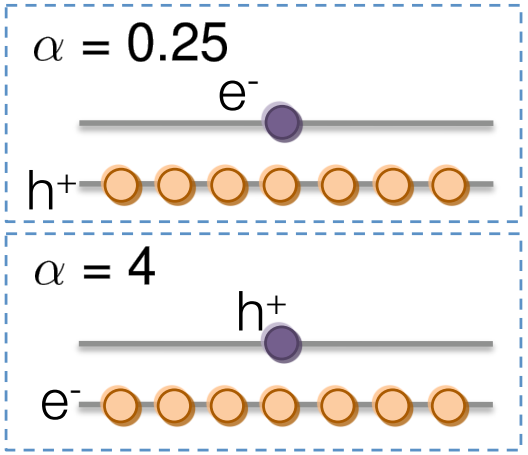


[arxiv/cond.mat/1009.1420]

$|\psi_0\rangle$

$|\psi(Q \neq 0)\rangle$

✧ GaAs:



FFLO region enhanced if minority particle lighter

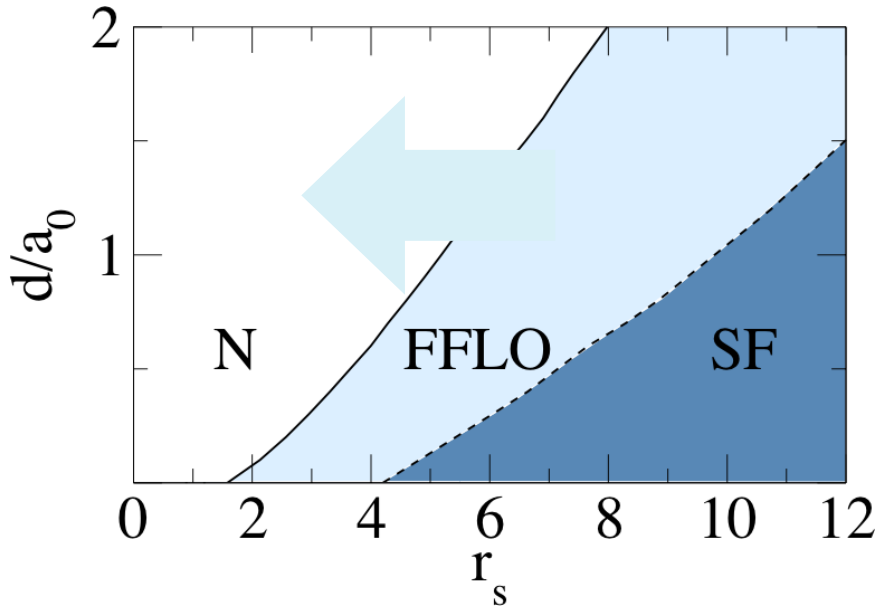
⇒ mass ratio $\alpha = \frac{m_2}{m_1}$

⇒ interlayer distance d/a_0

⇒ dimensionless density $k_F a_0 \equiv \frac{1}{r_s}$

Effect of screening on the LOFF phase

✧ Unscreened case

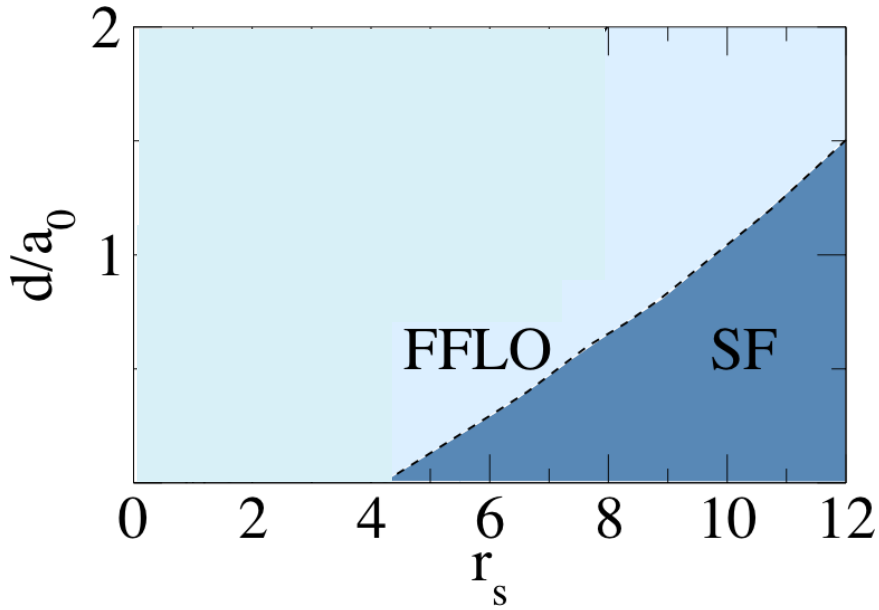


⇒ Numerically

⇒ Analytically

Effect of screening on the LOFF phase

✧ Unscreened case



⇒ Numerically

⇒ Analytically

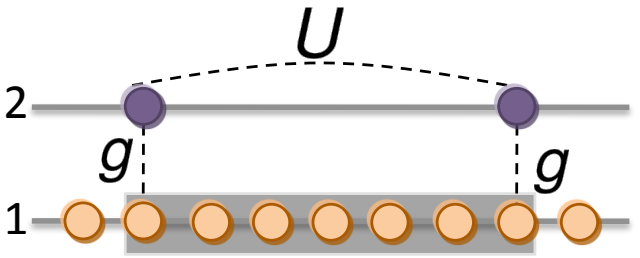
Which kind of FFLO phase?

