Behavior of the Lattice Schrodinger Equation

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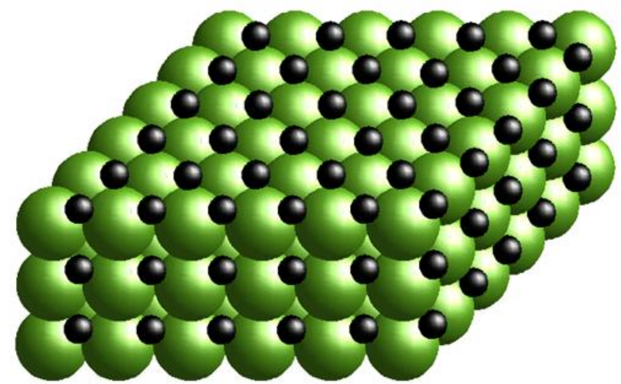
CE 291F

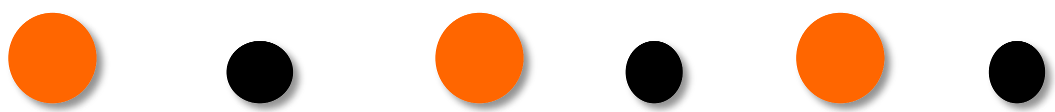
Diffusion on a Lattice

Dynamical localization of waves for the nonlinear Schrodinger equation with random potential on a lattice

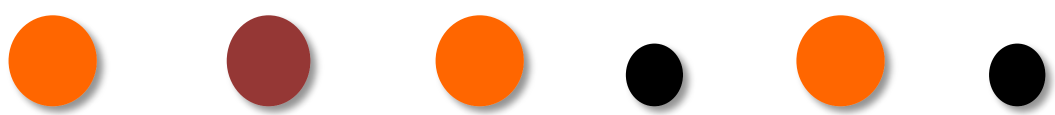
Example:

- Metal alloy
- Defects (disorder) impede diffusion
- Nonlinearity models attraction/repulsion





1-D No Disorder



1-D Disorder

Relevant Equations

1-D Discretized NLSE: $$i\dot{u}_n(t) = V_n u_n(t) + u_{n+1}(t) + u_{n-1}(t) + \alpha u_n(t)|u_n(t)|^2$$

Diffusion Equation: $$D(t) = \sum_{n=-\infty}^{\infty} (1+n^2)|u_n(t)|^2$$

Potentials:

Peierls model (for linear case): $V_n = \lambda \cos(2\pi\phi n)$

Fishman model: $V_n = \lambda \cos(2\pi\phi n^2)$

Anderson model: $V_n = \lambda \cos(2\pi\phi_n)$, where ϕ_n is a random number between 0 and 1.

Initial Conditions: $$u_n(0) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

Boundary Conditions: Dirichlet

n: site

λ : disorder

u: wavefunction

D(t): diffusion value

V: potential

ϕ : random number

α : nonlinearity, attractive if positive, repulsive if negative

Numerical Solution

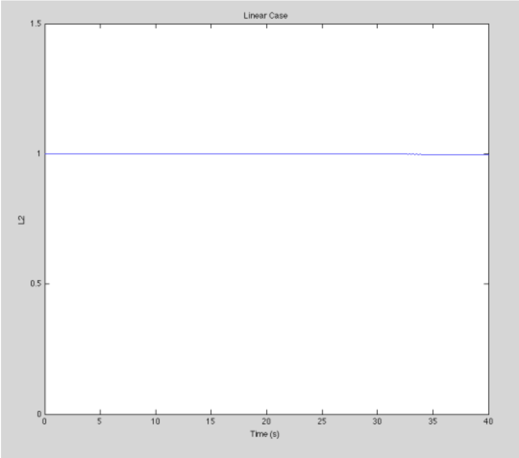
Algorithms:

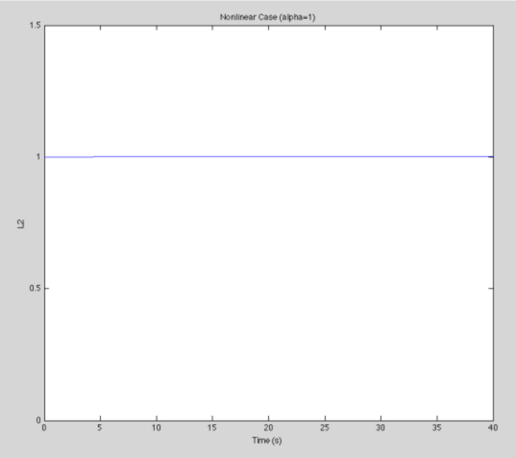
- Adaptive fourth-order Runge-Kutta
- Variable order Adams-Bashforth-Moulton PECE

