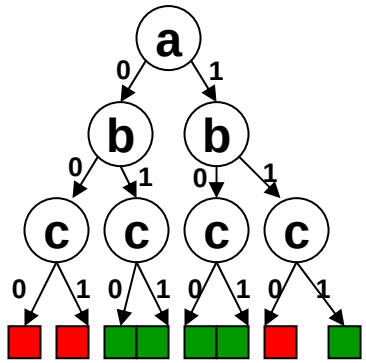
Satisfiability of Propositional Formulas

Mooly Sagiv

Based on a presentation by Sharad Malik & Ohad Shacham

The SAT Problem

- Given a propositional formula (Boolean function)
  - φ **= (a** $\vee$ **b)** $\wedge$**(**$\neg$ **a**$\vee$ $\neg$**b** $\vee$ **c)**
- Determine if φ is valid
- Determine if φ is satisfiable
  - Find a satisfying assignment or report that such does not exit
- For *n* variables, there are 2^n possible truth assignments to be checked



Why Bother?

- Core computational engine for major applications
  - Artificial Intelligence
    - Knowledge base deduction
    - Automatic theorem proving
  - Electronic Design Automaton
    - Testing and Verification
    - Logic synthesis
    - FPGA routing
    - Path delay analysis
    - And more…

Problem Representation

- Represent the formulas in Conjunctive Normal Form (CNF)
- Conversion to CNF is straightforward
  - **$a \vee (b \wedge \neg(c \vee \neg\, d)) \equiv (a \vee (b \wedge \neg c \wedge \neg\neg\, d)) \equiv$**
    $(a \vee (b \wedge \neg c \wedge d)) \equiv (a \vee b) \wedge (a \vee \neg c) \wedge (a \vee d)$
  - **May need to add variables**
- Notations
  - Literals
    - Variable or its negation
  - Clauses
    - Disjunction of literals
  - φ **$= (a \vee b) \wedge (\neg\, a \vee \neg b \vee c) \equiv (a + b)(a' + b' + c)$**
- Advantages of CNF
  - Simple data structure
  - All the clauses need to be satisfied

Complexity Results

- First established NP-Complete problem
  - Even when at most 3 literals per clause (3-SAT)

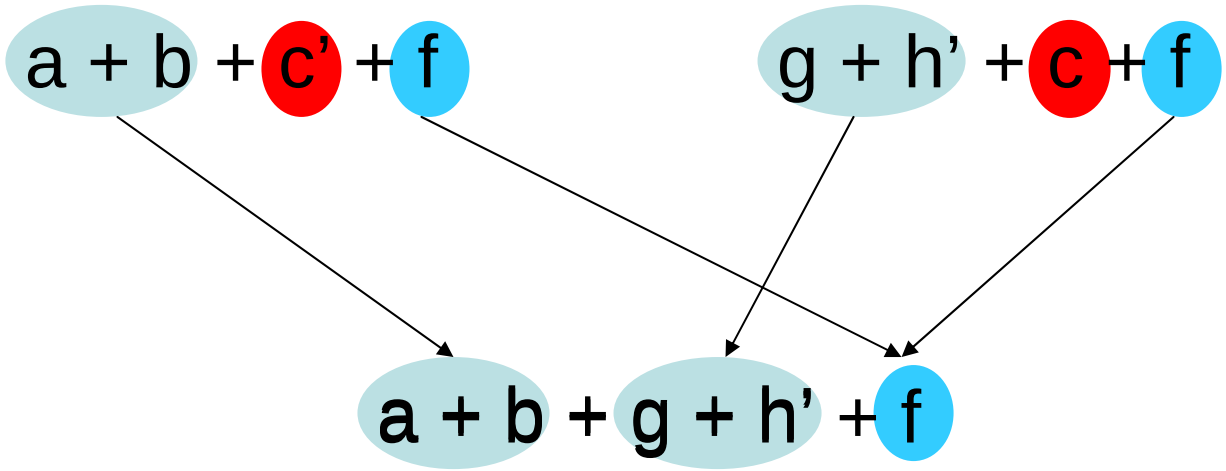
  S. A. Cook, The complexity of theorem proving procedures, *Proceedings, Third Annual ACM Symp. on the Theory of Computing*,1971, 151-158
  - No polynomial algorithm for all instances unless P = NP
- Becomes polynomial when
  - At most two literals per clause (2-SAT)
  - At most one positive literal in every clause (Horn)

Goals

- Develop algorithms which solve all SAT instances
- Exponential worst case complexity
- But works well on many instances
  - Interesting Heuristics
  - Annual SAT conferences
  - SAT competitions
    - Randomly, Handmade, Industrial, AI
  - 10 Millions variables!

Resolution

- Resolution of a pair of clauses with exactly ONE incompatible variable

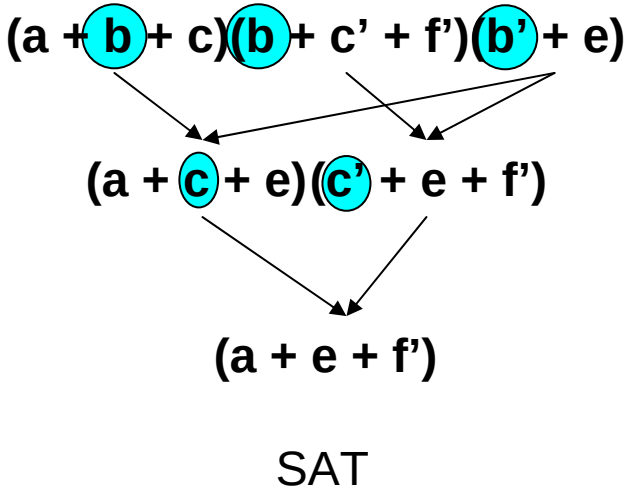


- What if more than one incompatible variables?

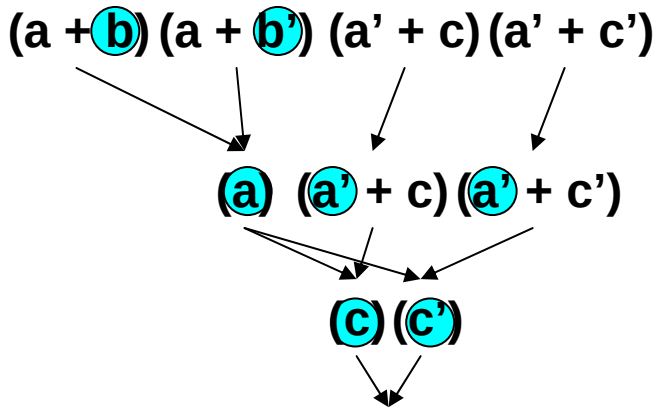
Davis Putnam Algorithm

M .Davis, H. Putnam, “A computing procedure for quantification theory", *J. of ACM*, Vol. 7, pp. 201-214, 1960

- Iteratively select a variable for resolution till no more variables are left
- Report UNSAT when the empty clause occurs
- Can discard resolved clauses after each iteration



SAT



UNSAT

Potential memory explosion problem!

Can we avoid using exponential space?

DLL Algorithm

- Davis, Logemann and Loveland

  M. Davis, G. Logemann and D. Loveland, “A Machine Program for Theorem-Proving", *Communications of ACM*, Vol. 5, No. 7, pp. 394-397, 1962

- Basic framework for many modern SAT solvers
- Also known as DPLL for historical reasons

Basic DLL Procedure - DFS

(a’ + b + c)
(a + c + d)
(a + c + d’)
(a + c’ + d)
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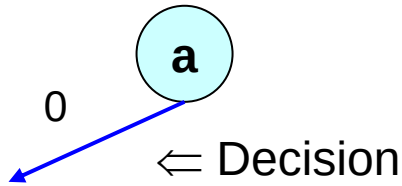

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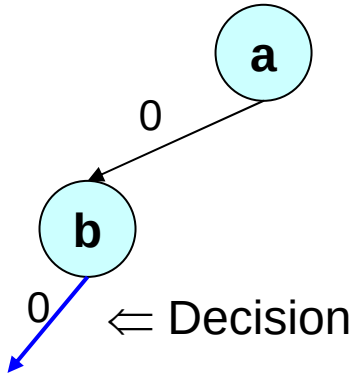

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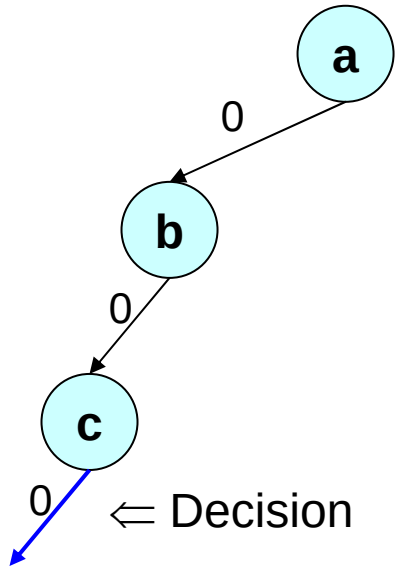

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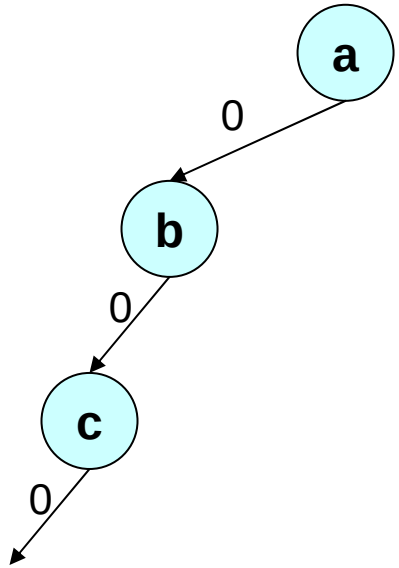
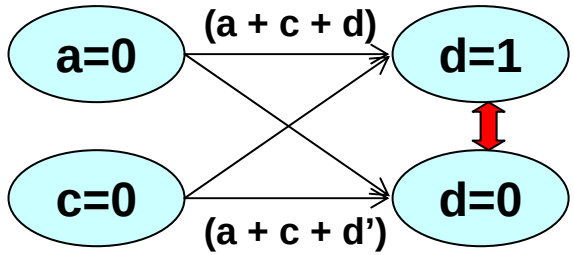


Implication Graph



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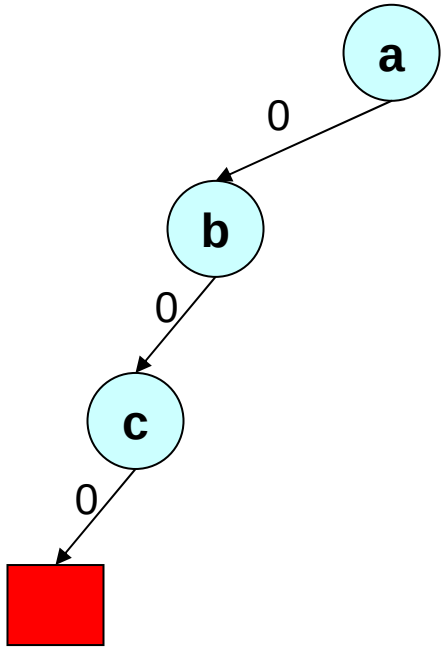

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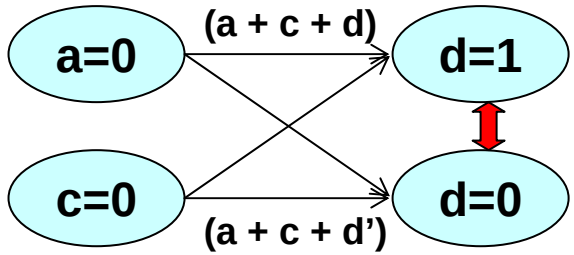

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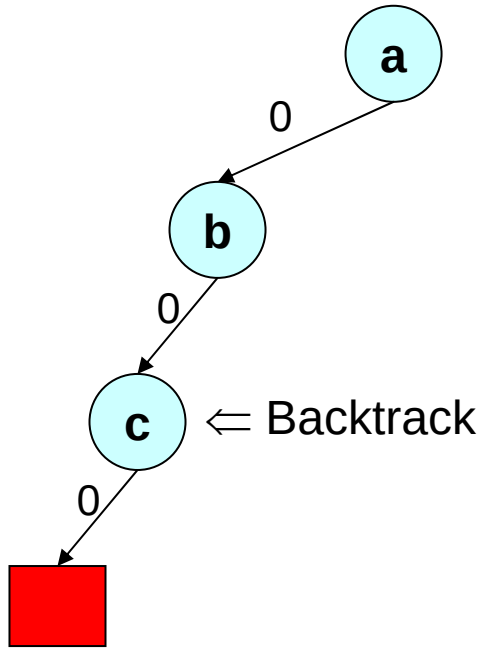