Dilute gas of minority particles: Interactions

✧ Normal phase:

effective interaction between two unbound minority particles (RPA)

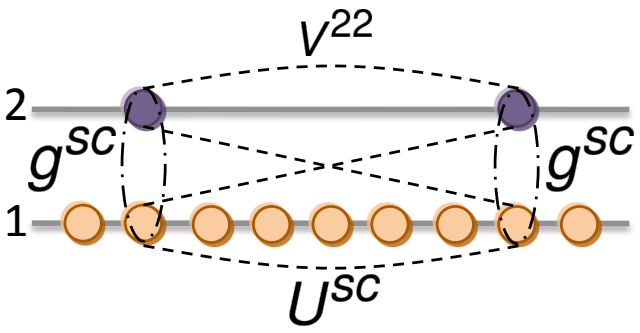
⇒ repulsive and dipolar $V^{22}(r) \to \frac{1}{r^3}$:
the dilute gas is a Fermi liquid



✧ Excitonic phase:

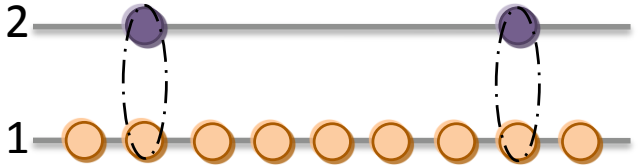
well separated excitons (dipoles)

⇒ also repulsive and dipolar $V^{ex}(r) \to \frac{1}{r^3}$:
no phase separation
(biexciton formation suppressed for single spin species & $\frac{d}{a_0} \gtrsim 0.25$)



Phenomenology of ‘bosonic’ FFLO

✧ Landau theory for $|\psi|^2$ = exciton density

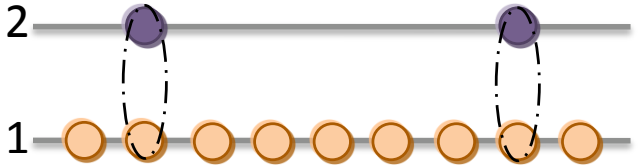


$$F[\psi] = \int d\mathbf{r} \left[-\mu|\psi|^2 + \frac{\lambda}{2}|\psi|^4 \right]$$

chemical potential

minimum energy at $|\mathbf{q}| = Q_{min}$

Phenomenology of ‘bosonic’ FFLO



- Landau theory for $|\psi|^2$ = exciton density (weak crystalisation theory)

$$F[\psi] = \int d\mathbf{r} \left[-\mu|\psi|^2 + \gamma|(\nabla^2 + Q_{min}^2)\psi|^2 + \frac{\lambda}{2}|\psi|^4 \right]$$

chemical potential

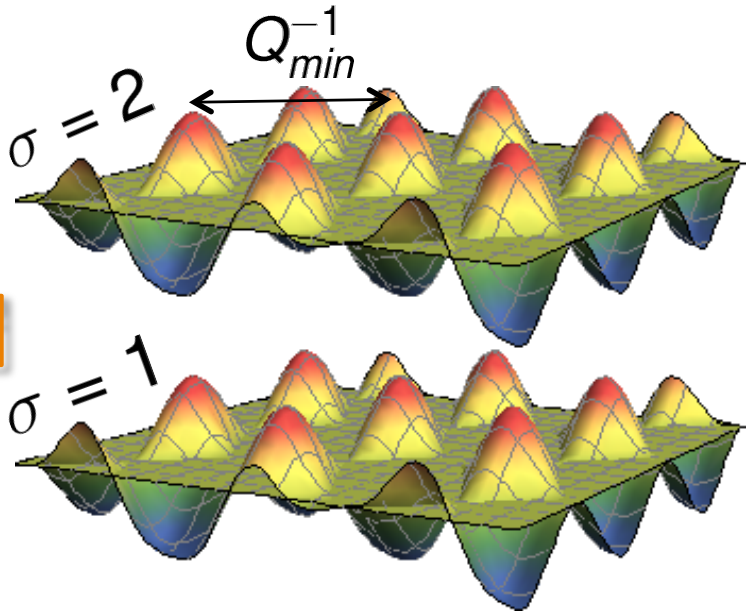
minimum energy at $|\mathbf{q}| = Q_{min}$

- Complex order parameter

$$\psi(\mathbf{r}) = \sum_n a_n e^{i\mathbf{q}_n \cdot \mathbf{r}} \Big|_{|\mathbf{q}_n| = Q_{min}}$$

- Minimal energy solution

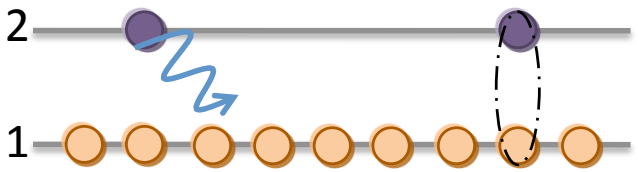
$$|\psi(\mathbf{r})|^2 \propto |\cos(\mathbf{q}_1 \cdot \mathbf{r}) - i\cos(\mathbf{q}_2 \cdot \mathbf{r} + \varphi)|^2$$



