Quantum quenches in a spinor condensate

Austen Lamacraft

cond-mat/0611017

Warwick, 19 December 2006

New phases of atomic matter

- Dilute gases of atoms ~ 10^{14} cm^{-3}

  Degenerate at 100nK - few μK

- But: wonderful *tunability*

- Tuning the interaction

- Different lattice structures, dimension, BF mixtures…

New phases ➡ new *phase ordering*

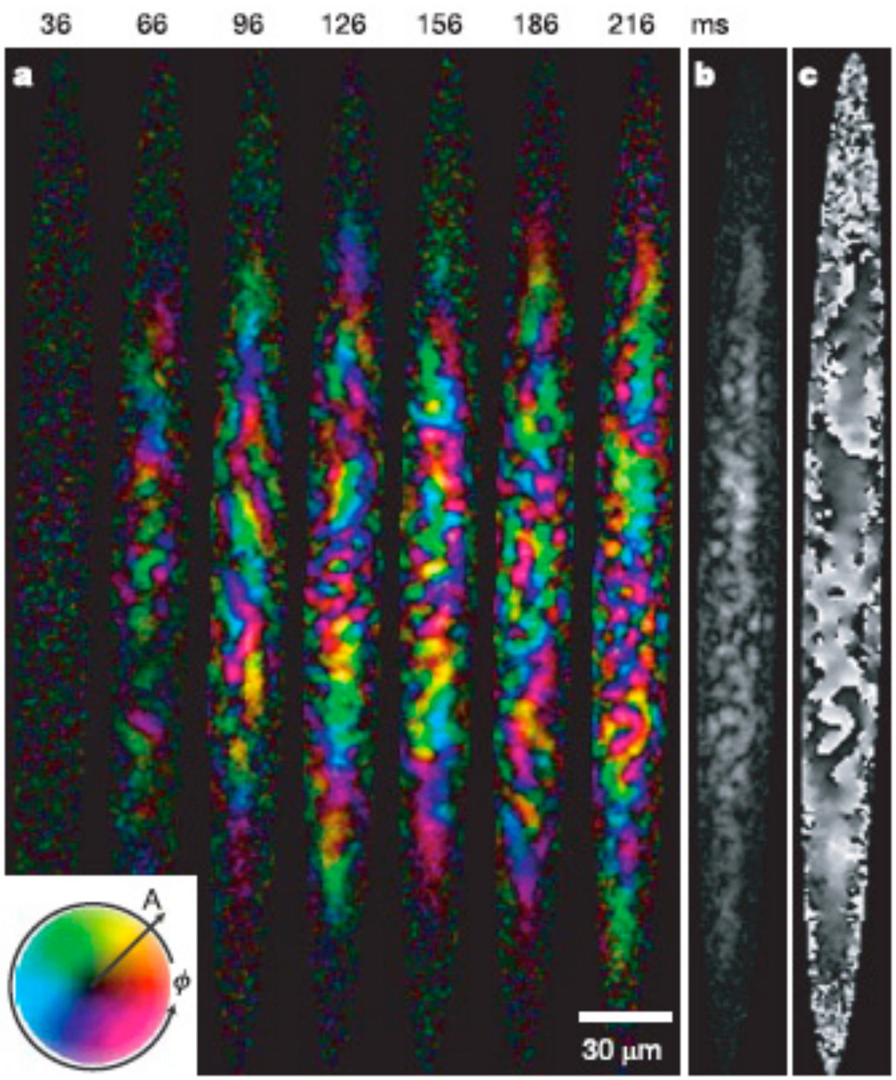
Domains in Cobalt film

What laws govern evolution of correlations, size of domains, etc.?

Outline

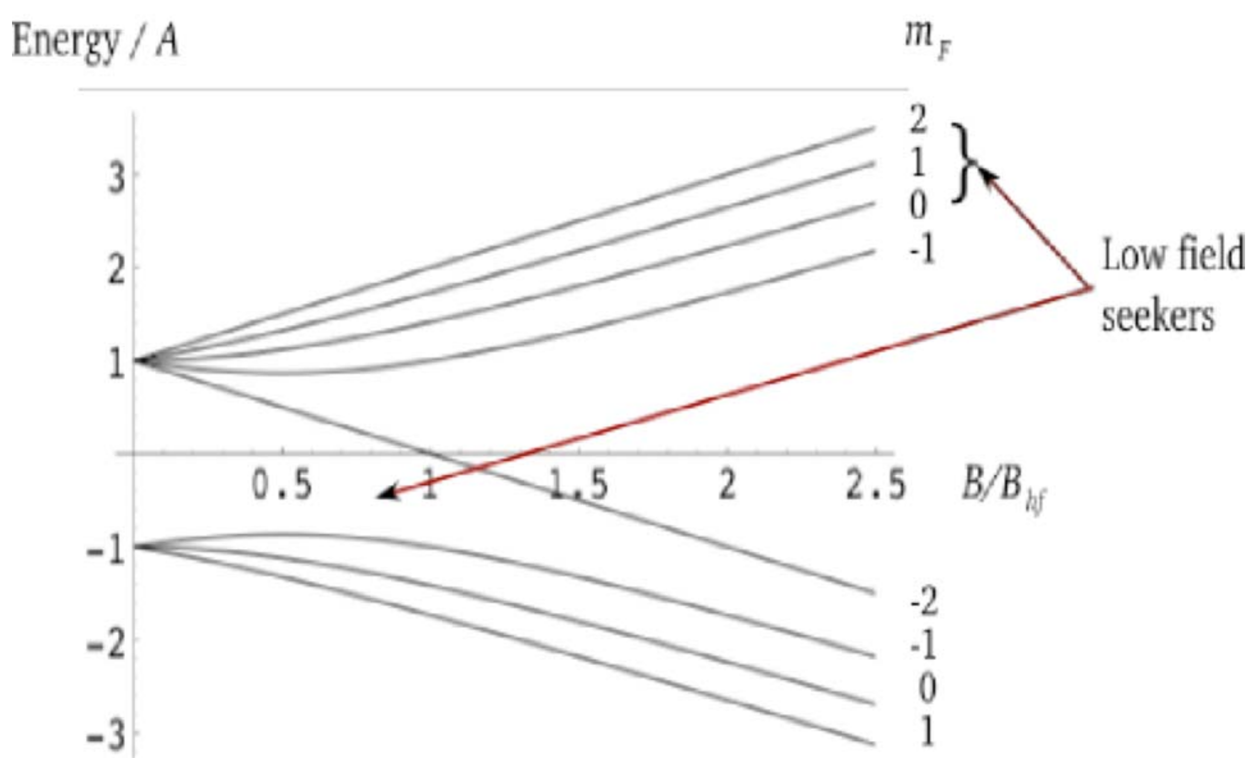
- The Berkeley experiment
- Spin 1 condensates
- XY ordering transition
- Quench dynamics
- Vortex density
- Late time behaviour

