Fracture and Fragmentation of Simplicial Finite Element Meshes using Graphs

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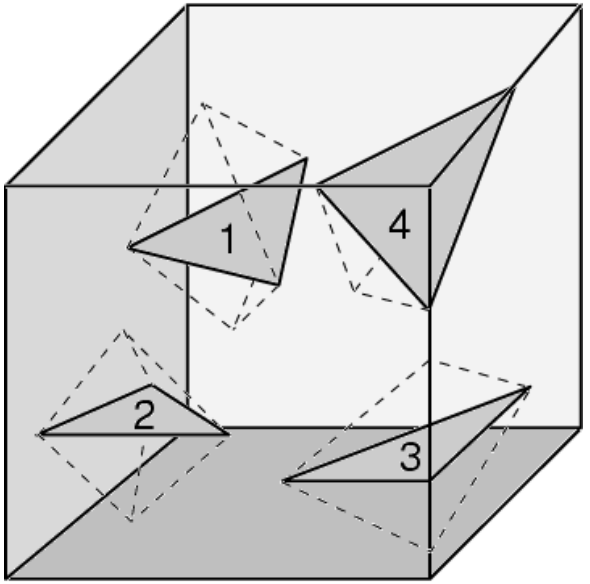
Existing Fracture Algorithm

Implemented using a 4-way linked-list data structure.



Pandolfi and Ortiz (1999)

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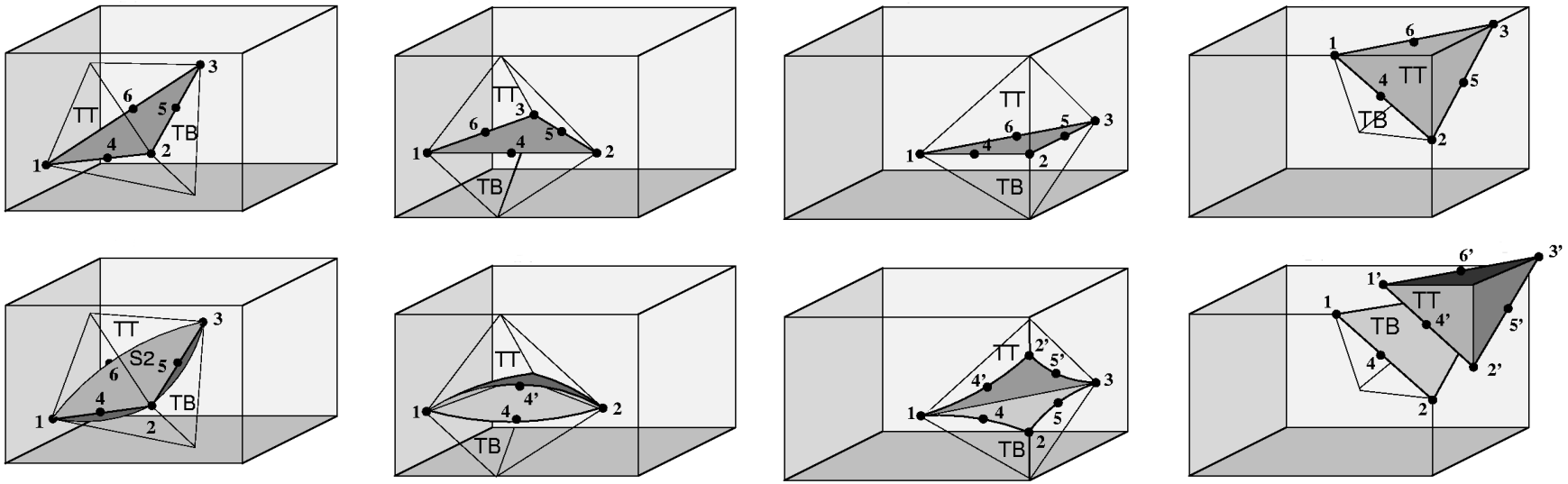
Fragmentation Cases

Change of topology according to the number of boundary sides of the triangle to be duplicated:



0 sides 1 side 2 sides 3 sides

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Motivation

- Parallel 3D fracture and fragmentation.
- Parallel 3D contact.
- Same topology representation for both.
- Reuse for 2D if possible.
- Better performance for large meshes.
- Independence of interpolation scheme.
- Correctness.

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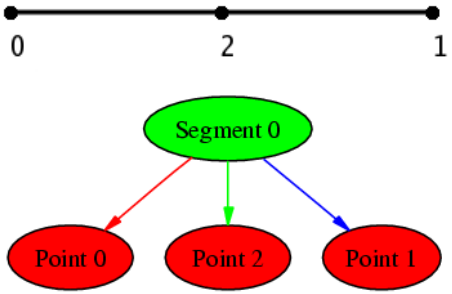
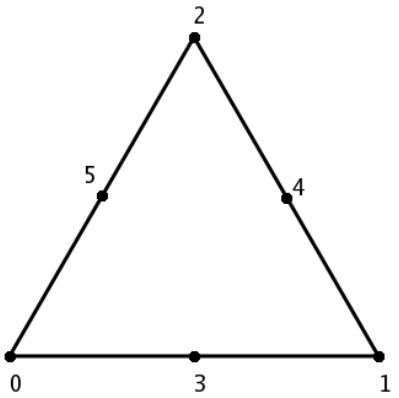
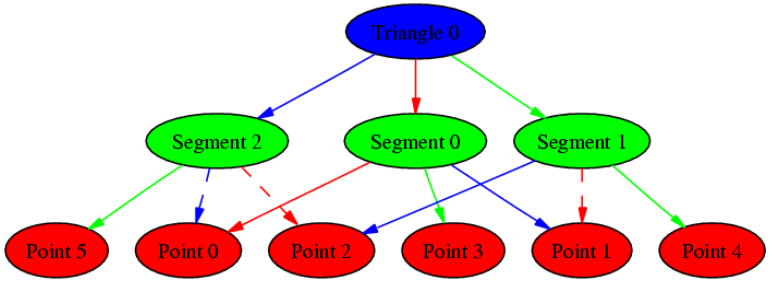
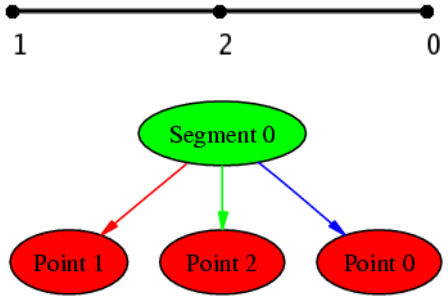
Simplices as Graphs

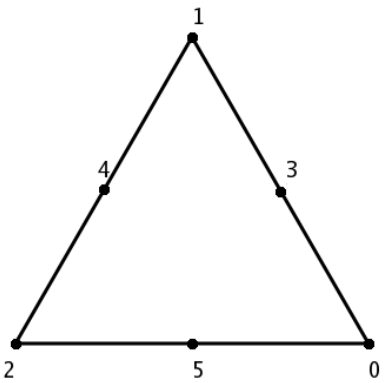
Simplices are graphs vertices, and colored edges represent connectivity and orientation. Graphs are directed.

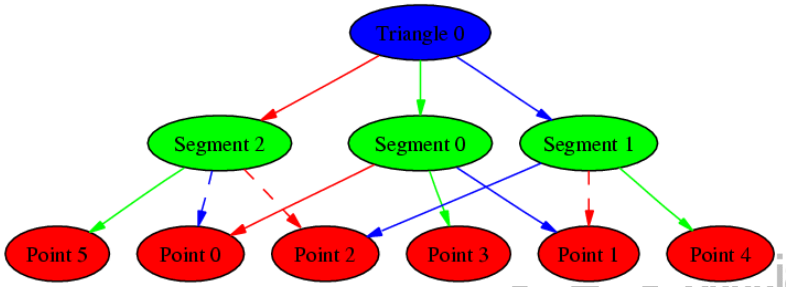












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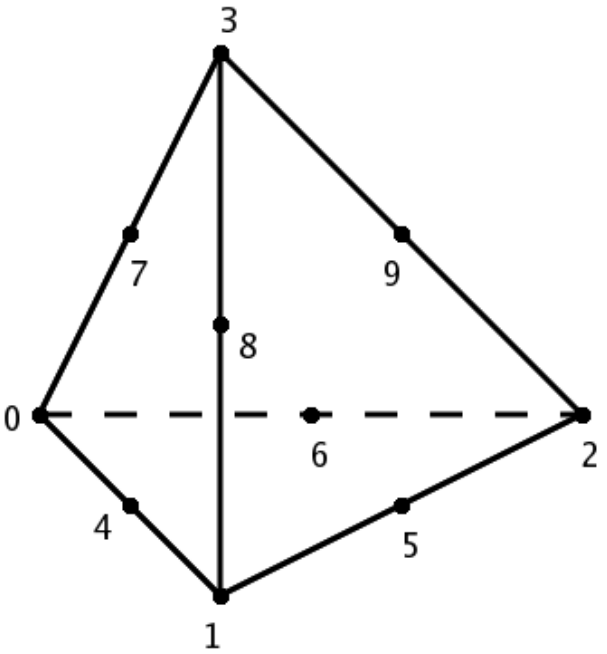
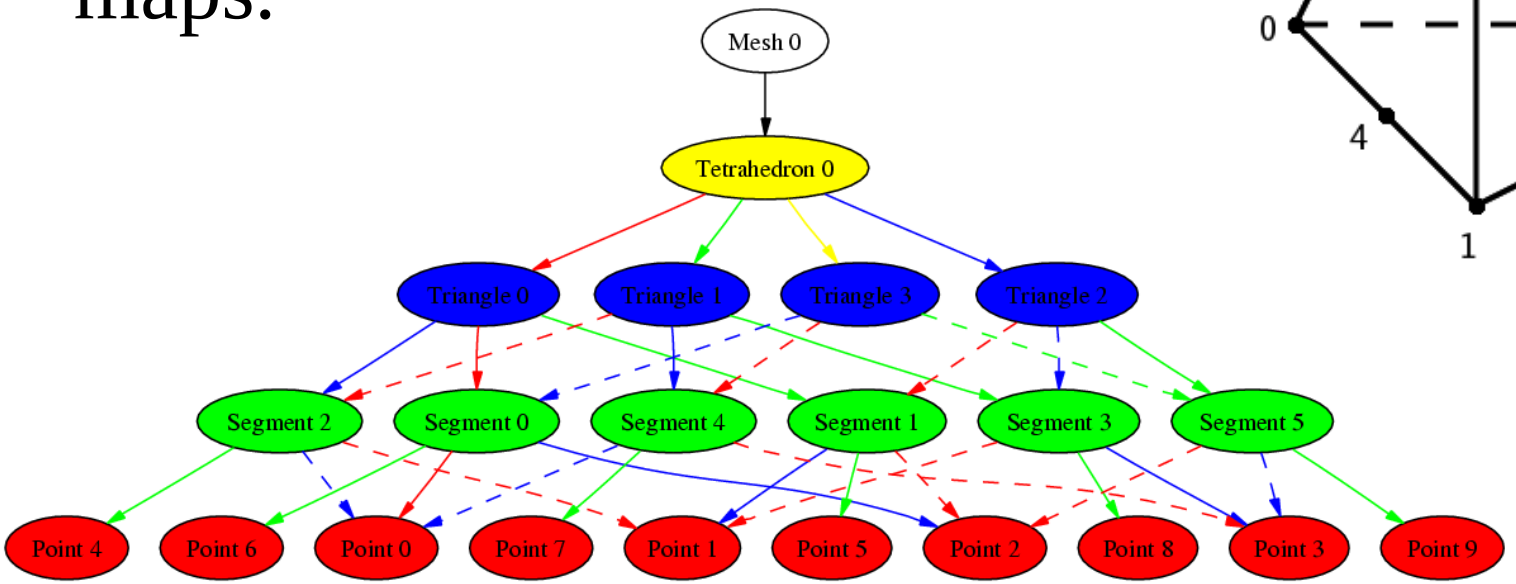
FE Meshes as Graphs

- Building of graph is top-down.
- Vertices are shared.
- Original orientation kept by color maps.





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FE Meshes as Graphs

Simplex:

$$\sigma = [x_0, \ldots, x_n]$$

Face operator:

$$d_i(\sigma) := [x_0, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n]$$

Incidence number:

$$[\sigma^p, \sigma^{p-1}] := \begin{cases} 0 & \text{if } \sigma^p \cap \sigma^{p-1} = \emptyset \\ 1 & \text{if } \sigma^{p-1} = d_i(\sigma^p) \\ -1 & \text{if } \sigma^{p-1} = -d_i(\sigma^p) \end{cases}$$

Graph:

$$V = \{v | v = f(\sigma) \in \mathbb{N}, \sigma \in K\}$$

$$E = \{e | e = (u, v), u = f(\sigma^p) \in V, v = f(\sigma^{p-1}) \in V, \sigma^p \in K, \sigma^{p-1} \in K, [\sigma^p, \sigma^{p-1}] \neq 0\}$$

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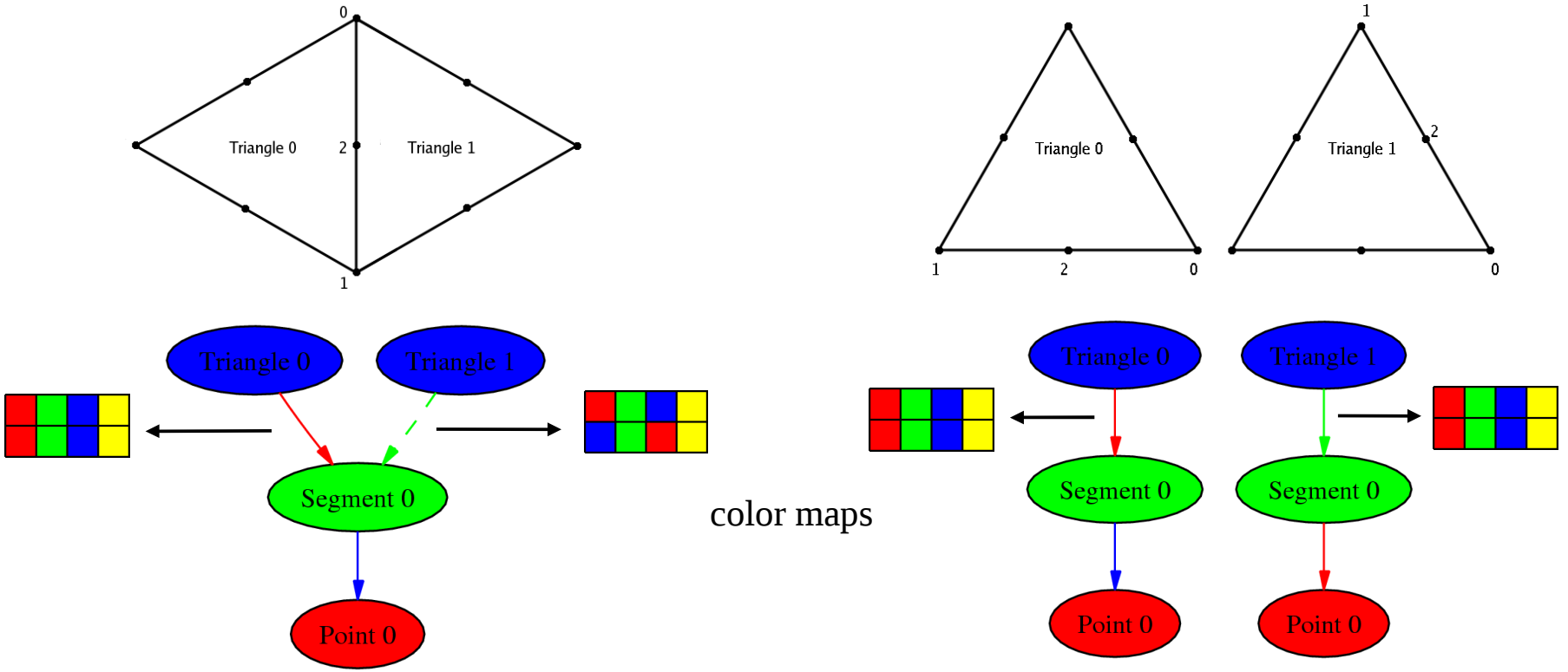
Local Ordering as Color Maps

Triangle 0 sees point 0 as the blue point in segment 0. Triangle 1 sees it as red.



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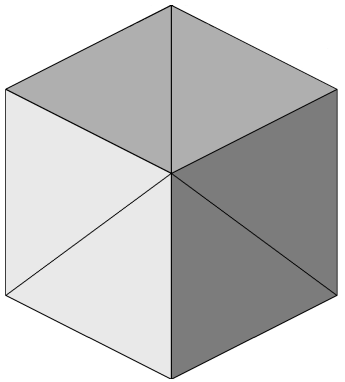
Cube Mesh as Graph

Graphs are quite complex even for meshes with a few elements.