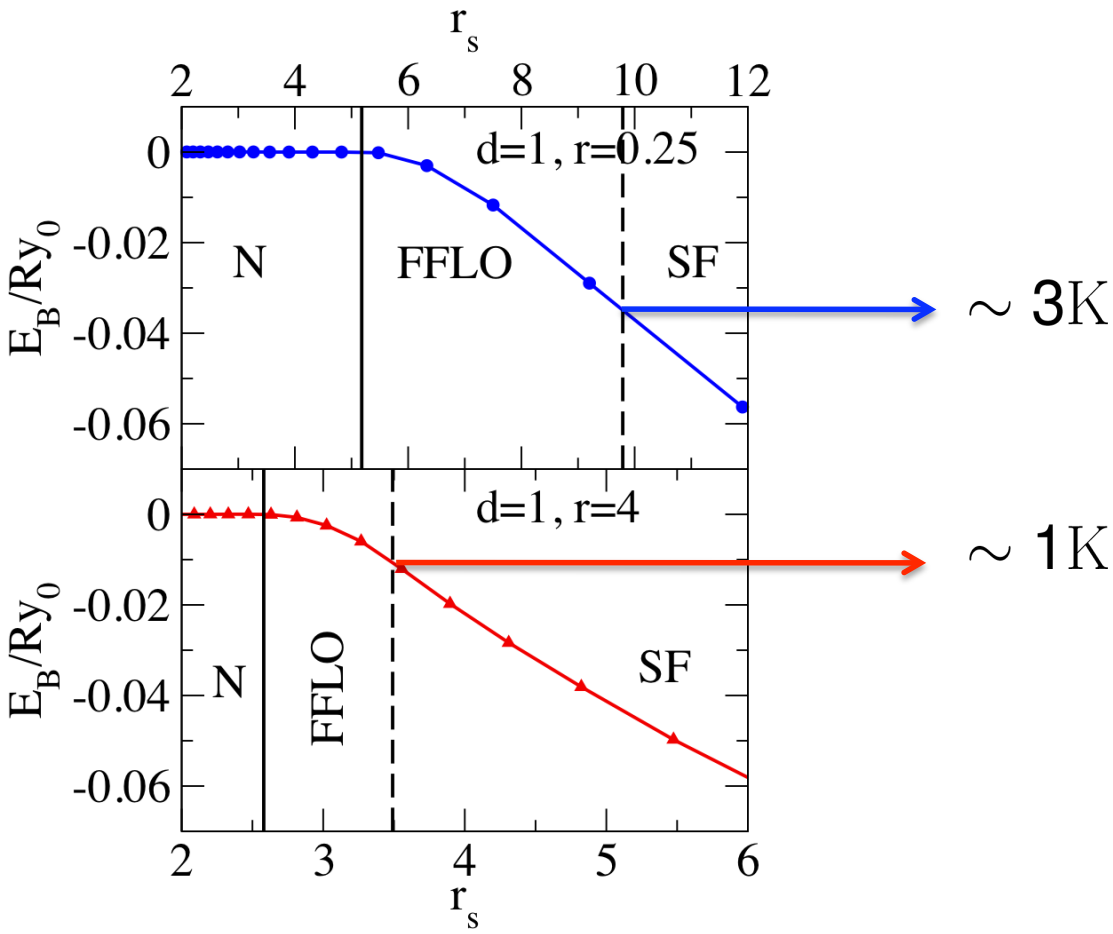
⇒ supersolid: exciton condesate with 2D spatial modulation (diagonal and off-diagonal order)

Observing FFLO

1. Light scattering off the spatial modulations
2. Photon angular emission



(electron hole recombining): finite momentum pairing ($\pm\mathbf{q}_1, \pm\mathbf{q}_2$)



$\sim 3\mathrm{K}$

$\sim 1\mathrm{K}$

Exciton binding energy in GaAs
= upper bound for the FFLO critical temperature

GaAs

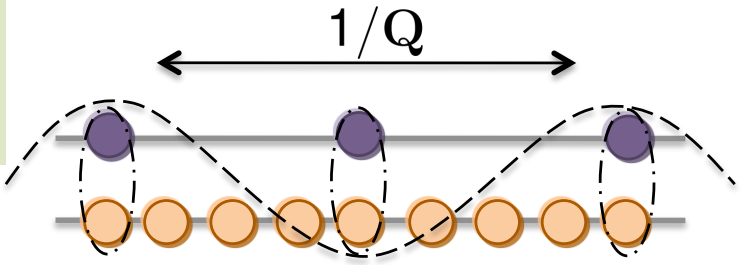
$\alpha = 0.25$

$a_0 = 7$ nm

$Ry_0 = 9$ meV

Conclusions

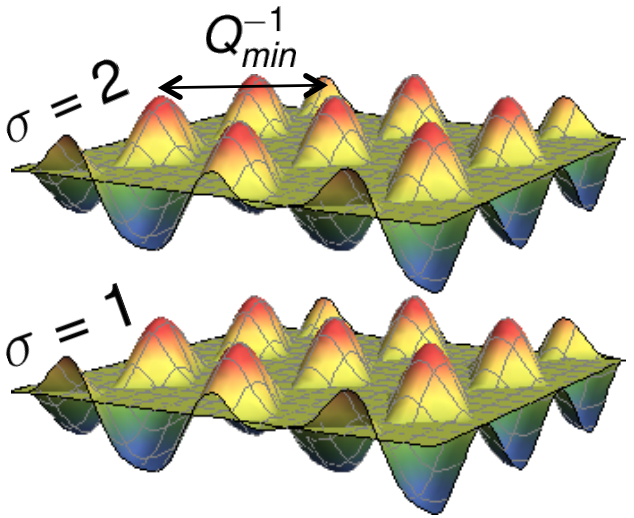
- ✧ Electron-hole bilayers:
  promising for observing exotic pairing phenomena