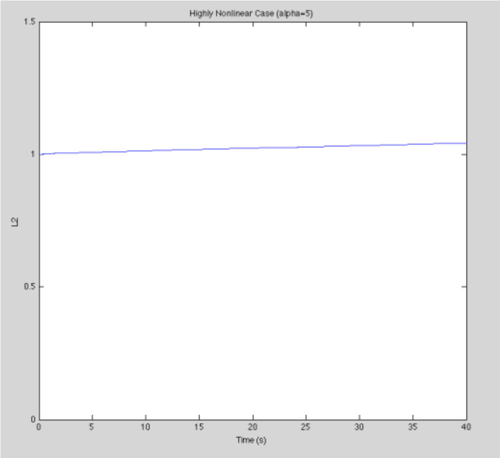
Box Size:

- Determine range of *n* (10^2-10^3)
- Determine number of time steps (10^5-10^8)

Verify size of the L^2:



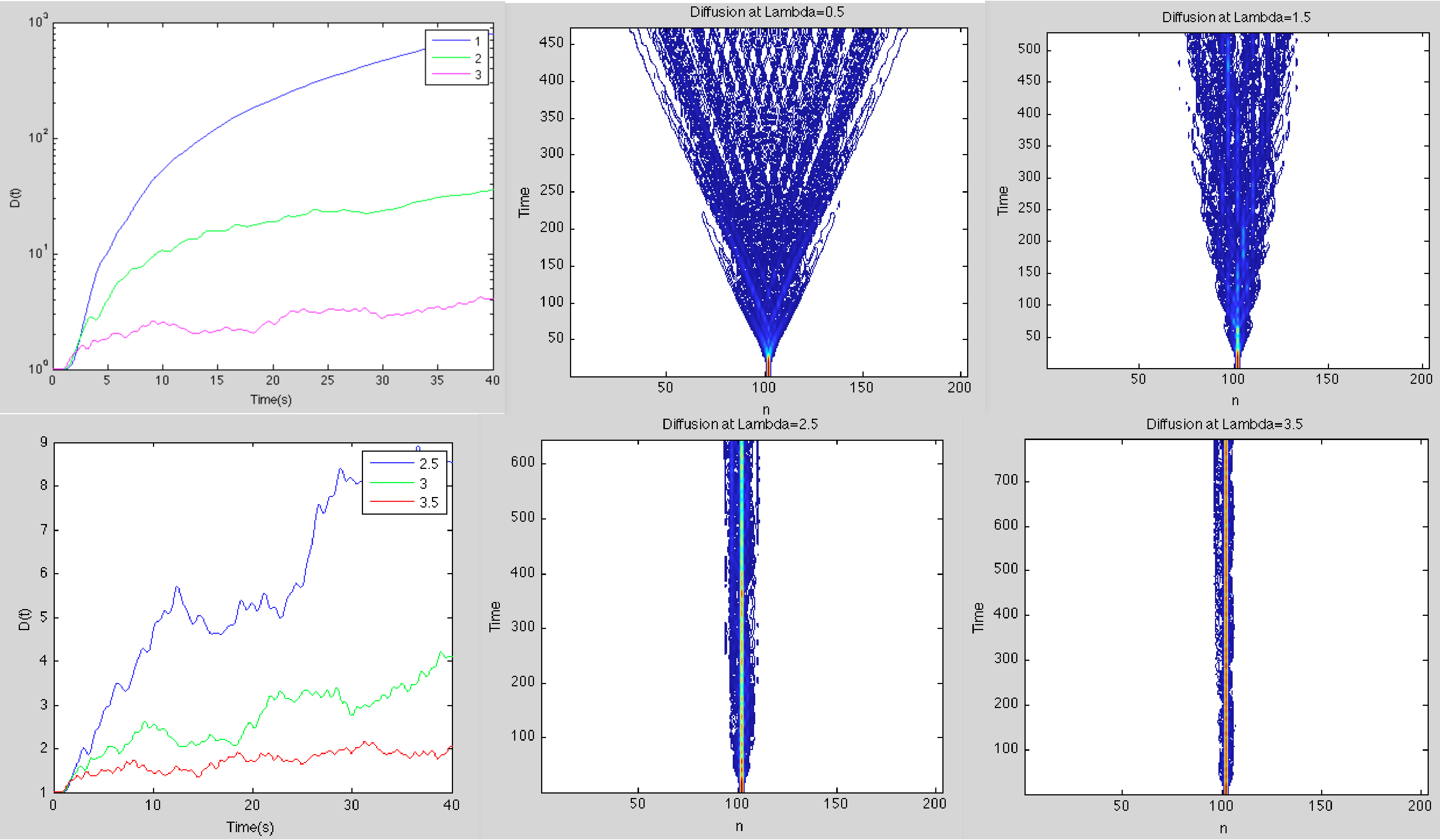




All with small disorder (lambda=1)