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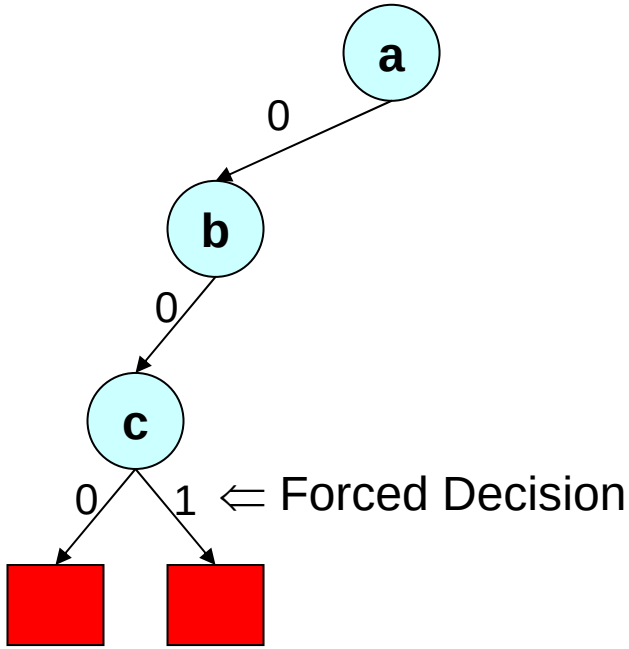

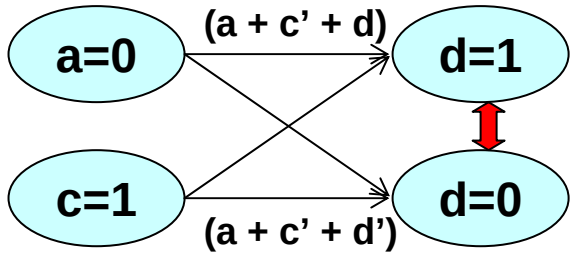

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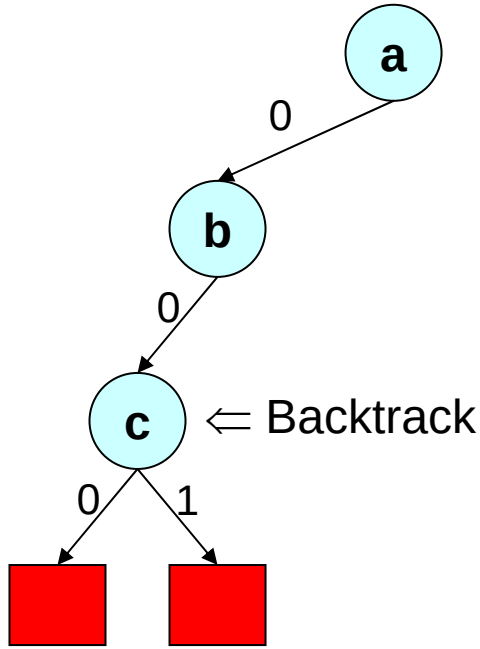

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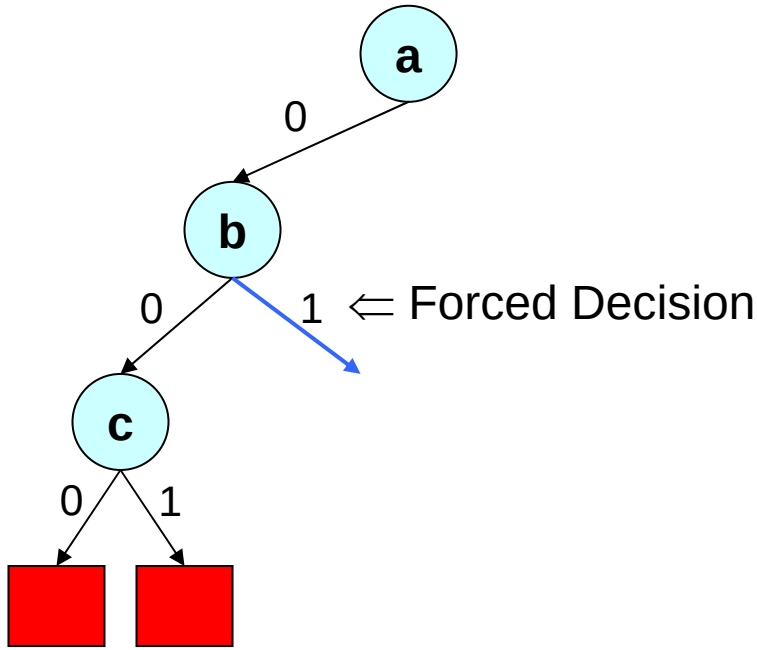

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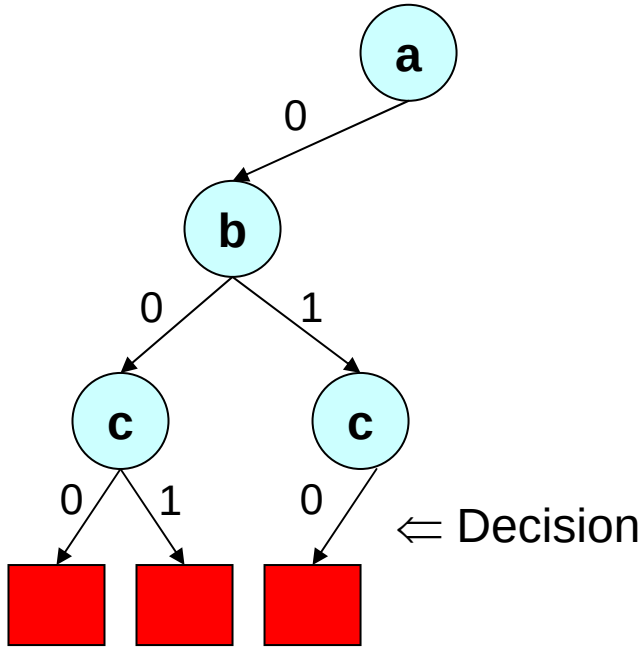

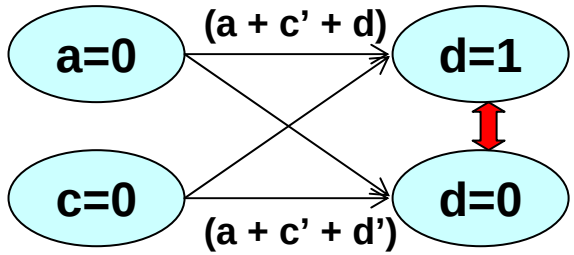

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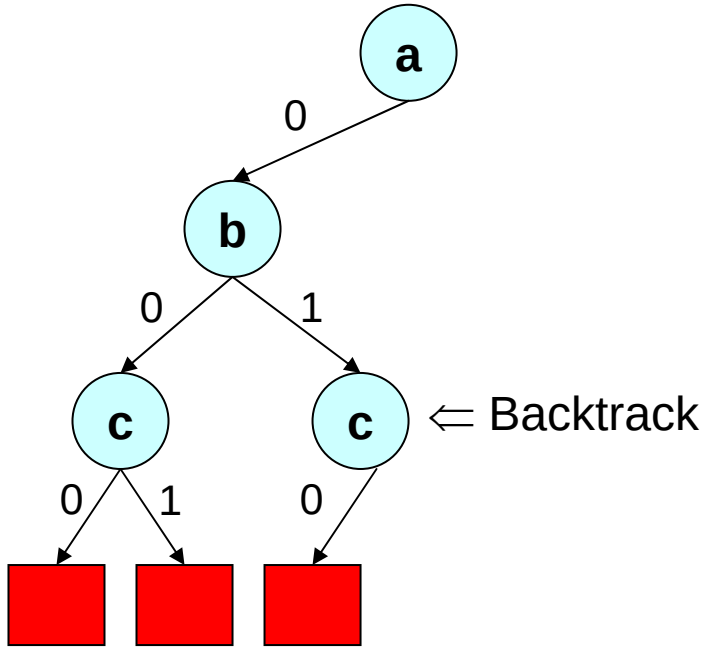

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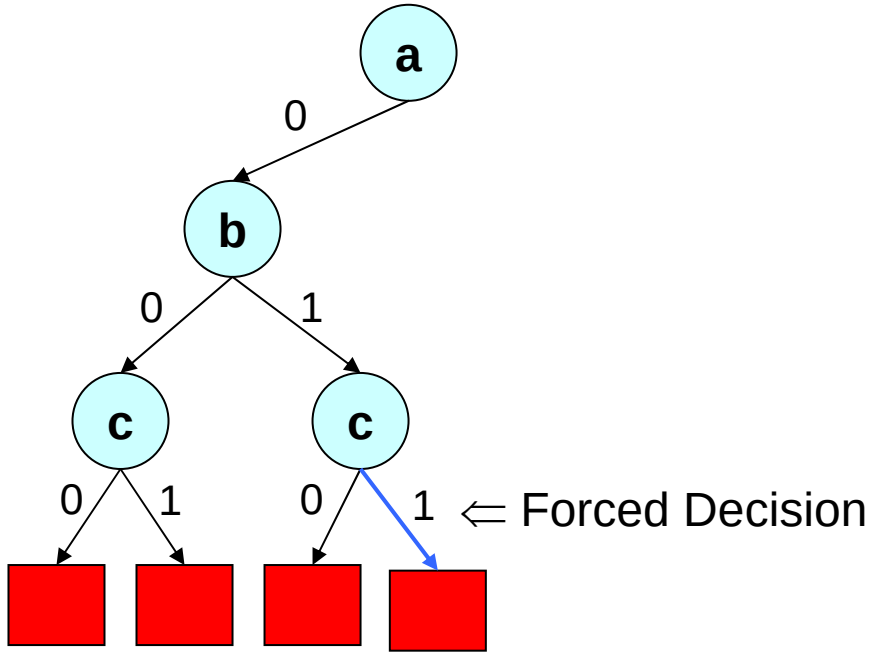

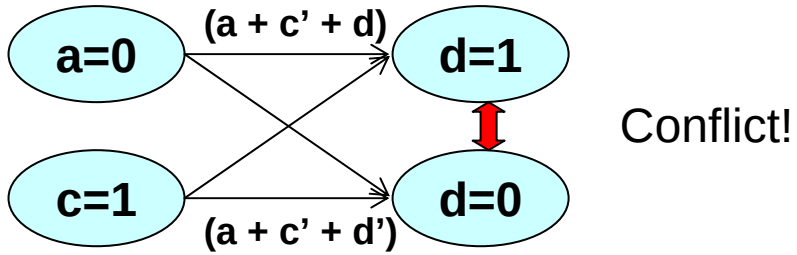

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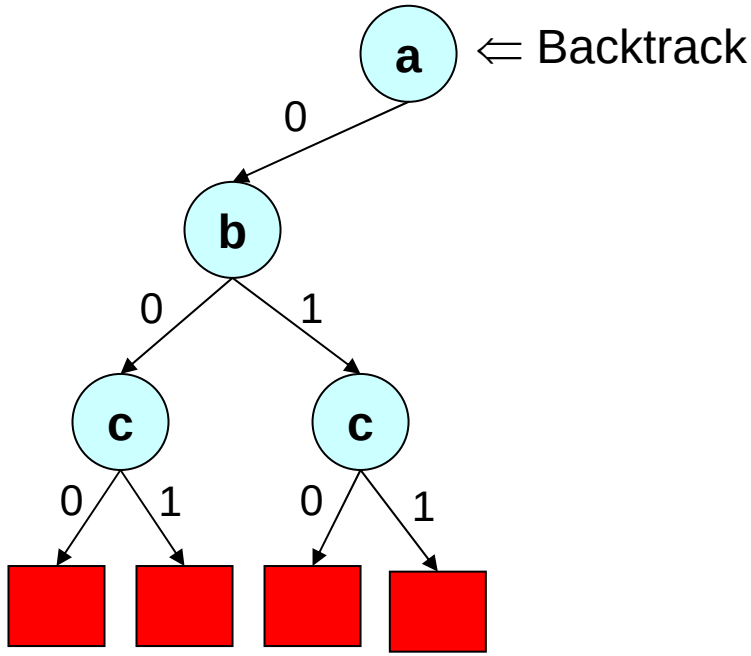

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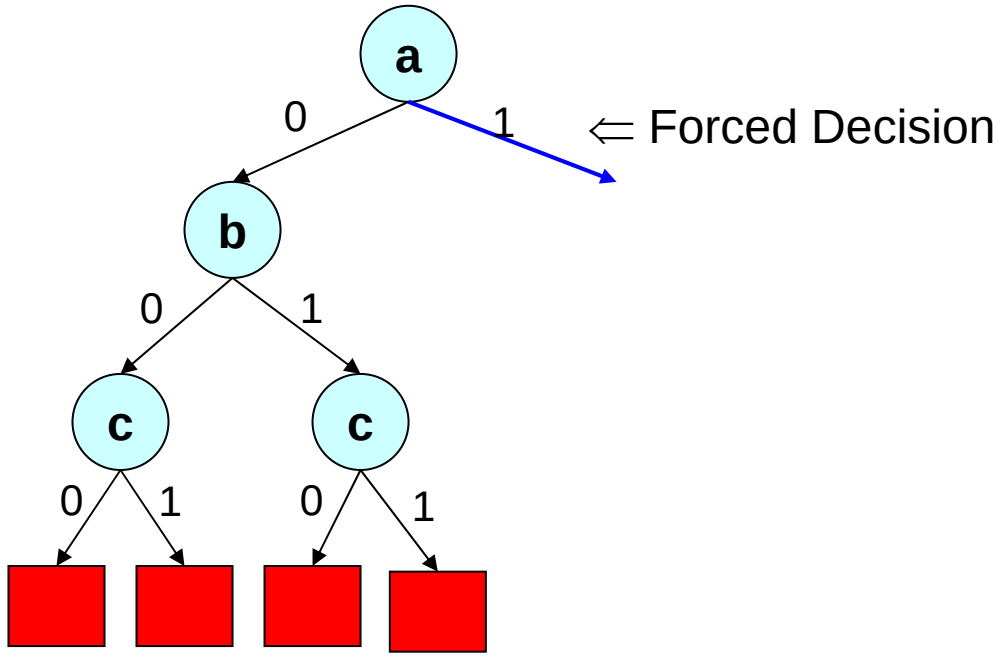