- ✧ Evidence of FFLO phase at large imbalance:
  finite-Q exciton in presence of a fermi sea

- ✧ Dilute gas of finite-Q excitons:
  condensation with 2D spatial modulation (a supersolid)

  ⇒ bosonic limit of FFLO



- ✧ Prospects for experimental observation

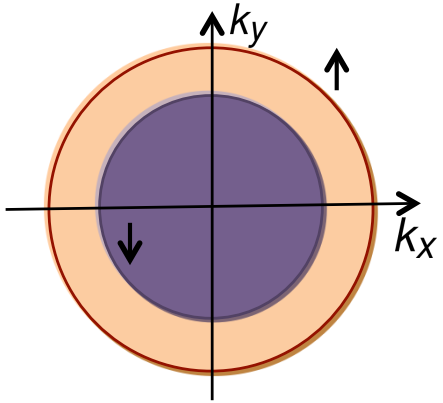
Additional slides

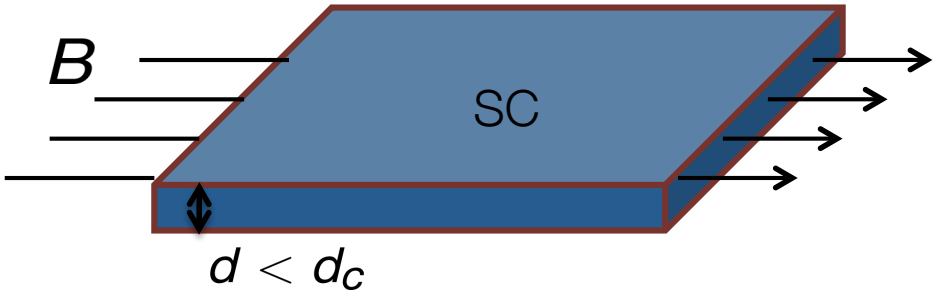
Motivation: Relevance to other systems

- Magnetised superconductors

$$H_{\mathrm{BdG}} = \begin{pmatrix} \epsilon_{\mathbf{k}} - \mu - h & -\Delta \\ -\Delta & -(\epsilon_{\mathbf{k}} - \mu) - h \end{pmatrix}$$

Zeeman term



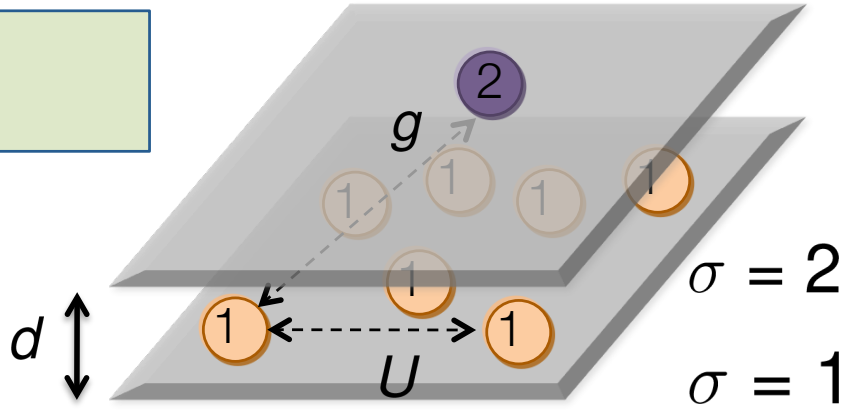


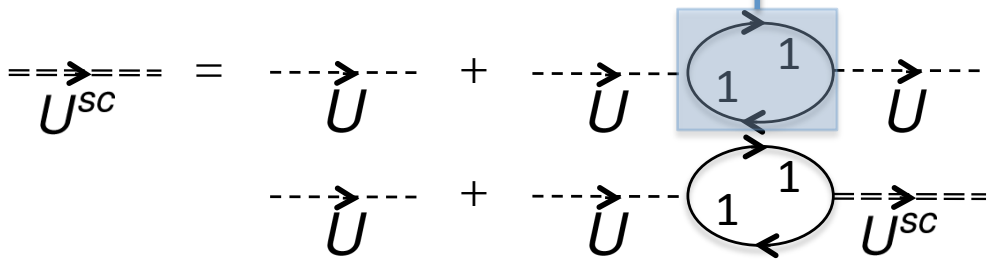
Include p-h excitation (in within RPA)

✧ Interpolate between high and low density via RPA

✧ Full imbalance limit:

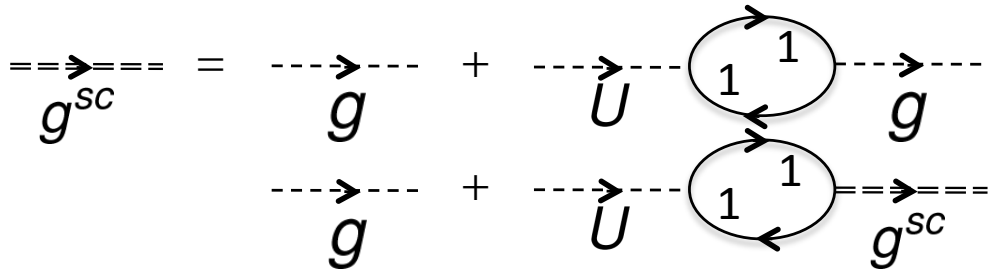
polarisation operator (Linhard function) $\Pi_1(q)$





$$U_q^{sc} = \frac{U_q}{1 - U_q \Pi_1(q)}$$

✧ Screening between 12 through 11



$$g_q^{sc} = -U_q^{sc} e^{-qd}$$

Phase diagram

1. Minimise $\langle \Psi(\mathbf{Q})|H|\Psi(\mathbf{Q})\rangle$
2. Eigenvalue equation

$$E\varphi_{\mathbf{kQ}} = \left(\epsilon_{\mathbf{Q}-\mathbf{k},2} + \epsilon_{\mathbf{k},1} - \frac{1}{\Omega}\sum_{\mathbf{k}'<k_F} U^{sc}_{\mathbf{k}-\mathbf{k}'}\right)\varphi_{\mathbf{kQ}} + \frac{1}{\Omega}\sum_{\mathbf{k}'>k_F} g^{sc}_{\mathbf{k}-\mathbf{k}'}\varphi_{\mathbf{k}'\mathbf{Q}}$$

$\Delta_{\mathbf{kQ}}$

3. Unbinding transition: compare with $\langle \Psi_0|H|\Psi_0\rangle$

$$E < \frac{k_F^2}{2m_1} - \frac{1}{\Omega}\sum_{k'<k_F} U_{k_F\hat{k}-k'}$$

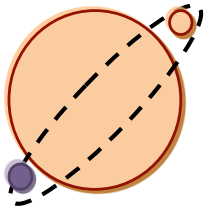
✧ Read also as mean-field (linearised) gap equation (BEC-BCS crossover)

$$\Delta_{\mathbf{kQ}} = \frac{1}{\Omega}\sum_{\mathbf{k}'>k_F} g^{sc}_{\mathbf{k}-\mathbf{k}'}\frac{\Delta_{\mathbf{k}'\mathbf{Q}}}{E - \epsilon_{\mathbf{Q}-\mathbf{k},2} - \epsilon_{\mathbf{k},1}}$$

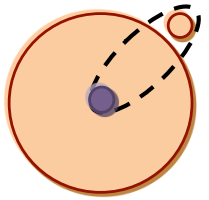
[Yamashita et al. J Phys Soc Japan (2010)]
[Pieri et al. PRB(2007)]
[Subasi et al. PRB (2010)]

⇒ minority particle chemical potential $\mu_2 = E - \mu_1 = E - \frac{k_F^2}{2m_1} < 0$

✧ SF & FFLO



$|\psi(\mathbf{Q}=0)\rangle$



$|\psi(\mathbf{Q}\neq 0)\rangle$

Comparison with the polaron approach

✧ Our approach

$$|\Psi(\mathbf{Q})\rangle = \sum_{\mathbf{k}>k_F} \varphi_{\mathbf{kQ}}\, c^\dagger_{\mathbf{Q}-\mathbf{k},2} c^\dagger_{\mathbf{k},1} |FS\rangle$$ bound state

$$|\Psi_0\rangle = c^\dagger_{0,2} c^\dagger_{k_F\hat{\mathbf{k}},1} |FS\rangle$$ unbound state

+ screened Coulomb interactions (infinite particle-hole excitations)

✧ In cold atoms (majority Fermi sea non-interacting and interspecies interaction short range): POLARON ANSATZ

$$|\Psi(\mathbf{Q})\rangle = \sum_{\mathbf{k}>k_F} \varphi_{\mathbf{kQ}}\, c^\dagger_{\mathbf{Q}-\mathbf{k},2} c^\dagger_{\mathbf{k},1} |FS\rangle$$

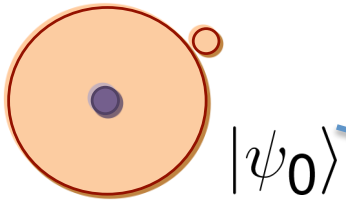
$$|\Psi_0\rangle = c^\dagger_{0,2} c^\dagger_{k_F\hat{\mathbf{k}},1} |FS\rangle$$

add a single particle-hole excitation

✧ Not profitable to just excite *one* particle-hole pair when interactions are long-range and the majority Fermi sea is interacting

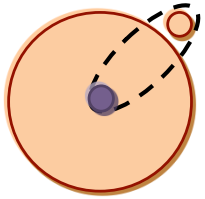
Phase diagram of fully imbalanced EH bilayer

- Low density: Q=0 exciton (SF)
  (two-body limit)
- High density:
  screening causes unbinding (N)