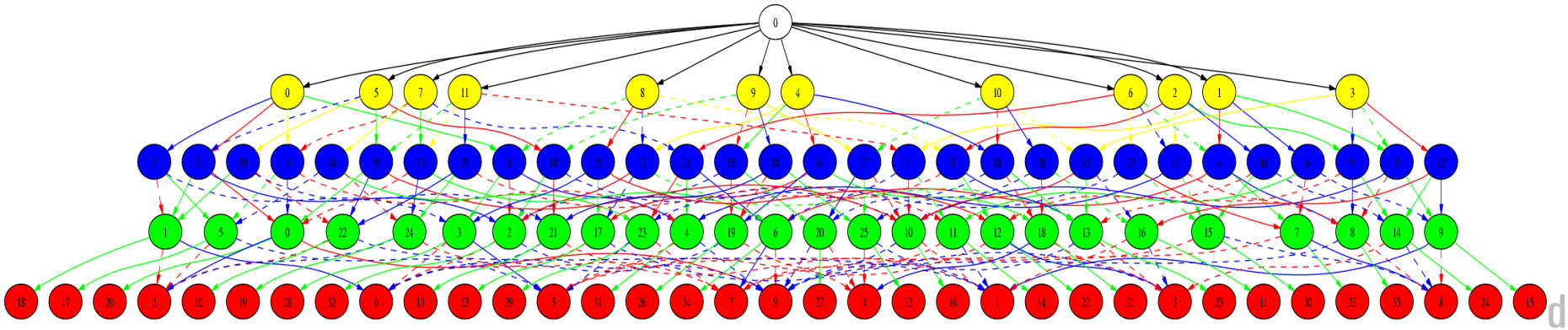


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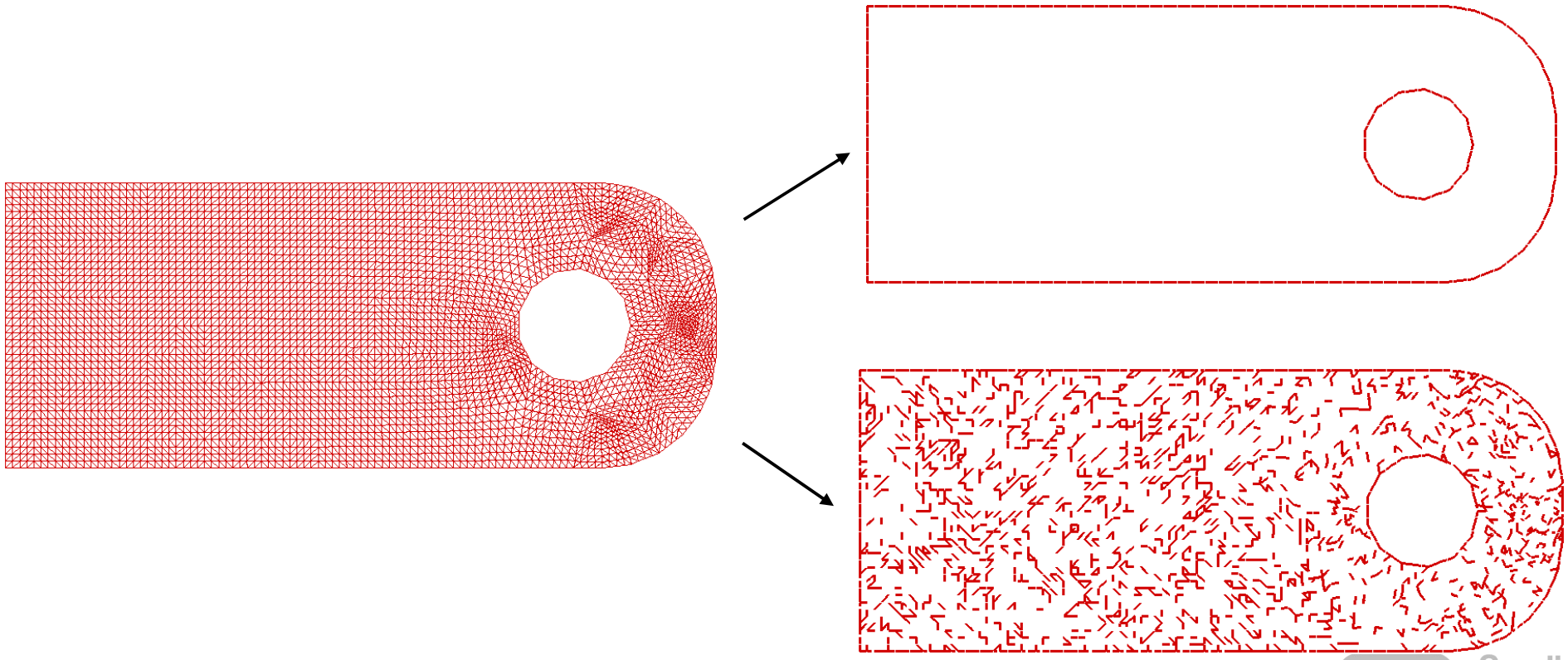
Plate with Hole Mesh

Same graph representation works for 2D. Easy to extract useful features.



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Simplex Graph Properties

- Simplices are vertices.
- Edges represent connectivity.
- Directed graph.
- Built top-down.
- Edge color is local order of n-simplex wrt to (n+1)-simplex.
- An n-simplex may be shared by many (n+1)-simplices.

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Simplex Graph Properties (2)

- Edge color map modifies color of edge directly below if target vertex is shared.
- Color maps are linear maps akin to reflections and rotations, or single and double permutations.
- Graph depth represents the dimension of the mesh. Works for N dimensions.
- Connectivity array can always be recovered from graph.

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Graph Fracture Algorithm

- Mark open triangles, segments, and corner points.
- Build segment subgraphs.
- Clone open triangles.
- Split segment vertices.
- Build point subgraphs.
- Split point vertices.

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Graph Fracture Algorithm

Algorithm 1. SPLIT(G, U, n, i) Split articulation points.

Require: $U \subset V_i$, $i \leqslant n-1$

for all $v \in U$ **do**
  if $i < n-2$ **then**
    SPLIT$(G, D^-(v), n, i+1)$
  else
    CLONE$(G, D^-(v), n)$
  end if
  $G'' \leftarrow G'(v) \setminus v$ // Check whether v is an articulation point
  for all $j \in \{2, \ldots, N(G'')\}$ **do**
    $Y \leftarrow \{u | u \in G''_j,\ g(u) = i+1\}$
    $V' \leftarrow \{V', z\}$ // Split the vertex in the subgraph and graph
    for all $u \in Y$ **do**
      $E' \leftarrow E' \setminus (u, v)$
      $E' \leftarrow \{E', (u, z)\}$
    end for
  end for
end for

Algorithm 2. CLONE(G, U, n) Duplicate fractured interface simplices.

Require: $U \subset V_{\mathrm{I},n-1}$

for all $v \in U$ **do**
  if $v \in V_{\mathrm{F},n-1}$ **then**
    $Y \leftarrow D^-(v)$ // Note that $d^-(v) = 2$, hence $|Y| = 2$
    $u_1 \leftarrow u \in Y$ s.t. $[f^{-1}(u_1), f^{-1}(v)] = 1$
    $u_2 \leftarrow u \in Y$ s.t. $[f^{-1}(u_2), f^{-1}(v)] = -1$
    $V \leftarrow \{V, w\}$
    $E \leftarrow E \setminus (u_2, v)$
    $E \leftarrow \{E, (u_2, w)\}$
    for all $z \in D^+(v)$ **do**
      $E \leftarrow \{E, (w, z)\}$
    end for
  end if
end for

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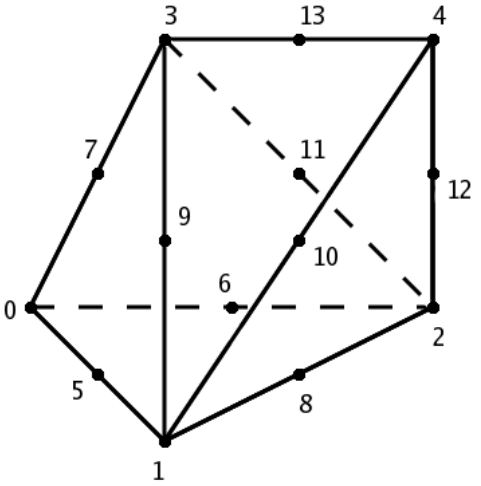
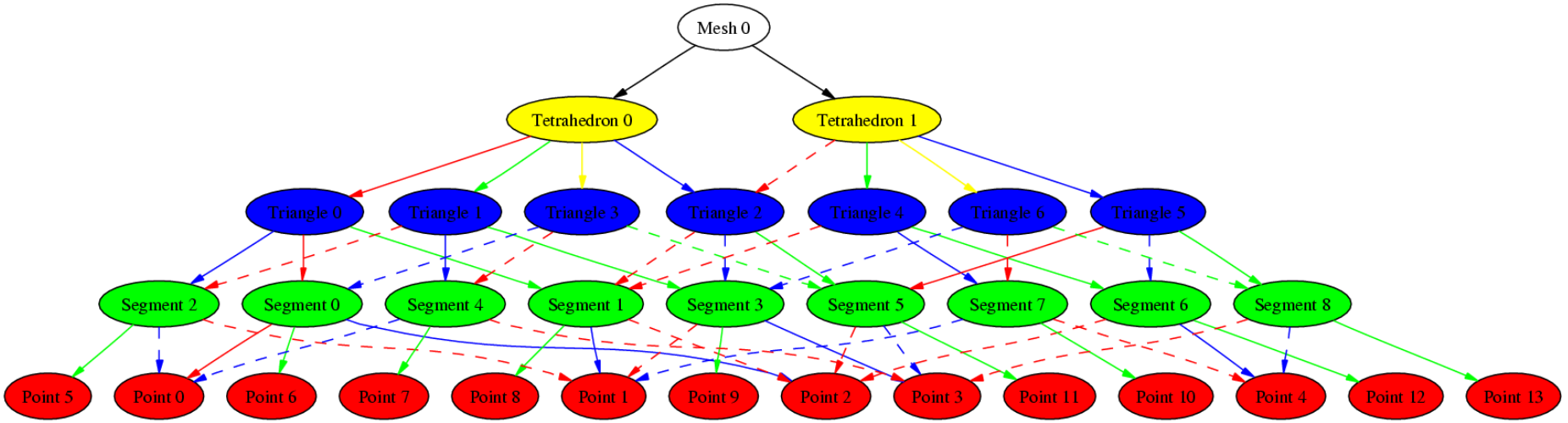
Two-Tetrahedra Mesh

- Single shared triangle.
- Open simplices are triangle 2, segments 1,3,5 and points 1,2,3.





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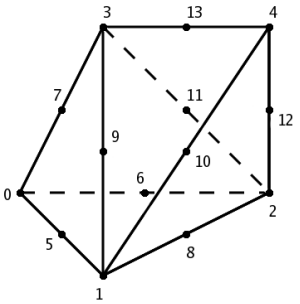
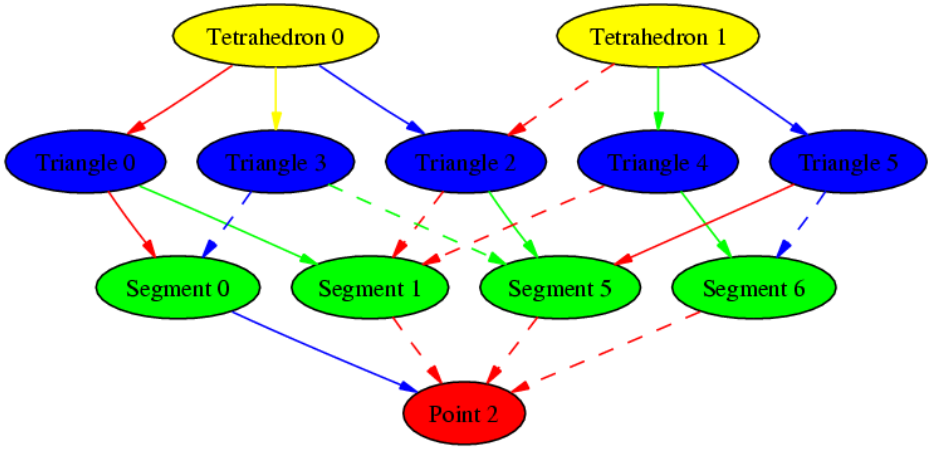
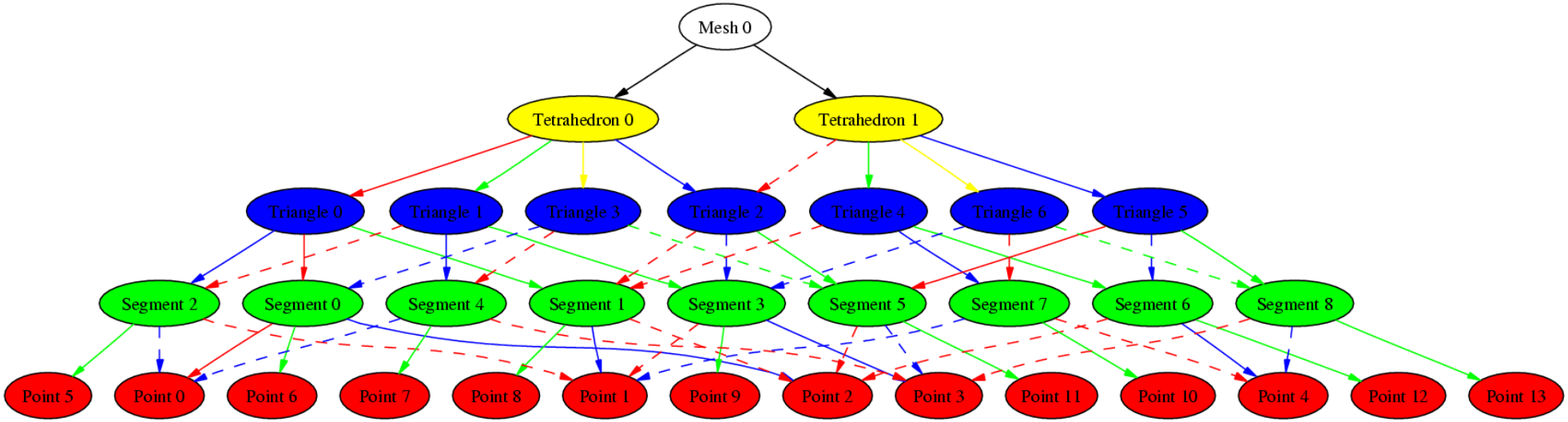
Fracture Algorithm (1)



First selected open point is 2, so process open segments 1 and 5 attached to it.