Berkeley experiment



Sadler et al, Nature 2006

Zeeman effect



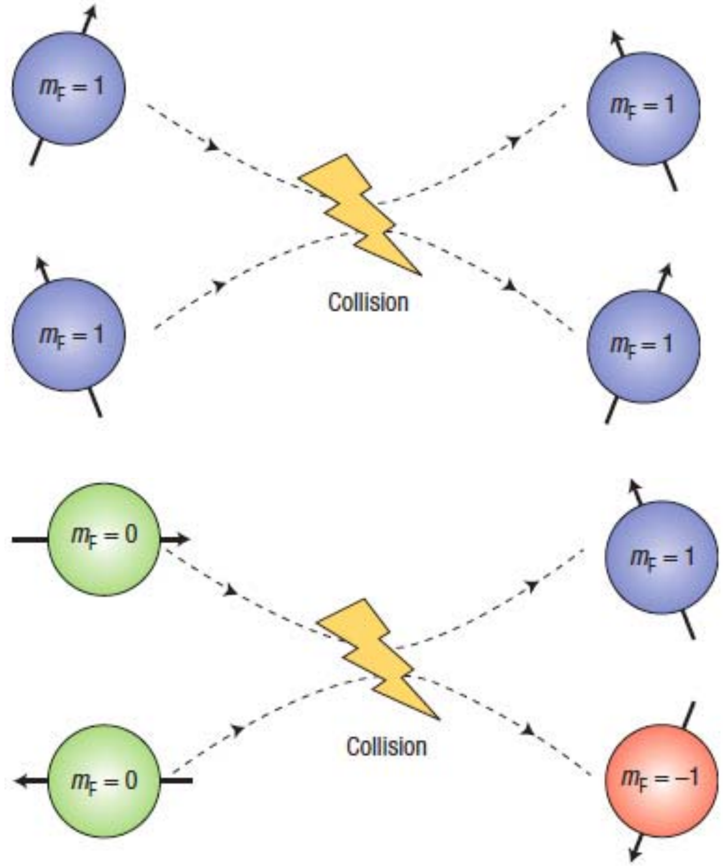
F=1: $$E_m^Z \equiv -\tilde{p}m + qm^2$$

…is most general

Interactions in an F=1 condensate

Total spin 2

Total spin 0



$$\boxed{H_{\text{int}} = \frac{1}{2} c_0 n^2 + \frac{1}{2} c_2 \mathbf{F} \cdot \mathbf{F}}$$

$$c_0 = (g_0 + 2g_2)/3 \qquad c_2 = g_2 - g_0$$

Crucially, $c_2 < 0$ in ^{87}Rb

Gross-Pitaevskii approximation

$$
\begin{aligned}
H &= H_0 + H_{\text{Int}} \\
H_0 &= \int d\mathbf{r}\, \phi_m^\dagger \left[-\frac{1}{2}\nabla^2 + U_{\text{ext}}(\mathbf{r}) + E_m^Z \right] \phi_m \\
H_{\text{Int}} &= \frac{1}{2} \int d\mathbf{r}\, c_0 \phi_m^\dagger \phi_{m'}^\dagger \phi_{m'} \phi_m + \\
&\quad c_2 \phi_m^\dagger \phi_{m'}^\dagger \mathbf{F}_{mn} \cdot \mathbf{F}_{m'n'} \phi_{n'} \phi_n
\end{aligned}
$$

$\phi_m \to \varphi_m = \sqrt{n}\widehat{\chi}_m,$ $\widehat{\chi}_m$ Normalized spinor

$$
\boxed{
\begin{aligned}
\frac{e_{GP}}{n} &= \frac{c_0 n}{2} + \frac{c_2 n}{2} \left[\left(|\chi_1|^2 - |\chi_{-1}|^2 \right)^2 \right. \\
&\quad \left. + 2 \left(|\chi_0 \chi_1|^2 + |\chi_0 \chi_{-1}|^2 + \chi_0^{*2} \chi_1 \chi_{-1} + \chi_0^2 \chi_1^* \chi_{-1}^* \right) \right] \\
&\quad + q \left[|\chi_1|^2 + |\chi_{-1}|^2 \right] - p \left[|\chi_1|^2 - |\chi_{-1}|^2 \right]
\end{aligned}
}
$$

GP approx. (cont.)

$$\frac{e_{GP}}{n} = \frac{c_0 n}{2} + \frac{c_2 n}{2}\left[\left(|\chi_1|^2 - |\chi_{-1}|^2\right)^2 \right.$$
$$\left. +2\left(|\chi_0\chi_1|^2 + |\chi_0\chi_{-1}|^2 + \chi_0^{*2}\chi_1\chi_{-1} + \chi_0^2\chi_1^*\chi_{-1}^*\right)\right]$$
$$+q\left[|\chi_1|^2 + |\chi_{-1}|^2\right] - p\left[|\chi_1|^2 - |\chi_{-1}|^2\right]$$

Consider first *$p=0$*

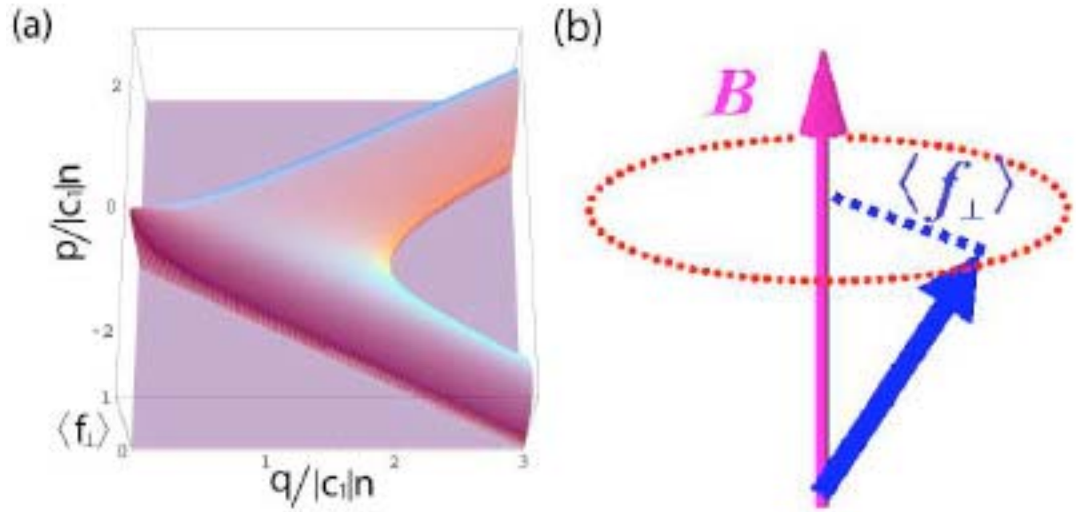
- $q > q_0 \equiv 2|c_2|n$ - only χ_0 occupied
- $q < q_0 \equiv 2|c_2|n$ - $\chi_{\pm 1}$ begin to fill

(choose phase freely)