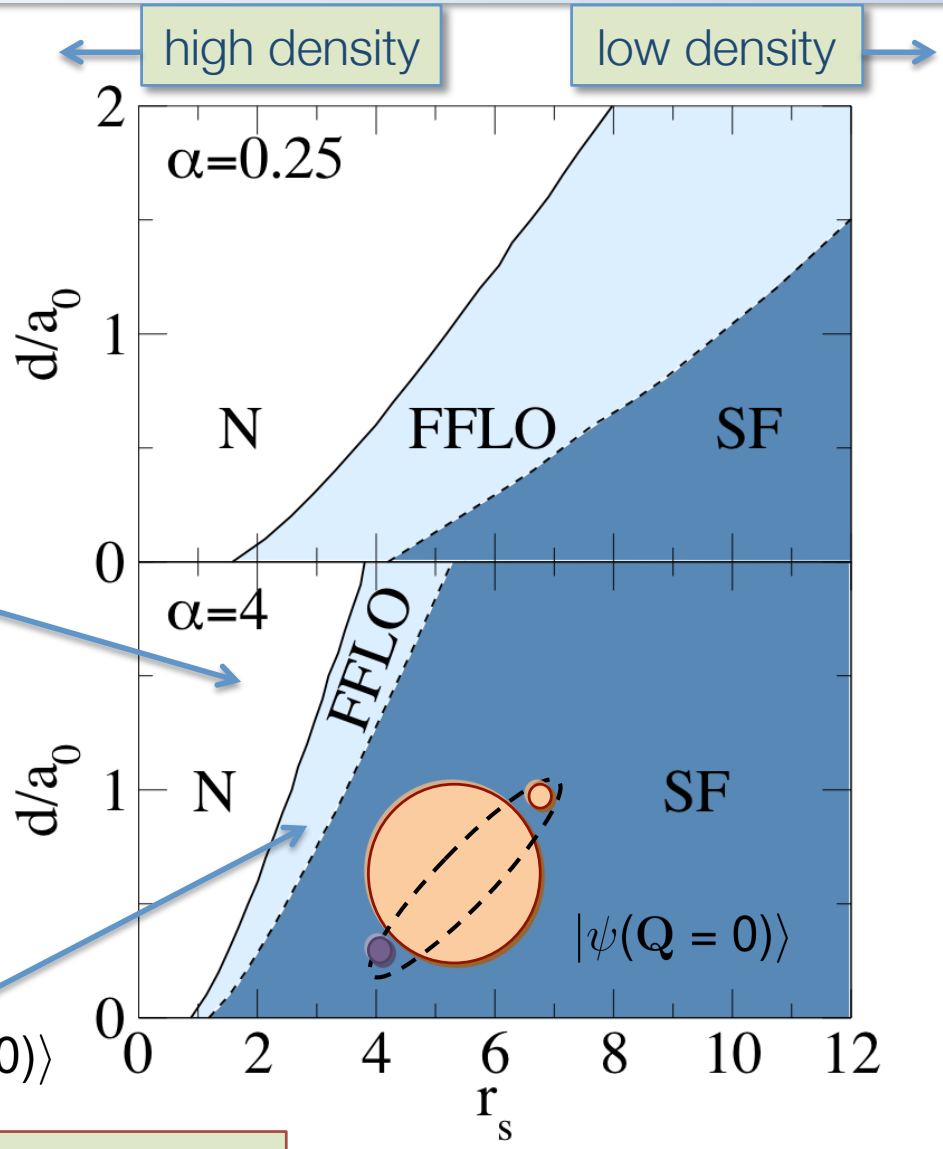


$|\psi_0\rangle$

- Intermediate density:
  Significant region of FFLO



$|\psi(\mathbf{Q} \neq 0)\rangle$

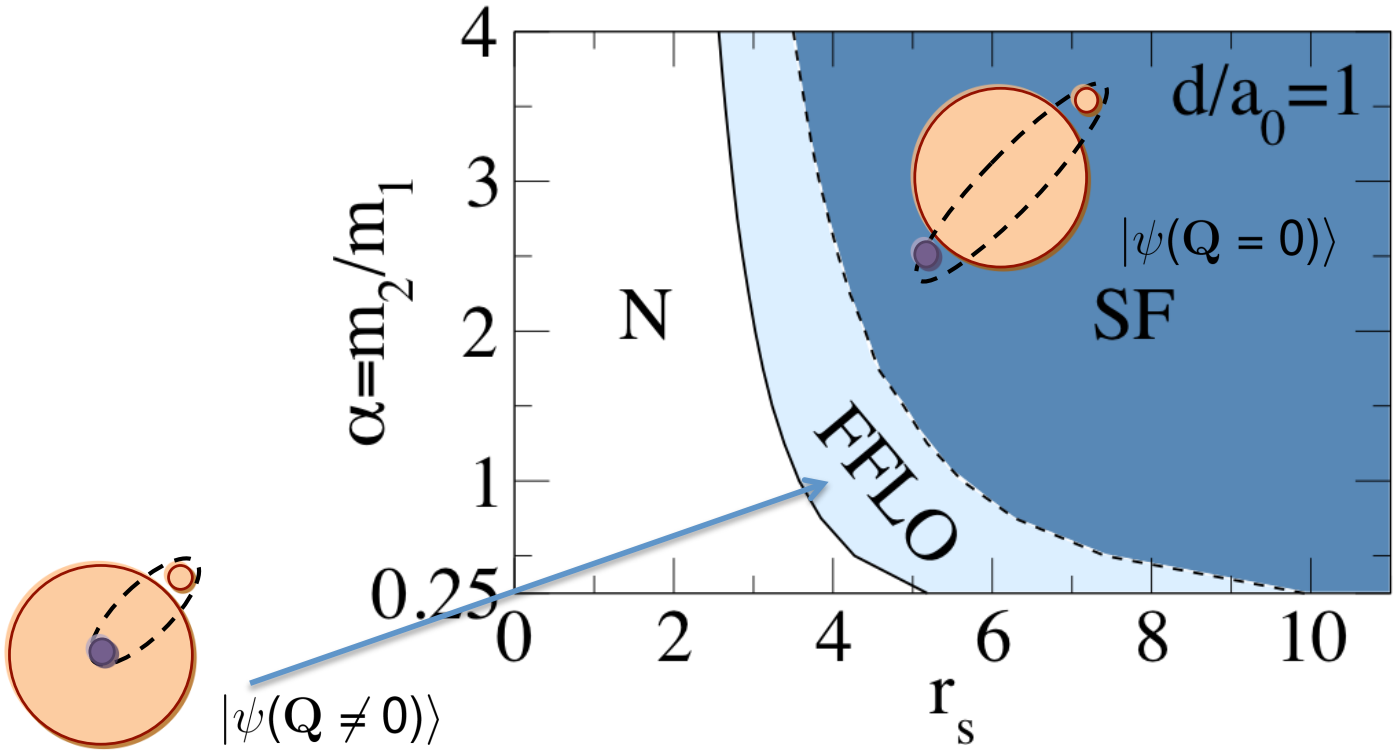


[arxiv/cond.mat/1009.1420]

- FFLO region enhanced if minority particle lighter