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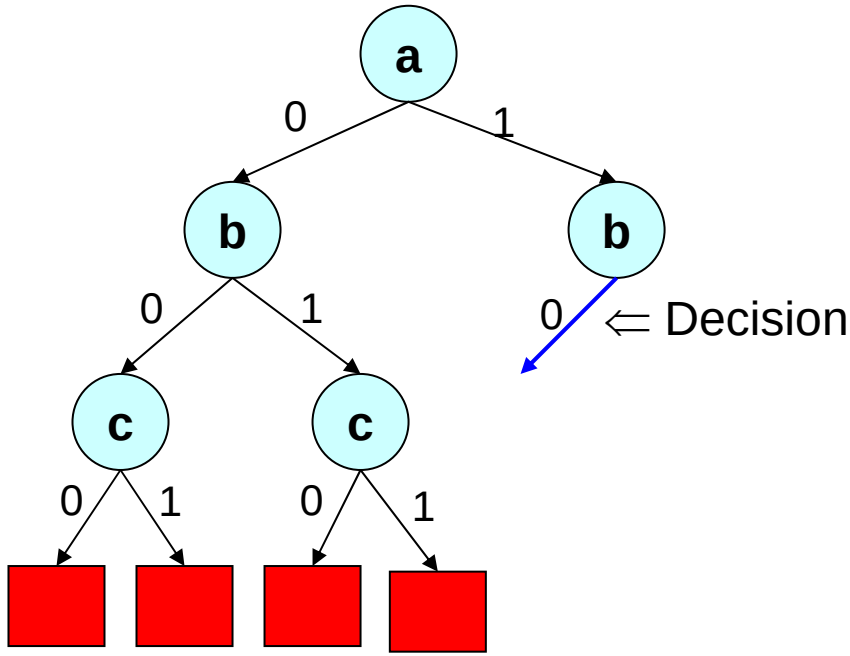

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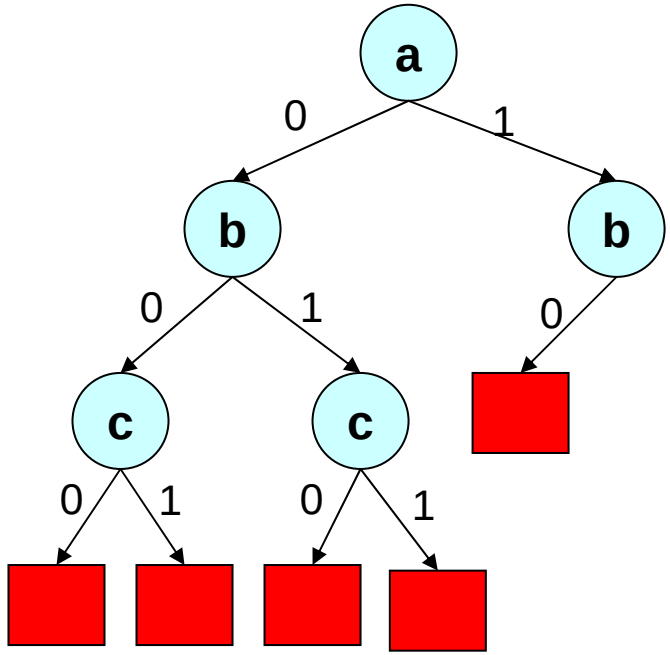

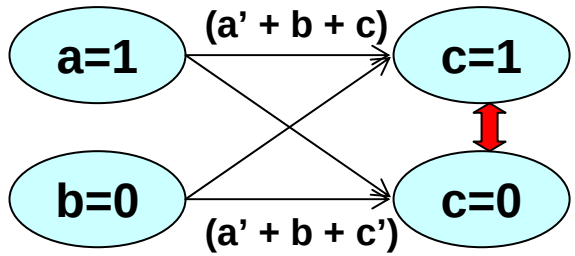

Conflict!

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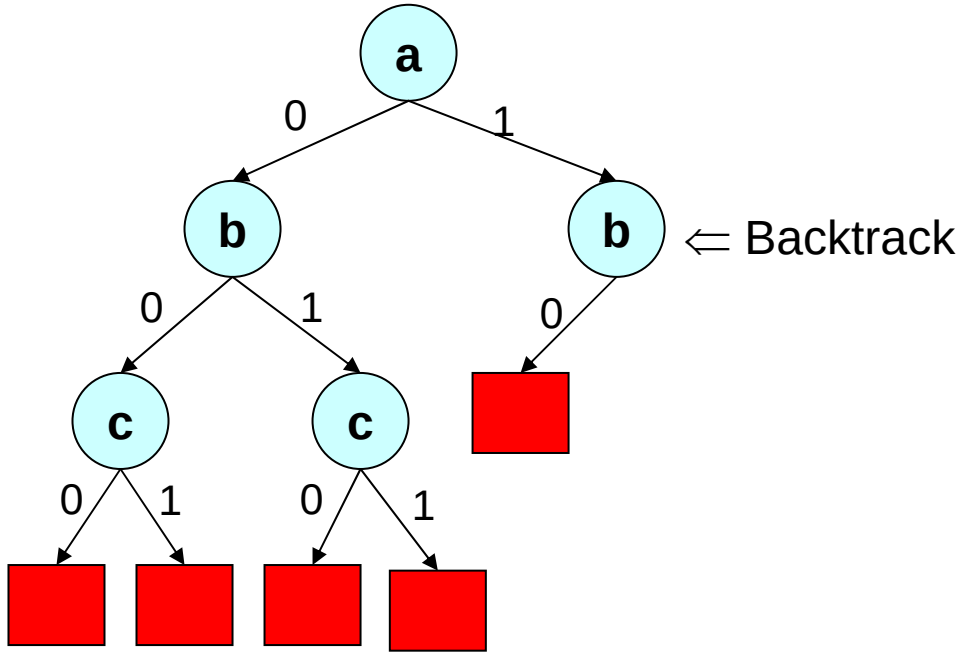

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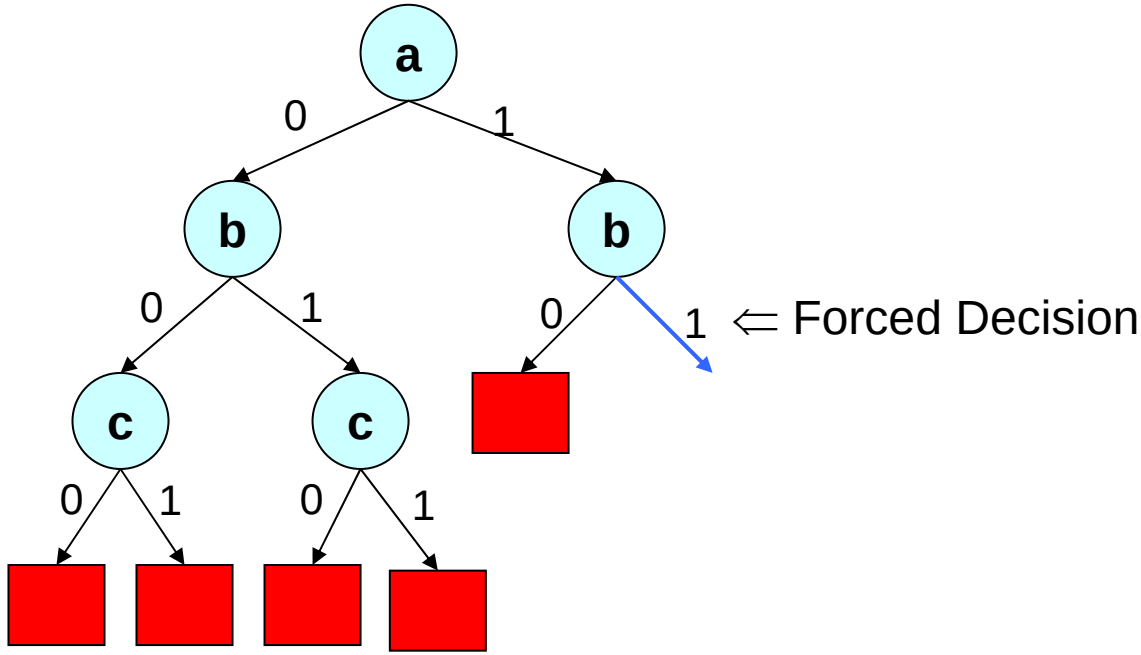

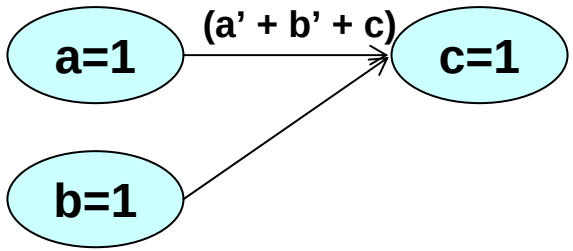

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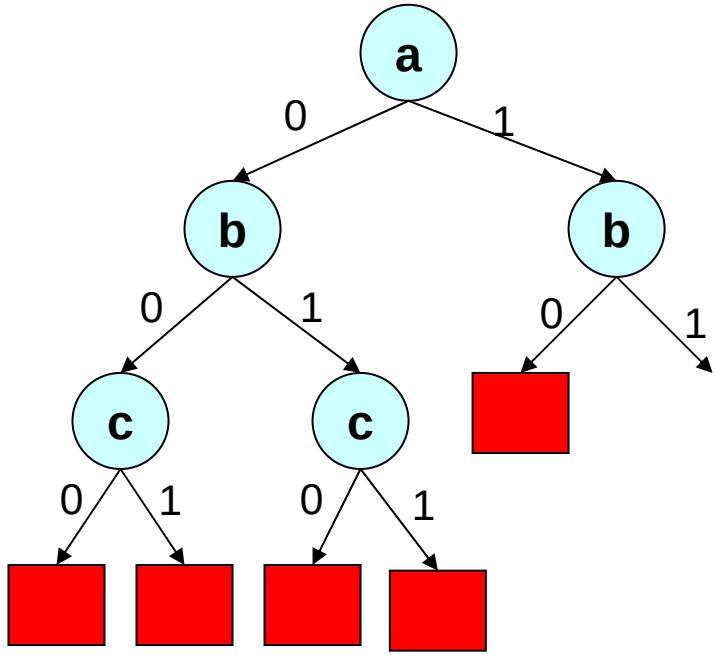

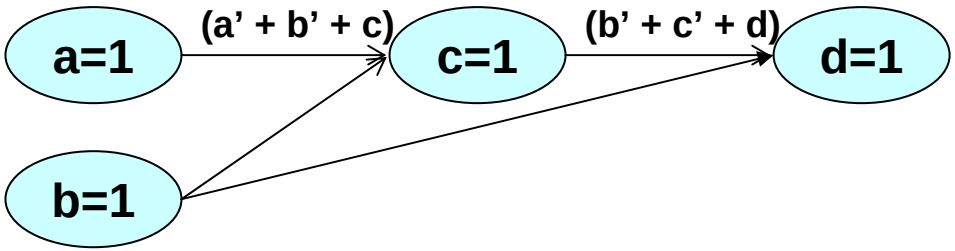

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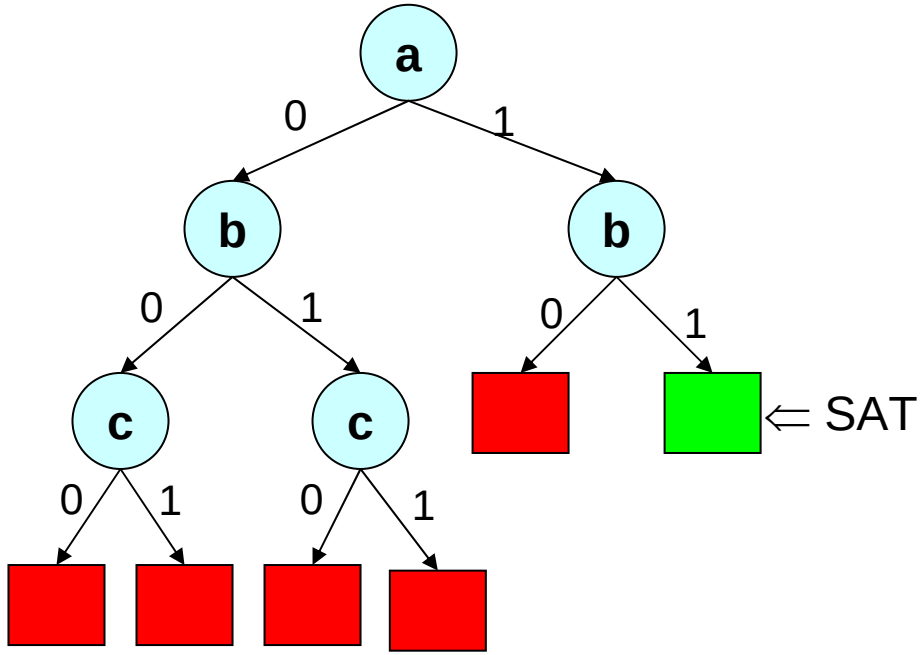
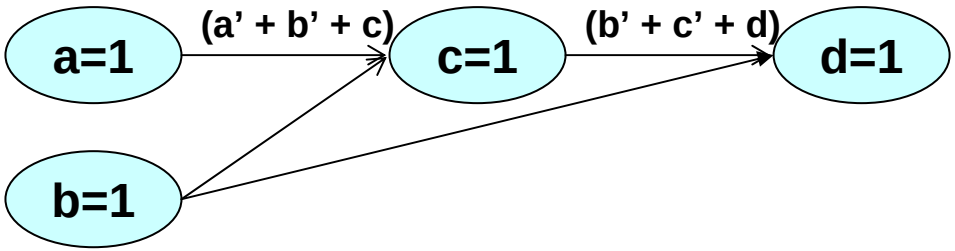