Result: *XY symmetry broken*

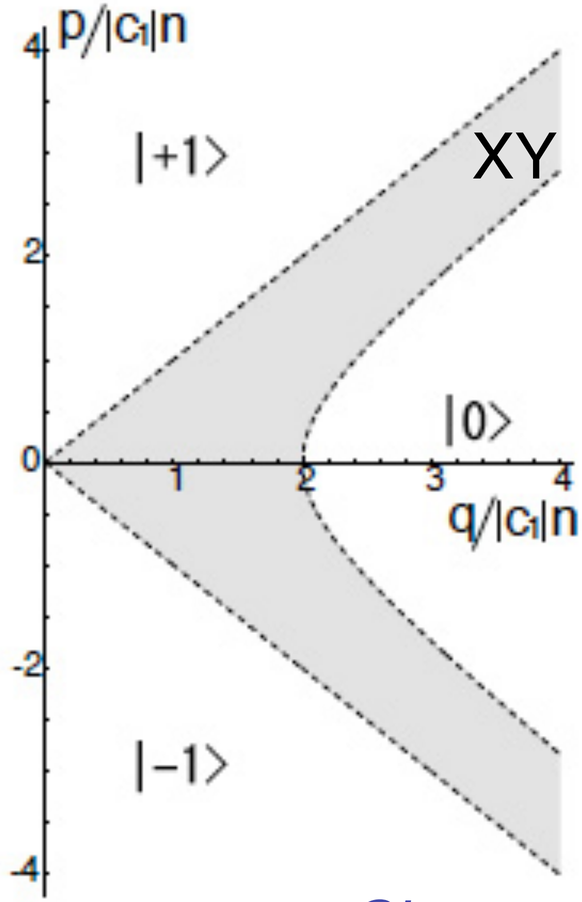
XY Ordering

General case ($p \neq 0$)



$$|\langle f_\perp \rangle| = n \frac{\sqrt{q^2 - p^2}\sqrt{(p^2 + q_0 q)^2 - q^4}}{q_0 q^2}$$

Phase Diagram



Character of transition?

Bogoliubov description

$$H_{\text{Int}} = \frac{1}{2}\int d\mathbf{r}\, c_0 \phi^\dagger_m \phi^\dagger_{m'} \phi_{m'} \phi_m + c_2 \phi^\dagger_m \phi^\dagger_{m'} \mathbf{F}_{mn} \cdot \mathbf{F}_{m'n'} \phi_{n'} \phi_n$$

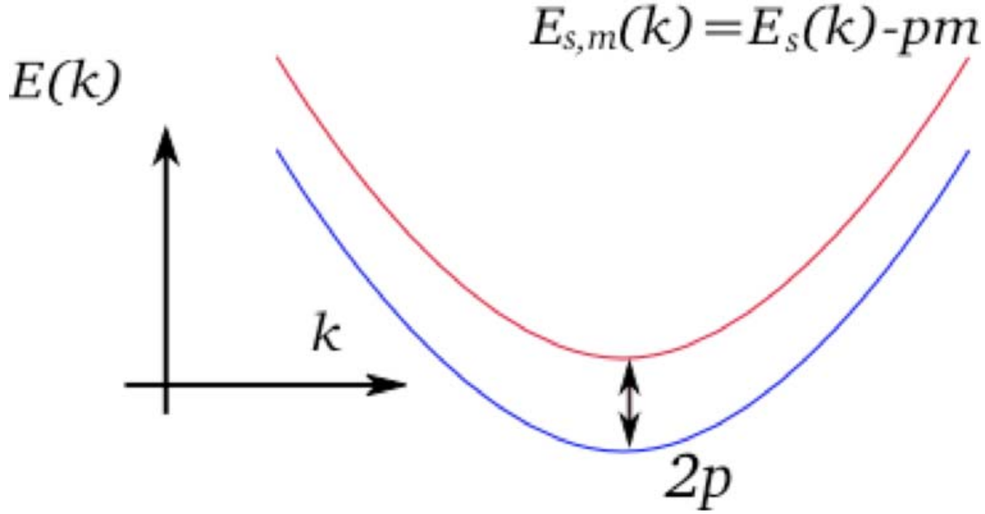
Shift $\phi_m \to \varphi_m + \phi_m, \qquad \varphi^\dagger = (0, \sqrt{n}, 0)$

$$\boxed{H_B \equiv \sum_{k,m=\pm 1} \phi^\dagger_{k,m} \left[\varepsilon_k - pm + q + c_2 n\right] \phi_{k,m} + c_2 n \sum_k \phi^\dagger_{k,1} \phi^\dagger_{-k,-1} + \phi_{k,1} \phi_{-k,-1}}$$

$$\boxed{H_B = \sum_k E_{s,+}(k) a^\dagger_k a_k + E_{s,-}(k) b^\dagger_k b_k}$$

Magnon dispersions

$$E_{s,m}(k) \equiv E_s(k) - pm$$
$$E_s^2(k) \equiv (\varepsilon_k + q)(\varepsilon_k + q + 2c_2 n)$$



- Band crosses zero when $p^2 = p_c^2 \equiv q(q - q_0)$
- $p = 0$ is a special point

Effective theory

$$z_k \equiv \frac{1}{\sqrt{E_s(k)}}\left(a_k + b^\dagger_{-k}\right)$$

$$\pi_k \equiv i\sqrt{E_s(k)}\left(a^\dagger_k - b_{-k}\right)$$

$$H_B = \tfrac{1}{2}\left(\pi^\dagger + ipz\right)\left(\pi - ipz^\dagger\right) + \tfrac{1}{2}\left[E_s^2 - p^2\right] z^\dagger z$$

Including quartic terms

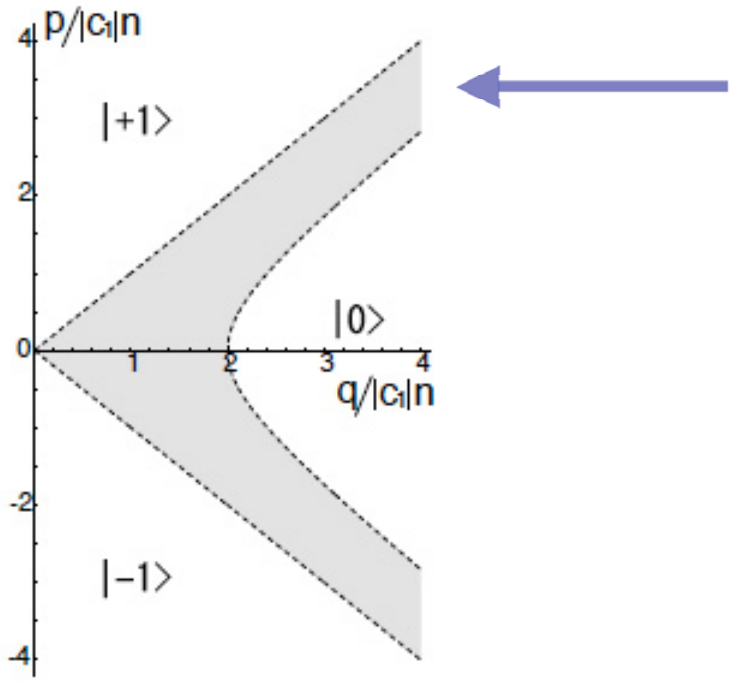
$$\boxed{\begin{aligned} S_{\text{eff.}} &= \int d\mathbf{r}\, dt\, \frac{1}{2}\left[\dot{z}^\dagger \dot{z} - c_s^2 \nabla z^\dagger \nabla z \right. \\ &\quad \left. - \left(p_c^2 - p^2\right) z^\dagger z\right] - ipz^\dagger \dot{z} - \frac{|c_2| q^2}{2} |z|^4 \end{aligned}}$$