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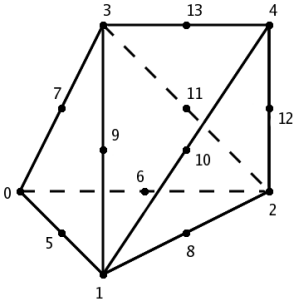
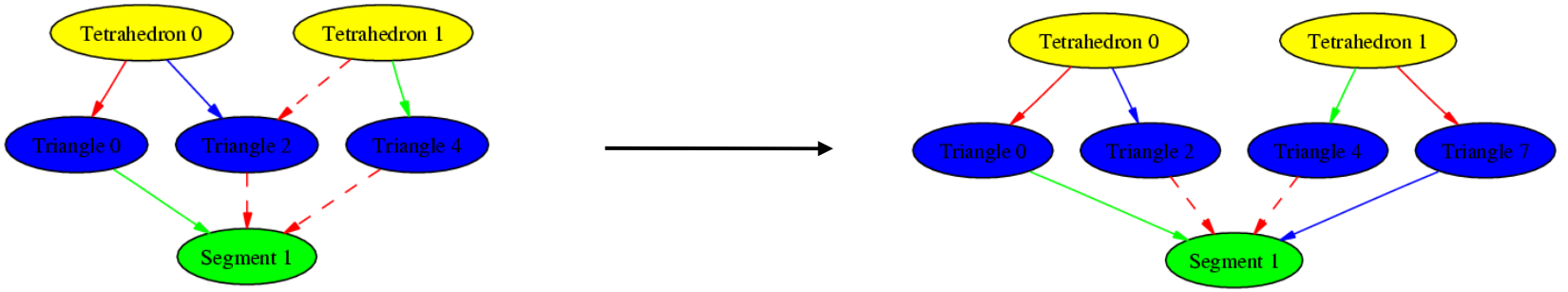
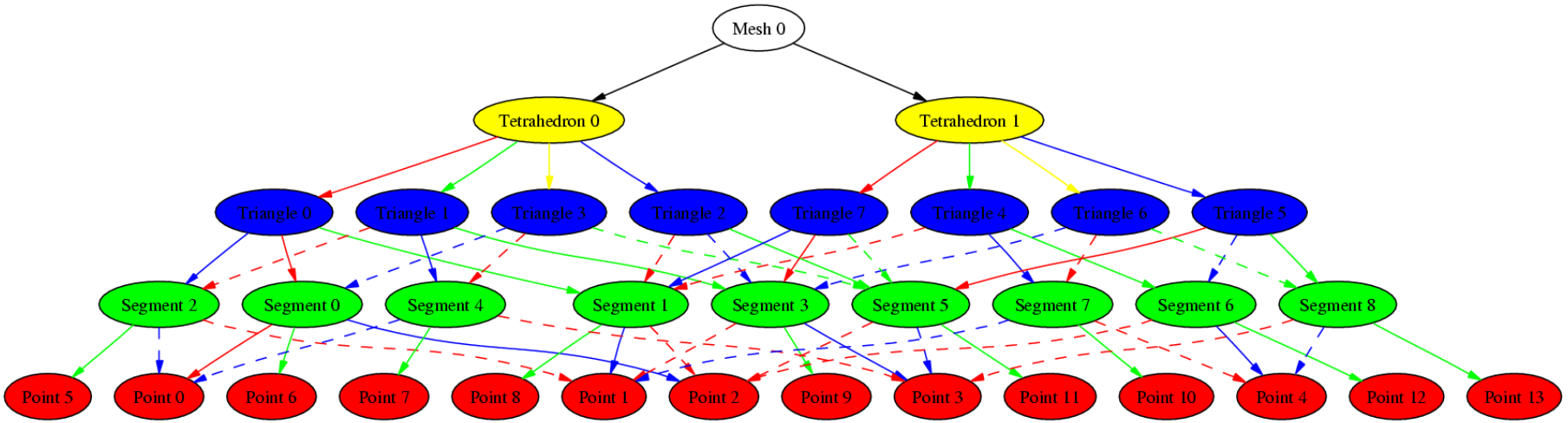
Fracture Algorithm (2)



Extract subgraph for segment 1, which clones open triangle 2, creating triangle 7.





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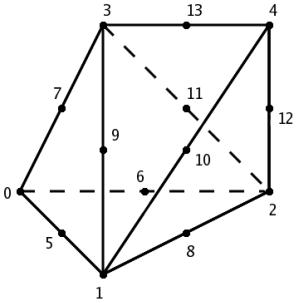
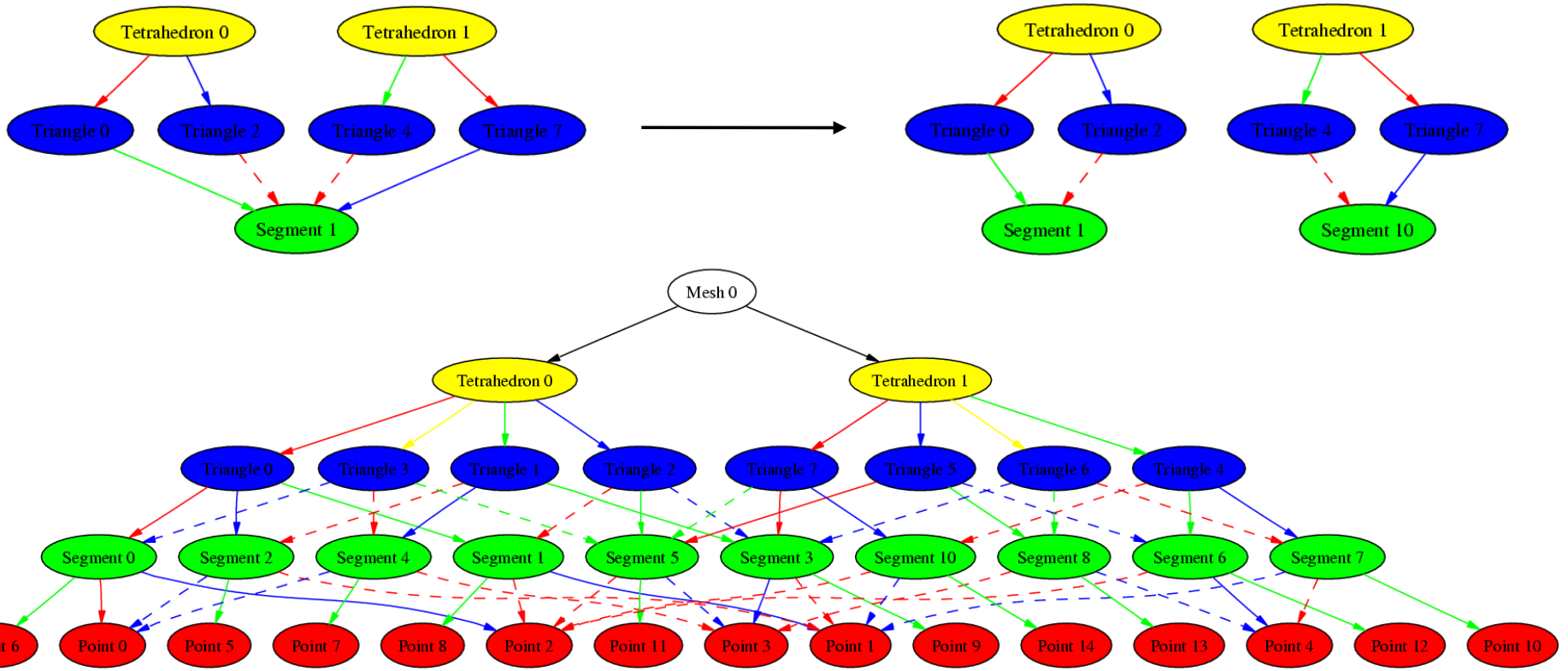
Fracture Algorithm (3)



There are 2 branches in segment 1's subgraph, so the segment is split.



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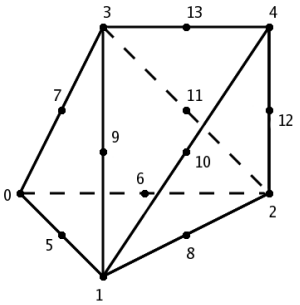
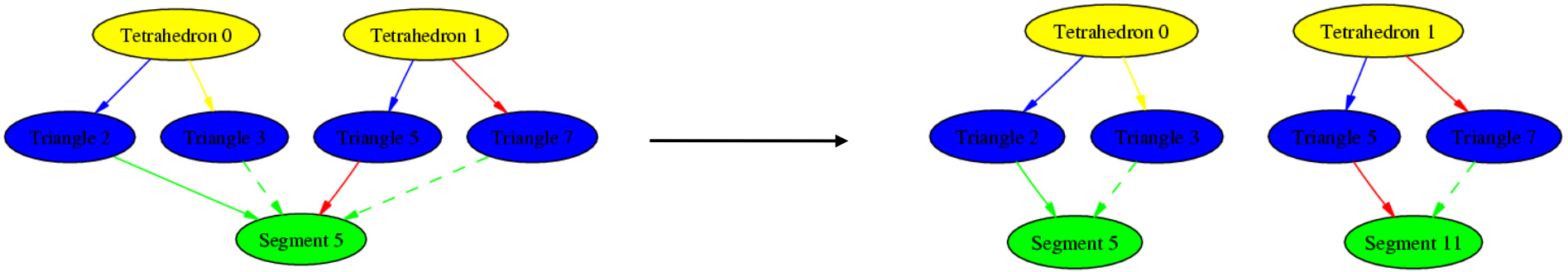
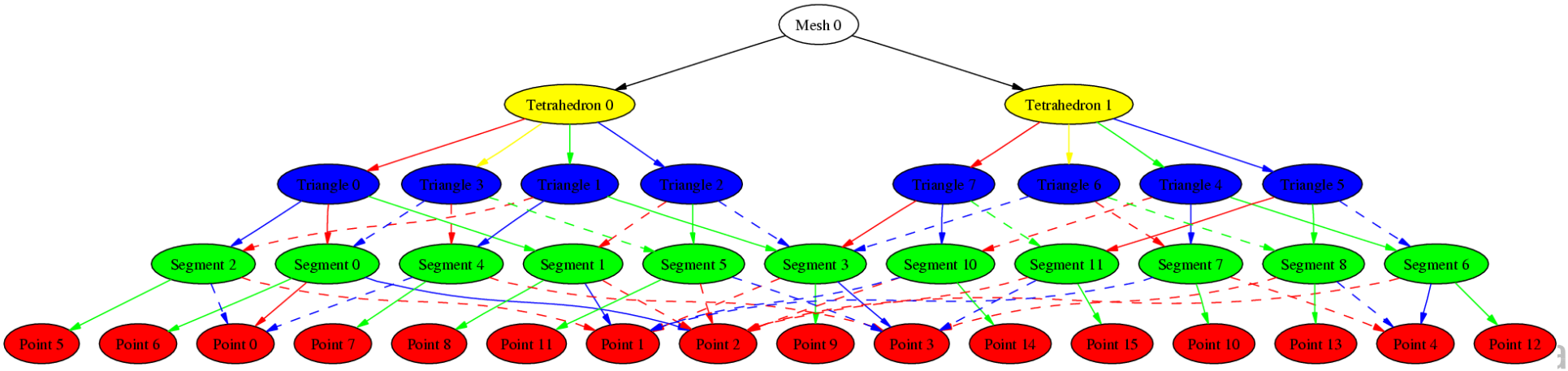
Fracture Algorithm (4)



Next is segment 5, now attached to 0 open triangles.
Its subgraph has 2 branches.





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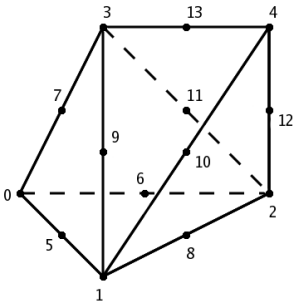
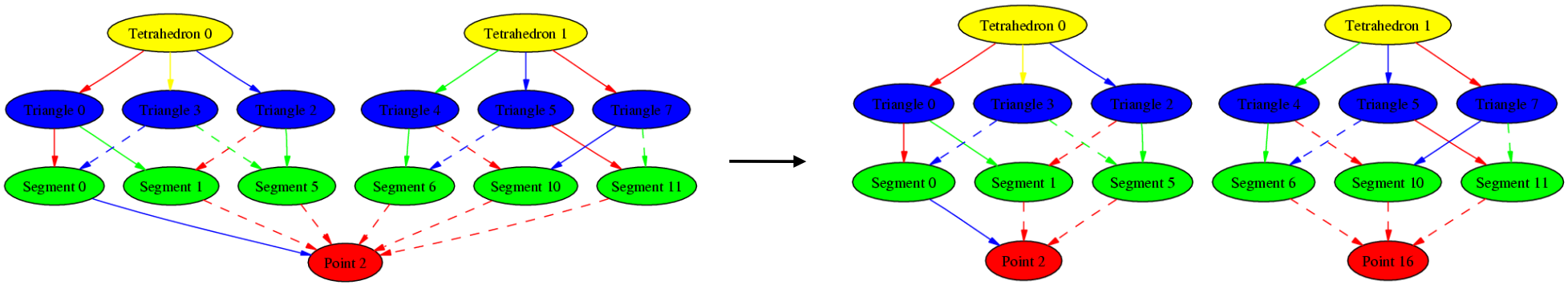
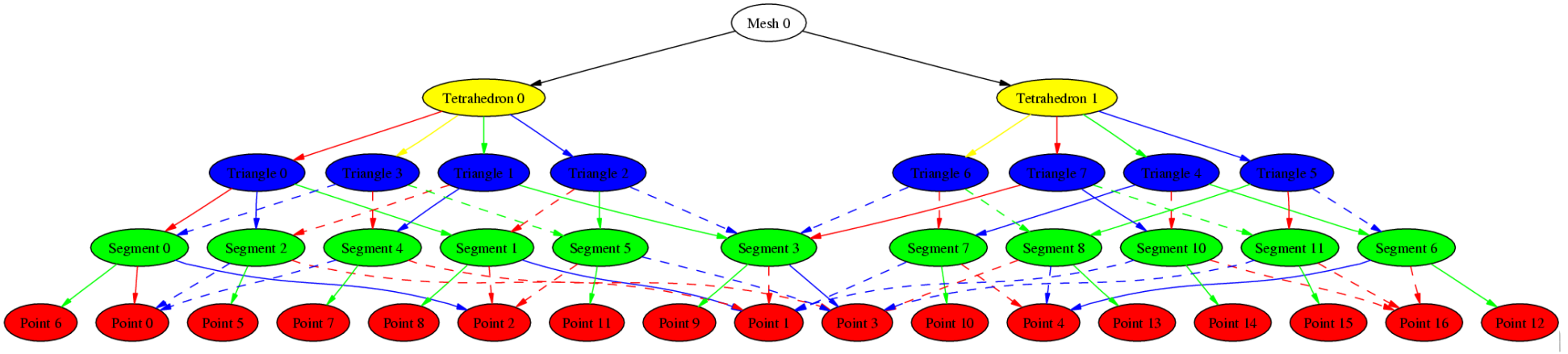
Fracture Algorithm (5)

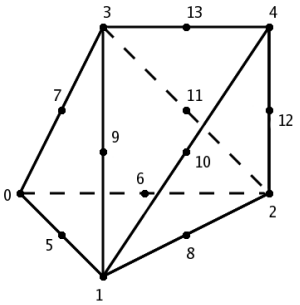


Both open segments for point 2 were split, its subgraph has 2 branches now.

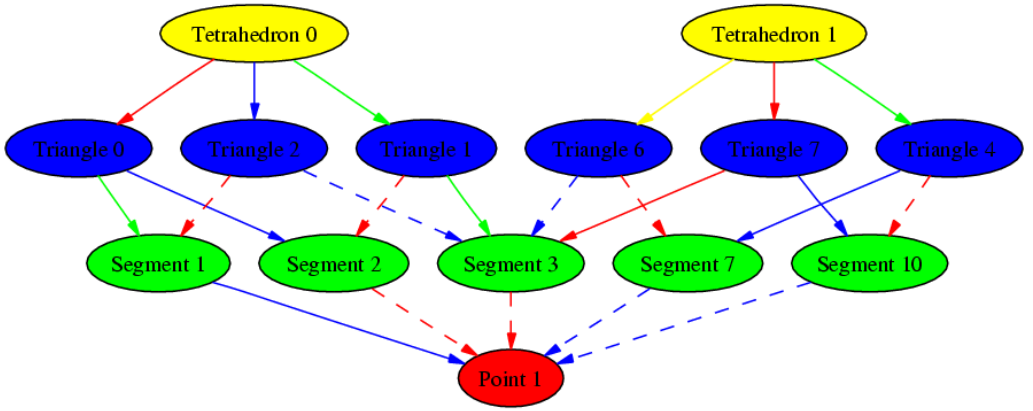


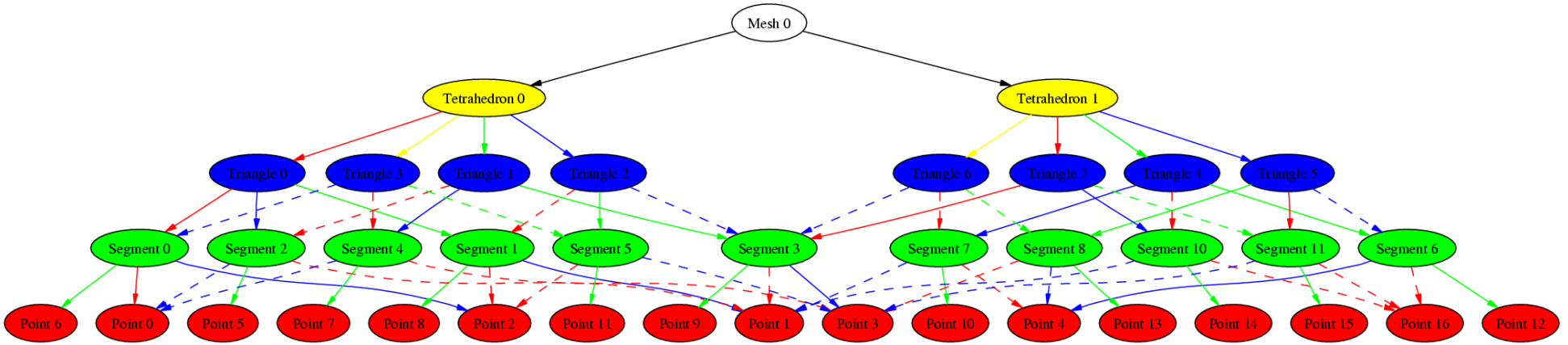


Fracture Algorithm (6)



Next is point 1, with segment 3 the only remaining open segment.