Features of DLL

- Eliminates the exponential memory requirements of DP
- Exponential time is still a problem
- Limited practical applicability – largest use seen in automatic theorem proving
- Very limited size of problems are allowed
  - 32K word memory
  - Problem size limited by total size of clauses (1300 clauses)

Implications and Boolean Constraint Propagation

- Implication
  - A variable is forced to be assigned to be True or False based on previous assignments
- Unit clause rule (rule for elimination of one literal clauses)
  - An unsatisfied clause is a unit clause if it has exactly one unassigned literal

Satisfied Literal

Unsatisfied Literal

Unassigned Literal

$$(a + b' + c)(b + c')(a' + c')$$

a = T, b = T, c is unassigned

  - The unassigned literal is implied because of the unit clause
- Boolean Constraint Propagation (BCP)
  - Iteratively apply the unit clause rule until there is no unit clause available.
- Workhorse of DLL based algorithms

GRASP

- Marques-Silva and Sakallah [SS96,SS99]

  J. P. Marques-Silva and Karem A. Sakallah, “GRASP: A Search Algorithm for Propositional Satisfiability”, *IEEE Trans. Computers*, C-48, 5:506-521, 1999.

- Incorporates conflict driven learning and non-chronological backtracking
- Practical SAT instances can be solved in reasonable time
- Bayardo and Schrag’s RelSAT also proposed conflict driven learning [BS97]

  R. J. Bayardo Jr. and R. C. Schrag “Using CSP look-back techniques to solve real world SAT instances.” *Proc. AAAI*, pp. 203-208, 1997

Conflict Driven Learning and Non-chronological Backtracking

x1 + x4

x1 + x3’ + x8’

x1 + x8 + x12

x2 + x11

x7’ + x3’ + x9

x7’ + x8 + x9’

x7 + x8 + x10’

x7 + x10 + x12’

Conflict Driven Learning and Non-chronological Backtracking

x1 + x4

x1 + x3’ + x8’

x1 + x8 + x12

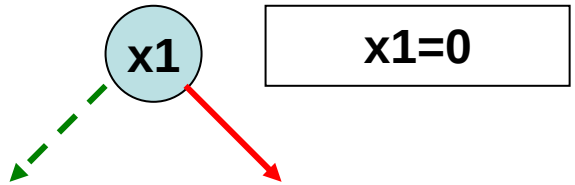
x2 + x11

x7’ + x3’ + x9

x7’ + x8 + x9’

x7 + x8 + x10’

x7 + x10 + x12’



x1=0

Conflict Driven Learning and Non-chronological Backtracking

x1 + x4

x1 + x3’ + x8’

x1 + x8 + x12

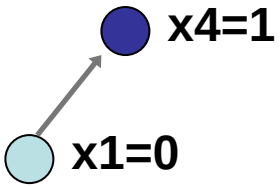
x2 + x11

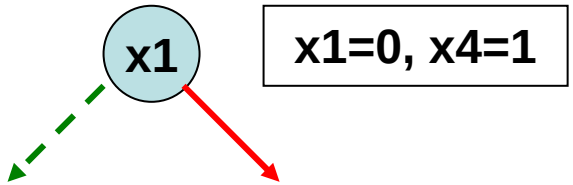
x7’ + x3’ + x9

x7’ + x8 + x9’

x7 + x8 + x10’

x7 + x10 + x12’





Conflict Driven Learning and Non-chronological Backtracking

x1 + x4

x1 + x3' + x8'

x1 + x8 + x12

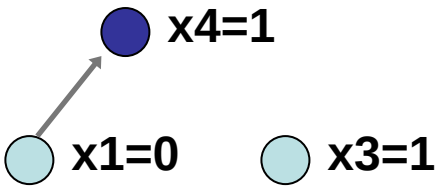
x2 + x11

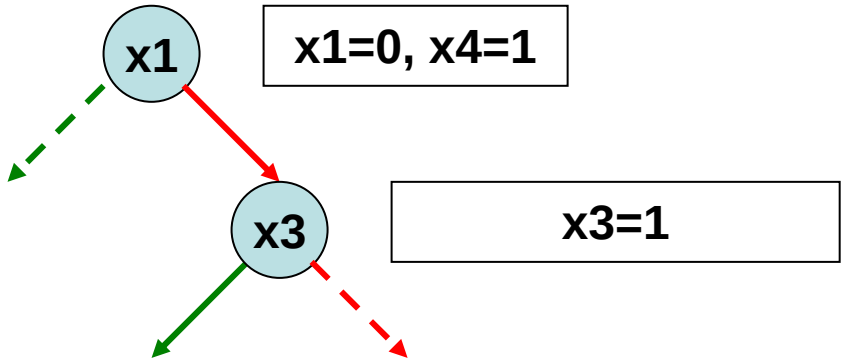
x7' + x3' + x9

x7' + x8 + x9'

x7 + x8 + x10'

x7 + x10 + x12'





Conflict Driven Learning and Non-chronological Backtracking

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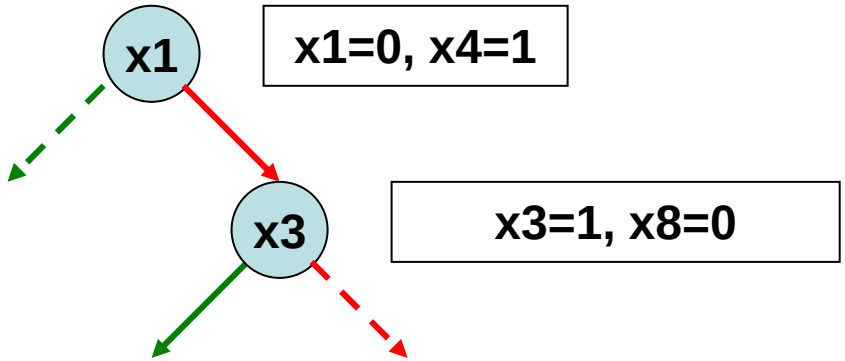

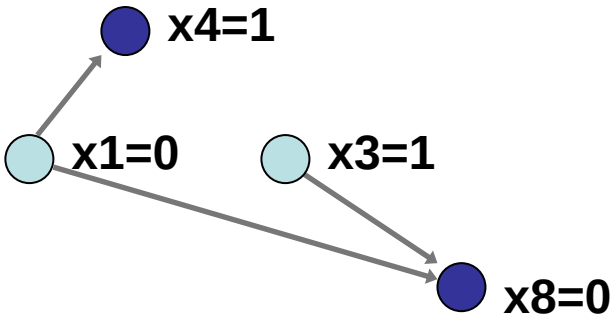

Conflict Driven Learning and Non-chronological Backtracking

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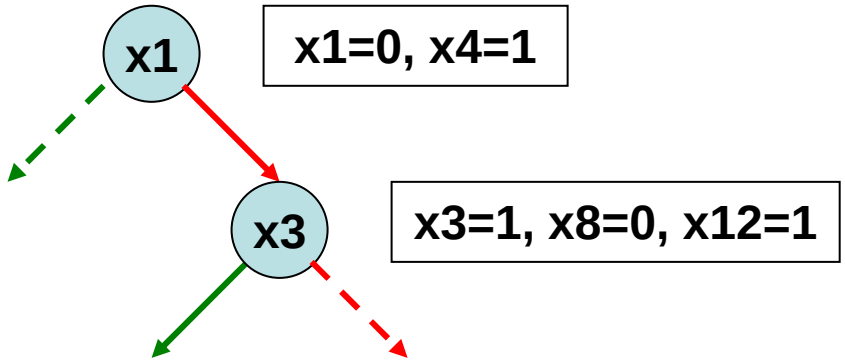

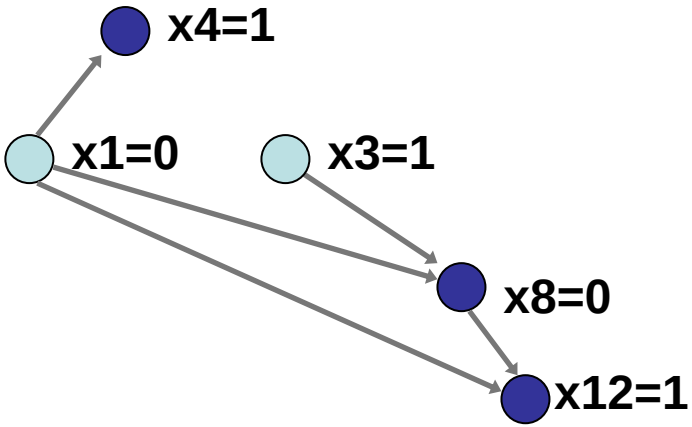

Conflict Driven Learning and Non-chronological Backtracking

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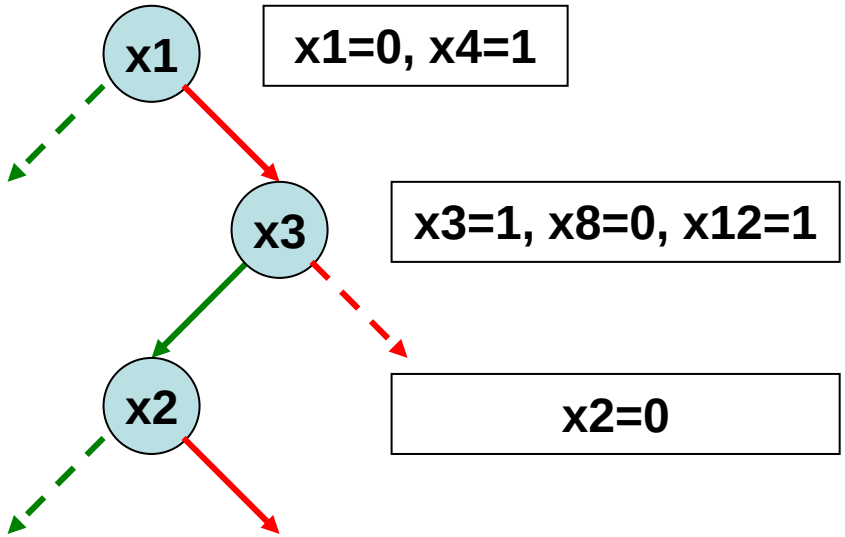
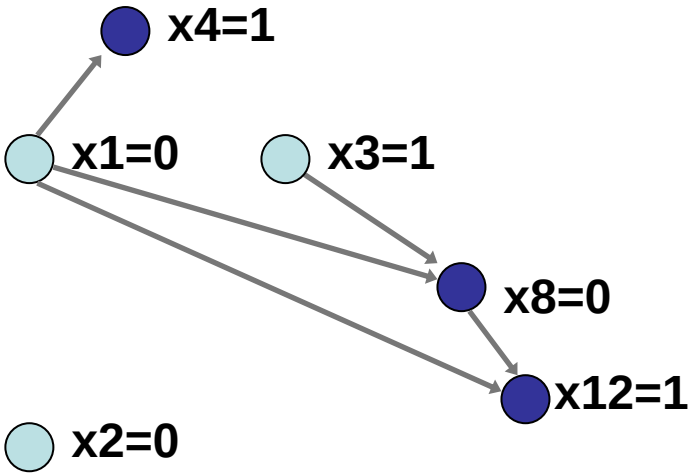
x7 + x10 + x12'





Conflict Driven Learning and Non-chronological Backtracking

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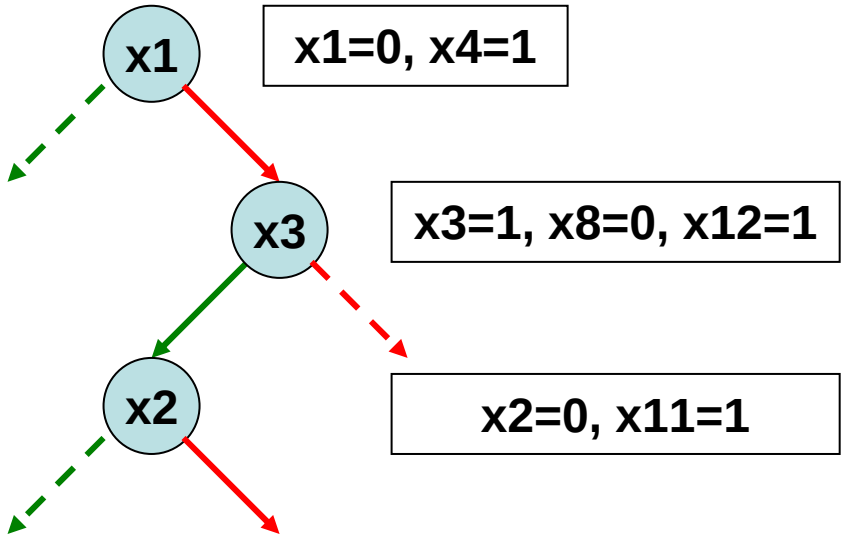