$$E_s^2(k) \sim c_s^2 k^2 + p_c^2 \qquad c_s^2 \equiv q - q_0/2$$

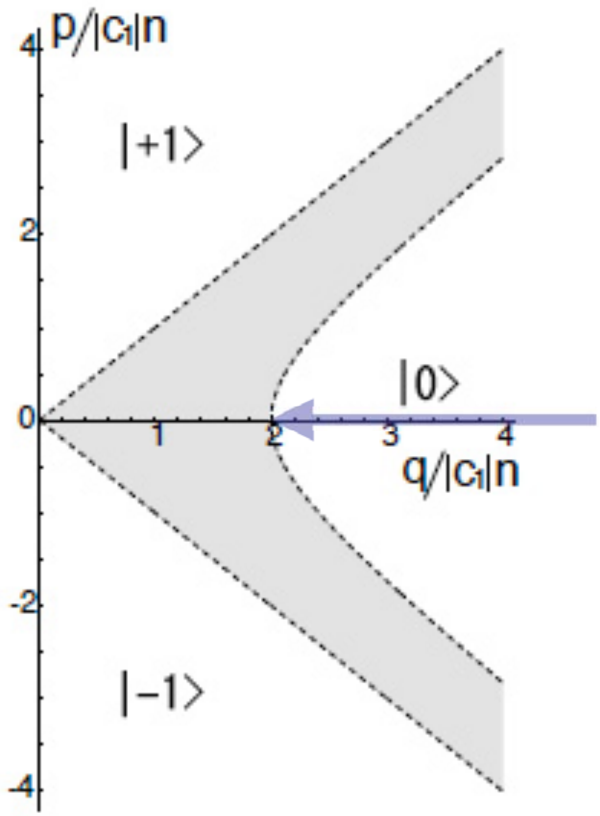
$p \neq 0$: BEC of Magnons

Non-zero magnetization ➡ finite density of bosons

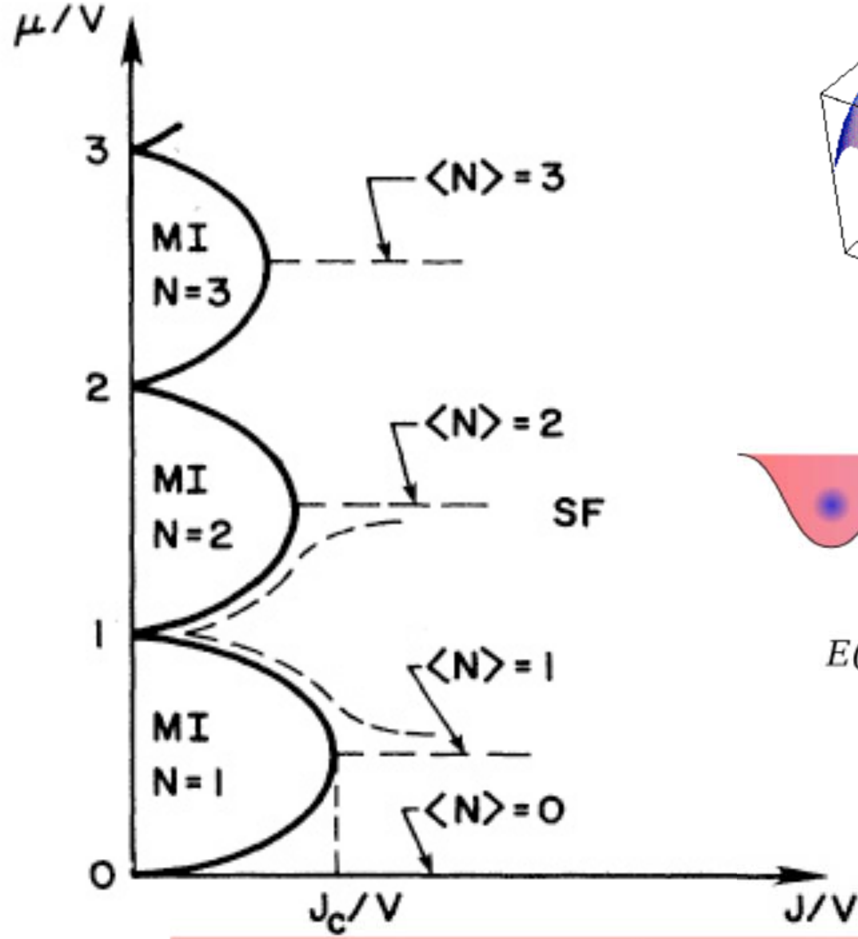
Always ordered at zero temp

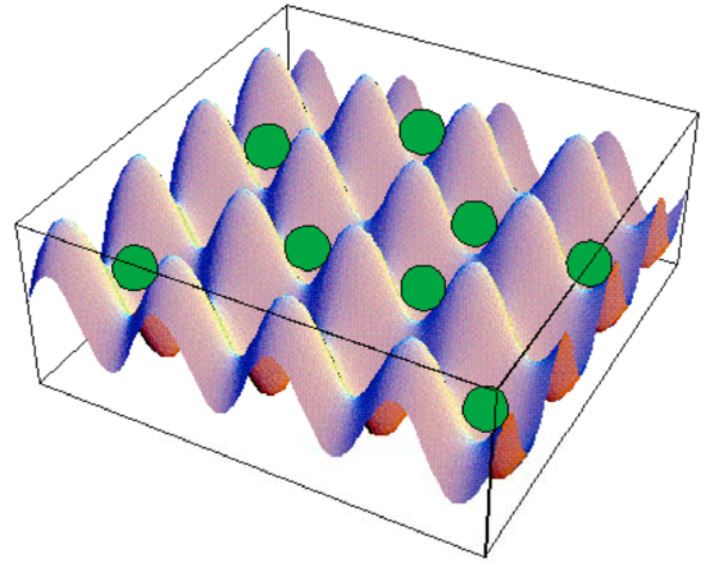


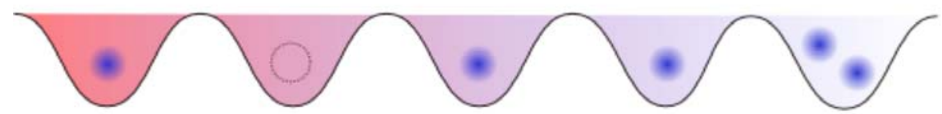
$p=0$: $(d+1)$-XY model

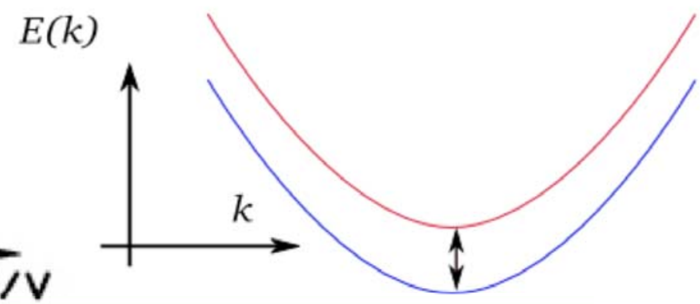


Bose Hubbard model



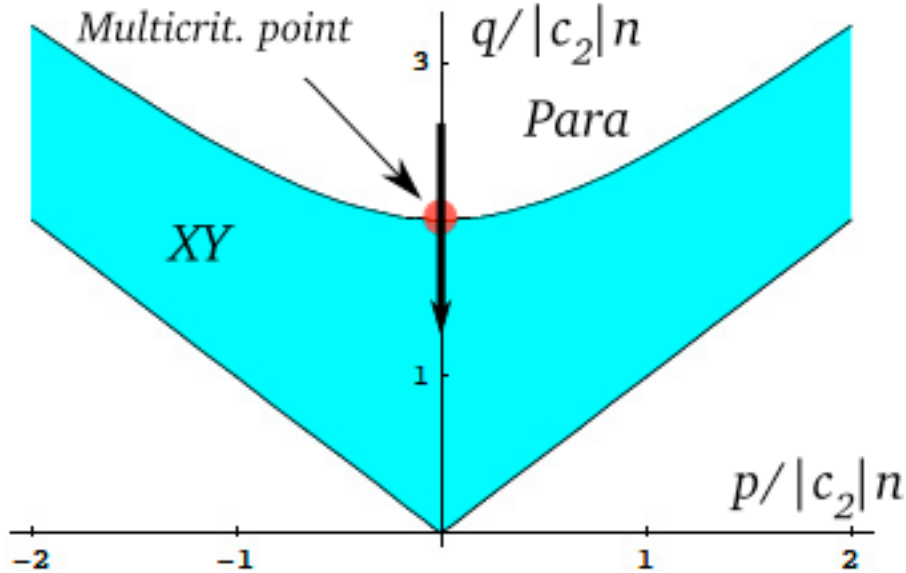






$$\boxed{H = \sum_{ij} J_{ij} b_i^\dagger b_j + V \sum_i N_i (N_i - 1)}$$

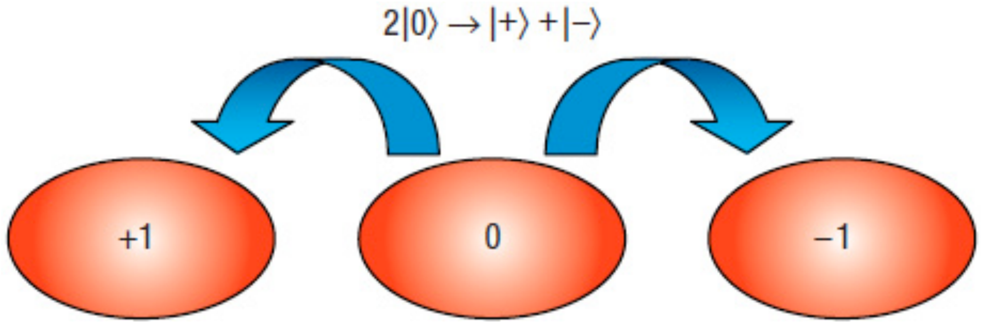
Quenching through the transition



Instability

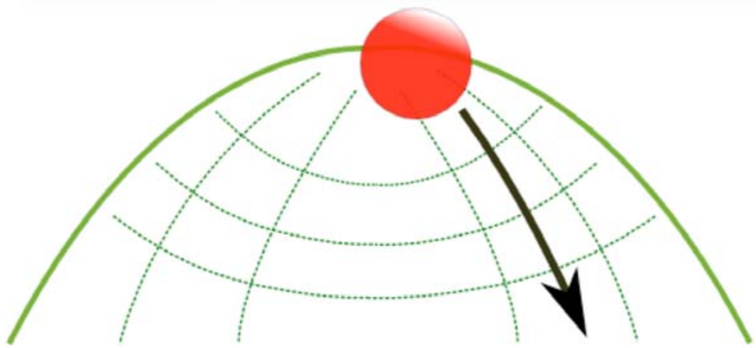
What happens for $q < q_0$?

$$H_B \equiv \sum_{k,m=\pm 1} \phi^\dagger_{k,m} \left[\varepsilon_k - pm + q + c_2 n\right] \phi_{k,m} + c_2 n \sum_k \phi^\dagger_{k,1} \phi^\dagger_{-k,-1} + \phi_{k,1} \phi_{-k,-1}$$



Instability

$$\boxed{H_B = \sum_k \frac{1}{2}\pi_k^\dagger \pi_k + \frac{1}{2}E_s^2(k) z_k^\dagger z_k}$$