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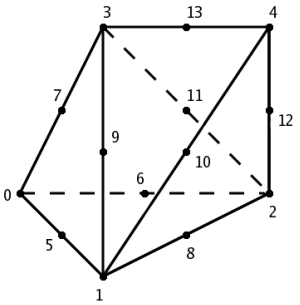
Fracture Algorithm (6)



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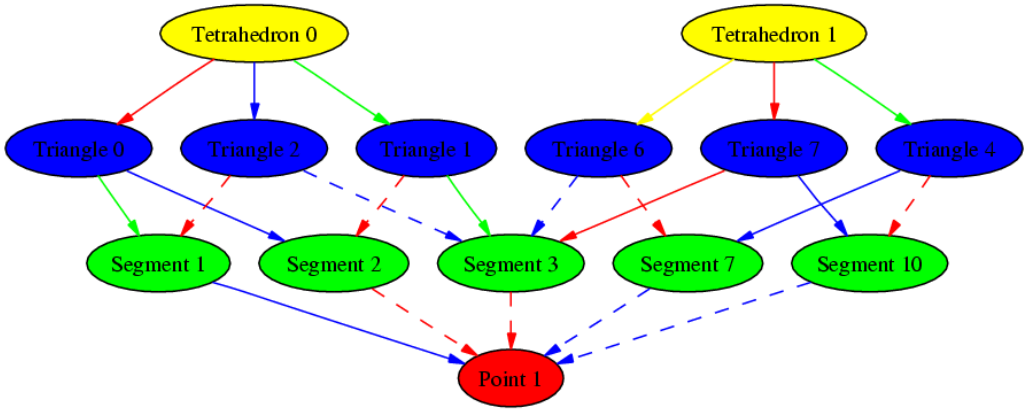

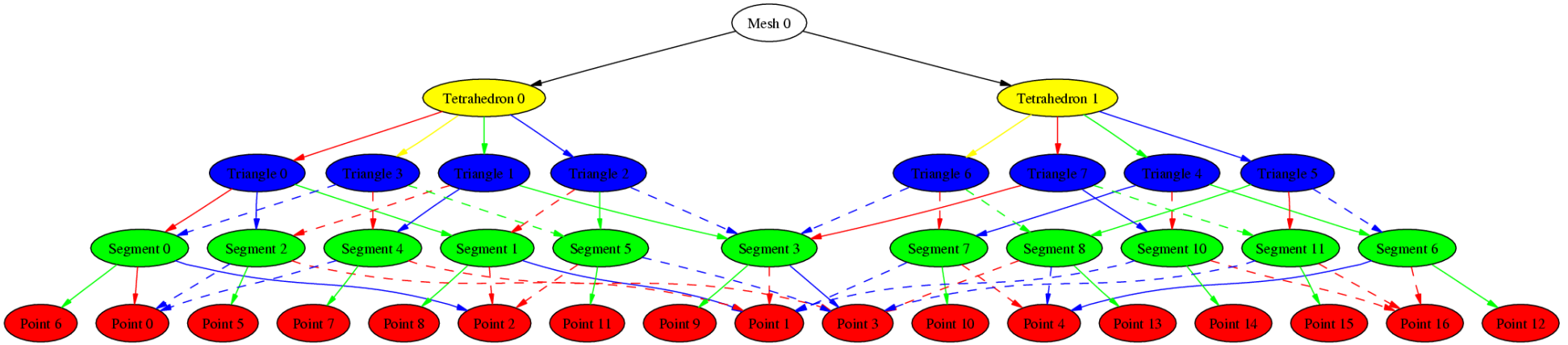

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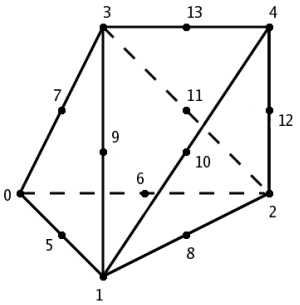
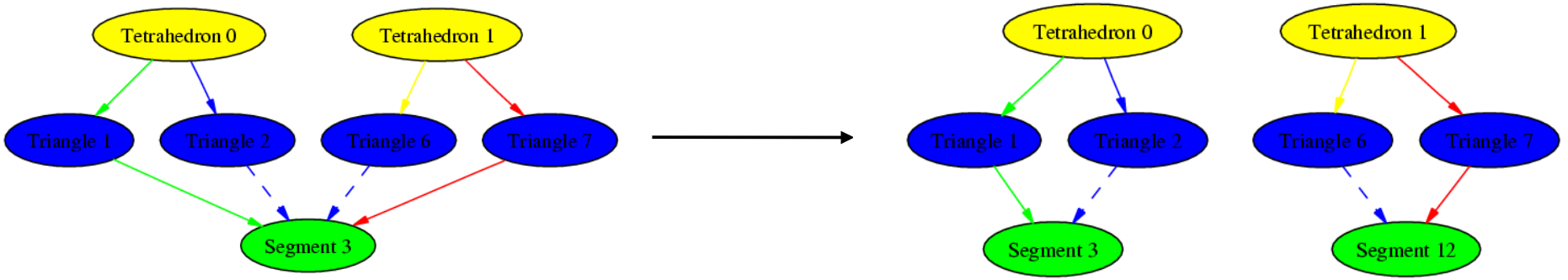
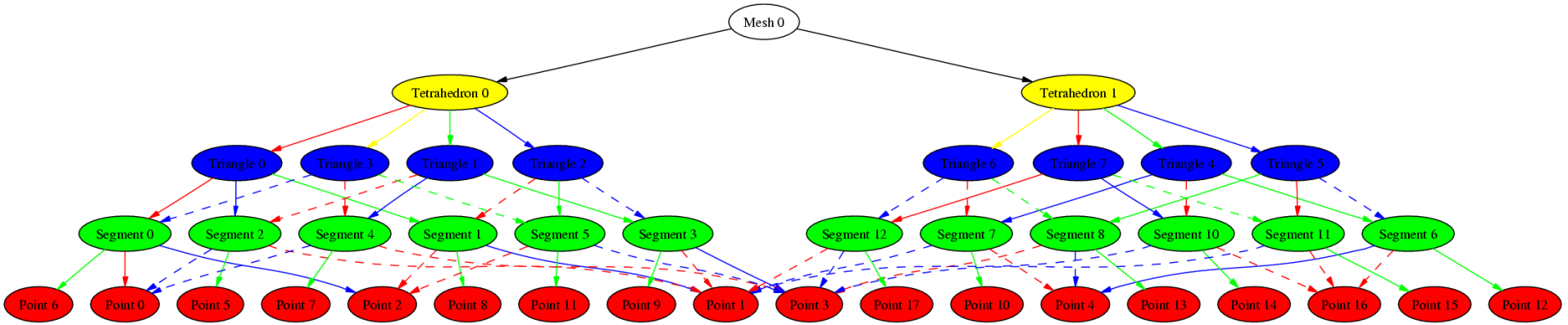
Fracture Algorithm (7)



Segment 3 is split.





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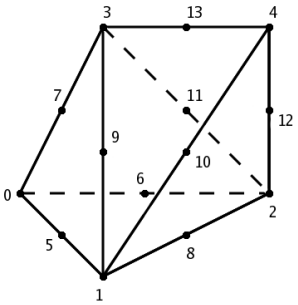
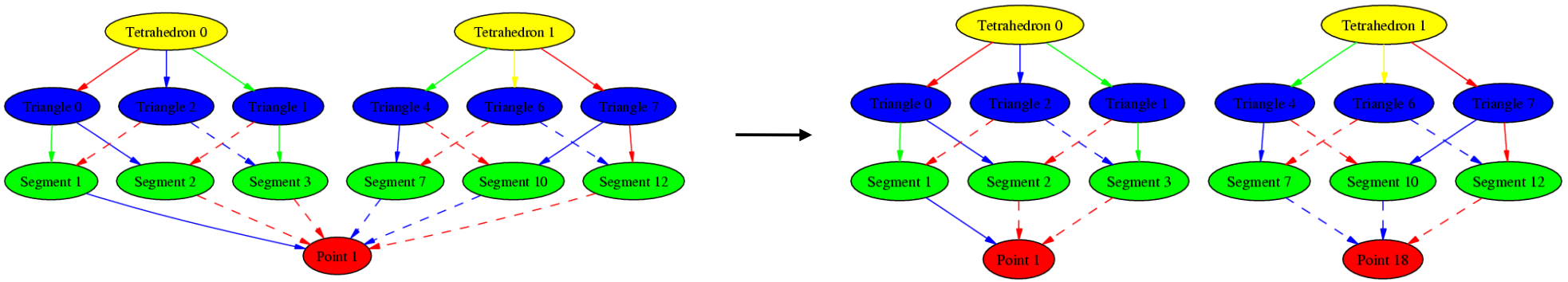
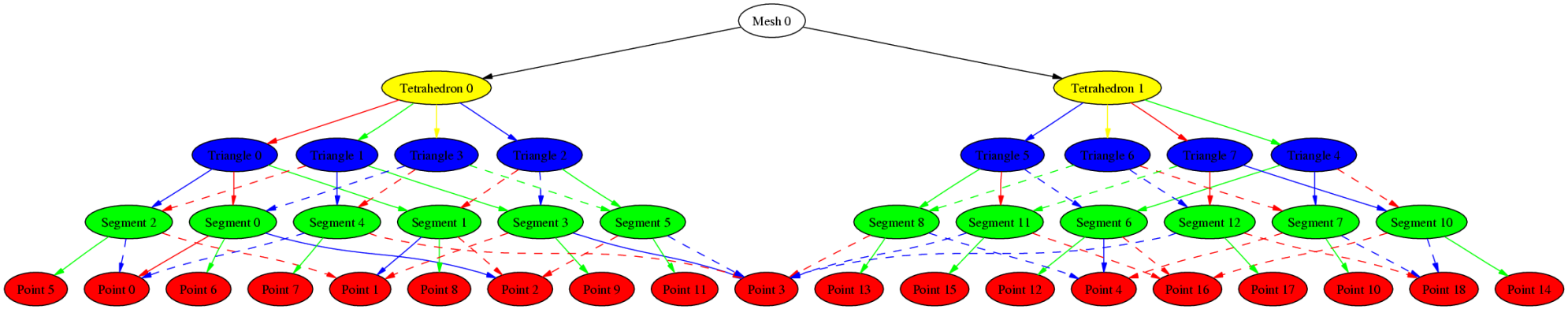
Fracture Algorithm (8)



Point 1 is split.





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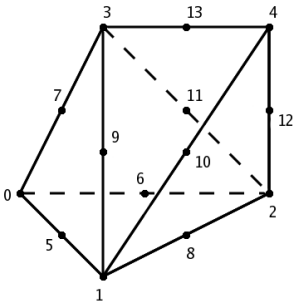
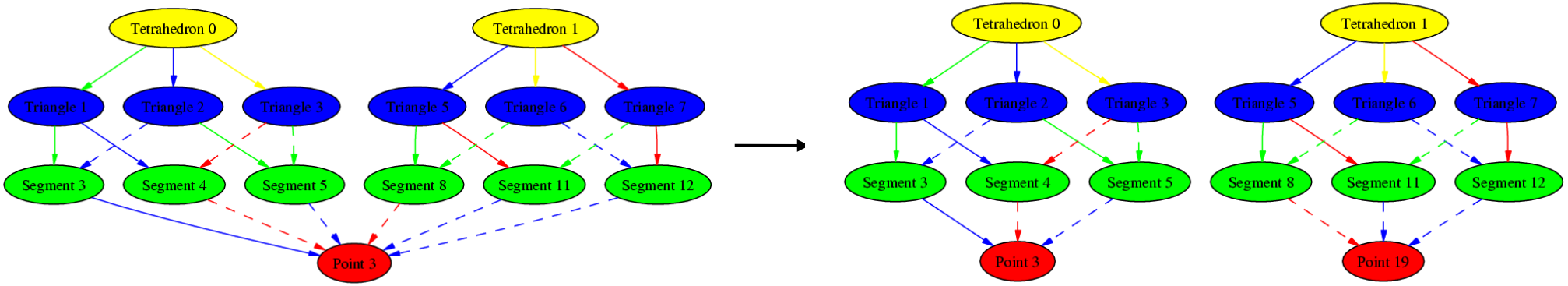
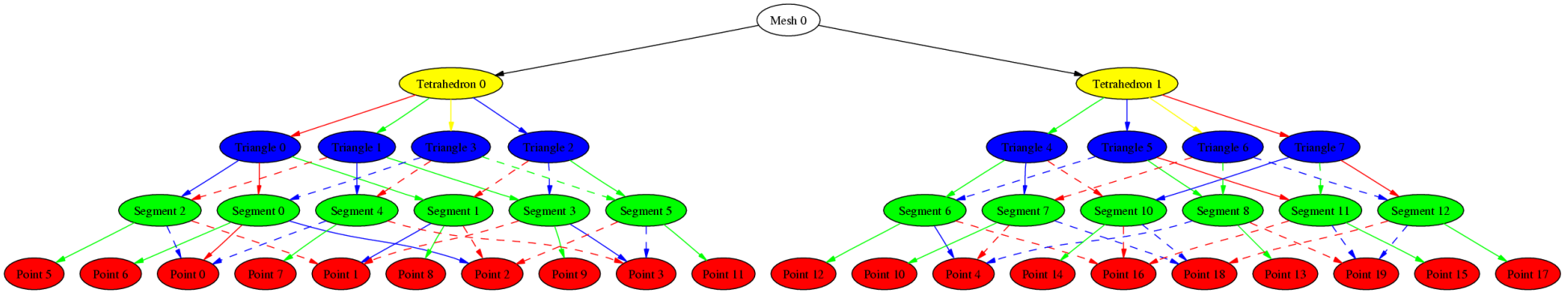
Fracture Algorithm (9)



Finally point 3, with 0 open segments, is split, obtaining 2 separate tetrahedra.





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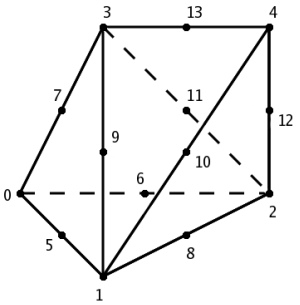
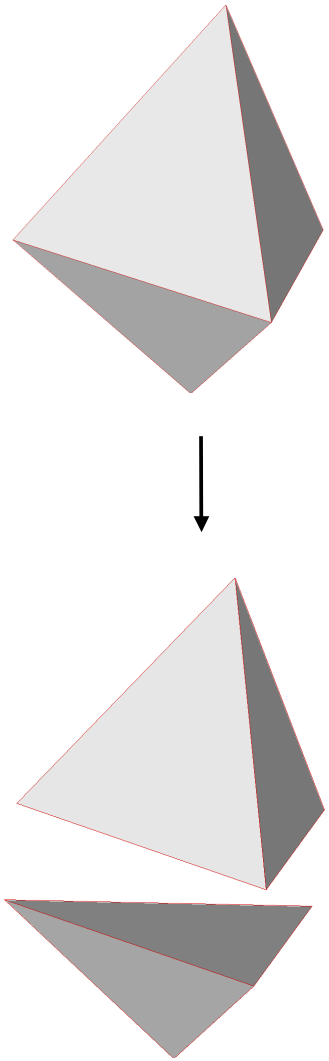
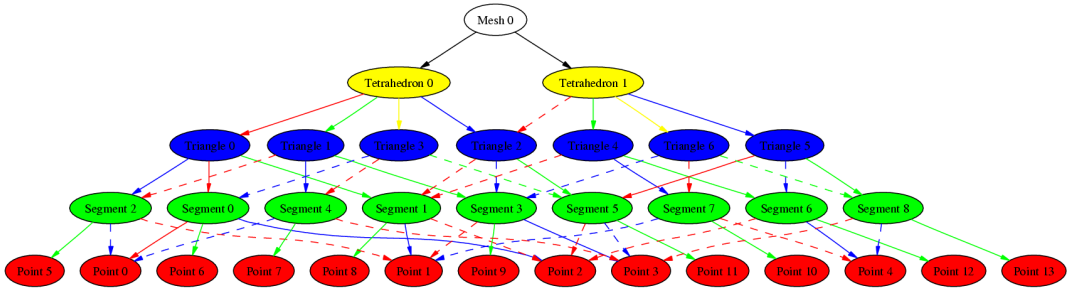
Two-Tet Mesh and Graphs

