Dynamic Pricing and Inventory Management under Fluctuating Procurement Costs

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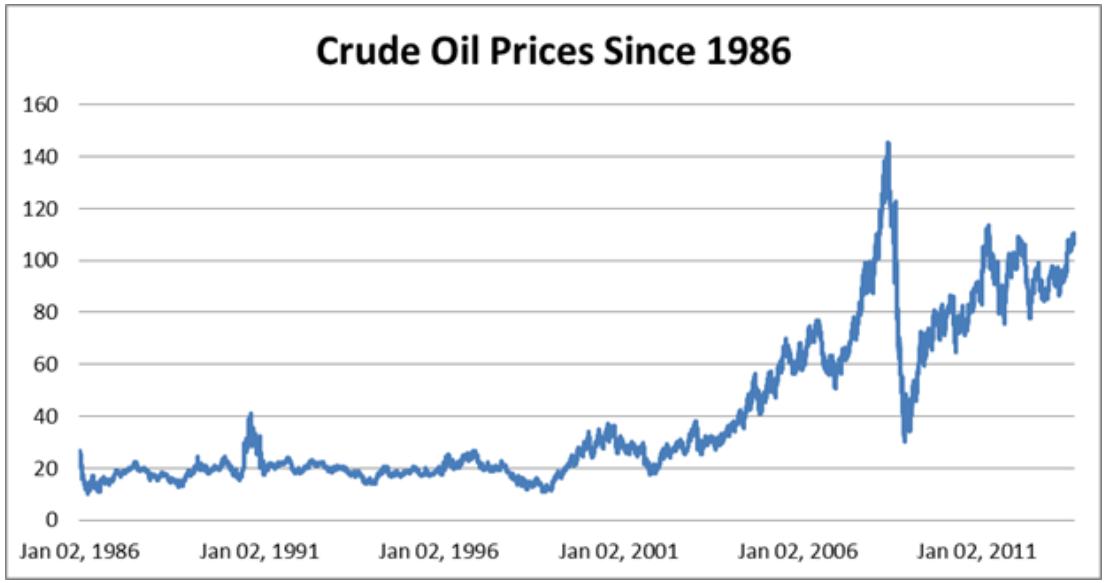
October 9, 2013

Motivation

Current global market: Prices of many commodities are now fluctuating as much in a single day as they did in a year in the early 1990s (Wiggins and Blas 2008).



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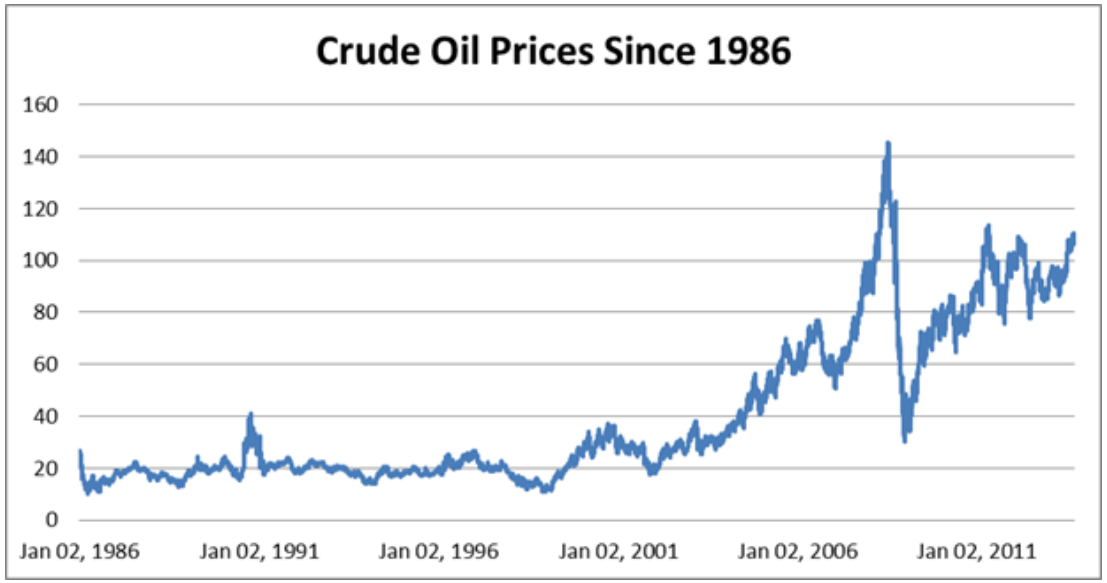

Goal of our paper: To study the optimal joint pricing and inventory policy of a firm under fluctuating procurement costs.