Appearance of unstable modes $E_s^2(k) < 0$

$$f_{\perp k} = \sqrt{2n(\varepsilon_k + q)} z_k^\dagger + \cdots$$

Growth of z_k ➡ $f_{\perp k}$

Rapid quenches

$$q_i \to q_f \quad \text{at} \quad t = 0$$

For unstable modes

$$\boxed{z_k(t) = z_k(0)\cosh\omega_k t + \pi_k(0)\omega_k^{-1}\frac{\varepsilon_k + q_f}{\varepsilon_k + q_i}\sinh\omega_k t}$$

$$\omega_k^2 \equiv -E_s^2(k)$$

Initial conditions

$$\langle z_k^\dagger(0) z_k(0)\rangle = (E_s(k))^{-1},\ \langle \pi_k^\dagger(0)\pi_k(0)\rangle = E_s(k)$$

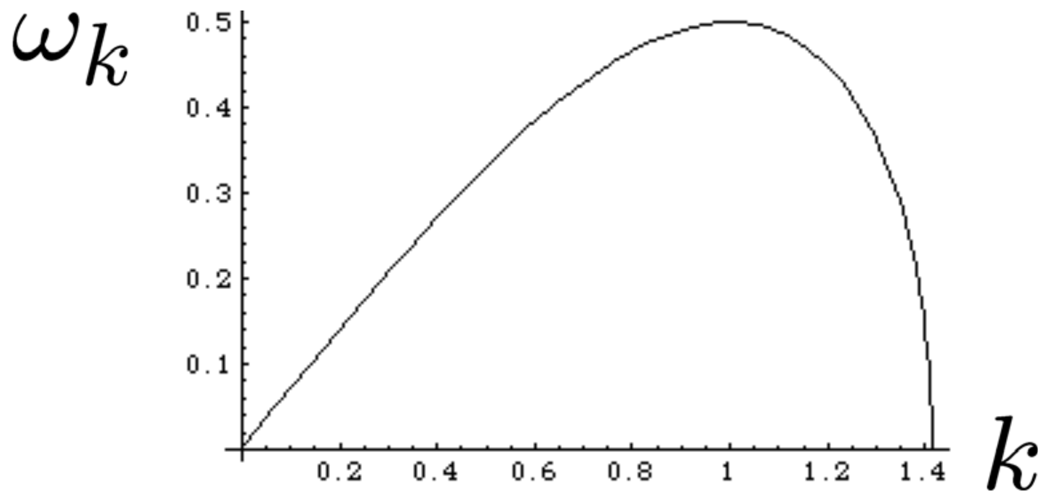
$$\boxed{\langle f_{\perp k}(t) f_{\perp -k}^\dagger(t)\rangle = 2n\left[\cosh^2\omega_q t + \left(\frac{\varepsilon_k + q_f}{\omega_k}\right)^2 \sinh^2\omega_q t\right]}$$

Shot noise limit $q_i \to \infty$

Deep quenches ($q_f<|c_2|n$)

Consider $q_f = 0$

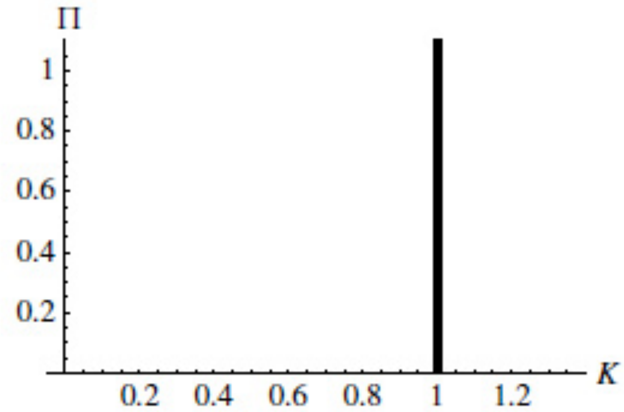
Spectrum of unstable modes $\omega_k^2 = \varepsilon_k(q_0 - \varepsilon_k)$

