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A globally asymptotically stable plasticity rule for firing rate homeostasis

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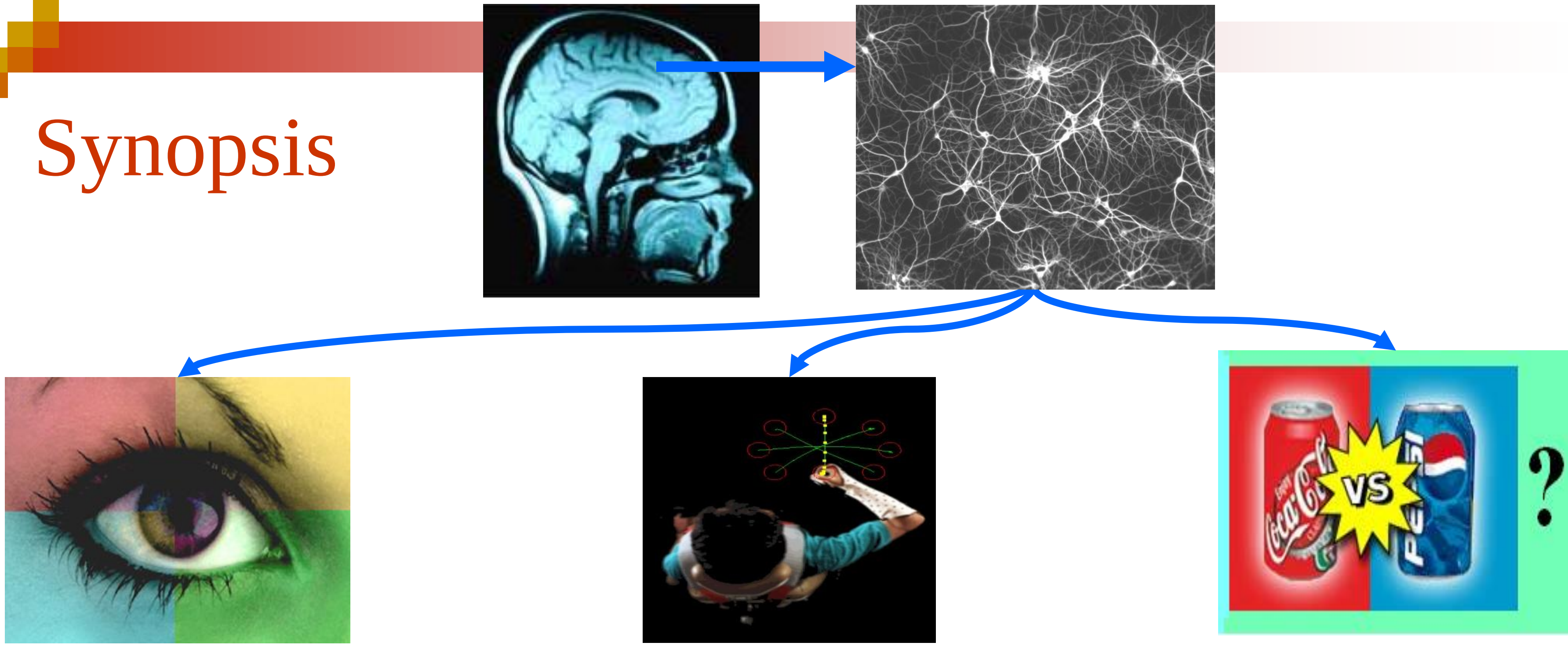
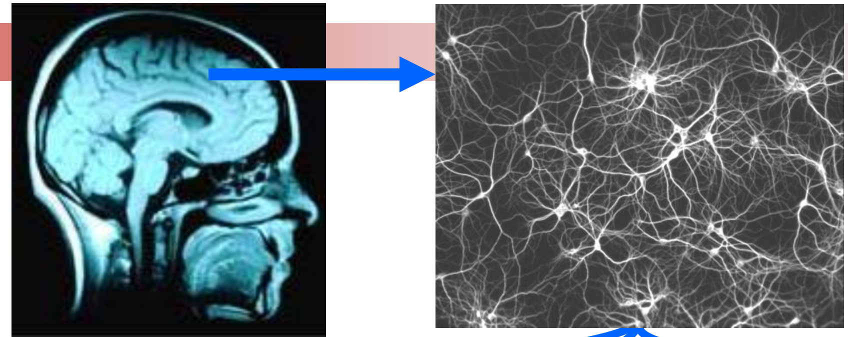
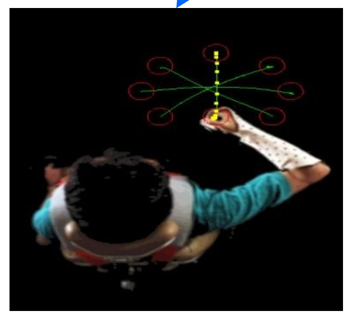
Synopsis