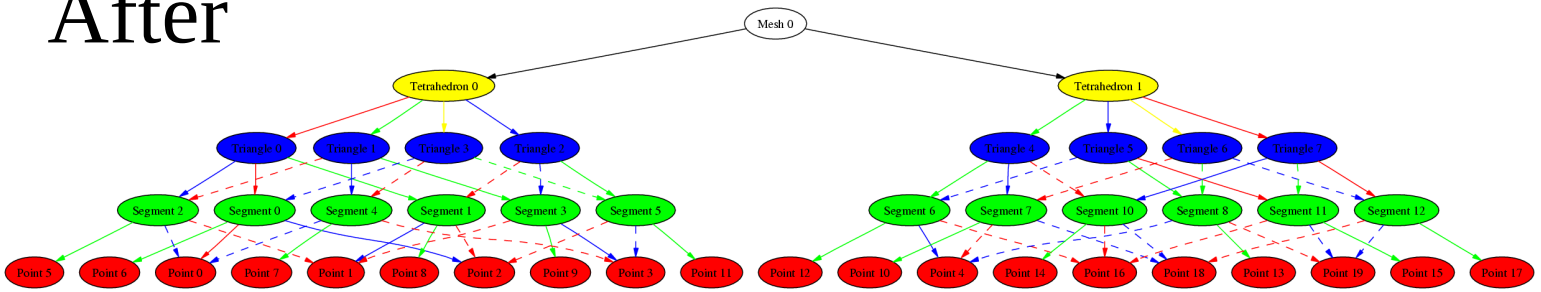




Before



After



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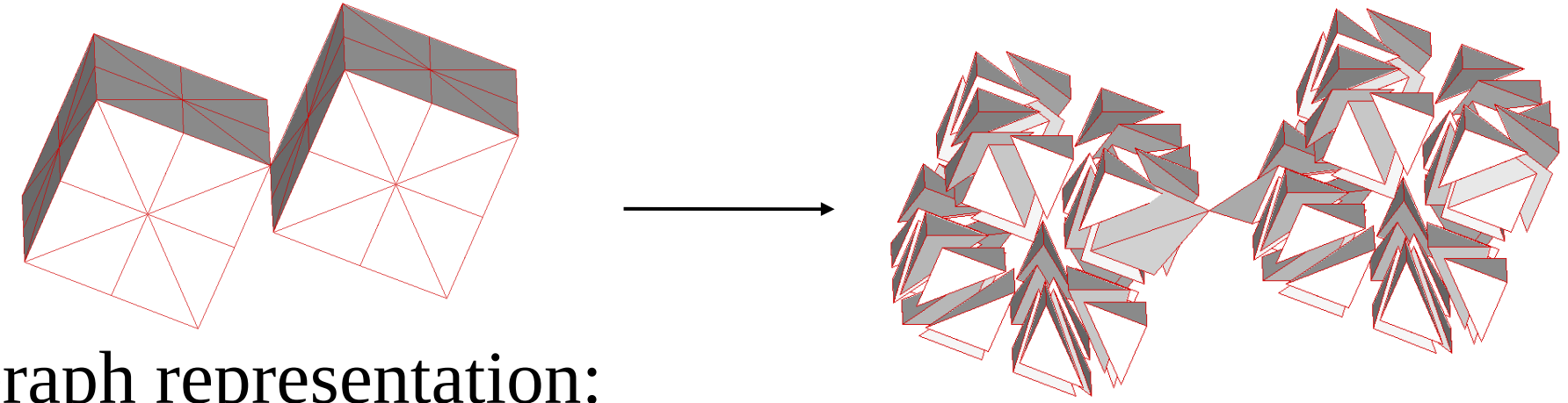
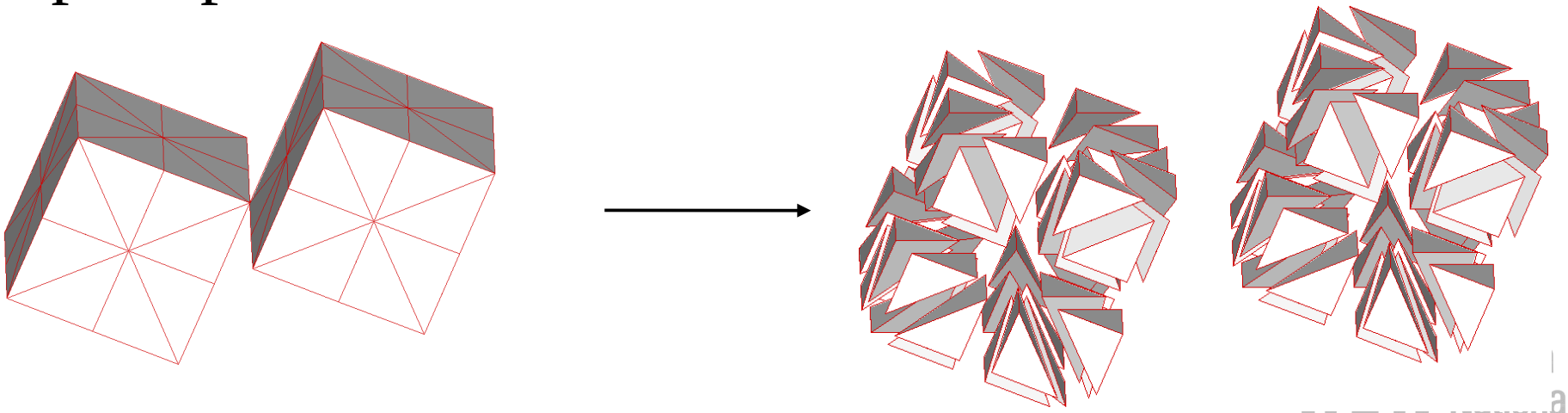
Correctness – Point Cubes

Two cubes joined by a point. Fracture all internal triangles. Current algorithm:

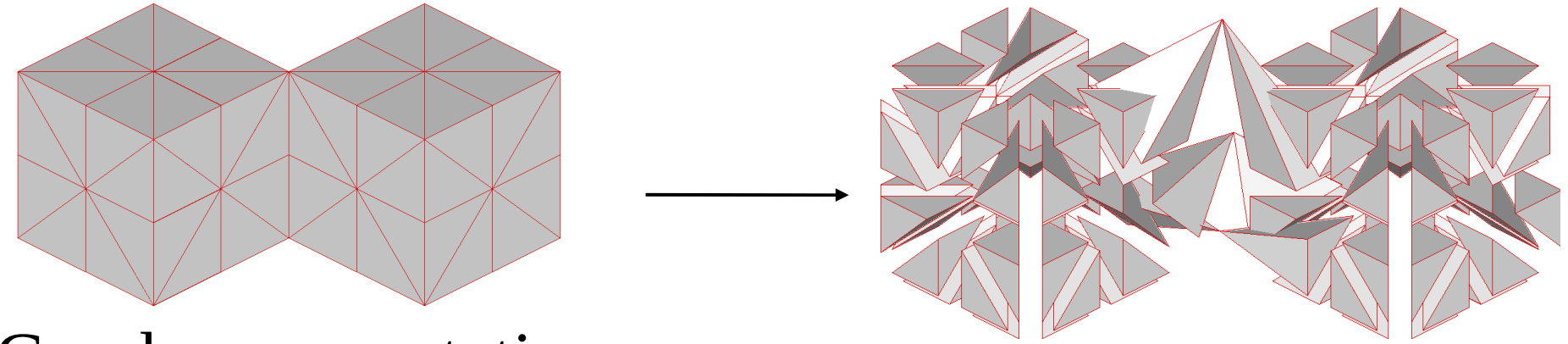


Graph representation:

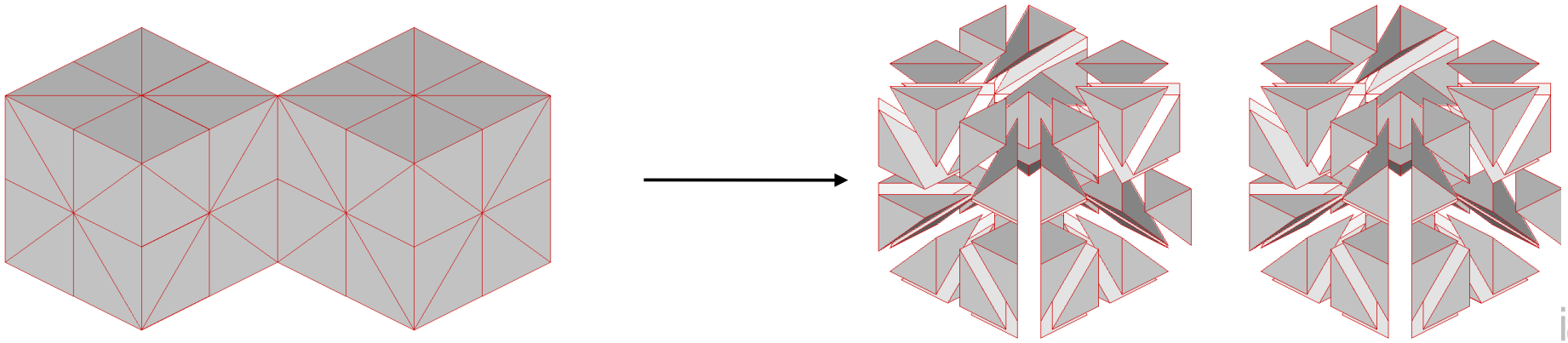


Correctness – Edge Cubes

Two cubes joined by an edge. Fracture all internal triangles. Current algorithm:



Graph representation:



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Graph Fracture Properties

- Subgraphs greatly simplify fracture.
- Operations on subgraphs are mirrored on parent graph.
- Localized operations confined to subgraphs, essential for parallelization.
- Time complexity of initialization linear with number of elements.
- Time complexity of fracture linear with number of open simplices.

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Graph Fracture Properties (2)

- Non-manifold cases handled correctly.
- Significant reuse of code.
- Marking of open simplices is top-down.
- Building of subgraphs is bottom-up.
- Fracture is top-down.
- Works for both 2D and 3D.
- Recursive with each level in the graph.

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C++ Boost Graph Library

- Free peer-reviewed portable C++ source library.
- Works well with the C++ Standard Library.
- Generic, STL-like interface for manipulating and traversing graphs.
- Hides details of the graph data structure implementation.

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Performance Evaluation

- Reference fracture implementation is the one used in the ARES FE code.
- Tested 20 3D meshes of various sizes and geometries.
- Initialization time plotted as a function of the mesh size.
- Fracture time plotted as a function of triangles to fracture.

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Performance Evaluation (2)

- Improvements in C++ Boost library result in immediate performance gain.
- First implementation.
- Some performance tuning.
- No thorough profiling and extensive performance tuning yet.
- Tests performed using GCC on an Intel Xeon 1.5 GHz machine.

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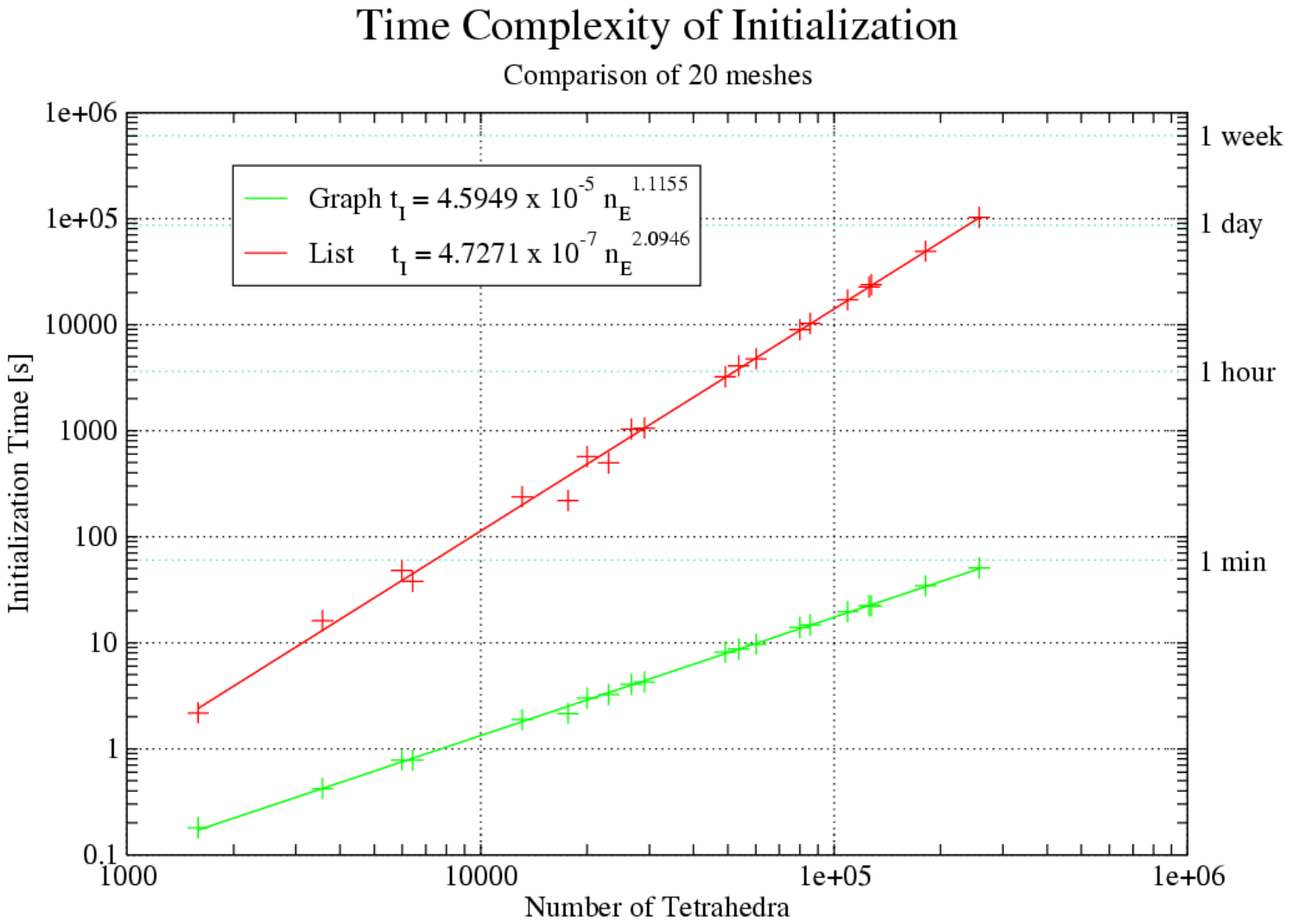
Initialization Time



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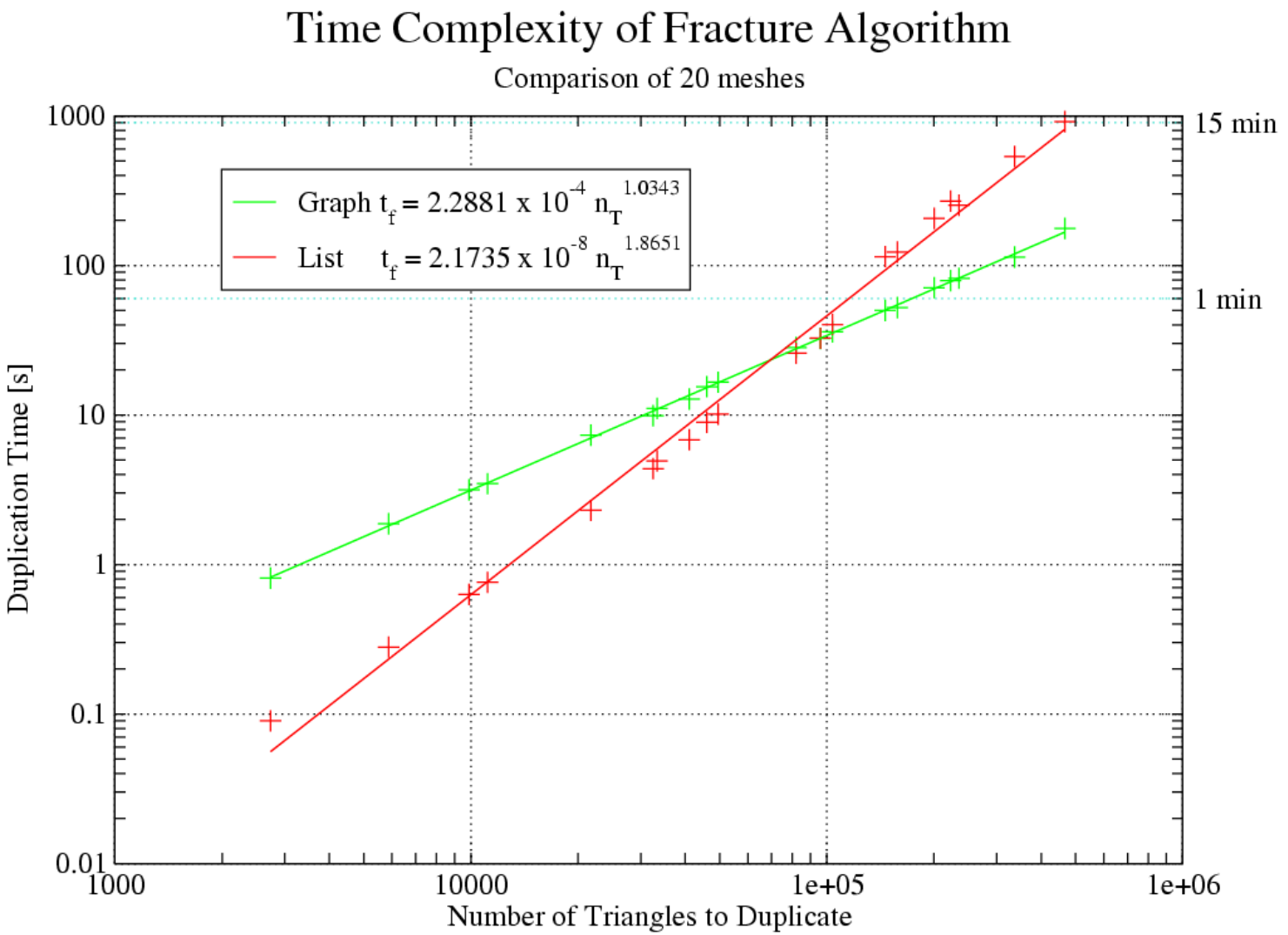
Fracture Time