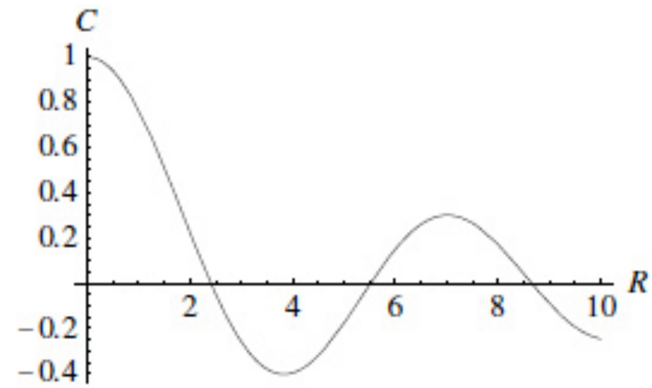


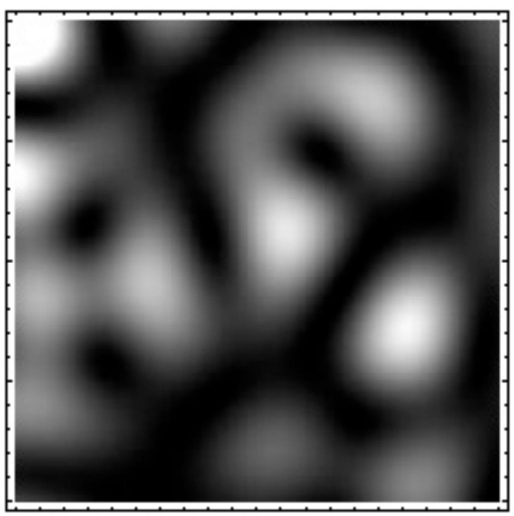
Consequence of spin conservation

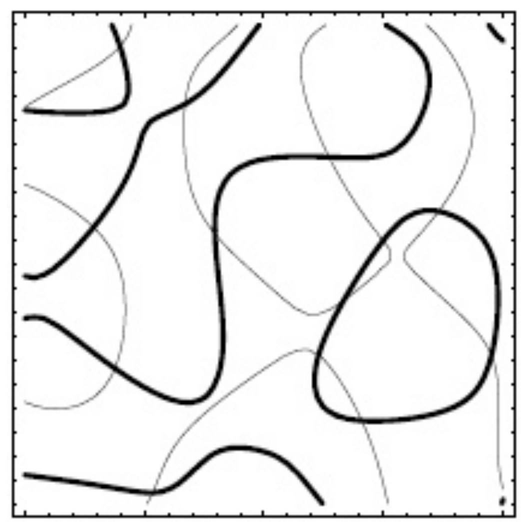
$$\boxed{\langle f_\perp(\mathrm{r},t) f_\perp^\dagger(\mathrm{r}',t)\rangle \to \frac{n}{2L_\perp}\sqrt{\frac{q_0}{2\pi t}} J_0(\sqrt{q_0}|\mathrm{r}-\mathrm{r}'|)e^{q_0 t}}$$

Random monochromatic waves



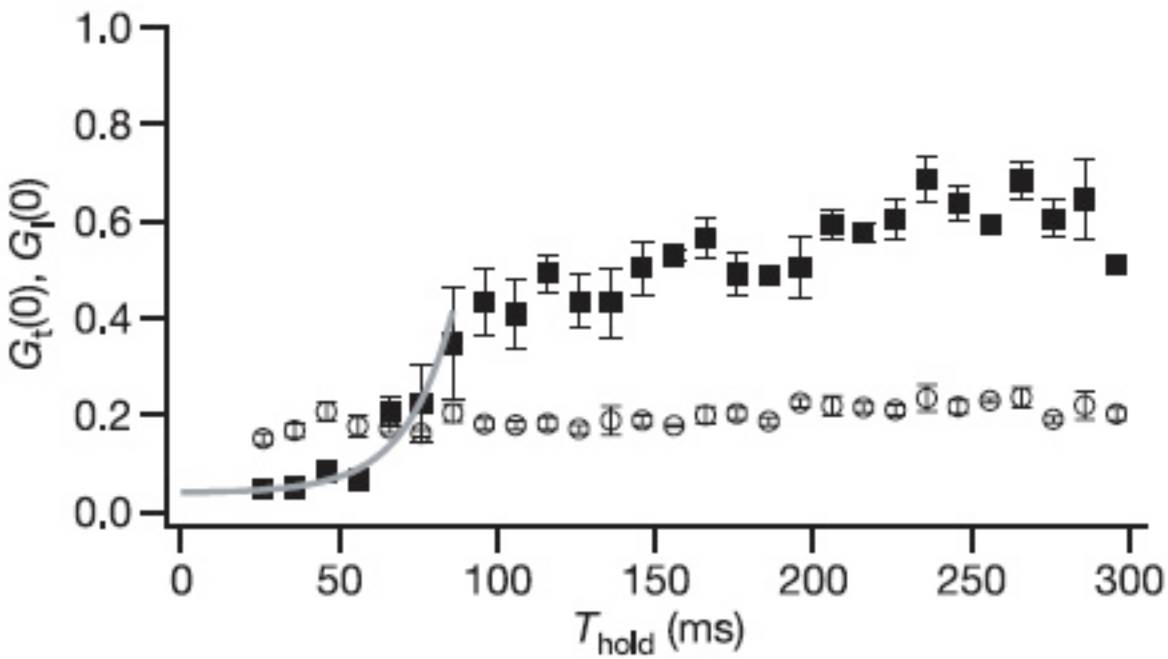






M.R. Dennis, Ph.D. thesis

Experiment

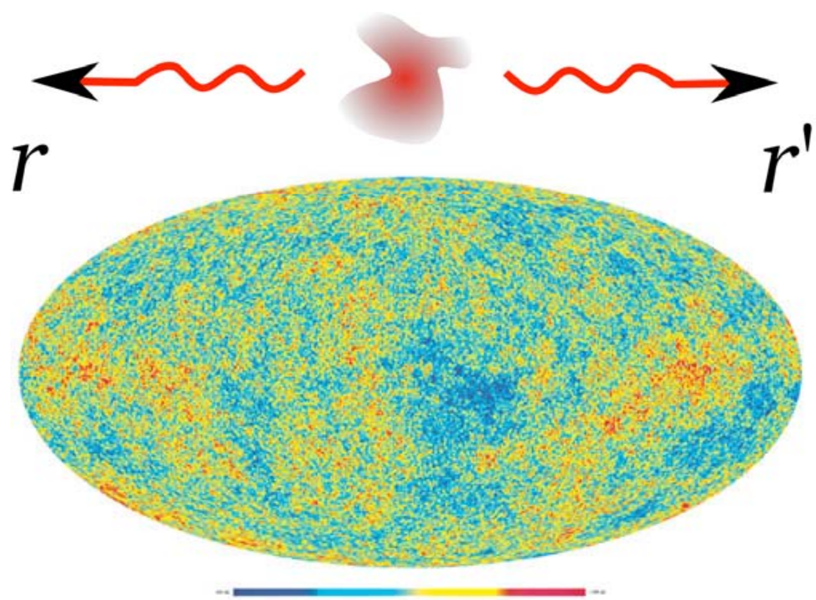


Time constant 15ms, in rough agreement with $\hbar/q_0 \sim 10\mathrm{ms}$

Shallow quenches (q_f~$2|c_2|n$)

$$\omega_k^2 = c_s^2(k^2 - k_c^2), \qquad k_c^2 \equiv q(q_0 - q)/c_s^2$$

$$\boxed{\langle f_\perp(\mathbf{r},t) f_\perp^\dagger(\mathbf{r}',t)\rangle \to \frac{nq_0}{4\pi L_\perp}\frac{1}{c_s k_c t} e^{k_c(4c_s^2 t^2 - |\mathbf{r}-\mathbf{r}'|^2)^{1/2}}}$$