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  2. Inventory becomes a risky asset to invest on.
- Dynamic Pricing
  1. Share the cost risk with customers.
  2. Control demand to consume its inventory in a most profitable way.
- Supply Diversification
  1. Procure inventory from a portfolio of complementary sources.
  2. Enjoy both the responsiveness and risk reduction.

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4. How significant is the value of dynamic pricing and dual-sourcing? Are they strategic complements or substitutes?

5. What is the risk attitude of a firm towards the procurement cost fluctuation?

Outline

- Contributions and Related Literature
- Model formulation
- Analytical and Numerical results
- Conclusion: Managerial Insights

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3. We show that the value of dynamic pricing and dual-sourcing under cost volatility is significant and that they are strategic complements.

4. We find the risk-seeking attitude of a risk-neutral firm under procurement cost fluctuation.

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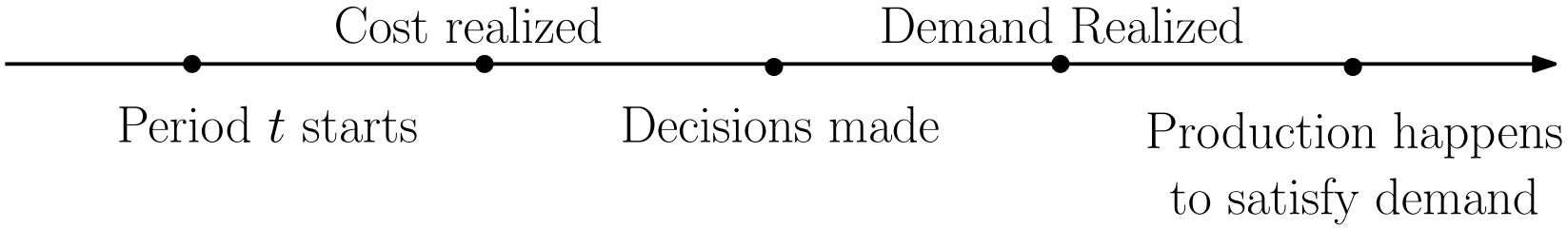
  - Federgruen and Heching (1999, 2002).
- Our paper: Joint pricing&inventory management under demand uncertainty and cost fluctuation.

Model Formulation: Basics

- A risk-neutral firm procures raw material inventory from a spot market, produces an end-product in an MTO fashion, and prices the product in a final market.

- T periods in total, labeled backwards, full backorder.

- Sequence of events in period t:



Model Formulation: Demand

$$D_t = d(p_t) + \epsilon_t.$$

- ϵ_t: $i.i.d.$ continuous random variables, with $\mathbb{E}\{\epsilon_t\} = 0$.
- $d(\cdot)$: strictly decreasing function of p_t, the sales price of the product, with strictly decreasing inverse $p(\cdot)$ in the expected demand, d_t.

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Assumption 1

$R(d_t) := p(d_t)d_t$ is continuously differentiable and strictly concave.

Remark: The concavity of $R(\cdot)$ implies the decreasing marginal revenue with respect to demand.

Model Formulation: Cost Process

$$c_{t-1} = s_t(c_t, \xi_t).$$

- $s_t(\cdot, \cdot) > 0$ *a.s.*, increasing in c_t for any given ξ_t.
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- $\mu_t(c_t) := \mathbb{E}\{s_t(c_t, \xi_t)\} < +\infty$ is increasing in c_t.

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- Model Primitives
  - $\alpha = \frac{1}{1+r_f}$ = discount factor, where r_f is the risk-free interest rate.
  - b = backlogging cost per unit backlogged at the end of a period.
  - h = holding cost per unit stocked at the end of a period.

Sole-sourcing Model

- The firm procures from a spot market only and cannot resell the excess inventory to the spot market.

$$V_t(I_t|c_t) = \text{the maximal expected discounted profits in periods } t, t-1, \cdots 1 \text{ with starting inventory } I_t \text{ and cost } c_t \text{ in period } t.$$

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- Terminal condition: $V_0(I_0|c_0) = 0$, with $c_0 = s_1(c_1, \xi_1)$. i,e., excess inventory is discarded without any salvage value in the last period.