- Network of neurons in brain perform diverse cortical computations in parallel
- External environment and experiences modify these neuronal circuits via synaptic plasticity mechanisms
- Correlation based Hebbian plasticity forms the basis of much of the research done on the role of synaptic plasticity in learning and memory
- What is wrong with Hebbian plasticity?

Synopsis

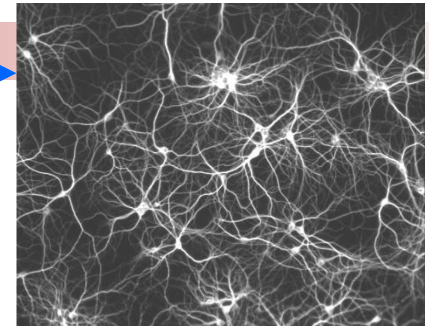


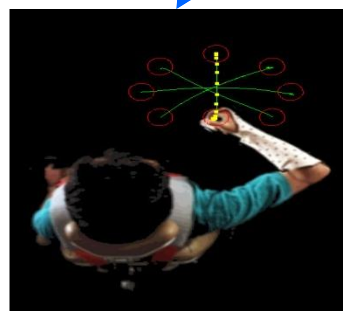


Classical Hebbian plasticity leads to unstable activity regimes in the absence of some kind of regulatory mechanism (positive feedback process)

In the absence of such regulatory mechanism, Hebbian learning will lead the circuit into hyper- or hypo-activity regimes

Synopsis





How can neural circuits maintain stable activity states when they are constantly being modified by Hebbian processes that are notorious for being unstable?

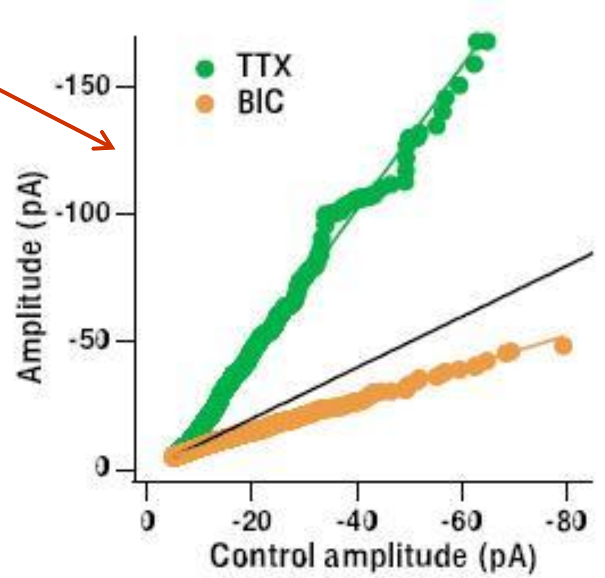
- A new synaptic plasticity mechanism is presented
  - Enables a neuron to maintain homeostasis of its firing rate over longer timescales
  - Leaves the neuron free to exhibit fluctuating dynamics in response to external inputs
  - Is globally asymptotically stable
  - Simulation results are presented from single neuron to network level for sigmoidal as well as spiking neurons