Finite time quenches

$$q(t) = q_0 \left(1 - t/\tau_Q\right)$$

$$\ddot{z}_k + \left(c_s^2 k^2 - q_0^2 t/\tau_Q\right) z_k = 0$$

$$\boxed{\langle f_\perp(\mathbf{r},t) f_\perp^\dagger(\mathbf{r}',t)\rangle = \frac{n q_0}{2 c_s^2 t_{\mathrm{KZ}} L_\perp} \mathcal{F}(t/t_{\mathrm{KZ}}, |\mathbf{r}-\mathbf{r}'|/(c_s t_{\mathrm{KZ}}))}$$

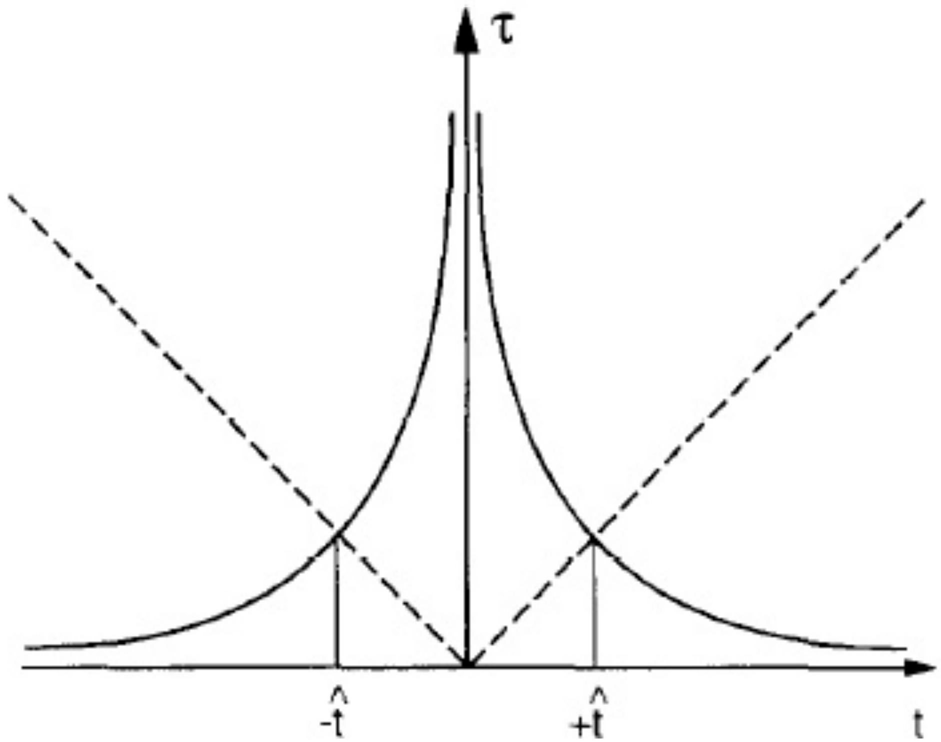
$$t_{\mathrm{KZ}} \equiv \left(\tau_Q/q_0^2\right)^{1/3}$$

Saturation at $t \propto t_{\mathrm{KZ}}$

$$\boxed{\xi(t_{\mathrm{KZ}}) \propto c_s t_{\mathrm{KZ}} = c_s \left(\tau_Q/q_0^2\right)^{1/3}}$$

