Particle-Based Simulation of Granular Materials

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What is it all about?

What is it all about? (contd.)

Physical Modeling
(Sand Pile)

3D dynamic Motion
(Falling Sand)

Granular Materials

Granular Phenomena
(Splashing, Avalanches)

Issues

- Need to incorporate sufficient *spatial* and *physical* accuracy
- Simulate *collisions* and *friction* among the particles
- Simulate *interaction* with other large scale objects

Uniqueness of the Problem

- Different from fluids
  - No viscosity
  - Static friction
  - Contact forces
- Should reliably reproduce granular phenomena

Approach

- Based upon both theoretical and experimental results in physics
- Particles are represented as discrete elements
- Interparticle interactions are selected based on computational costs
- Interactions are governed by a ***molecular dynamics*** based contact model
- Contact model derived from elasticity theory and experimental results

Rigid Bodies

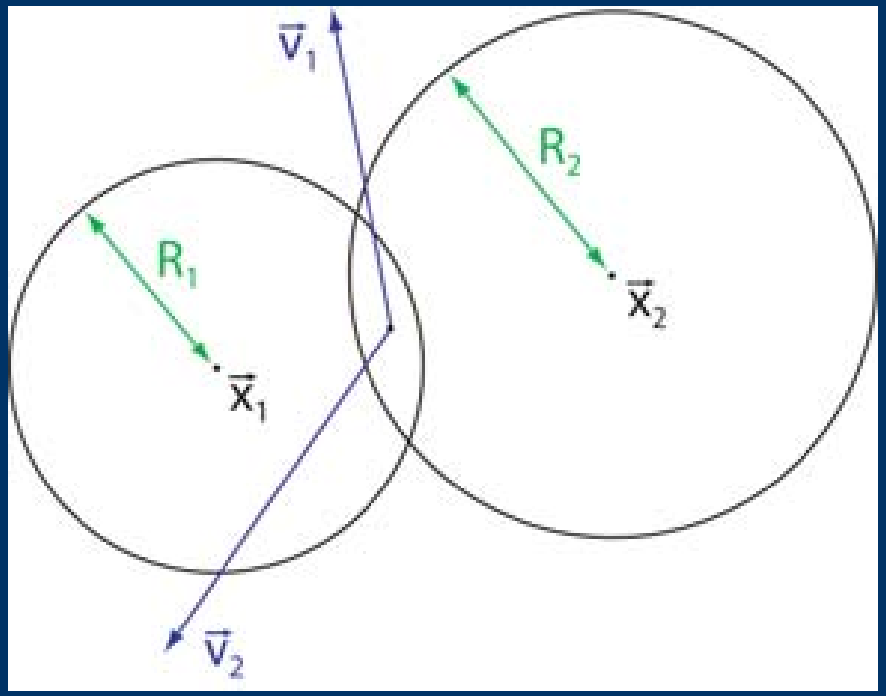
- Extend by covering their surface with particles
- Particles are placed at an offset from the original mesh
- Force and torque accumulated is integrated
- ***Two-way*** coupling is achieved

Advantage: interactions can be simulated using the same particle-based approach

Contact Forces

- Normal and sheer forces
- Obtained from ***relative velocity*** and ***overlap***

Contact Forces (contd.)