Changing the mass ratio

- A Q=0 exciton requires a minority particle to sit above the Fermi sea
  ⇒ Kinetic cost increases for lighter m_2 favouring FFLO

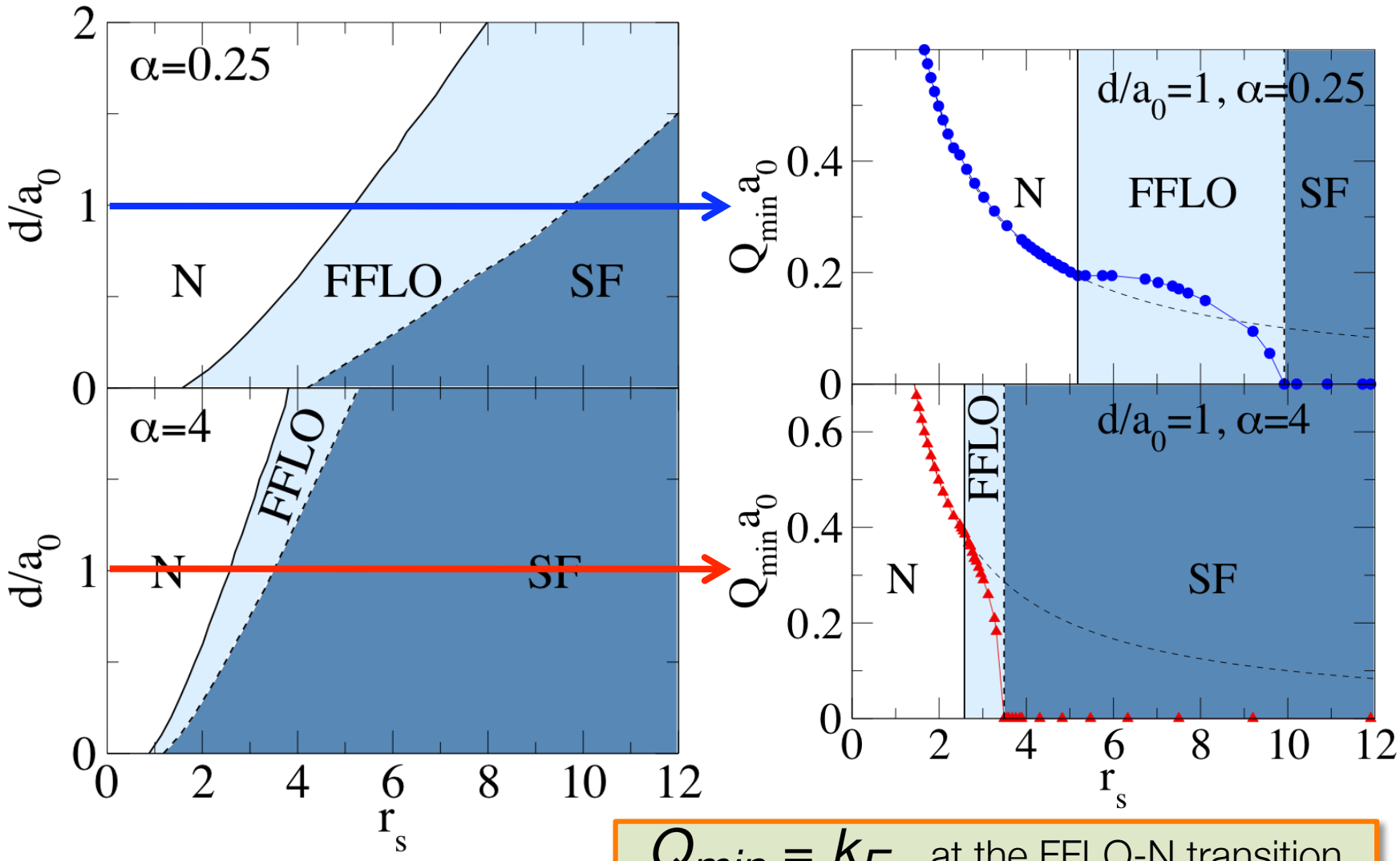


- FFLO region enhanced if minority particle lighter

[arxiv/cond.mat/1009.1420]