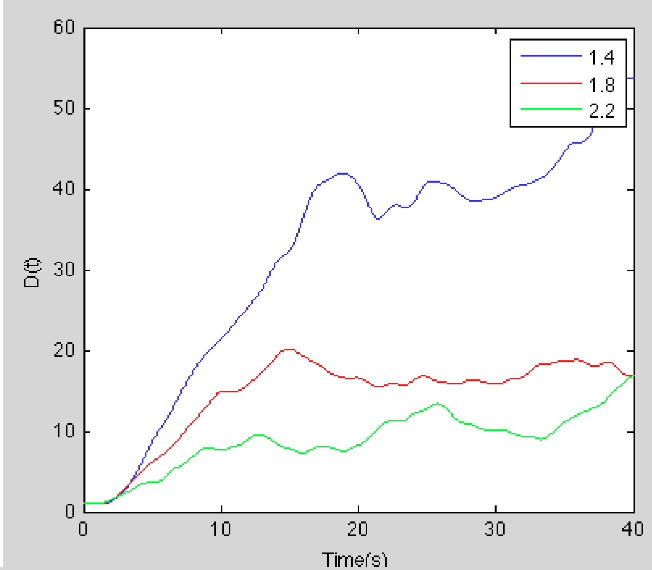
Peierls Model

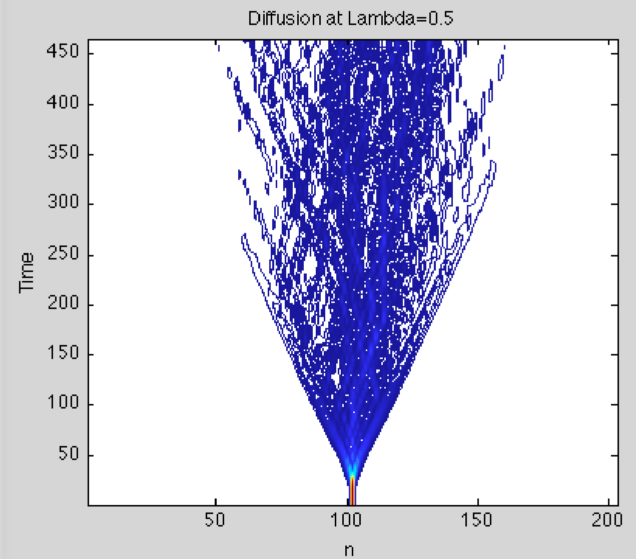
$$V_n = \lambda \cos(2\pi\phi n)$$

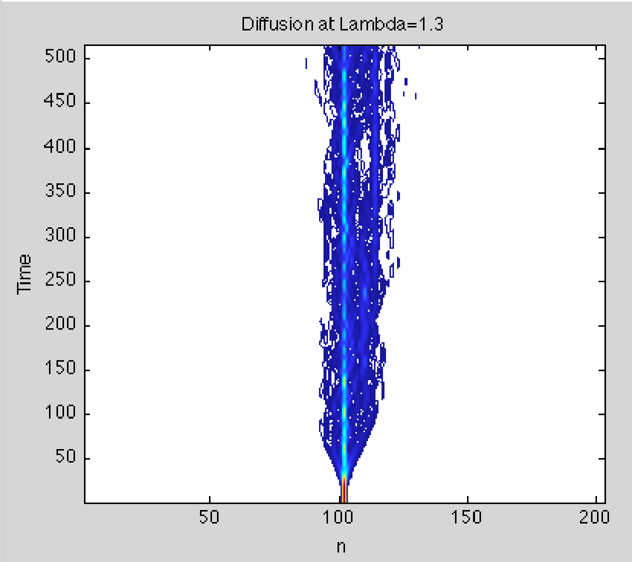


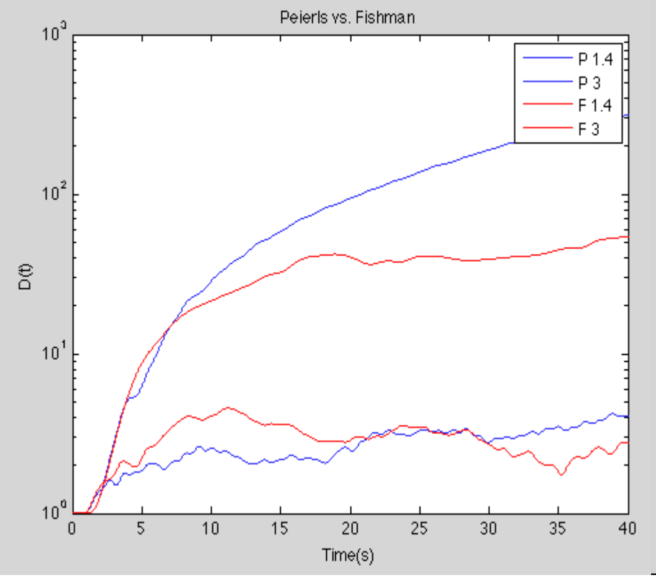