Outline

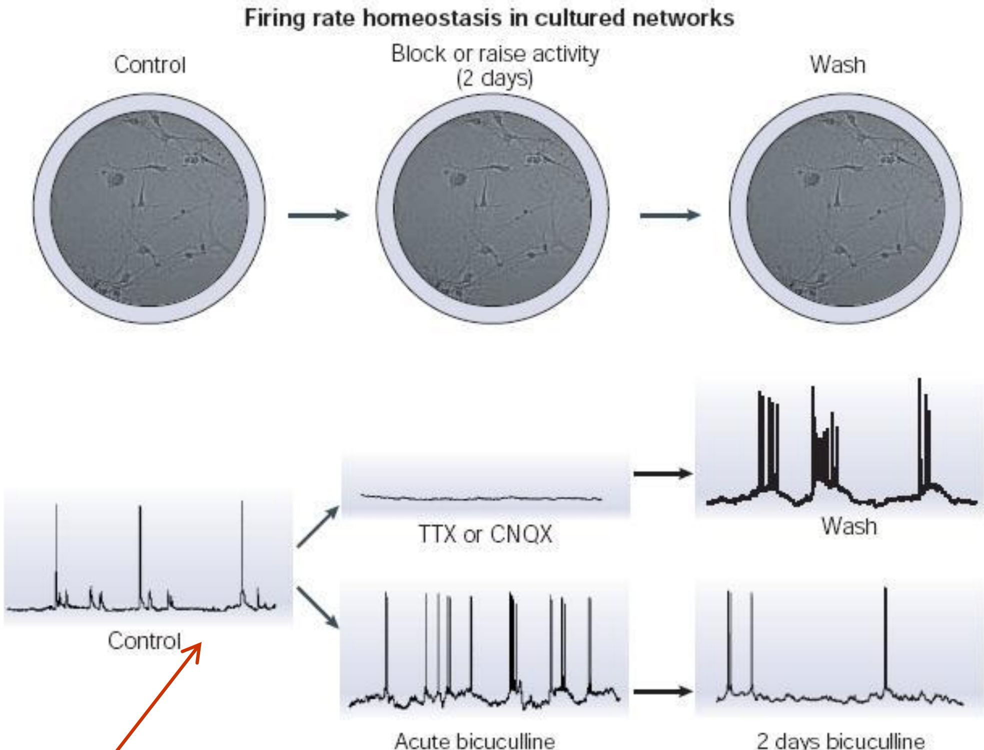
- Homeostatic mechanisms in biology
- Computational theory and learning rule
- Simulation results
- Conclusion

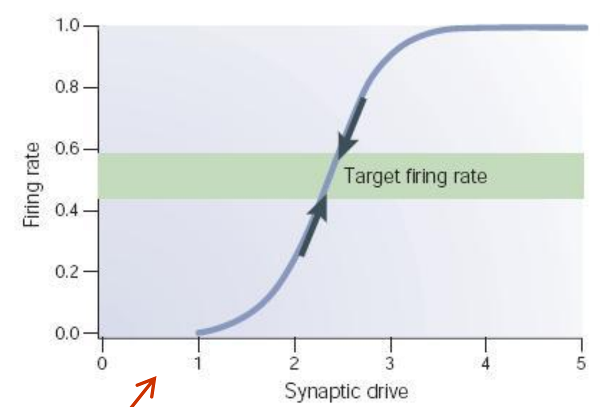
Homeostatic mechanisms in biology

Abott, L.F., Nelson S. B.: Synaptic plasticity: taming the beast. Nature Neurosci. 3, 1178-1183 (2000)



- Slow homeostatic plasticity mechanisms enable the neurons to maintain average firing rate levels by dynamically modifying the synaptic strengths in the direction that promotes stability





Turrigiano, G.G., Nelson, S.B.: Homeostatic plasticity in the developing nervous system. Nature Neurosci. 5, 97-107 (2004)

Hebb and beyond (Computational theory and learning rule)

- Hebb's premise:

$$\tau_w \frac{dw}{dt} = v_{pre}(t) \bullet v_{post}(t)$$

Stable points:
$v_{pre}(t) = 0$ or $v_{post}(t) = 0$

- We can make the postsynaptic neuron achieve a baseline firing rate of v_{base} by adding a multiplicative term $(v_{base} - v_{post}(t))$

$$\tau_w \frac{dw}{dt} = v_{pre}(t) \bullet v_{post}(t) \bullet (v_{base} - v_{post}(t))$$

Some Math

- Learning rule:

$$\tau_w \frac{dw}{dt} = \nu_{pre}(t) \bullet \nu_{post}(t) \bullet (\nu_{base} - \nu_{post}(t)) \qquad (1)$$