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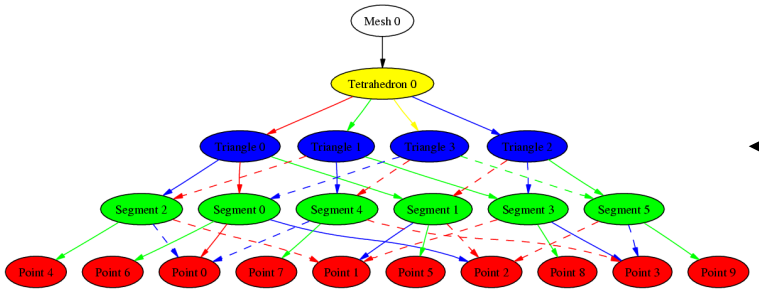
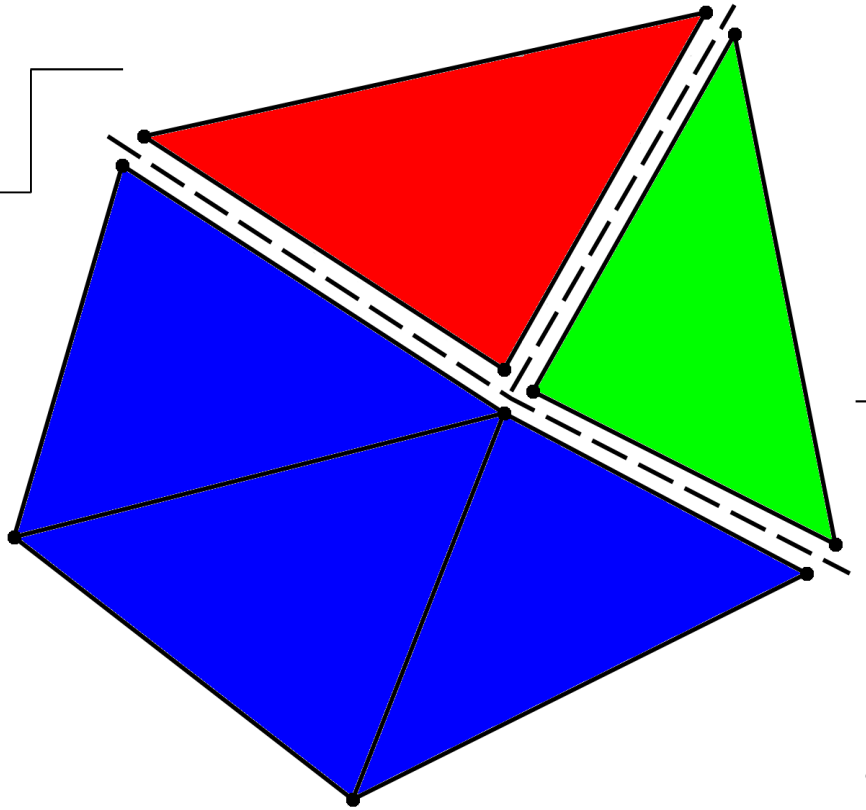
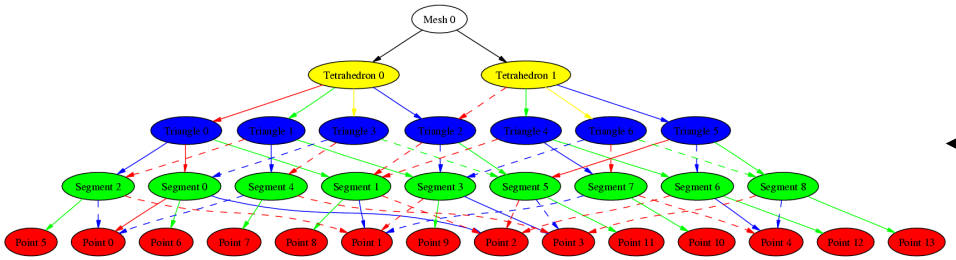
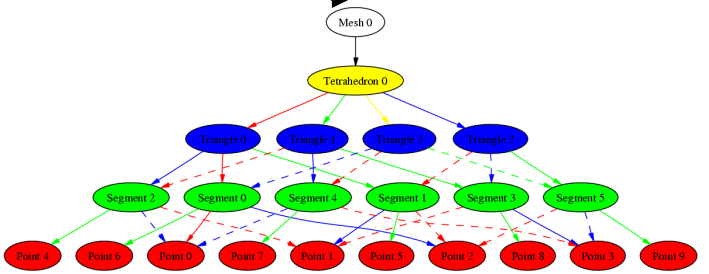
3D Graph Parallel Fracture

Partition mesh and create graph for each partition.



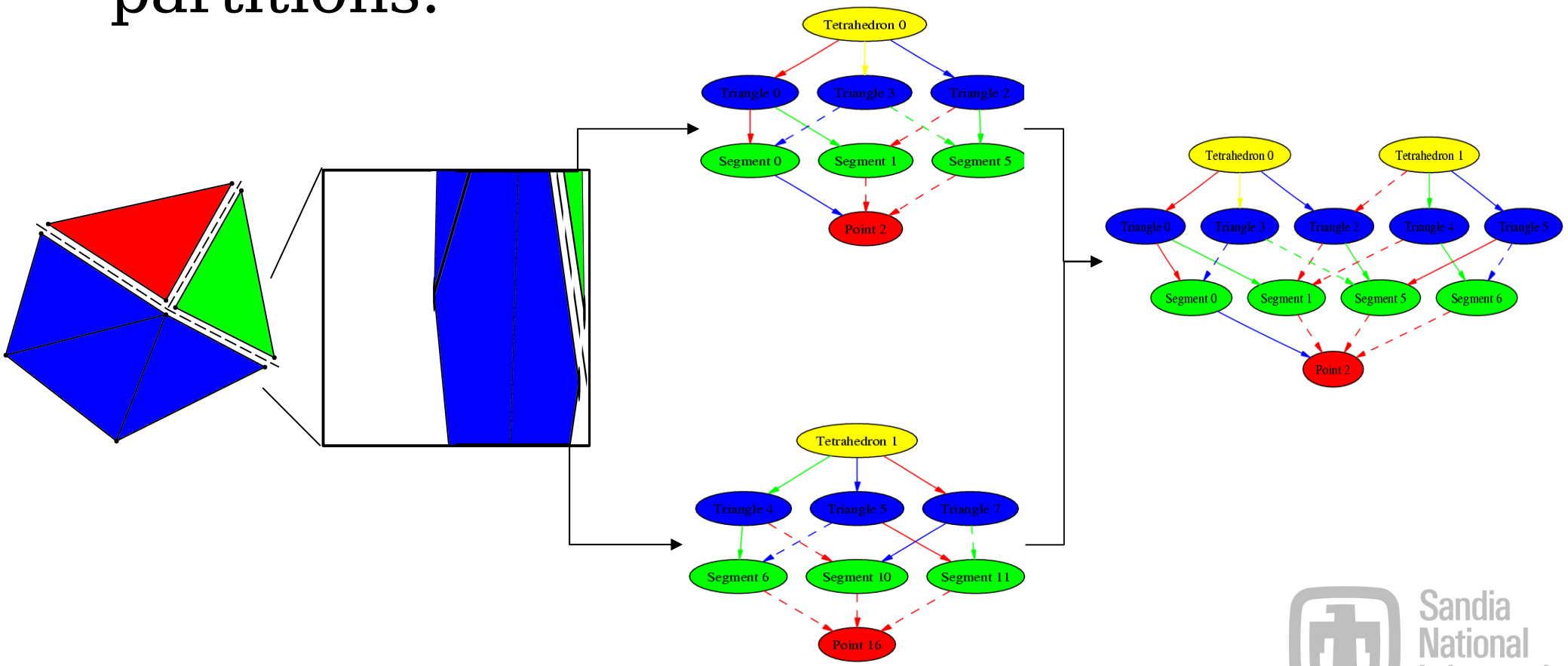






3D Graph Parallel Fracture

- On each fracture step mark open simplices.
- Partition-boundary points are missing parts of their subgraphs. Supplement from remote partitions.



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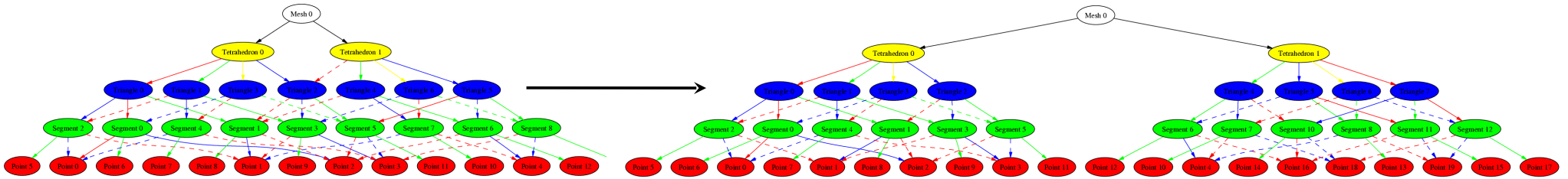
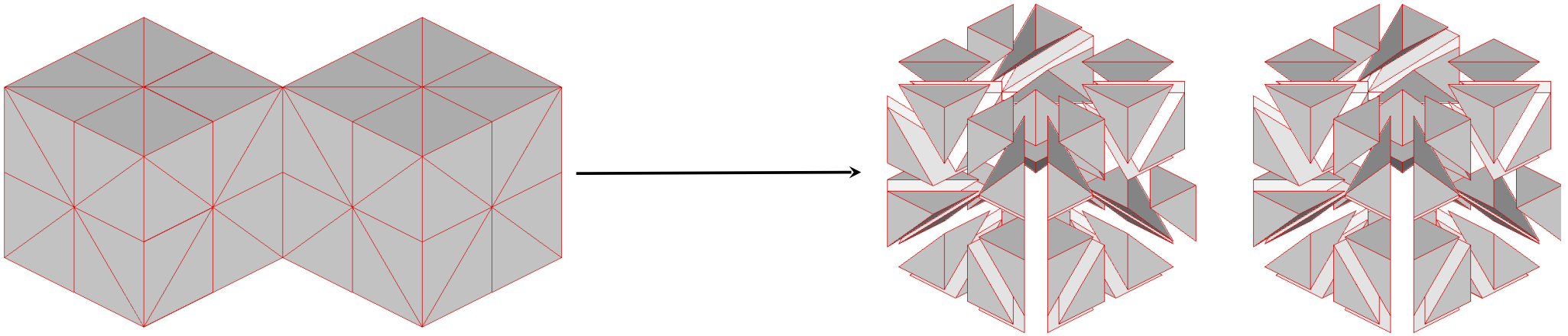
3D Graph Parallel Fracture

Apply serial fracture algorithm to each partition.





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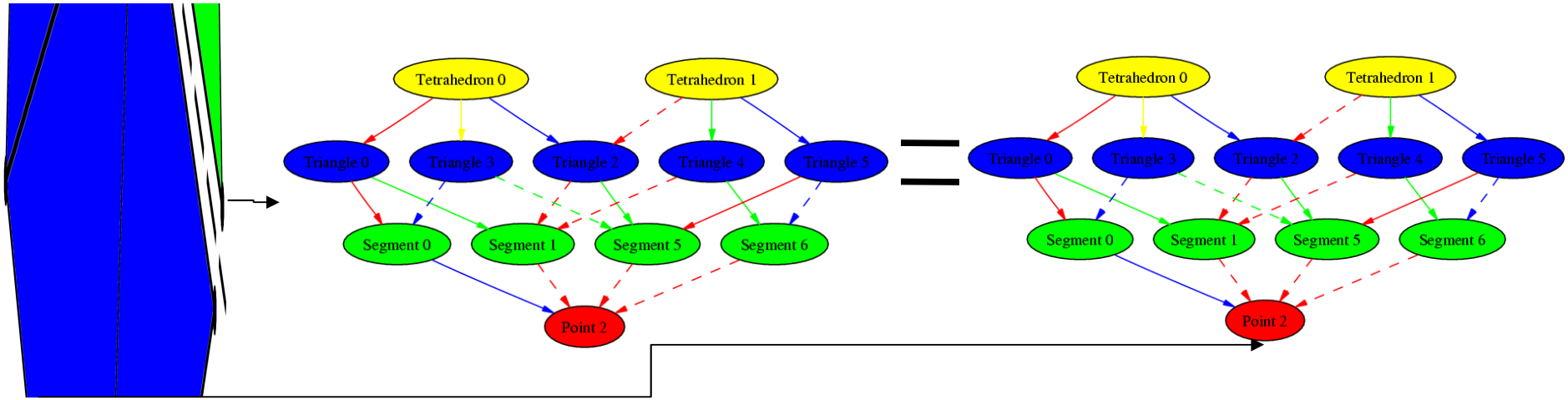
3D Graph Parallel Fracture

Subgraphs for partition-boundary points are in a consistent state across partition boundaries after serial fracture on each partition.



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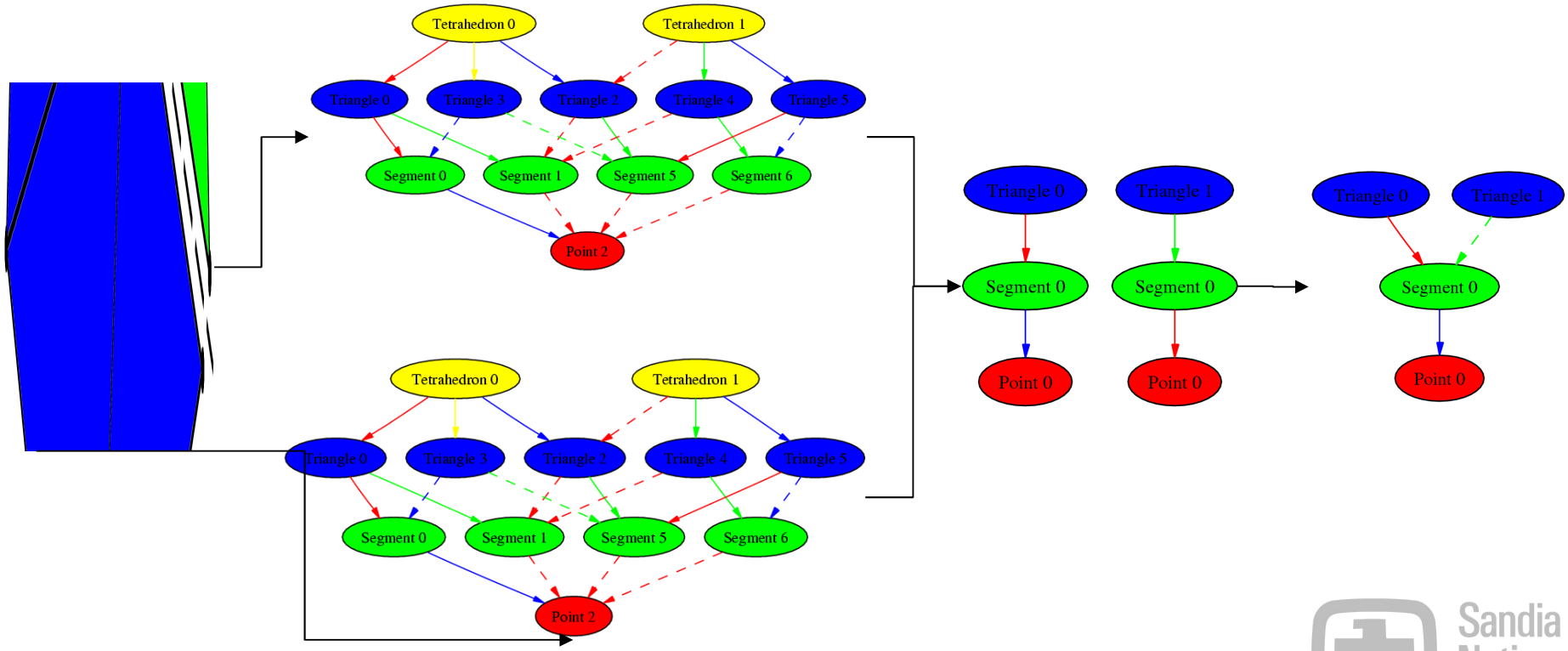
3D Graph Parallel Fracture

Exchange information between partition-boundary point subgraphs to identify and match newly created simplices across partition boundaries.