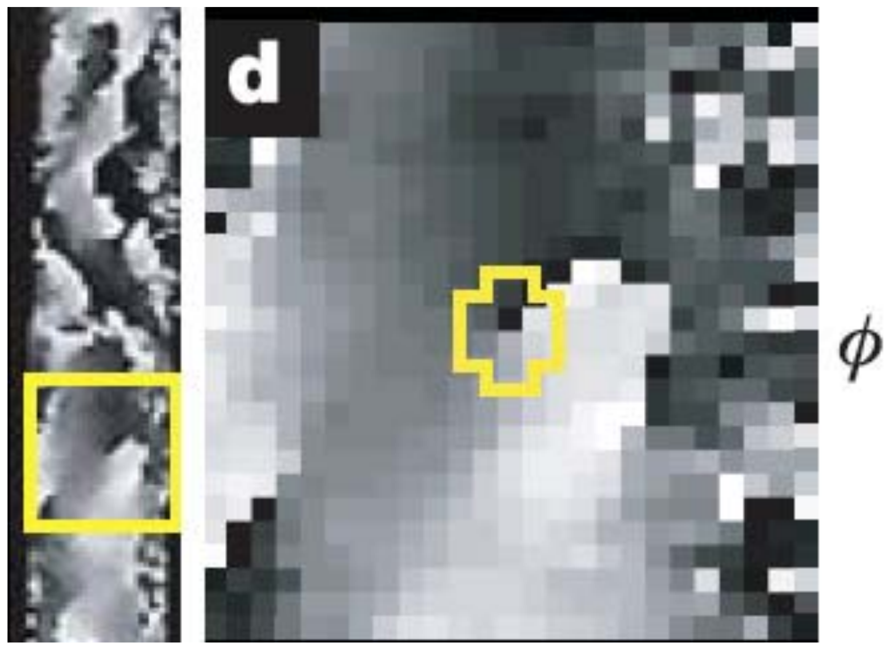
Kibble-Zurek mechanism

$$\boxed{\xi \propto \tau_Q^{\frac{\nu}{z\nu+1}}}$$



$$z = 1, \qquad \nu = 1/2$$ for us

Vortices

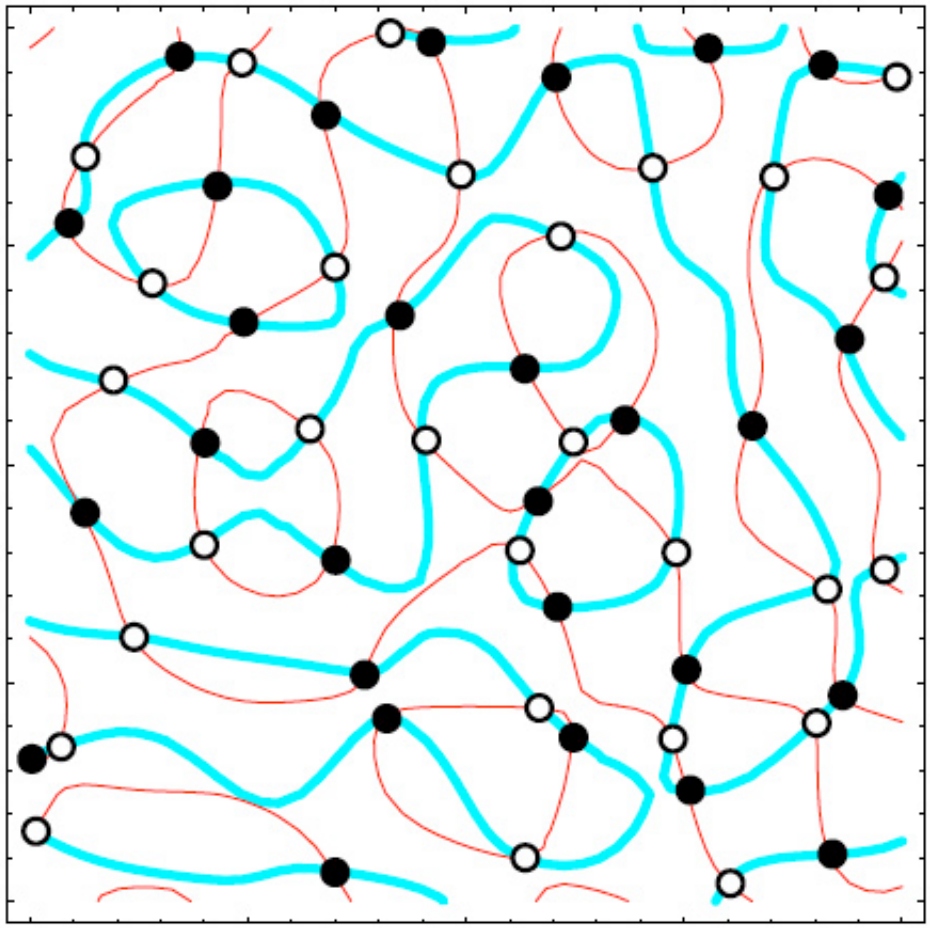


Spin vortices (no mass current) with `polar core'

What density of defects should we expect?

Vortices as zeroes

Treat growing modes as classical Gaussian fields



Vortices (cont.)

Halperin-Liu-Mazenko formula

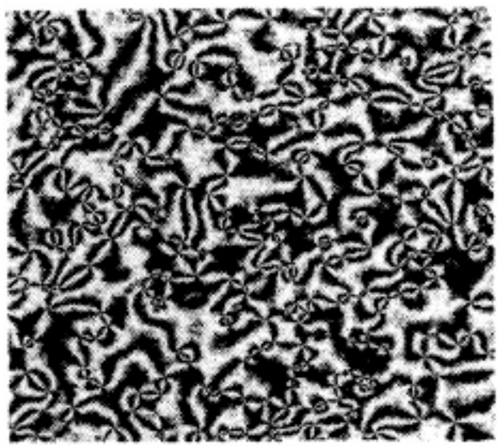
$$\boxed{n_V(t) = -\frac{1}{2\pi} g''(r,t)}$$

$$g(|\mathbf{r}-\mathbf{r}'|,t) \equiv \frac{\langle f_\perp(\mathbf{r},t) f_\perp^\dagger(\mathbf{r}',t)\rangle}{\langle f_\perp(0,t) f_\perp^\dagger(0,t)\rangle}$$

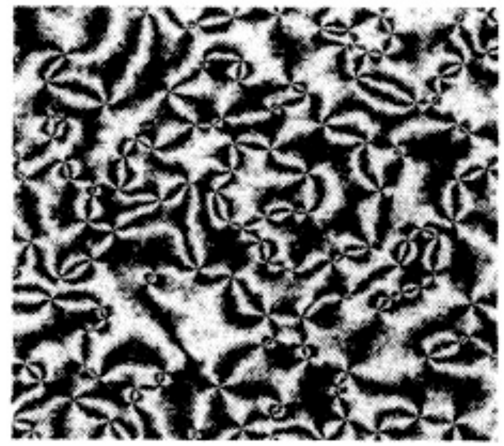
- Deep quench $\quad n_V(t) \to \frac{k_{\mathrm{max}}^2}{4\pi}$
- Shallow quench $\quad \to \frac{1}{4\pi}\frac{k_c}{c_s t}$
- Finite time $\quad \propto \xi^{-2}(t_{\mathrm{KZ}})$

Coarsening

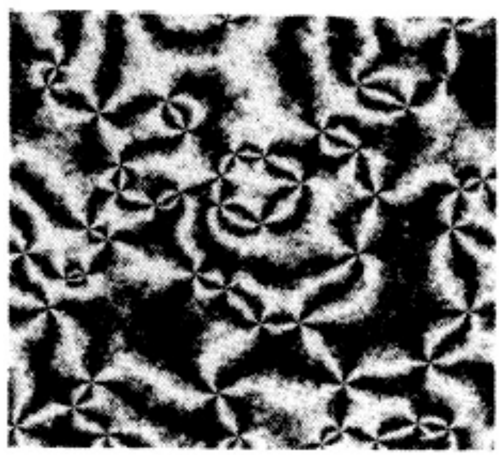
What happens at late times?



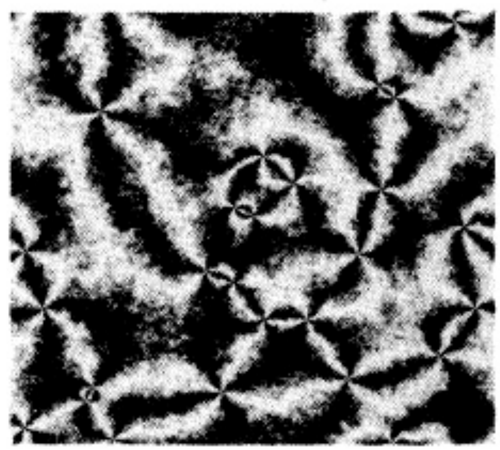
N = 1000



N = 3000



N = 10 000



N = 30 000

Scaling and conservation laws

Scaling at long times $\xi(t) \propto t^{1/z}$

Two universality classes

- OP not conserved $z = 2$ ($q_f \neq 0$)
- OP conserved $z = 3$ ($q_f = 0$)

But: Assumes disspative dynamics

emerges after coarse-graining

Models of coarsening

$$\begin{aligned} S_{\text{eff.}} &= \int d\mathbf{r}\, dt\, \frac{1}{2}\left[\dot{z}^\dagger \dot{z} - c_s^2 \nabla z^\dagger \nabla z \right. \\ &\quad \left. - \left(p_c^2 - p^2\right) z^\dagger z\right] - ipz^\dagger \dot{z} - \frac{|c_2|q^2}{2}|z|^4 \end{aligned}$$

Model F dynamics (Model E for *p=0*)?

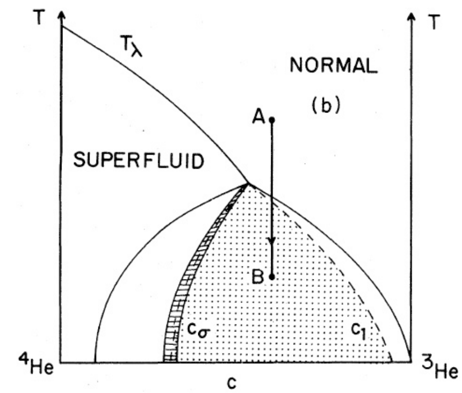
What is effect of Hamiltonian dynamics?

Conclusions

New states of atomic matter…

…give new questions about *phase ordering*

Outlook



- Quantum spinodal decomp.

e.g. in BCS-BEC crossover (AL and F. Marchetti, to appear)

- Coarsening in strongly coupled systems

Access to more decades