# Belief Revision in Description Logic

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# Motivation

- Study the dynamics of ontologies, specially "OWL-like" DL ontologies.
- AGM Belief Revision deals with the problem of adding/removing information in a consistent way.
- AGM is most commonly applied to propositional classical logic and cannot be directly used with DLs.
- How can we adapt AGM so that it can deal with interesting DLs?

# In this work

- Show reasons why AGM fails to apply to DLs.
- Adapt Contraction (easy).
- Adapt Revision (less easy).

# Outline of the Talk

1. Motivation
2. The AGM paradigm
3. Contraction and DLs
4. Revision and DLs
5. Conclusions and Future Work

# AGM Belief Revision

Three operations defined to deal with knowledge base dynamics:

- **Expansion** - adding knowledge (possibly inconsistent)

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- **Expansion** - adding knowledge (possibly inconsistent)
- **Contraction** - removing knowledge
- **Revision** - adding knowledge consistently

Revision usually defined in terms of contraction:
$K * \alpha = (K - \neg\alpha) + \alpha$

# AGM Theory

For contraction and revision:

- **Rationality Postulates**

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**AGM Assumptions:** Tarskian, Compact, Deduction Theorem, Supraclassical.

# AGM contraction

**(closure)** $K - \alpha = Cn(K - \alpha)$

**(success)** If $\alpha \notin Cn(\emptyset)$ then $\alpha \notin K - \alpha$

**(inclusion)** $K - \alpha \subseteq K$

**(vacuity)** If $\alpha \notin K$ then $K - \alpha = K$

**(recovery)** $K \subseteq K - \alpha + \alpha$

**(extensionality)** If $Cn(\alpha) = Cn(\beta)$ then $K - \alpha = K - \beta$

# Applying to DL

- AGM cannot be applied to every logic. In particular it can not be applied to SHIF and SHOIN. [Flouris 2006]
- Solution: substitute recovery by relevance

  **(relevance)** If $\beta \in K \setminus K - \alpha$, then there is $K'$ s. t. $K - \alpha \subseteq K' \subseteq K$ and $\alpha \notin Cn(K')$, but $\alpha \in Cn(K' \cup \{\beta\})$.

- Good property: AGM assumptions + 5 postulates ⇒ recovery and relevance are equivalent.

# Results - contraction

**Representation Theorem [RW06]**

If the underlying logic is tarskian and compact, partial meet contraction is equivalent to the AGM postulates with relevance instead of recovery.

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Can we do the same for revision???

# AGM Revision

**(closure)** $K * \alpha = Cn(K * \alpha)$

**(success)** $\alpha \in K * a$

**(inclusion)** $K * \alpha \subseteq K + \alpha$

**(vacuity)** If $K + \alpha$ is consistent then $K * \alpha = K + \alpha$

**(consistency)** If $\alpha$ is consistent then $K * \alpha$ is consistent.

**(extensionality)** If $Cn(\alpha) = Cn(\beta)$ then $K * \alpha = K * \beta$

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## Definition (Revision without negation)

$$K *_\gamma \alpha = \bigcap \gamma(K \downarrow \alpha) + \alpha$$

where $\gamma$ selects at least one element of $K \downarrow \alpha$.

# Properties

1. Inconsistent explosion: Whenever $K$ is inconsistent, then for all formulas $\alpha$, $\alpha \in Cn(K)$
2. Distributivity: For all sets of formulas $X$, $Y$ and $W$, $Cn(X \cup (Cn(Y) \cap Cn(W))) = Cn(X \cup Y) \cap Cn(X \cup W)$

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## Representation Theorem [RW09]

If the logic is monotonic and compact and satisfies Inconsistent explosion and Distributivity, then * is a revision without negation iff it satisfies closure, success, inclusion, consistency, relevance and uniformity.

**(uniformity)** If for all $K' \subseteq K$, $K' \cup \{\alpha\}$ is inconsistent iff $K' \cup \{\beta\}$ is inconsistent then $K \cap K * \alpha = K \cap K * \beta$

# Which Logics Satisfy Distributivity?

- Classical logic does.
- But what about DLs?
  - $\mathcal{ALC}$ does not.
  - $\mathcal{ALC}$ with empty $\mathcal{A}$Box does.
  - not many more...

# New characterisation

## Representation Theorem [RW14]

If the logic is monotonic and compact and satisfies Inconsistent explosion ~~and Distributivity~~, then * is a revision without negation iff it satisfies closure, success, *strong* inclusion, consistency, relevance and uniformity.

**(strong inclusion)** $K * \alpha \subseteq (K \cap K * \alpha) + \alpha$

In classical logics this postulate is equivalent to inclusion.

# What was done

- Adapted AGM to DLs
    - Contraction - only 1 postulate changed
    - Revision - Contruction and postulates
- Provided representation results.

# What we want to do

- Study other forms of revision for DLs avoiding negation.
- Apply the solutions to other fragments
  - Horn
  - ???