Fishman Model

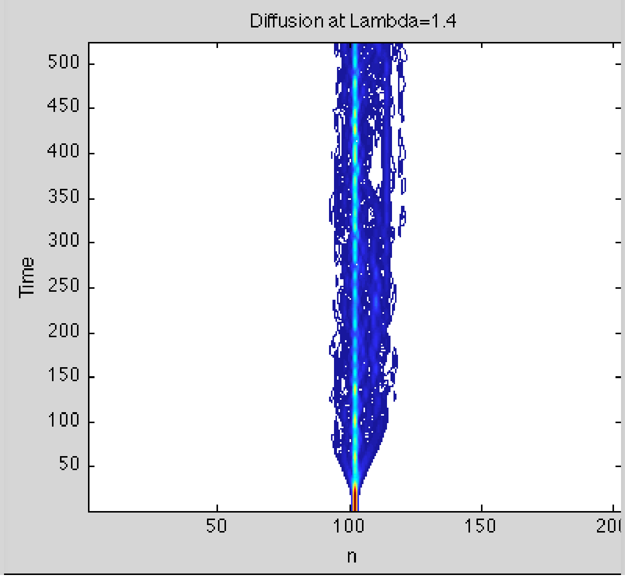
$$V_n = \lambda \cos(2\pi\phi n^2)$$

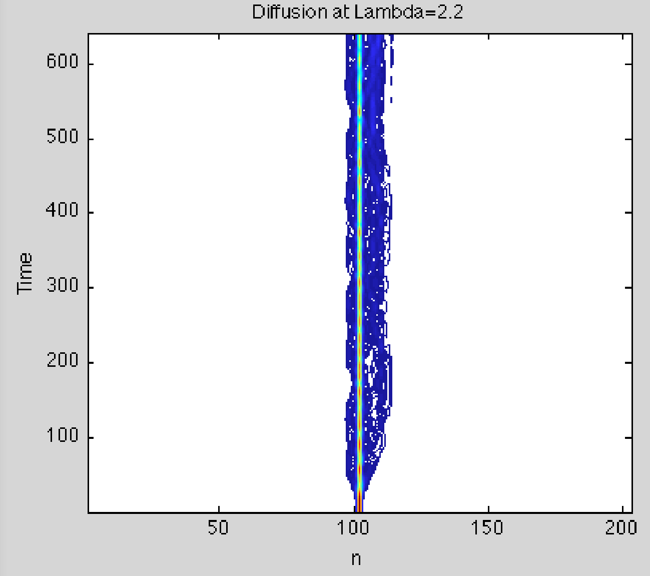






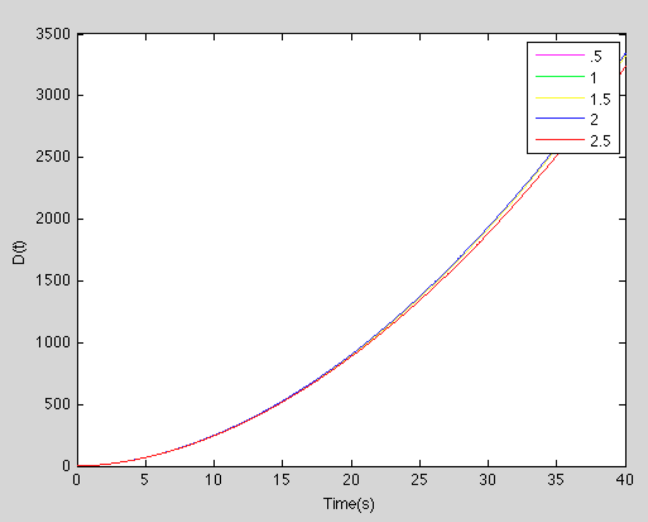


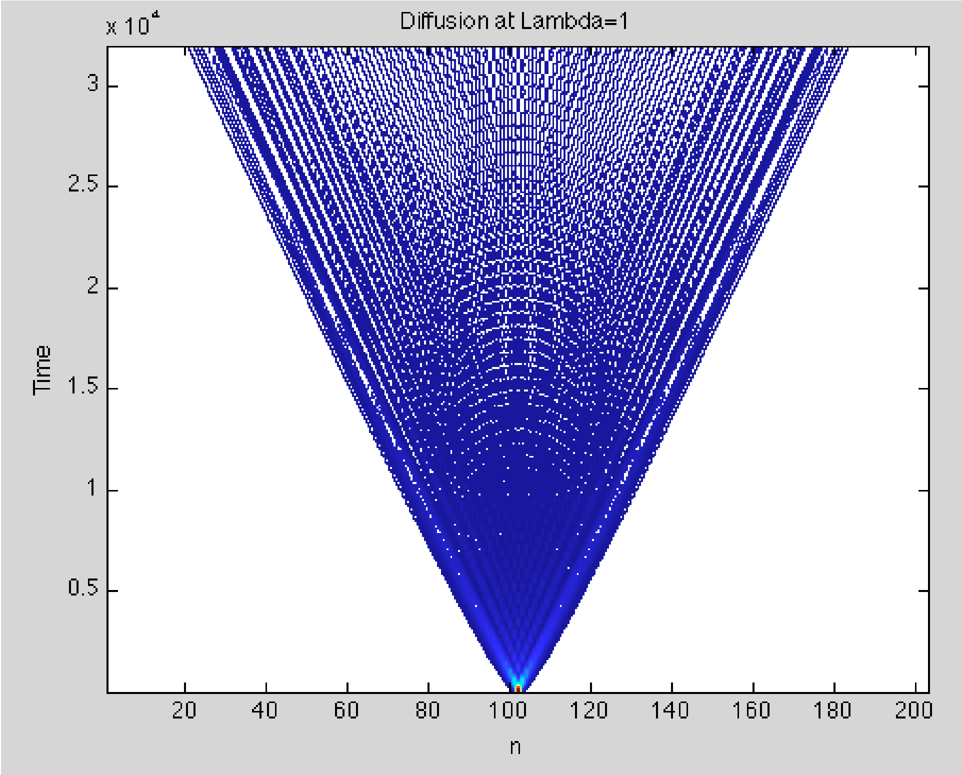