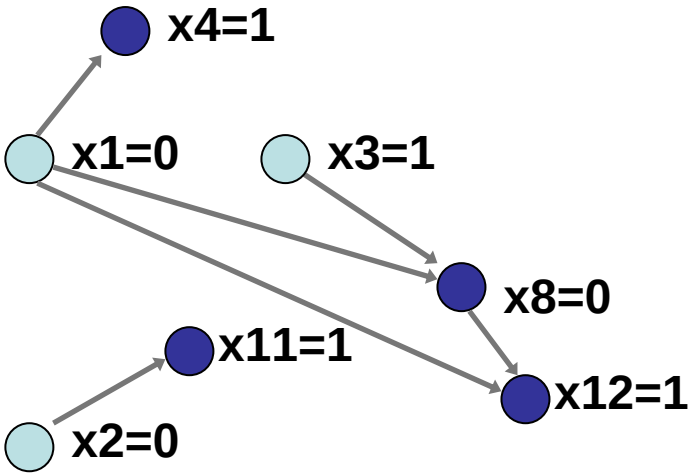

Conflict Driven Learning and Non-chronological Backtracking

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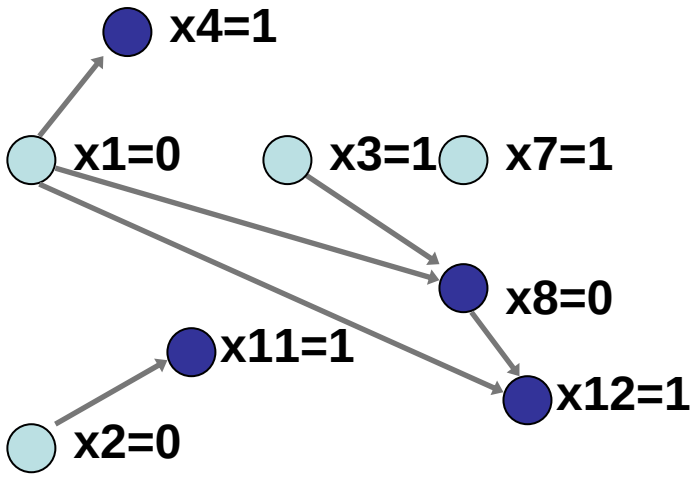
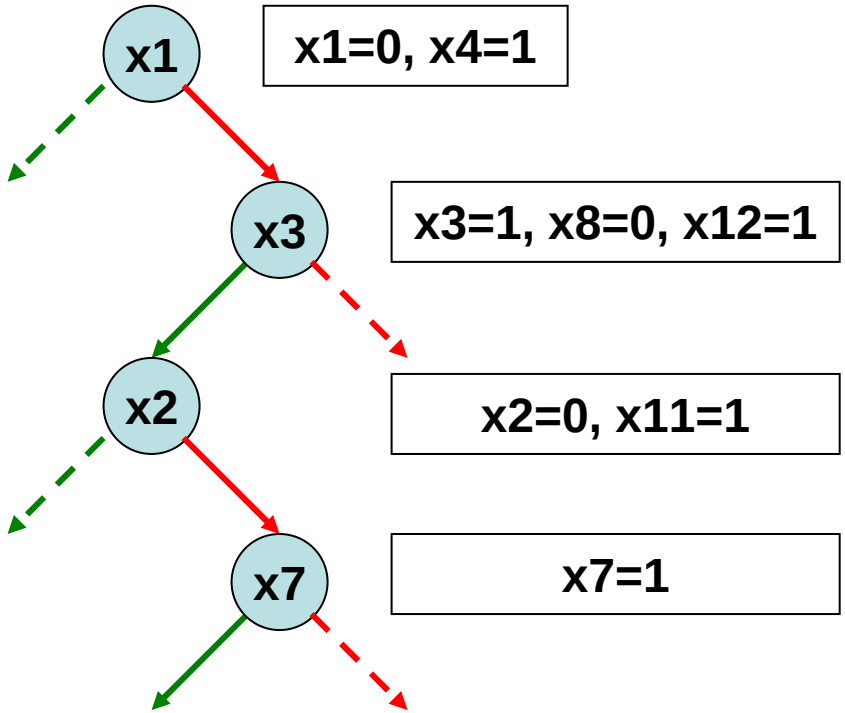
x7' + x8 + x9'

x7 + x8 + x10'

x7 + x10 + x12'





Conflict Driven Learning and Non-chronological Backtracking

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x1 + x8 + x12

x2 + x11

x7' + x3' + x9

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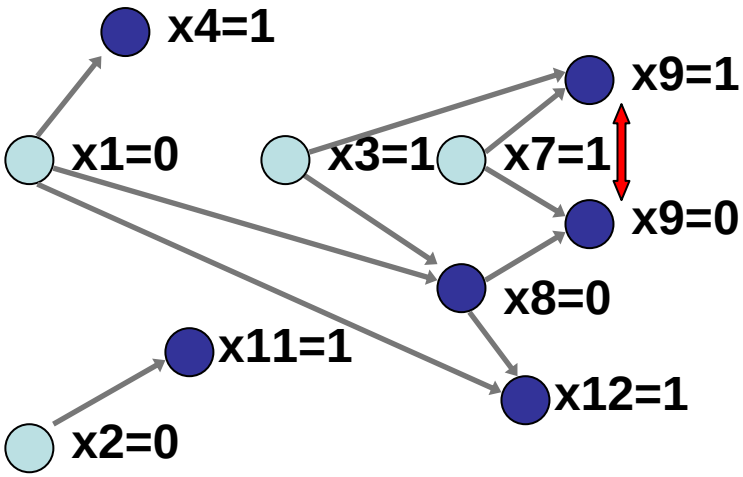
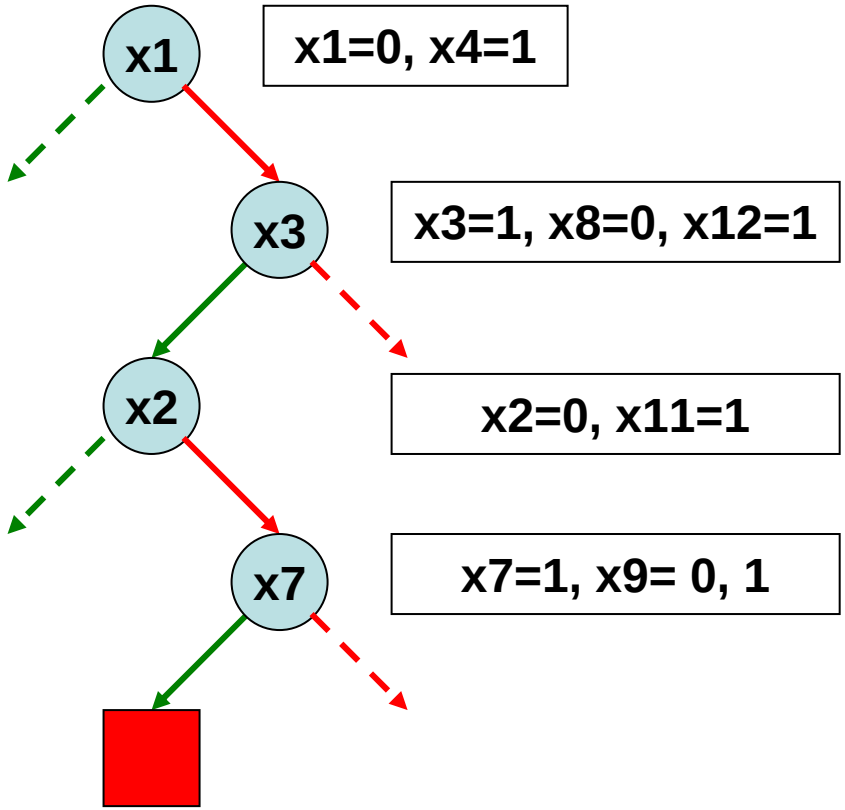




Conflict Driven Learning and Non-chronological Backtracking

x1 + x4

x1 + x3' + x8'

x1 + x8 + x12

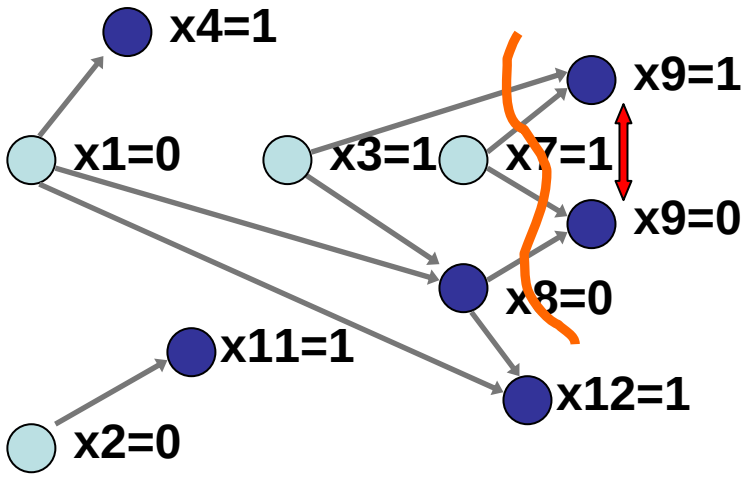
x2 + x11

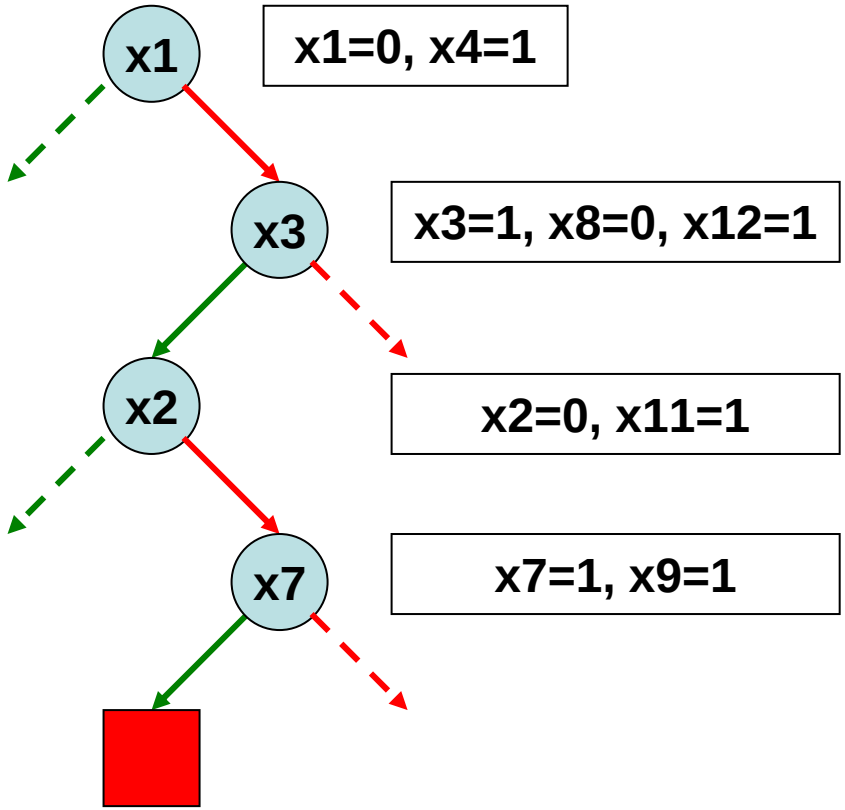
x7' + x3' + x9

x7' + x8 + x9'

x7 + x8 + x10'

x7 + x10 + x12'