- Basic Assumption – pre and post-synaptic neurons are linear

$$\nu_{pre}.w(t) = \nu_{post}(t) \qquad (2a)$$

- Differentiating equation 2a we get:

$$\nu_{pre}.\frac{dw(t)}{dt} = \frac{d\nu_{post}(t)}{dt} \qquad (2b)$$

- By substituting in equation (1):

$$\frac{d\nu_{post}(t)}{dt} = -\frac{\nu_{pre}^2}{\tau_w}.\nu_{post}(t).(\nu_{post}(t) - \nu_{base}) \qquad (3)$$

Theorem 1 (Stability)

For a SISO case, with the presynaptic input held constant at ν_{pre}, and the postsynaptic output having the value ν^0_{post} at time $t = 0$, and ν_{base} being the homeostatic firing rate of the postsynaptic neuron, the system describing the evolution of $\nu_{post}(.)$ is globally asymptotically stable. Further ν_{post} globally asymptotically converges to ν_{base}

$$\tau_w \frac{dw}{dt} = \nu_{pre}(t) \bullet \nu_{post}(t) \bullet (\nu_{base} - \nu_{post}(t)) \rightarrow \frac{d\nu_{post}(t)}{dt} = -\frac{\nu_{pre}^2}{\tau_w}.\nu_{post}(t).(\nu_{post}(t) - \nu_{base})$$

Hint for proof:

1. Use as Lyapunov function:

$$V(\nu_{post}) = \tau_w\left(\nu_{post} - \nu_{base} \ln(\nu_{post})\right)$$

2. The derivative of V is negative definite over the whole state space
3. Apply global invariant set theorem

Theorem 2

For a SISO case, with the presynaptic input held constant at ν_{pre}, and the postsynaptic output having the value ν^0_{post}, at time $t = 0$, and ν_{base} being the homeostatic firing rate of the postsynaptic neuron, the postsynaptic value at any time $t > 0$ is given by:

$$\nu_{post}(t) = \frac{\nu^0_{post}.\nu_{base}}{\nu^0_{post} + (\nu_{base} - \nu^0_{post}).\exp\left(\frac{-\nu^2_{pre}.\nu_{base}.t}{\tau_w}\right)}$$

Hint for proof:

Convert equation:

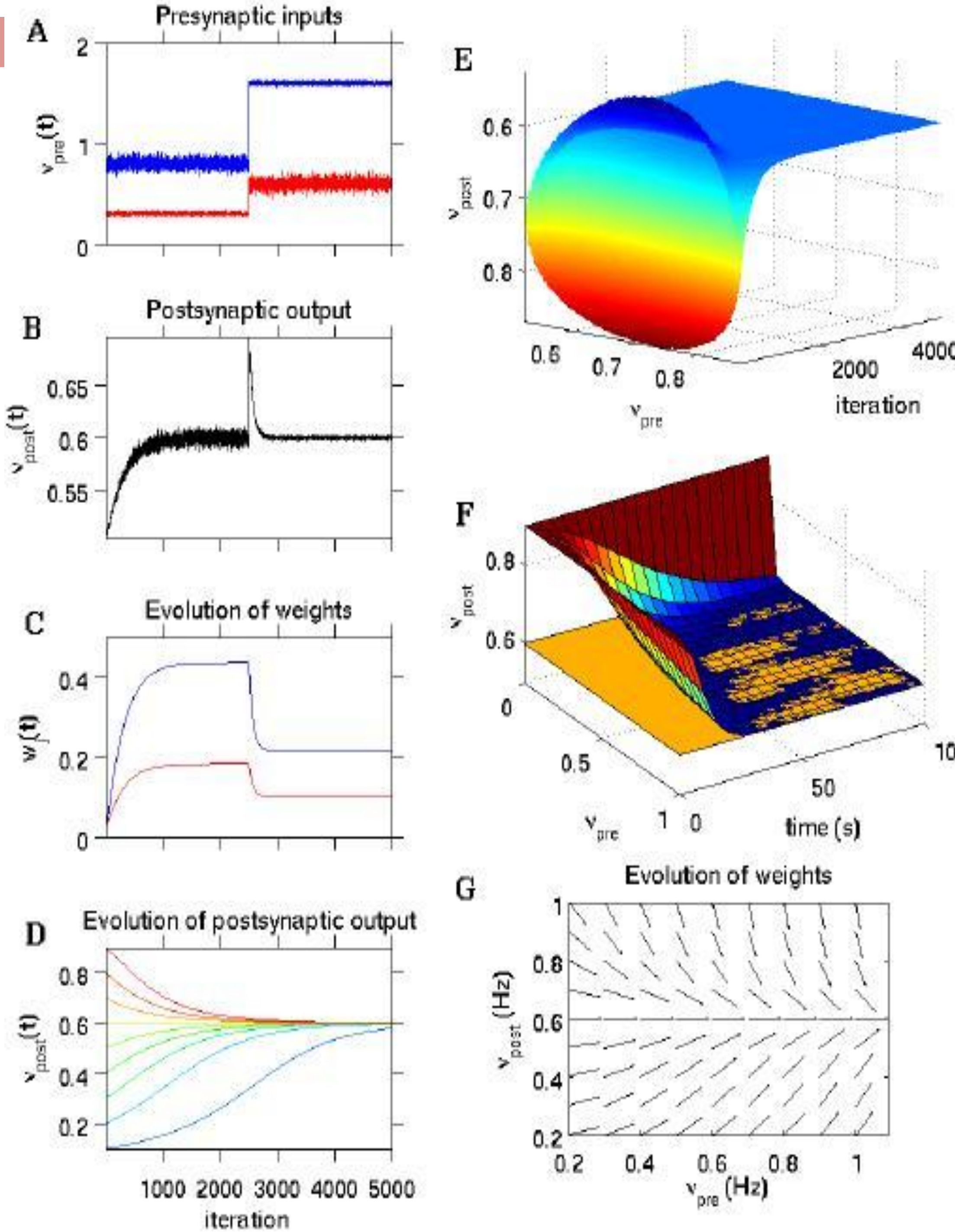
$$\frac{d\nu_{post}(t)}{dt} = -\frac{\nu^2_{pre}}{\tau_w}.\nu_{post}(t).(\nu_{post}(t) - \nu_{base})$$

into a linear form and solve it.