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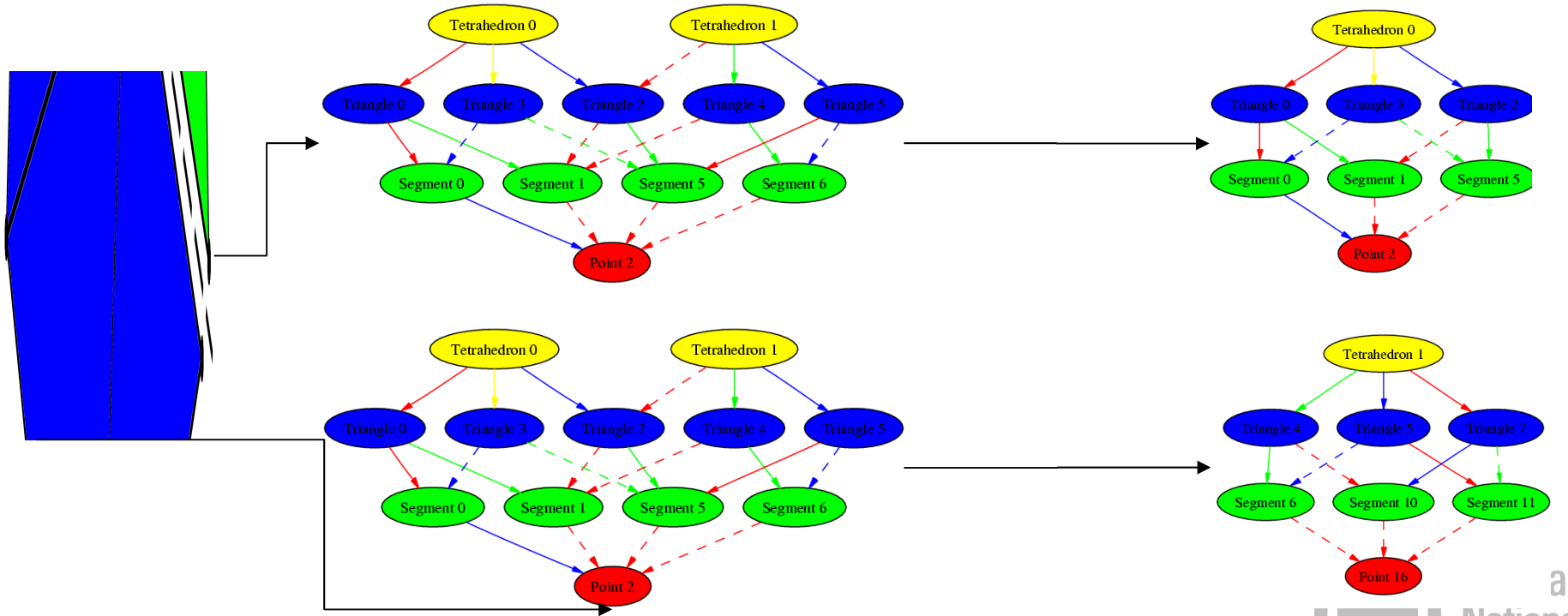
3D Graph Parallel Fracture

- Update boundary communicator.
- Discard the remote part of the graph for each partition and proceed with mechanics until next fracture step.



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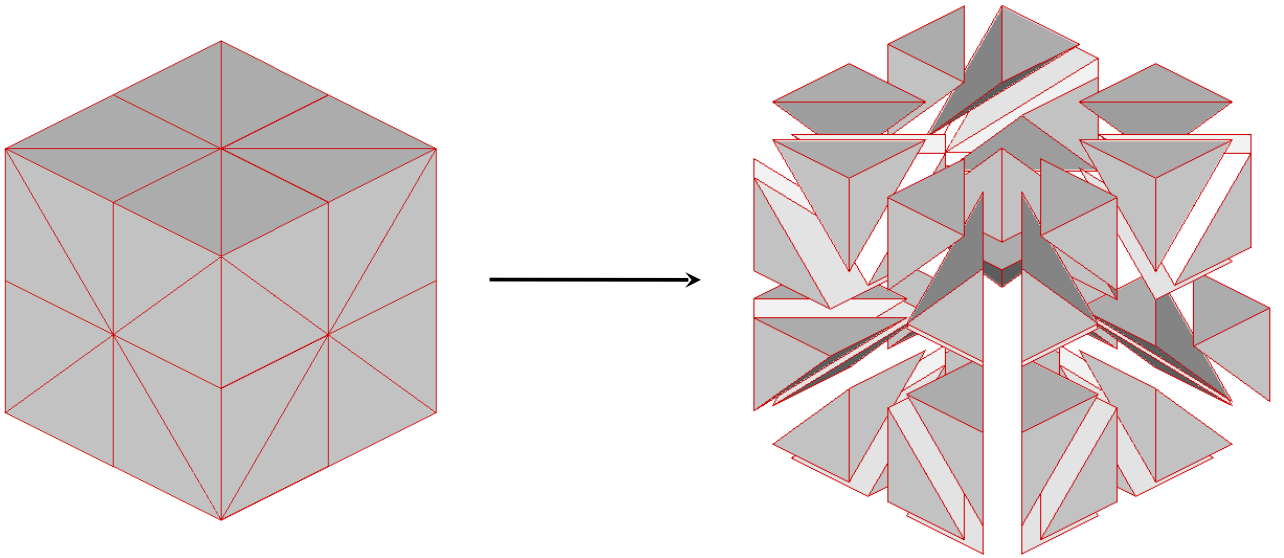
Testing

- Test on simple meshes yield the same results as serial fragmentation.
- The number of partitions is incremented until the partition algorithm (METIS) produces partitions with zero elements.



Cube example
2 to 14 partitions

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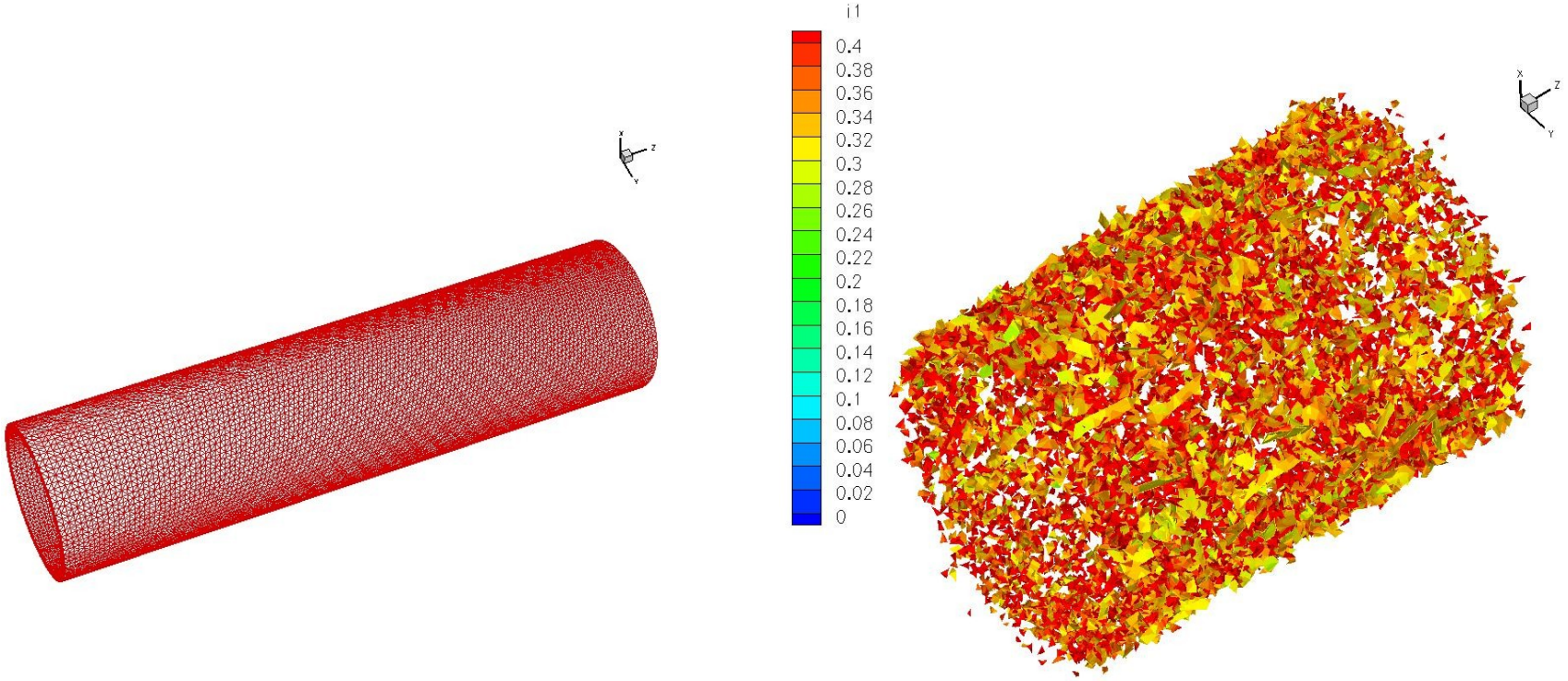
Testing

- Test on simple meshes yield the same results as serial fragmentation.



Tube example
2 to 8 partitions

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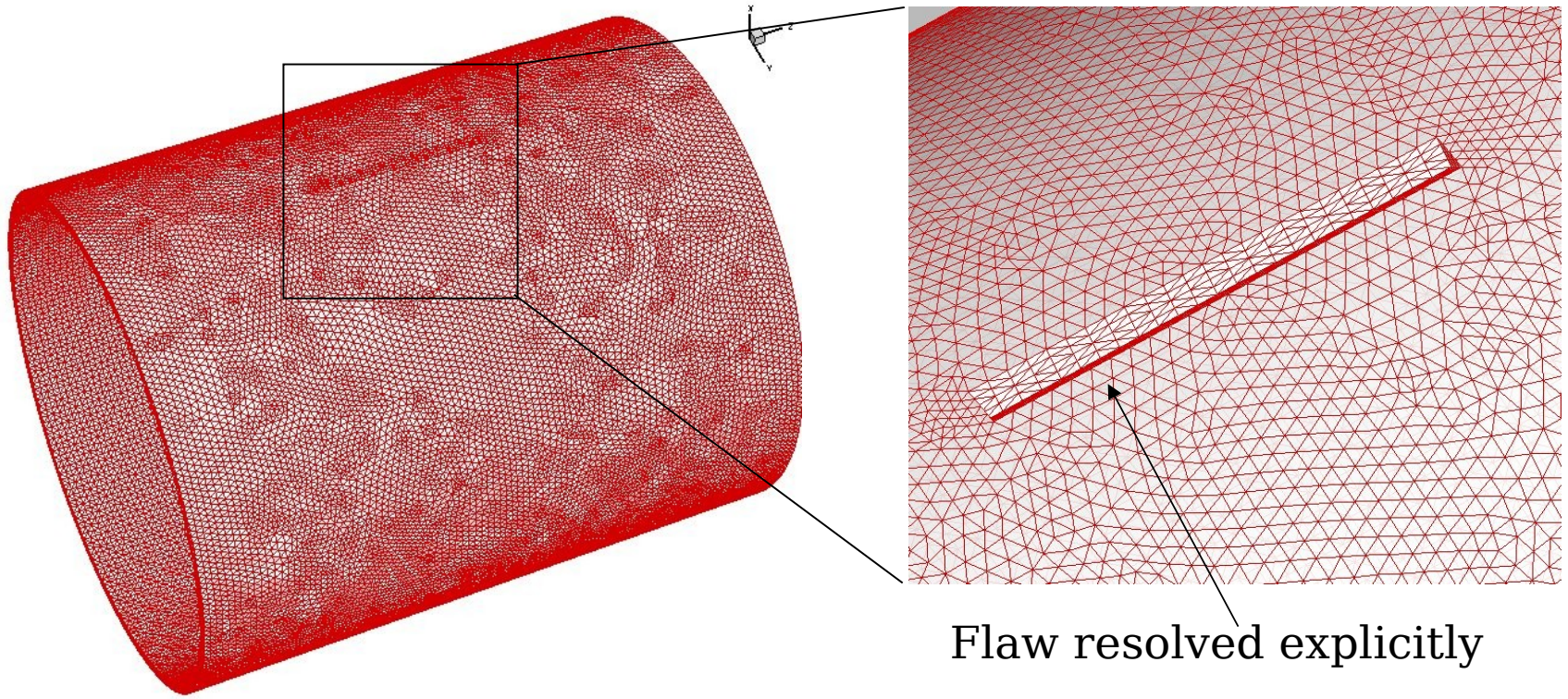
Coarse FE Mesh - Serial



Material model: Porous plasticity informed by QC
Interface model: Strain localization element
Failure criterion: critical plastic strain ~ 0.12
Mesh: 39150 elements, 78850 nodes
Machine: Single-processor Linux PC, ~1 week run time

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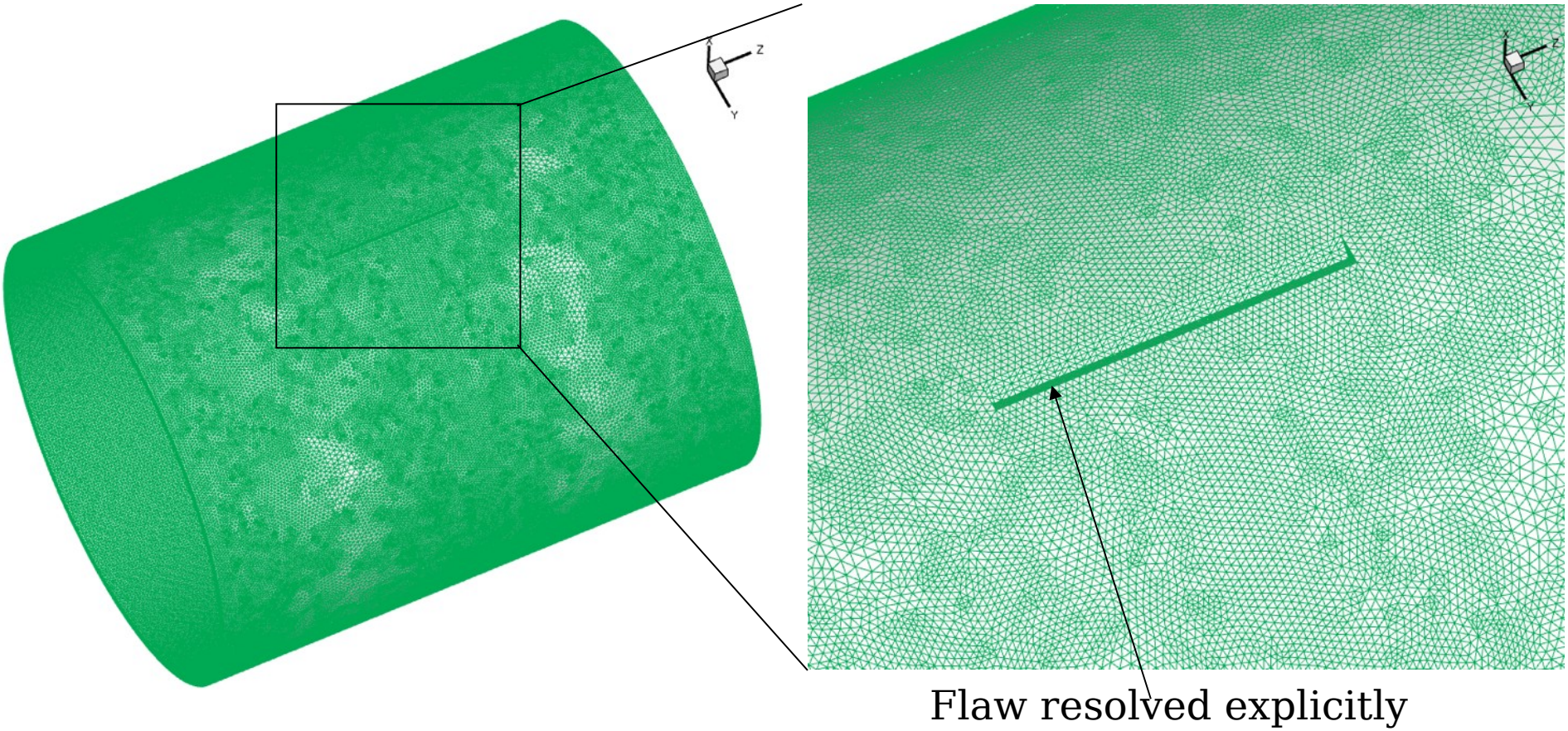
Fine FE Mesh - Parallel