$x3=1 \wedge x7=1 \wedge x8=0 \rightarrow$ conflict

Conflict Driven Learning and Non-chronological Backtracking

x1 + x4

x1 + x3' + x8'

x1 + x8 + x12

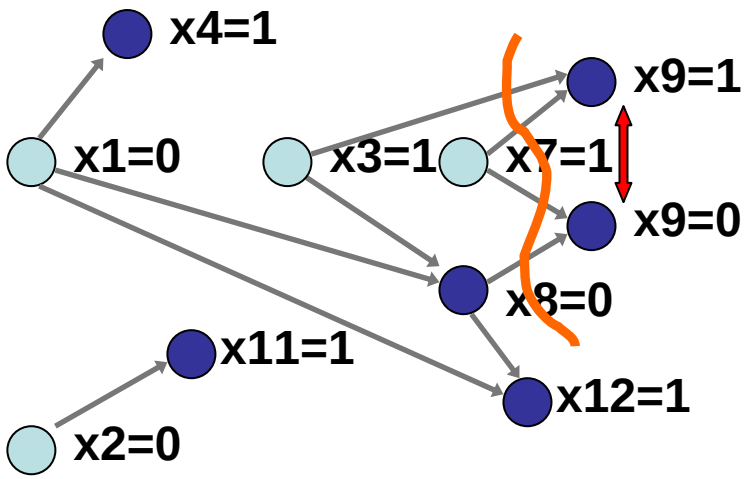
x2 + x11

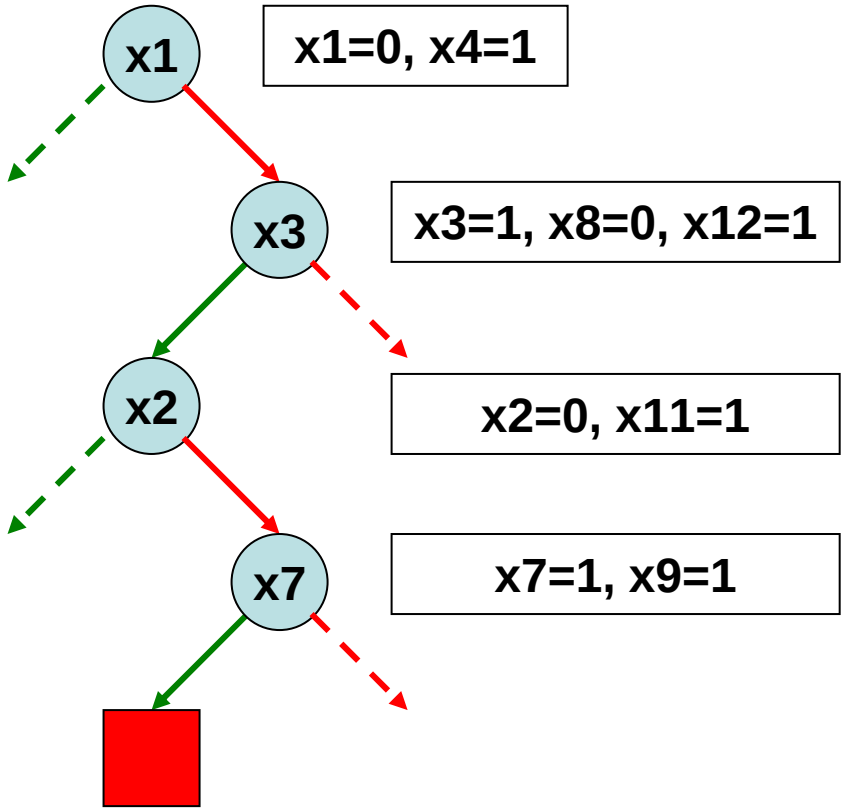
x7' + x3' + x9

x7' + x8 + x9'

x7 + x8 + x10'

x7 + x10 + x12'





$x3=1 \wedge x7=1 \wedge x8=0 \rightarrow$ conflict

Add conflict clause: x3'+x7'+x8

Conflict Driven Learning and Non-chronological Backtracking

x1 + x4

x1 + x3' + x8'

x1 + x8 + x12

x2 + x11

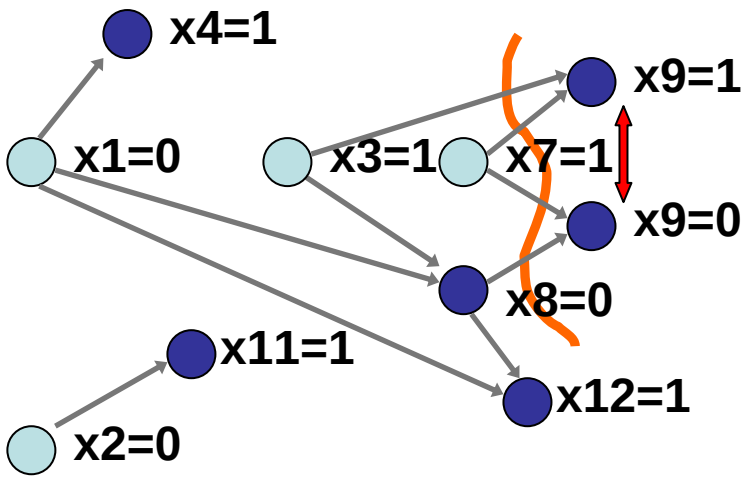
x7' + x3' + x9

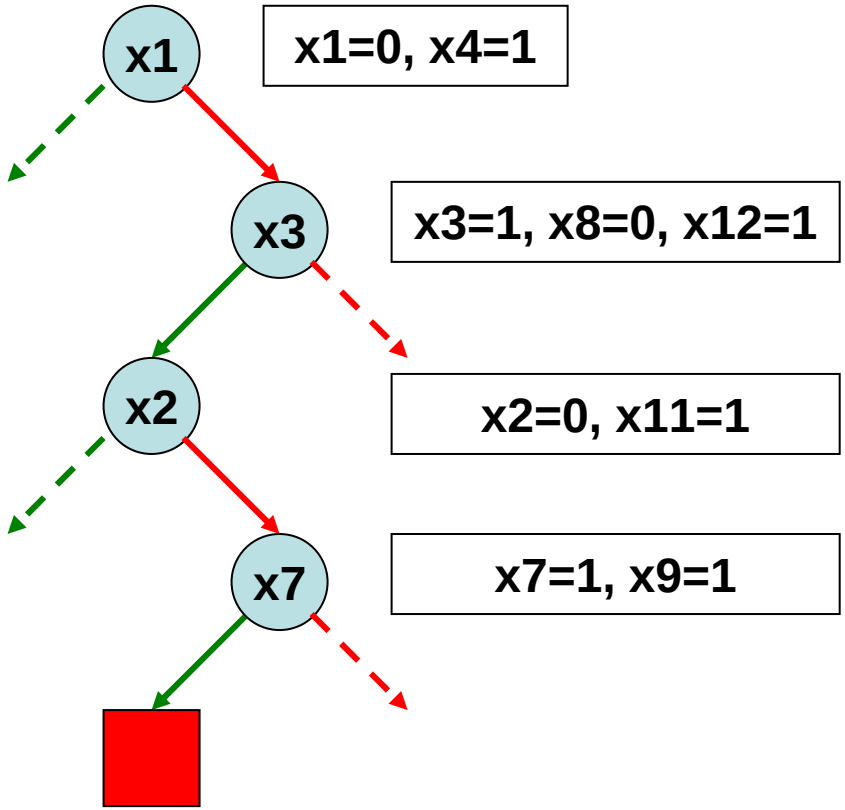
x7' + x8 + x9'

x7 + x8 + x10'

x7 + x10 + x12'

x3'+x7'+x8





x3=1∧x7=1∧x8=0 → conflict

Add conflict clause: x3'+x7'+x8

Conflict Driven Learning and Non-chronological Backtracking

x1 + x4

x1 + x3' + x8'

x1 + x8 + x12

x2 + x11

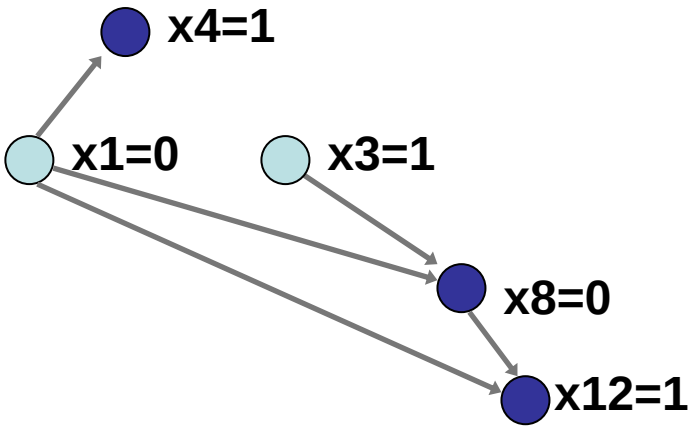
x7' + x3' + x9

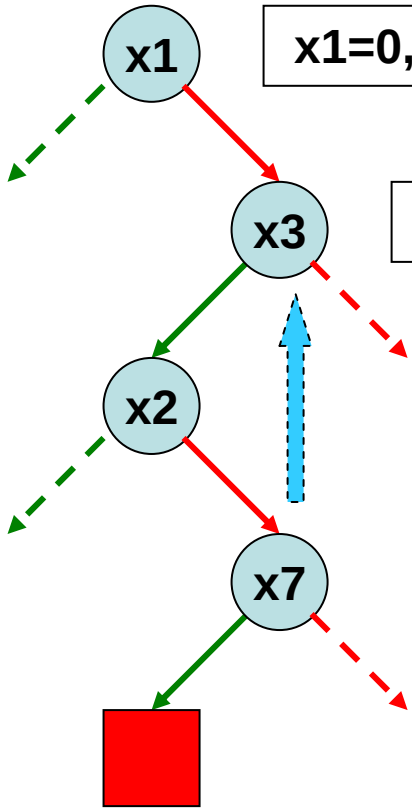
x7' + x8 + x9'

x7 + x8 + x10'

x7 + x10 + x12'

x3' + x8 + x7'





x1=0, x4=1

x3=1, x8=0, x12=1

Backtrack to the decision level of x3=1
x7 = 0

Conflict Driven Learning and Non-chronological Backtracking

x1 + x4

x1 + x3' + x8'

x1 + x8 + x12

x2 + x11

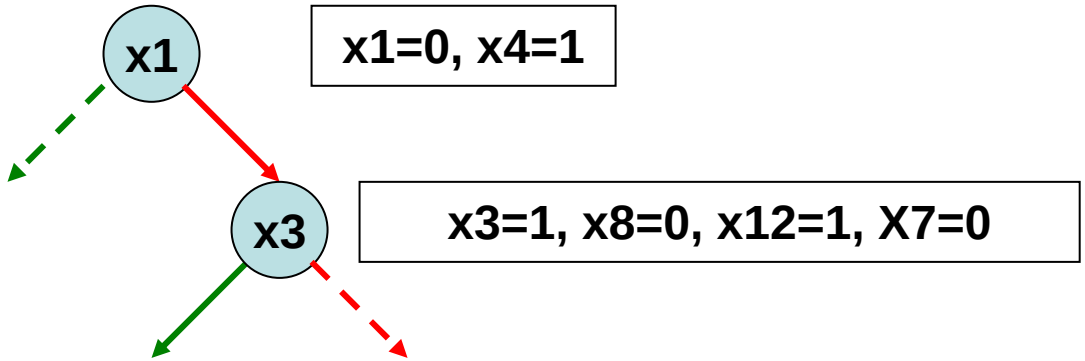
x7' + x3' + x9

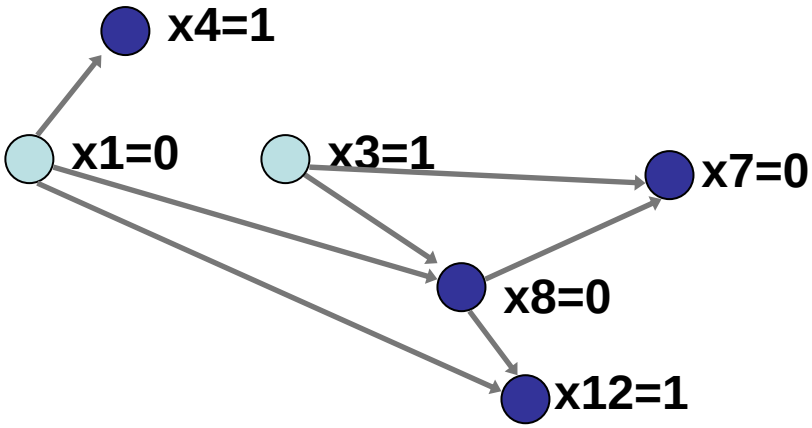
x7' + x8 + x9'

x7 + x8 + x10'

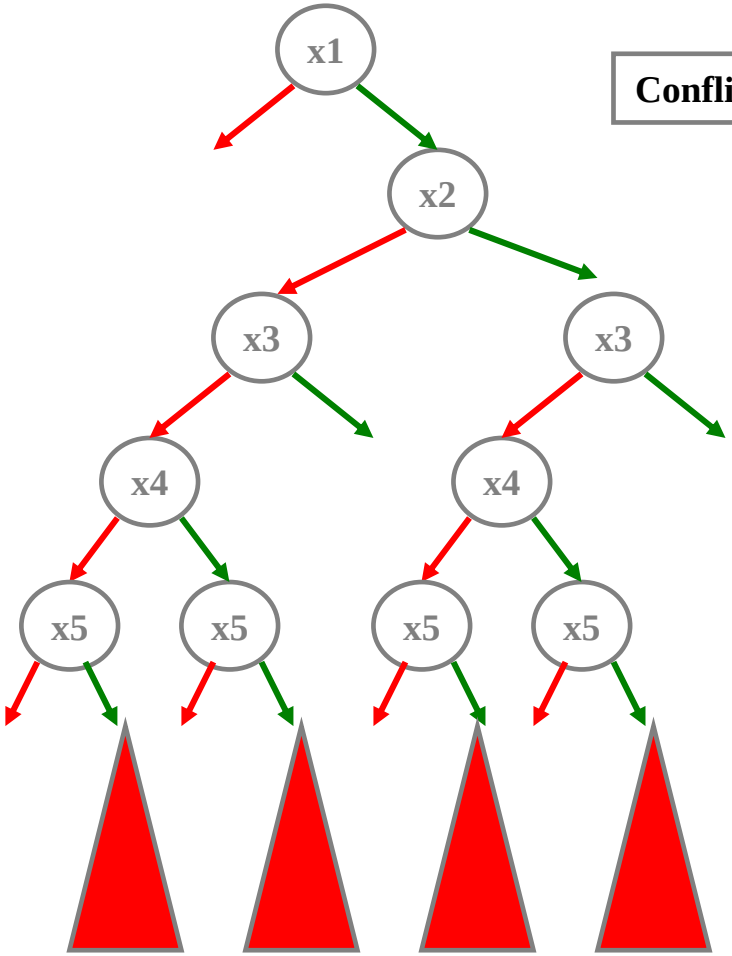
x7 + x10 + x12'

x3' + x8 + x7'





What's the big deal?