Bellman Equation

$V_t(I_t|c_t)$ satisfies the following Bellman equation:

$$V_t(I_t|c_t) = c_t I_t + \max_{x_t \geq I_t, d_t \in [\underline{d}, \bar{d}]} J_t(x_t, d_t|c_t), \text{ where}$$

$$\begin{aligned} J_t(x_t, d_t|c_t) = &- c_t I_t + \mathbb{E}\{p(d_t)D_t - c_t(x_t - I_t) - h(x_t - D_t)^+ - b(x_t - D_t)^- \\ &+ \alpha V_{t-1}(x_t - D_t|s_t(c_t, \xi_t))\} \\ =&R(d_t|c_t) + (b - c_t + \alpha\mu_t(c_t))x_t + G_t(x_t - d_t|c_t), \end{aligned}$$

with $R(d_t|c_t) := (p(d_t) - b - \alpha\mu_t(c_t))d_t$ and

$$G_t(y|c_t) := \mathbb{E}\{-(h+b)(y-\epsilon_t)^+ + \alpha[V_{t-1}(y-\epsilon_t|s_t(c_t, \xi_t)) - s_t(c_t, \xi_t)(y-\epsilon_t)]\}.$$

Dual-sourcing Model

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with $L(y) := \mathbb{E}[-(b+h)(y - \epsilon_t)^+]$ and
$H_t^d(y|c_t) := \alpha\mathbb{E}[V_{t-1}^d(y - \epsilon_t|s_t(c_t, \xi_t)) - s_t(c_t, \xi_t)(y - \epsilon_t)]$.

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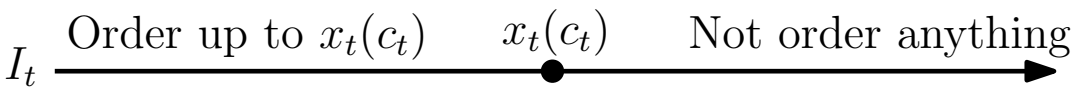

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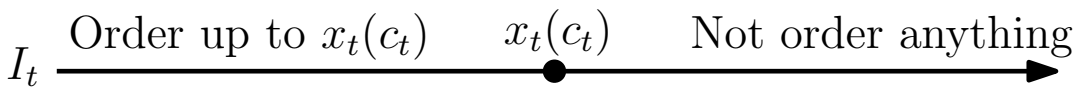
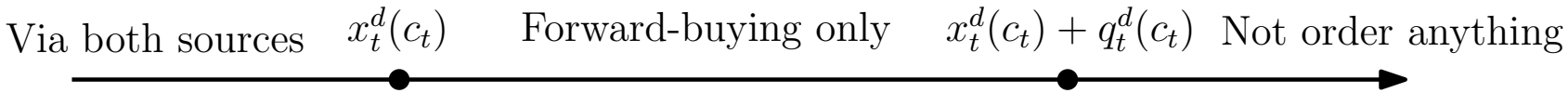


2. Cost-dependent order-up-to/pre-order-up-to list-price policy. (Dual-sourcing, without inventory reselling)



Optimal Policy Structure with Inventory Reselling

With inventory reselling:

1. Starting inventory does not influence the optimal policy.

2. Dual-sourcing model is reduced to the sole-sourcing model for a risk-neutral firm.

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2. The firm may respond to a higher spot market price by increasing or decreasing its safety-stock, order-up-to level and pre-order quantity (it's a tradeoff between current cost saving and future speculation opportunity).

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2. The firm should increase its safety-stock, order-up-to level and pre-order quantity with a higher future cost trend, i.e., the firm bets more on potentially higher future costs.

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2. Dual-sourcing motivates the firm to decrease its order-up-to level and sales price.

3. Dynamic pricing and dual-sourcing are strategic complements, i.e., the application of one strategy increases the value of the other.

Risk-seeking Attitude of the Risk-neutral Firm

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- If the future procurement cost is concavely increasing in the current cost (e.g., Geometric Brownian Motion and mean-reverting process), the firm will earn more profit under a more volatile procurement cost process, i.e., the risk-neural firm will exhibit a risk-seeking attitude towards the cost fluctuation risk.

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4. Procurement cost fluctuation creates more opportunities than risks for a risk-neutral profit-maximizing firm.
5. The feedback mechanism partially contributes to the wild fluctuation of commodity prices in recent years.

Q&A

Thank you!

Questions?