Results

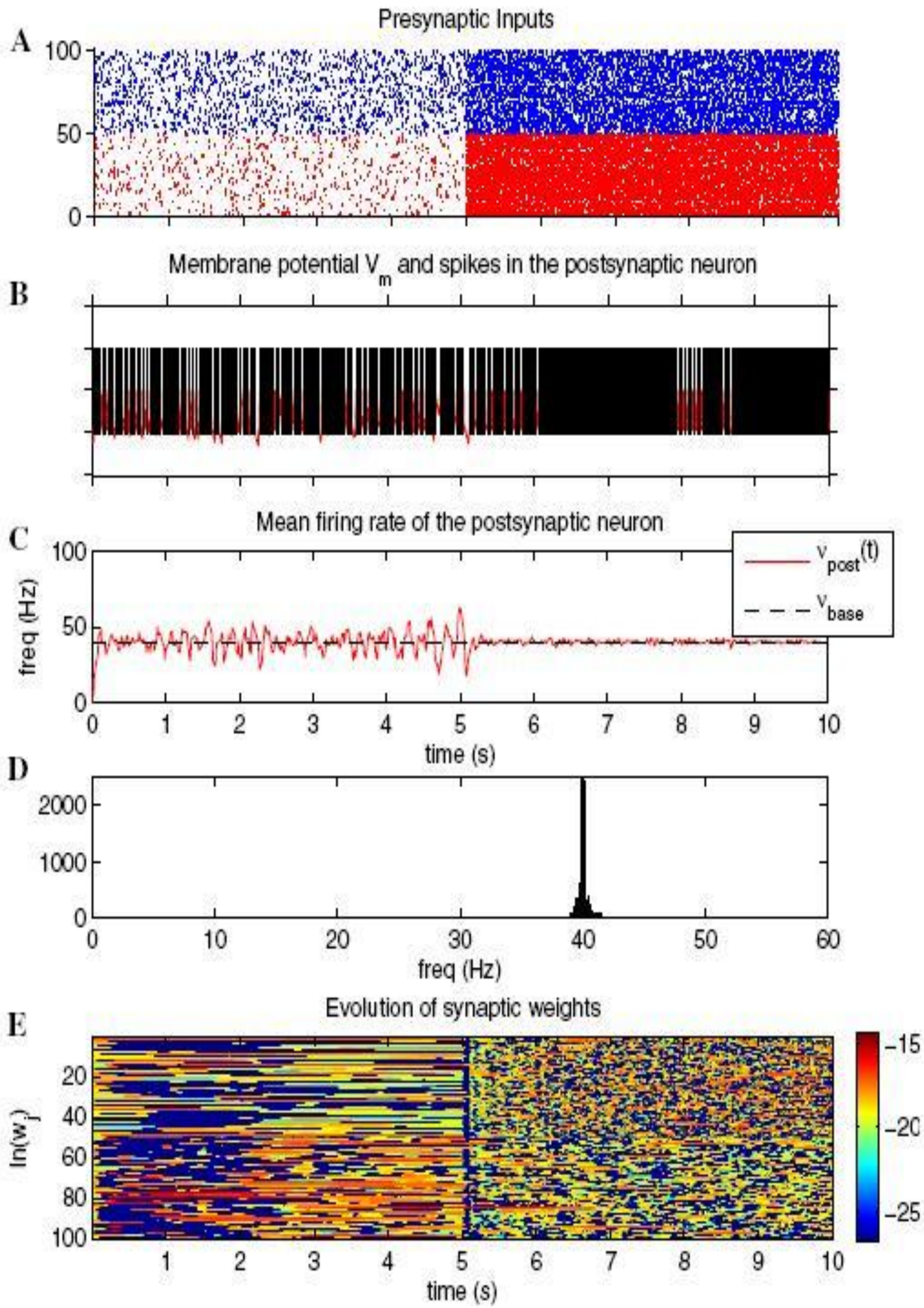
A sigmoidal postsynaptic neuron receiving presynaptic inputs from two different and independent Gaussian input streams