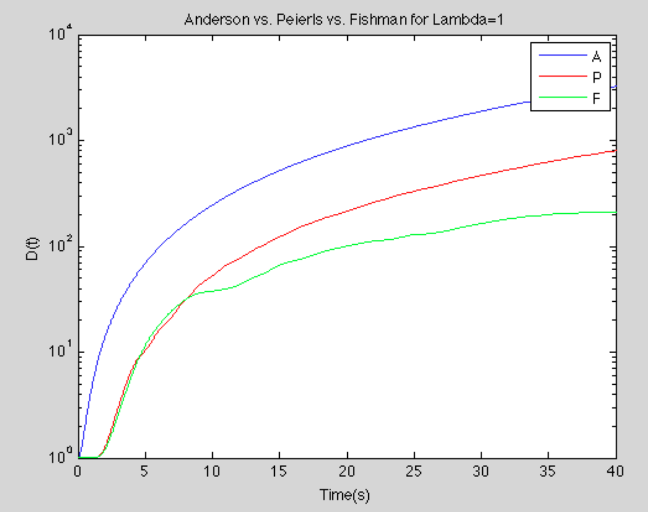


Anderson Model

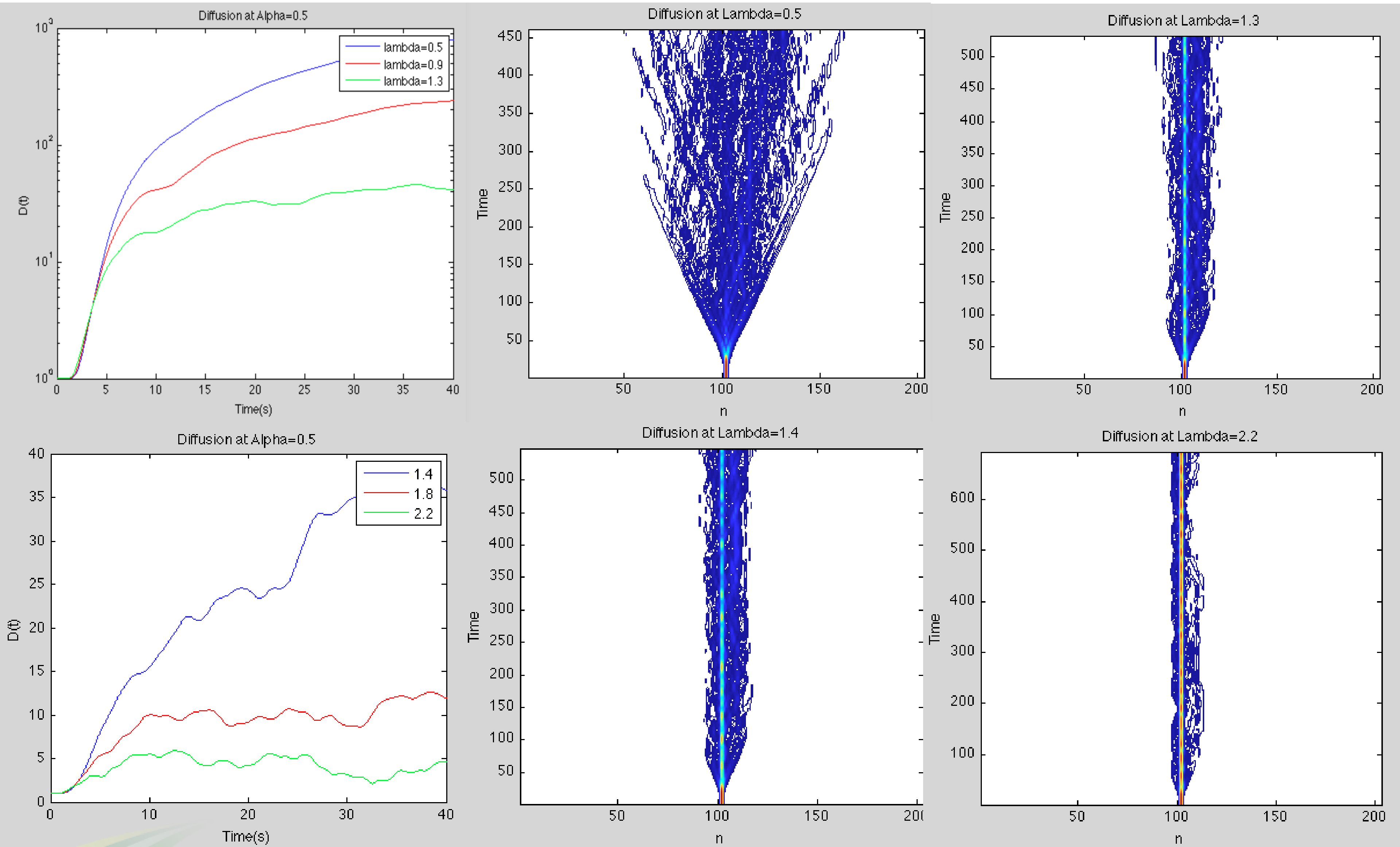
$$V_n = \lambda \cos(2\pi\phi_n), \text{ where } \phi_n \text{ is a random number between 0 and 1.}$$