$$\bar{F} = \bar{F}_n + \bar{F}_t$$

$$\xi = max(0, R_1 + R_2 - \|\bar{x}_2 - \bar{x}_1\|)$$

$$\bar{N} = x_2 - x_1 / \|x_2 - x_1\|$$

$$\bar{V} = \bar{v}_1 - \bar{v}_2$$

$$\xi = \bar{V} \cdot \bar{N}$$

$$\bar{V}_t = \bar{V} - \xi \bar{N}$$

Normal Forces

$$\bar{F}_n = f_n \bar{N}$$

$$f_n + k_d \xi^{\alpha} \dot{\xi} + k_r \xi^{\beta} = 0$$

$$when\ \alpha = 0, \beta = 1 \qquad f_n + k_d \dot{\xi} + k_r \xi = 0$$

$\mathbf{k_r}$: elastic restoration coefficient

controls particle stiffness

$\mathbf{k_d}$: viscous damping coefficient

controls dissipation during collisions

Sheer Forces

Opposes the tangential velocity

$$\bar{F}_t = -k_t \bar{V}_t$$

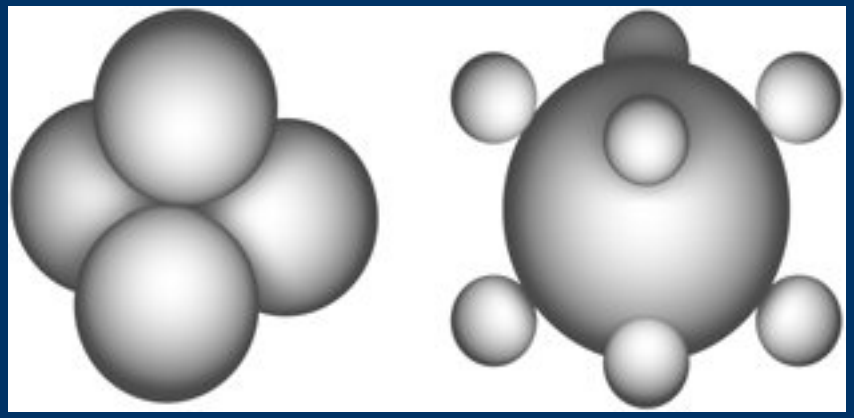
Including the friction coefficient **μ** and normal force $\mathbf{f_n}$

$$\bar{F}_t = -\mu f_n (\bar{V}_t / \|\bar{V}_t\|)$$

Simulation Details

The Model

- A large collection of granular particles
- External forces lead to ***relative motion*** and ***energy exchange*** through collisions
- Particles are non-spherical, a grain is a set of spheres constrained to move together

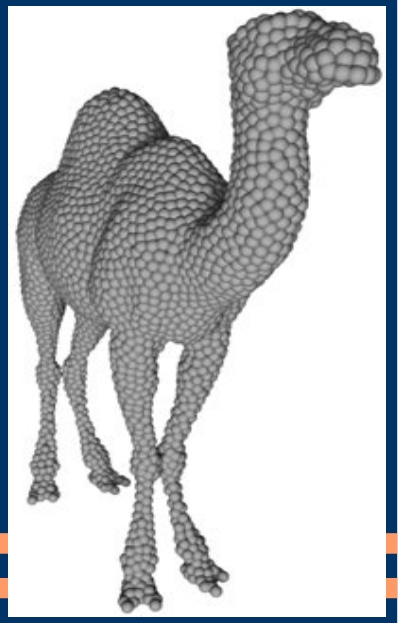


Contact Detection

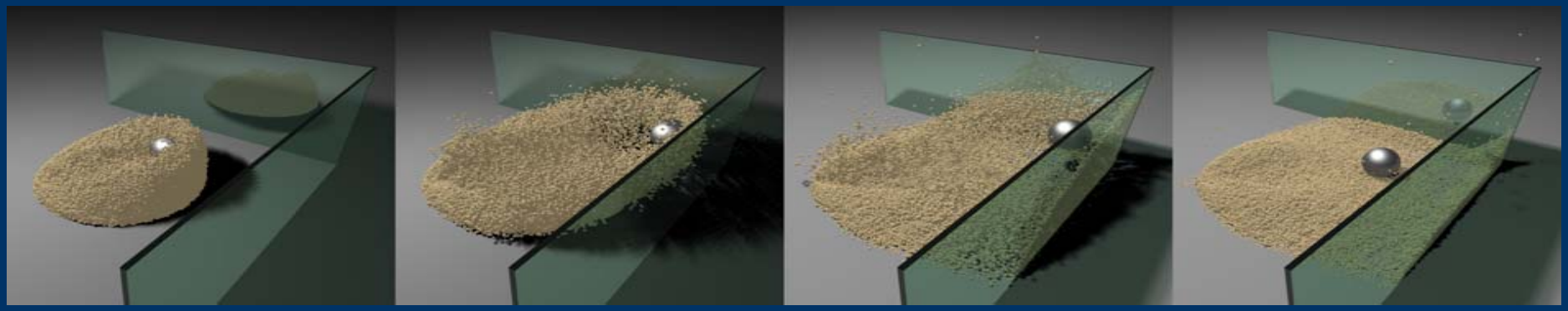
- Naive method will consider all pairs of particles $O(n^2)$
- Instead, assuming all particles are of similar size, search in a voxel, two times the maximum particle size
- The lookup reduced to $O(n)$
- Spatial hashing leads to more efficient contact detection

Interaction with Rigid Bodies

- Completely cover surface of the body with particles
- Creates a two-way coupling between the rigid body and the granular particles



Results: Steel ball and Sand Pile



- Represented by 45,494 tetrahedron-shaped particles
- Transitions from static to dynamic to static regimes

Results: Avalanche

- Demonstrates two-way coupling
- Structure composed of rigid bodies is destroyed

Results: Rings

Demonstrates varying degree of friction and dissipation

Results

Simulation	Round Particles	Frames	Min. / Frame
Hourglass	109,708	1600	3.18
1000 Rings	110,000	460	3.73 3.09
Splash	186,892	480	3.41
Avalanche	294,820	720	26.40
Bulldozer	310,149	300	17.40

Table 1: *Timing data collected on a set of 3Ghz class PCs.*

Future Work

- Include cohesive forces between the particles Ex: moist sand
- Coupling between granular materials and fluids
- Generate textures to hide the underlying granularity

Questions?