- Simulation time n = 5000 steps
- Initial weights uniformly drawn from [0, 0.1]
- $\nu_{base} = 0.6$, $\tau_w = 30$
- For n <= 2500
  - IP1: mean = 0.3, SD = 0.01
  - IP2: mean = 0.8, SD = 0.04
- For n > 2500
  - IP1: mean = 0.36 SD = 0.04
  - IP2: mean = 1.6, SD = 0.01



Results

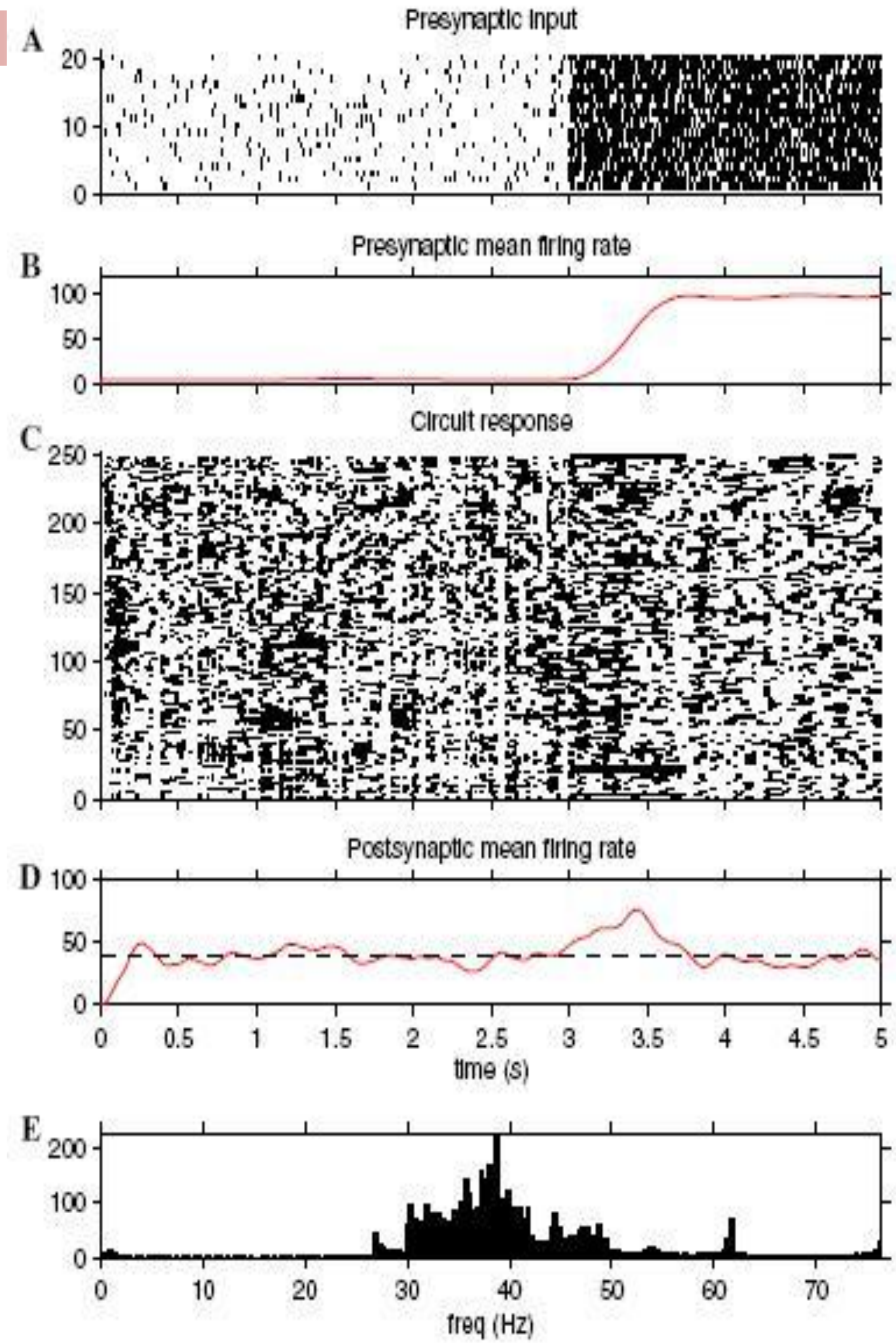
- Single postsynaptic integrate-and-fire neuron receiving presynaptic inputs from 100 Poisson spike trains via dynamic synapses
- Simulation time, t = 10 sec, dt = 1ms
- Initial weights = 10^{-8}
- ν_{base} = 40 Hz, τ_w = 3600
- For $0 < t <= 5$ sec:
  - First 50 spike trains : 3 Hz
  - Remaining 50 spike trains: 7 Hz
- For $t > 5$ sec:
  - First 50 spike trains: 60 Hz
  - Remaining 50 spike trains: 30 Hz



Results

Can synaptic homeostatic mechanisms be used to maintain stable ongoing activity in recurrent circuits?

- 250 I&F neurons, 80% E, 20%I with dynamic synapses
- 20 Poisson IP spike trains spiking at 5 Hz for $t <= 3$ sec, and at 100 Hz for $t > 3$ sec



Conclusion

- A new synaptic plasticity mechanism is presented that enables a neuron to maintain stable firing rates
- At the same time the rule leaves the neuron free to show moment-to-moment fluctuations based on variations in its presynaptic inputs
- The rule is completely local
- Globally asymptotically stable
- Able to achieve firing rate homeostasis from single neuron to network level


THANK YOU

References

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Lyapunov Function

Let V be a continuously differentiable function from $\Re^n$ to $\Re$. If G is any subset of $\Re^n$, we say that V is a Lyapunov function on G for the system $\frac{d\mathbf{a}}{dt} = g(\mathbf{a})$ if :

$$\frac{dV(\mathbf{a})}{dt} = (\nabla V(\mathbf{a}))^T g(\mathbf{a})$$

does not changes sign on G.

More precisely, it is not required that the function V be positive-definite (just continuously differentiable). The only requirement is on the derivative of V, which can not change sign anywhere on the set G.

Global Invariant Set Theorem

Consider the autonomous system d**x**/dt = **f**(**x**), with **f** continuous, and let **V**(**x**) be a scalar function with continuous first partial derivatives. Assume that

1. **V**(**x**) → ∞ as ||**x**|| → ∞
2. **V'(x**) <= 0 over the whole state space

Let **R** be the set of all points where **V'(x**) = 0, and **M** be the largest invariant set in **R**. Then all solutions of the system globally asymptotically converge to **M** as t → ∞