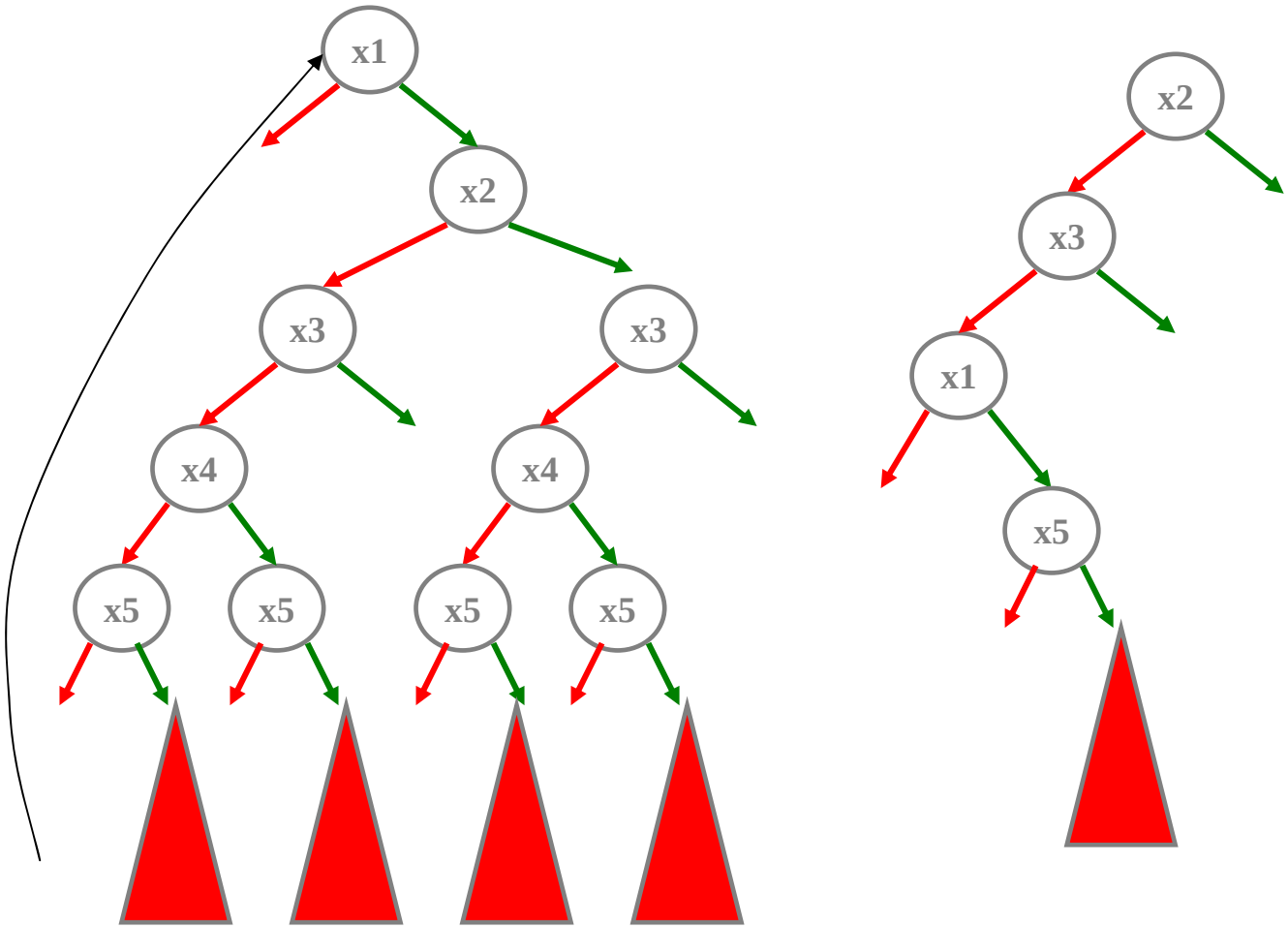
Conflict clause: x1'+x3+x5'

Significantly prune the search space – learned clause is useful forever!

Useful in generating future conflict clauses.

Restart

- Abandon the current search tree and reconstruct a new one
- The clauses learned prior to the restart are *still there* after the restart and can help pruning the search space
- Adds to robustness in the solver



SAT becomes practical!

- Conflict driven learning greatly increases the capacity of SAT solvers (several thousand variables) for structured problems
- Realistic applications become feasible
  - Usually thousands and even millions of variables
  - Typical EDA applications that can make use of SAT
    - circuit verification
    - FPGA routing
    - many other applications…
- Research direction changes towards more efficient implementations

Large Example: Tough

- Industrial Processor Verification
  - Bounded Model Checking, 14 cycle behavior
- Statistics
  - 1 million variables
  - 10 million literals initially
    - 200 million literals including added clauses
    - 30 million literals finally
  - 4 million clauses (initially)
    - 200K clauses added
  - 1.5 million decisions
  - 3 hours run time

Chaff

- One to two orders of magnitude faster than other solvers…

  M. Moskewicz, C. Madigan, Y. Zhao, L. Zhang, S. Malik,“Chaff: Engineering an Efficient SAT Solver” *Proc. DAC* 2001.

- Widely Used:
  - BlackBox – AI Planning
    - Henry Kautz (UW)
  - NuSMV – Symbolic Verification toolset

    A. Cimatti, et. al. **“**NuSMV 2: An Open Source Tool for Symbolic Model Checking**”** *Proc. CAV* 2002.
  - GrAnDe – Automatic theorem prover
  - Several industrial licenses

Chaff Philosophy

- Make the core operations fast
  - profiling driven, most time-consuming parts:
    - Boolean Constraint Propagation (BCP) and Decision
- Emphasis on coding efficiency and elegance
- Emphasis on optimizing data cache behavior
- As always, good search space pruning (i.e. conflict resolution and learning) is important

Motivating Metrics: Decisions, Instructions, Cache Performance and Run Time

	1dlx_c_mc_ex_bp_f
Num Variables	**776**
Num Clauses	**3725**
Num Literals	**10045**

	Z-Chaff	GRASP
# Decisions	3166	1795
# Instructions	86.6M	1415.9M
# L1/L2 accesses	24M / 1.7M	416M / 153M
% L1/L2 misses	4.8% / 4.6%	32.9% / 50.3%
# Seconds	0.22	11.78

BCP Algorithm

- What “causes” an implication? When can it occur?
  - All literals in a clause but one are assigned to F
    - (v1 + v2 + v3): implied cases: (0 + 0 + v3) or (0 + v2 + 0) or (v1 + 0 + 0)
  - For an N-literal clause, this can only occur after N-1 of the literals have been assigned to F
  - So, (theoretically) we could completely ignore the first N-2 assignments to this clause
  - In reality, we pick two literals in each clause to “watch” and thus can ignore any assignments to the other literals in the clause.
    - Example: (v1 + v2 + v3 + v4 + v5)
    - (**v1=X + v2=X** + v3=? {i.e. X or 0 or 1} + v4=? + v5=?)

BCP Algorithm

- Big Invariants
  - Each clause has two watched literals
  - If a clause can become newly implied via any sequence of assignments, then this sequence will include an assignment of one of the watched literals to F.
    - Example again: (v1 + v2 + v3 + v4 + v5)
    - (**v1=X** + **v2=X** + v3=? + v4=? + v5=?)
- BCP consists of identifying implied clauses (and the associated implications) while maintaining the “Big Invariants”
- No actions on backtracking

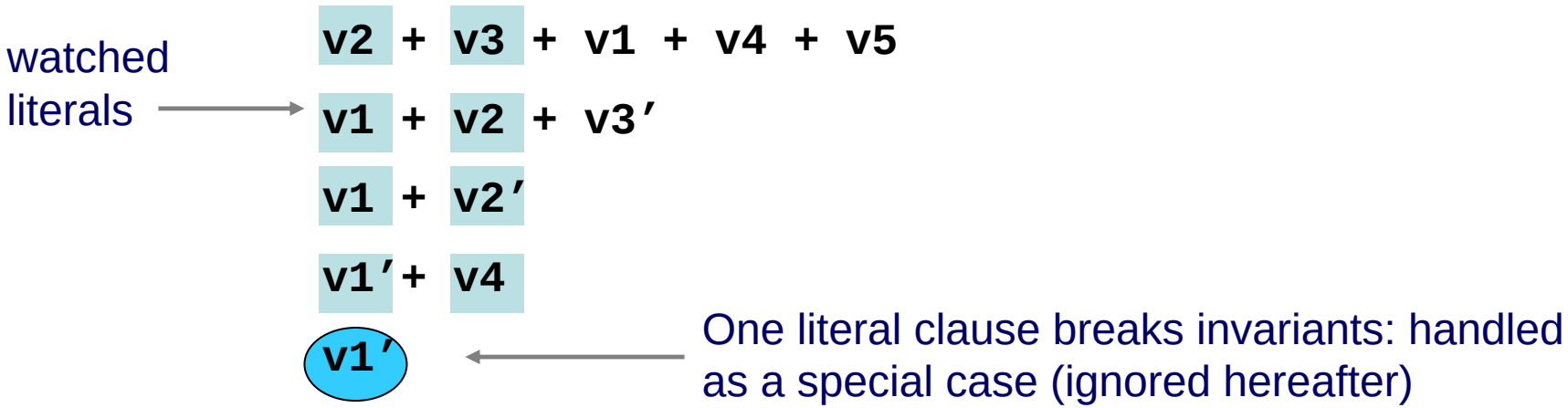
BCP Algorithm

- Let’s illustrate this with an example:

```
v2 + v3 + v1 + v4 + v5
v1 + v2 + v3’
v1 + v2’
v1’+ v4
v1’
```

BCP Algorithm

- Let's illustrate this with an example:



- Initially, we identify any two literals in each clause as the watched ones
- Clauses of size one are a special case

BCP Algorithm

- We begin by processing the assignment v1 = F (which is implied by the size one clause)

State:(v1=F)

Pending:

v2 + v3 + v1 + v4 + v5

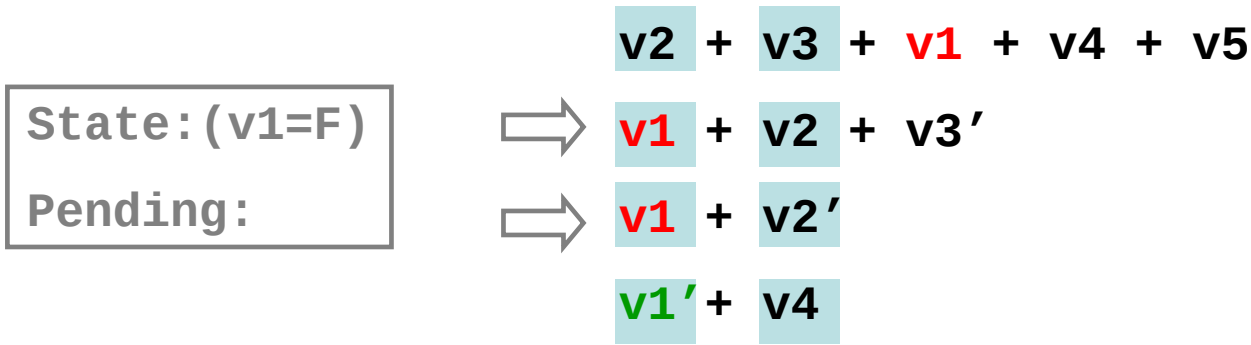
v1 + v2 + v3’

v1 + v2’

v1’ + v4

BCP Algorithm

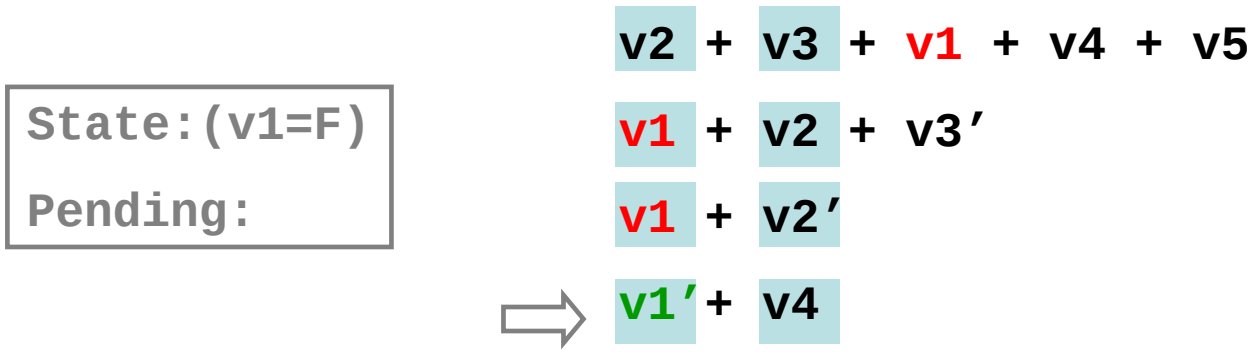
- We begin by processing the assignment v1 = F (which is implied by the size one clause)



- To maintain our invariants, we must examine each clause where the assignment being processed has set a watched literal to F

BCP Algorithm

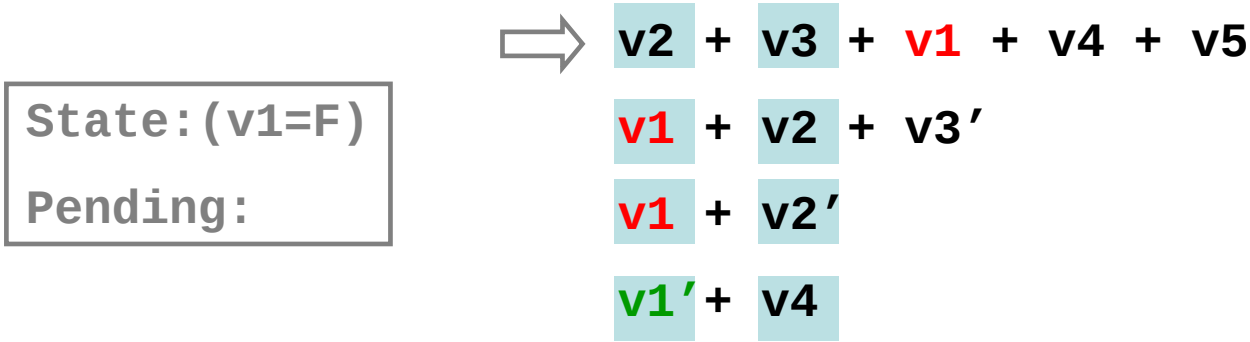
- We begin by processing the assignment v1 = F (which is implied by the size one clause)



- To maintain our invariants, we must examine each clause where the assignment being processed has set a watched literal to F.
- We need not process clauses where a watched literal has been set to T, because the clause is now satisfied and so can not become implied.