Material model: Porous plasticity informed by QC
Interface model: Strain localization element
Failure criterion: critical plastic strain ~ 0.12
Mesh: 220945 elements, 399998 nodes
Machine: LLNL ALC, 400 processors, 8 hours run time

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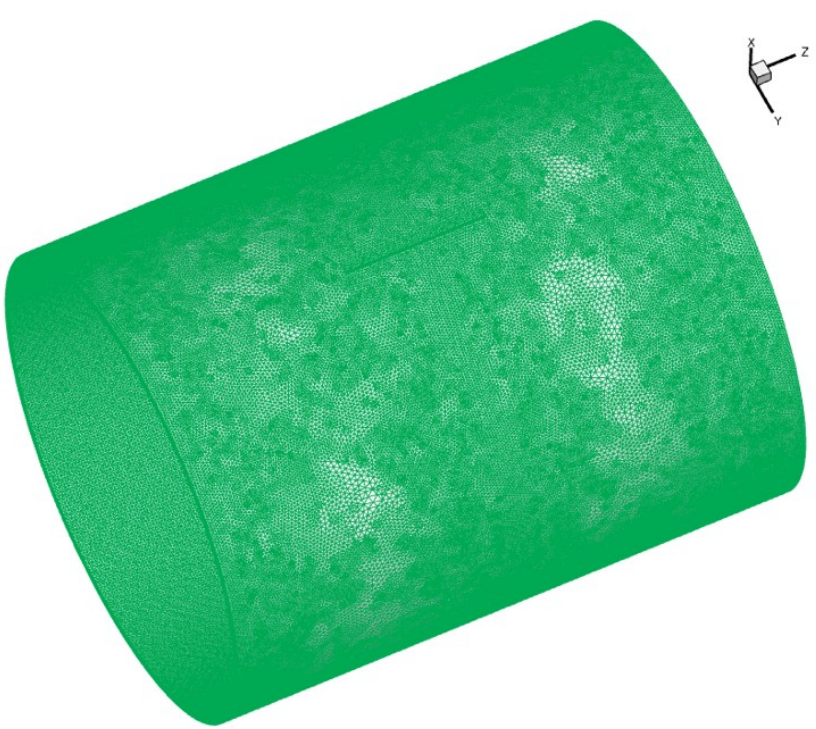
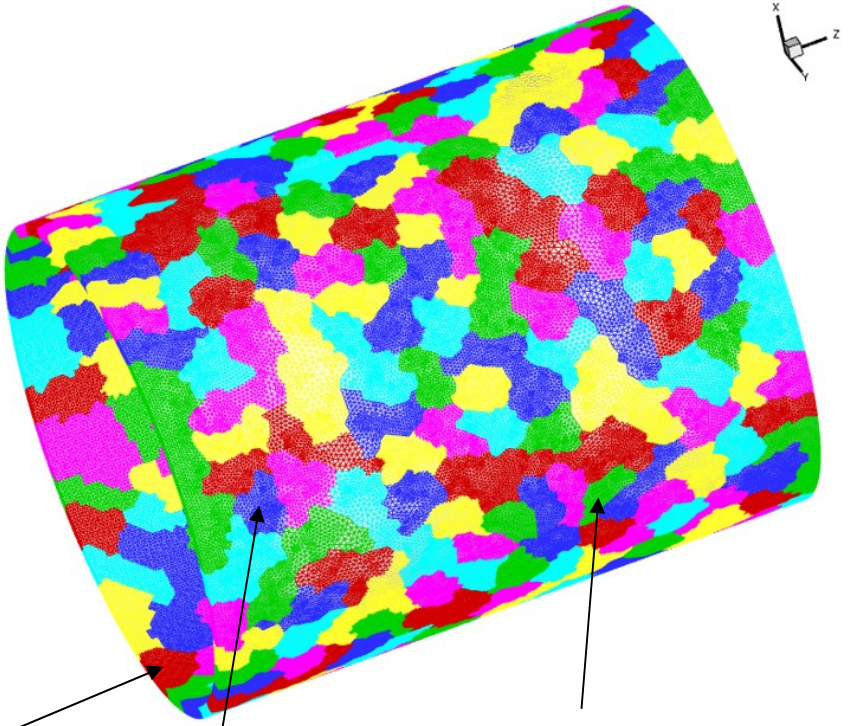
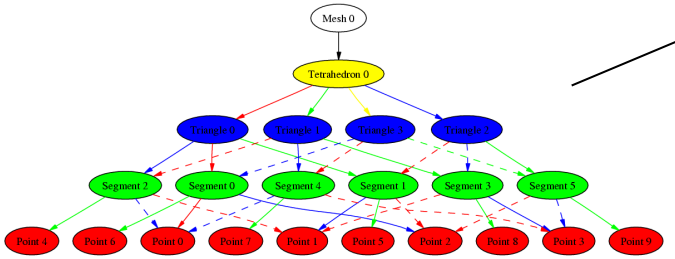
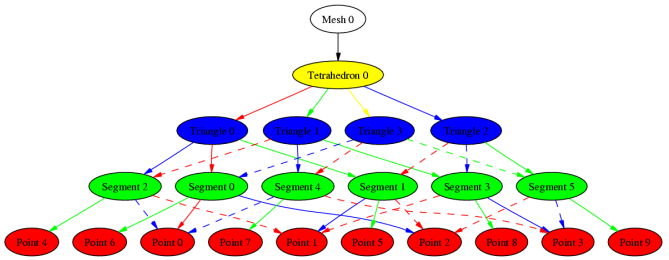
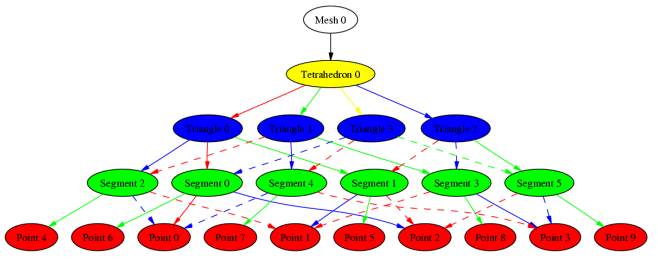
3D Graph Parallel Fracture











Partition mesh and create graph for each partition.

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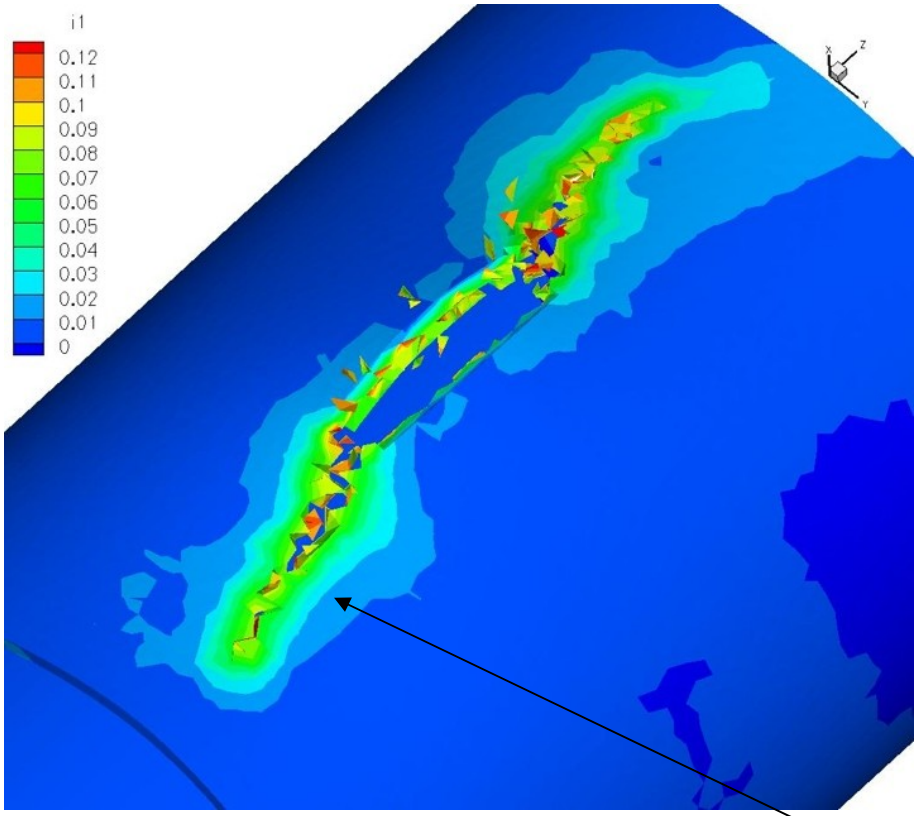
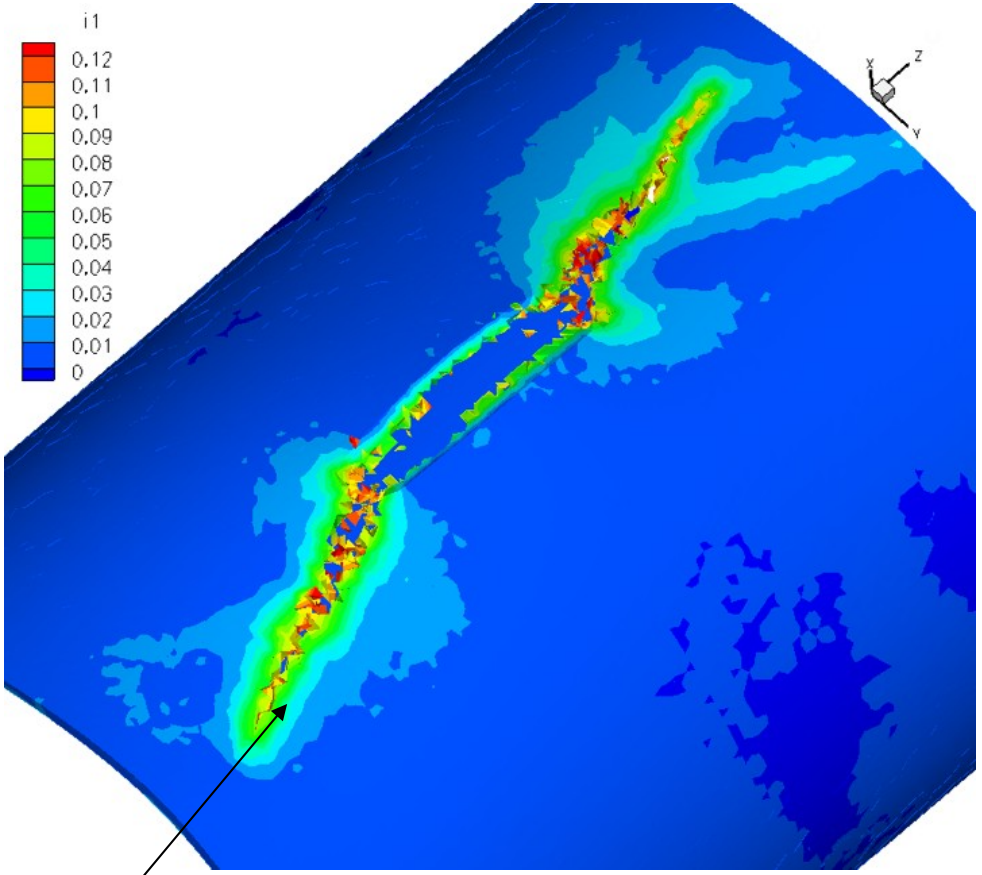
Results: Coarse, Fine Meshes





Coarse Mesh - Serial

Kinked cracks

Fine Mesh - Parallel

Configuration at 75 μs
with initial torsion

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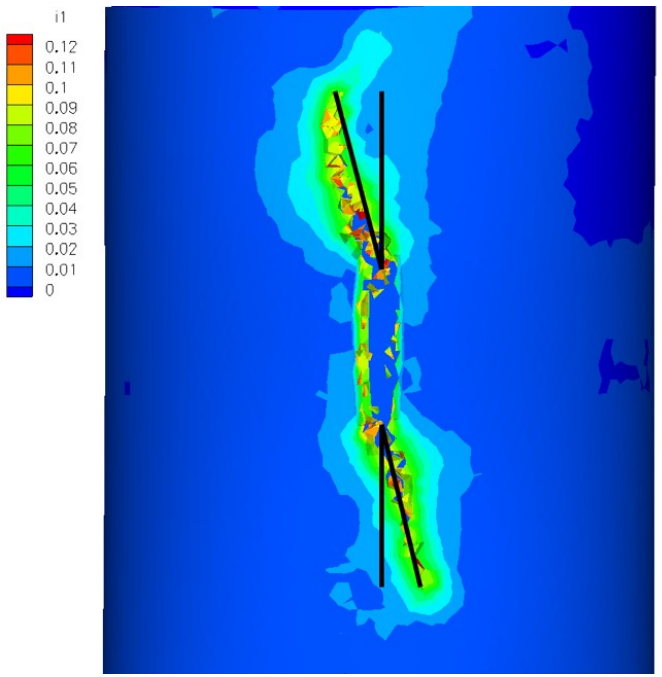
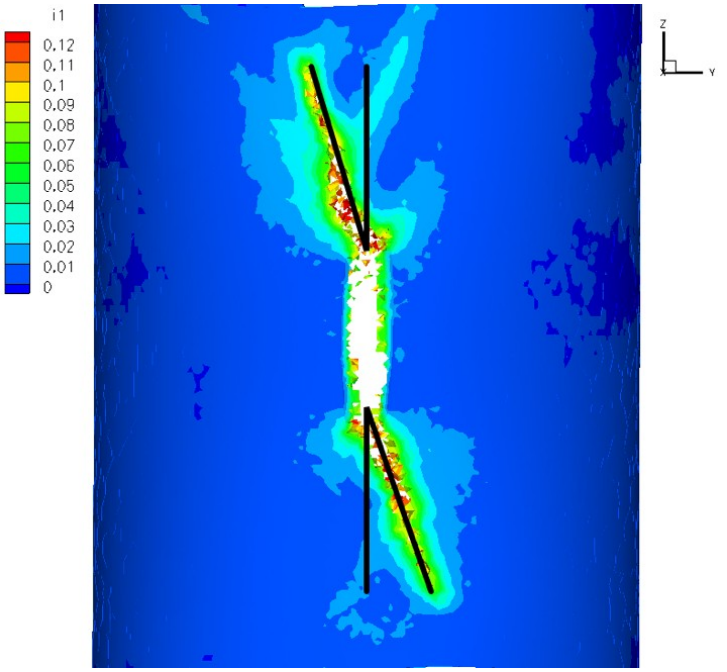
Crack Angles



Serial simulation
Peak pressure: 6.1 MPa
Crack kink angle ~ 14 degrees

Peak pressure [MPa]	Crack angle [degrees]
6.1	7
5.1	13
4.1	38
3.3	50

Experimental results
(Chao and Shepherd, 2004)



Parallel simulation
Peak pressure: 6.1 MPa
Crack kink angle ~ 16 degrees

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Conclusions

- Graphs allow fracture by simple, repetitive operations.
- The use of the Boost library reduced the work load significantly.
- Localized fracture operations suitable for parallel implementation.
- Time complexity reduced practically from quadratic to linear.
- Non-manifold cases handled correctly.

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Future Work

- Extend graph representation for CW complexes.
- Run very large fracture problems.
- Reuse graph mesh representation for serial and parallel contact.
- Replace connectivity arrays and use the graph for all FE computations.
- Mesh subdivision.

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