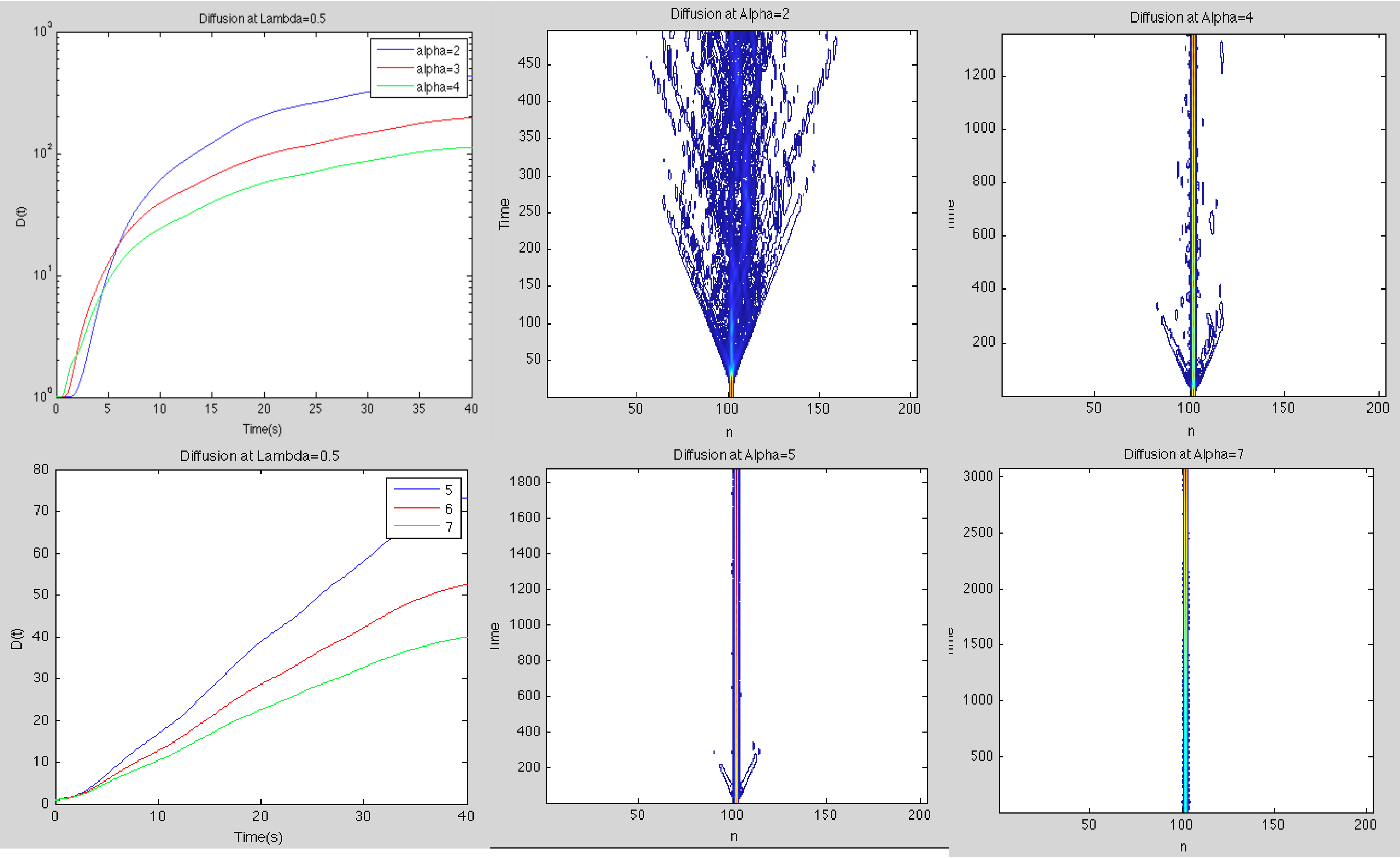




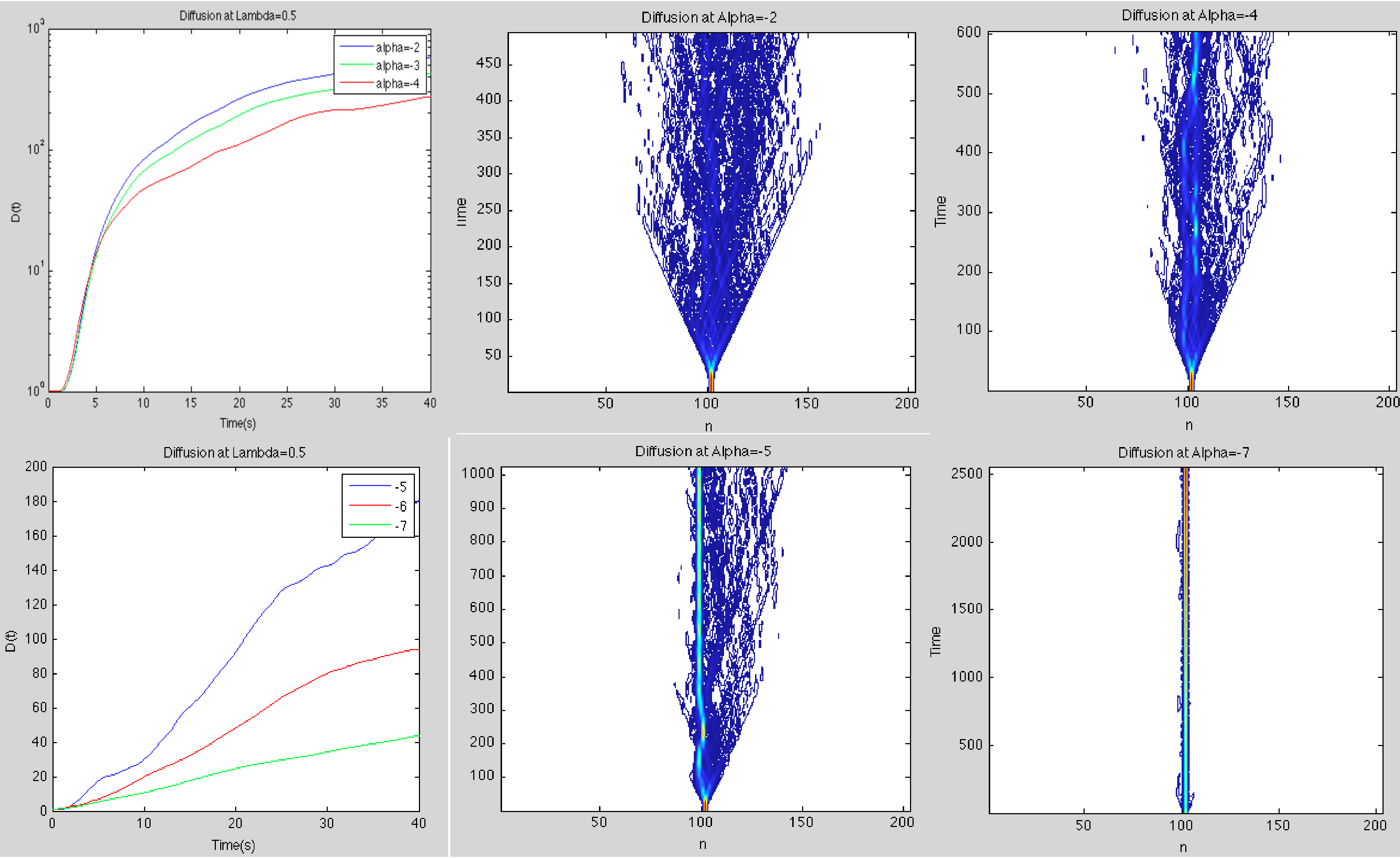
High Disorder, Low Nonlinearity



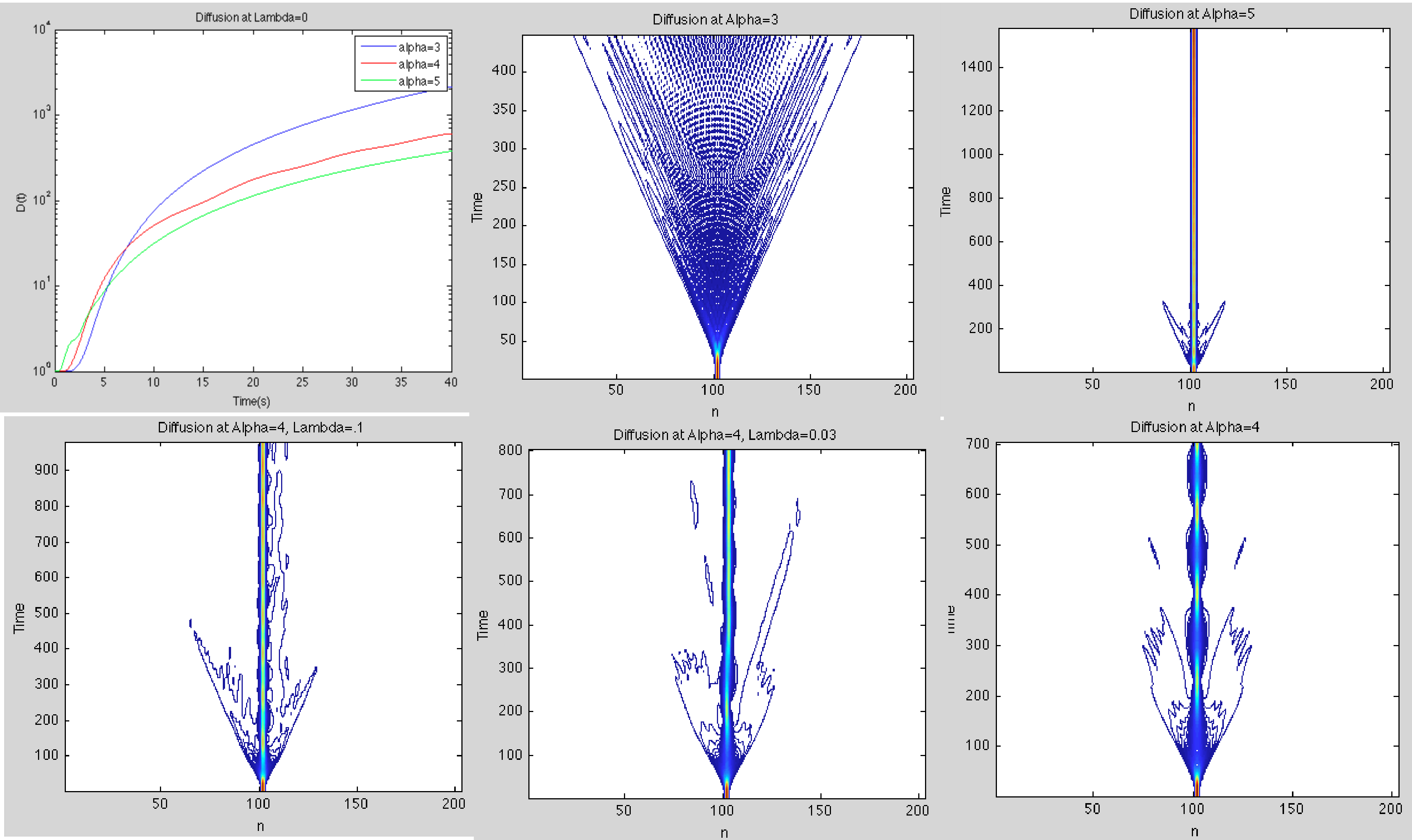
High Attractive Nonlinearity, Low Disorder



High Repulsive Nonlinearity, Low Disorder



Nonzero Nonlinearity, No Disorder



A Summary of a Summary

- Potentials for higher powers of *n* result in lower critical λ
- Anderson Model is independent of disorder for nonzero λ
- Attractive nonlinearity reduces diffusion more than repulsive nonlinearity
- Increasing disorder can reduce diffusion over time but increasing nonlinearity can't