Behaviour of Q_{min}

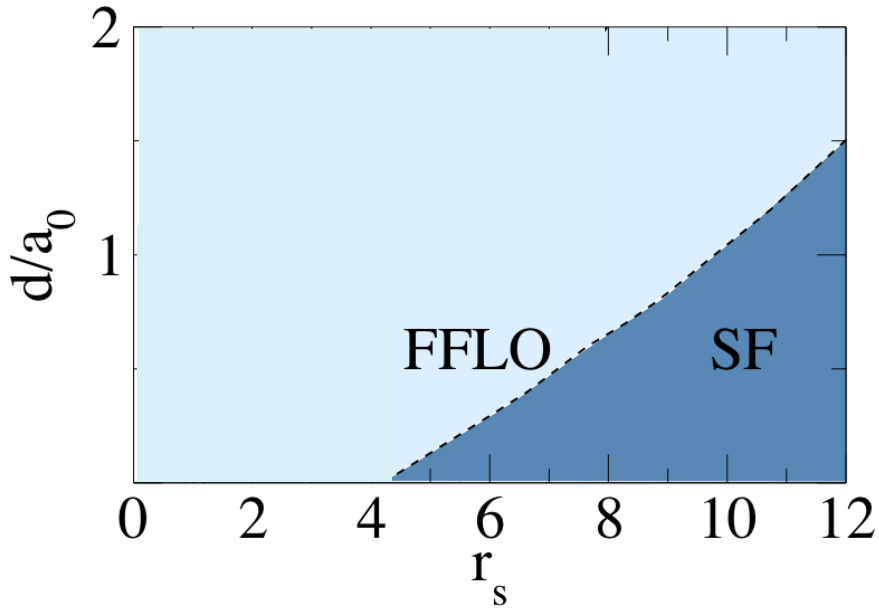
- Second order phase transitions



$Q_{min} = k_F$ at the FFLO-N transition

Effect of screening on the LOFF phase

✧ Unscreened case



⇒ Numerically

⇒ Analytically:

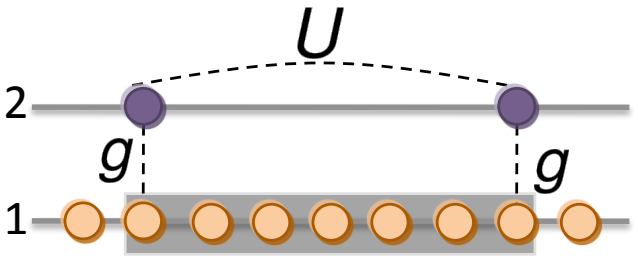
$$\Delta_{\mathbf{kQ}} = \int_{k' > \frac{1}{r_s}} \frac{d\mathbf{k}'}{2\pi} \frac{e^{-d|\mathbf{k}-\mathbf{k}'|}}{|\mathbf{k}-\mathbf{k}'|} \frac{\Delta_{\mathbf{k}'\mathbf{Q}}}{-E + \epsilon_{\mathbf{Q}-\mathbf{k}',2} + \epsilon_{\mathbf{k}',1}}$$

✧ At the unbinding transitions ($E = \varepsilon_F$) the integral is divergent for $\mathbf{k}' = \mathbf{Q} = \mathbf{k} = k_F\hat{\mathbf{k}}$, so the exciton with $Q = k_F$ is always bound ($r_s \to 0$)

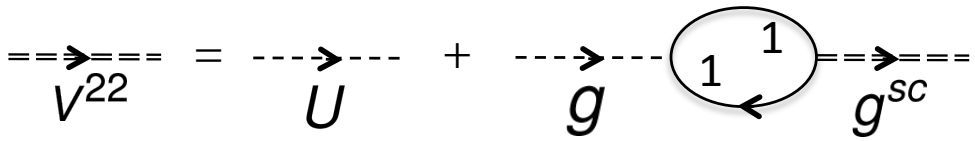
[Yamashita et al. J Phys Soc Japan (2010)]

Dilute gas of minority particles: Interactions

✧ Normal phase: effective interaction between two unbound minority particles (RPA)

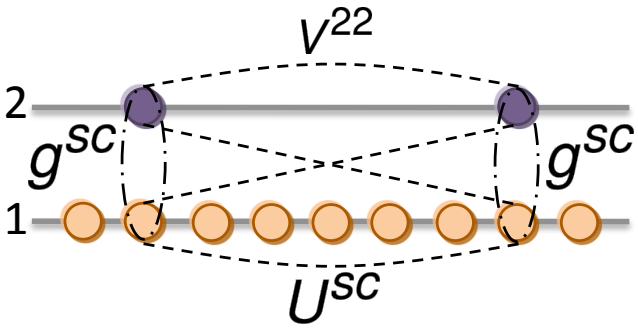


$$V_q^{22} = U_q + g_q \Pi_1(q) g_q^{sc}$$



⇒ repulsive and dipolar $V^{22}(r) \to \frac{1}{r^3}$ the dilute gas is a Fermi liquid

✧ Well separated excitons (dipoles)



$$V_q^{ex} = 2g_q^{sc} + U_q^{sc} + V_q^{22}$$

⇒ also repulsive and dipolar $V^{ex}(r) \to \frac{1}{r^3}$: no phase separation

(biexciton formation suppressed for single spin species & $\frac{d}{a_0} \gtrsim 0.25$)

Transparency title