BCP Algorithm

- We begin by processing the assignment v1 = F (which is implied by the size one clause)



- To maintain our invariants, we must examine each clause where the assignment being processed has set a watched literal to F.
- We need not process clauses where a watched literal has been set to T, because the clause is now satisfied and so can not become implied.
- We *certainly* need not process any clauses where neither watched literal changes state (in this example, where v1 is not watched).

BCP Algorithm

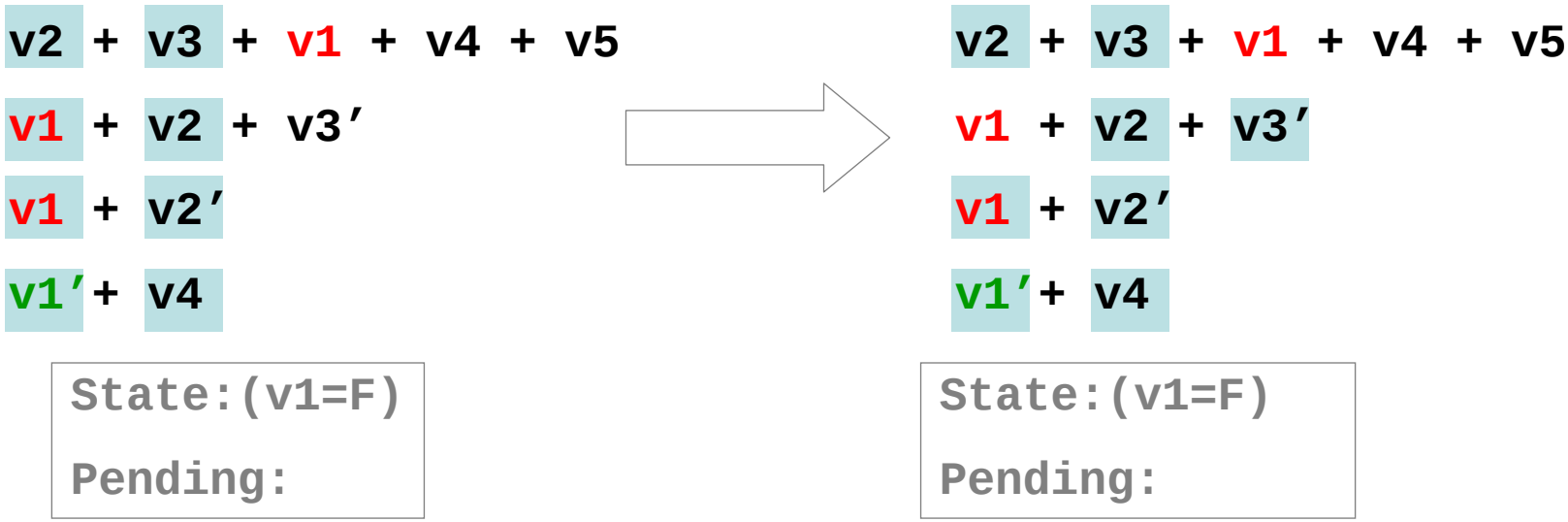
- Now let's actually process the second and third clauses:

```
v2 + v3 + v1 + v4 + v5
v1 + v2 + v3'
v1 + v2'
v1'+ v4
```

```
State:(v1=F)
Pending:
```

BCP Algorithm

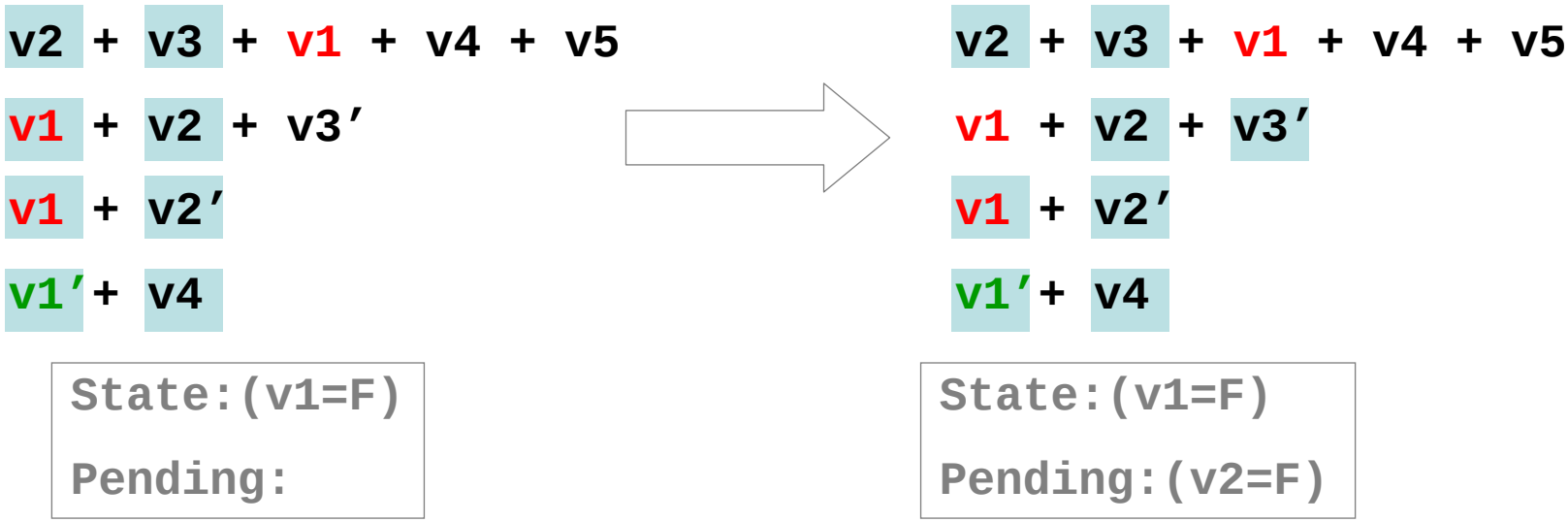
- Now let's actually process the second and third clauses:



- For the second clause, we replace v1 with v3' as a new watched literal. Since v3' is not assigned to F, this maintains our invariants.

BCP Algorithm

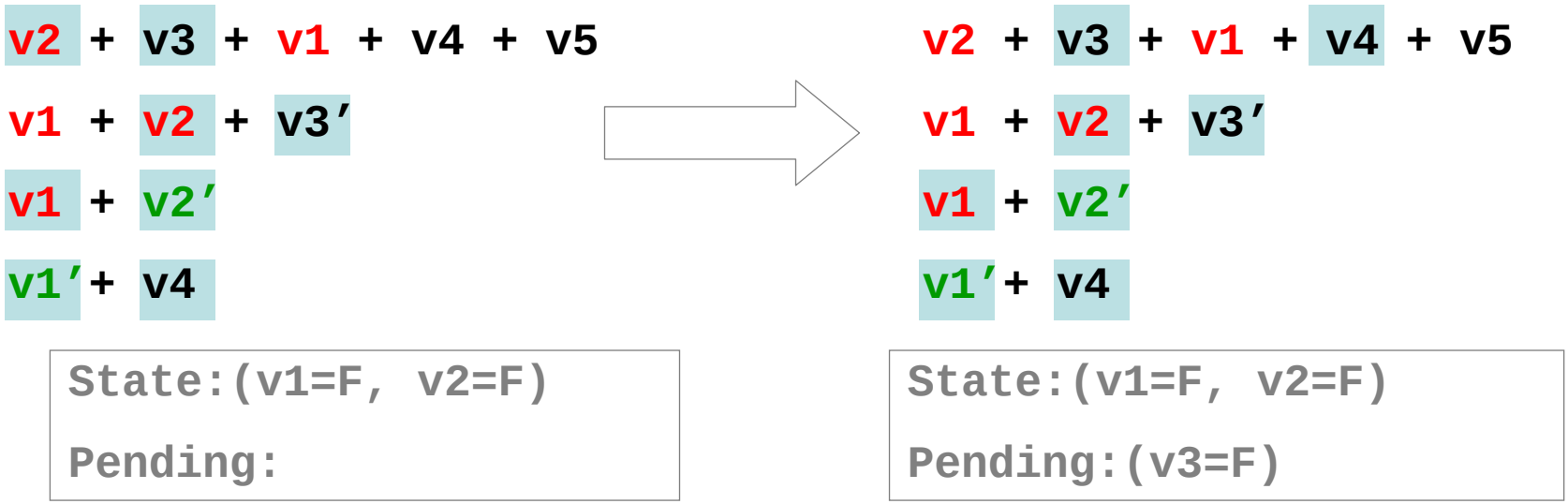
- Now let's actually process the second and third clauses:



- For the second clause, we replace v1 with v3' as a new watched literal. Since v3' is not assigned to F, this maintains our invariants.
- The third clause is implied. We record the new implication of v2', and add it to the queue of assignments to process. Since the clause cannot again become newly implied, our invariants are maintained.

BCP Algorithm

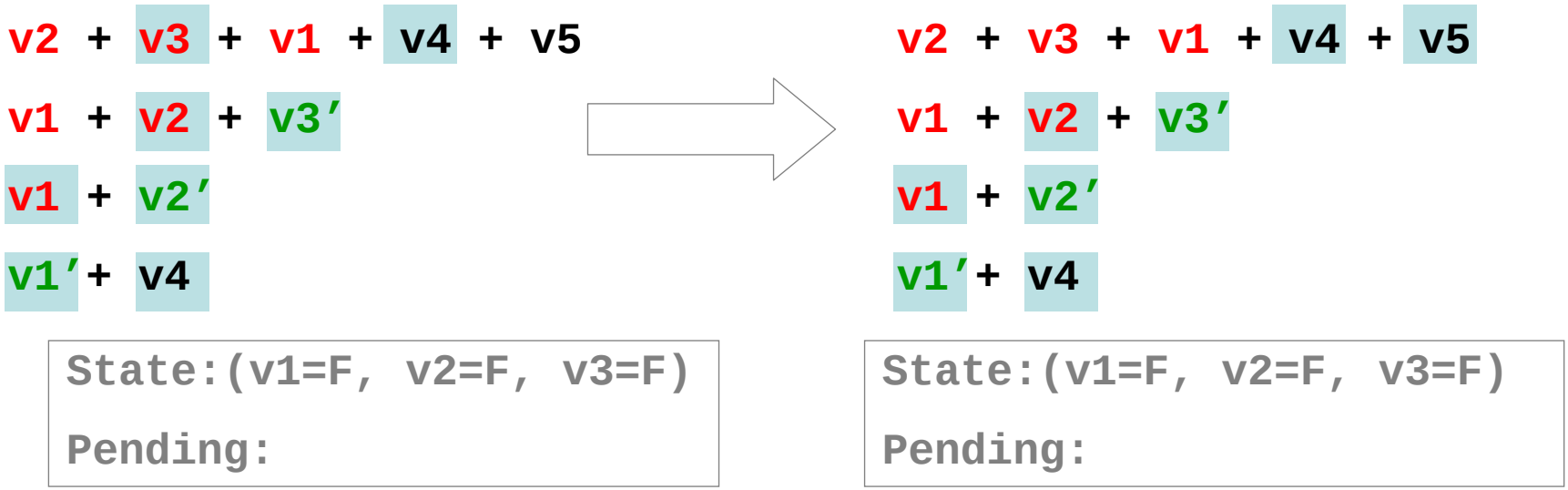
- Next, we process v2'. We only examine the first 2 clauses.



- For the first clause, we replace v2 with v4 as a new watched literal. Since v4 is not assigned to F, this maintains our invariants.
- The second clause is implied. We record the new implication of v3', and add it to the queue of assignments to process. Since the clause cannot again become newly implied, our invariants are maintained.

BCP Algorithm

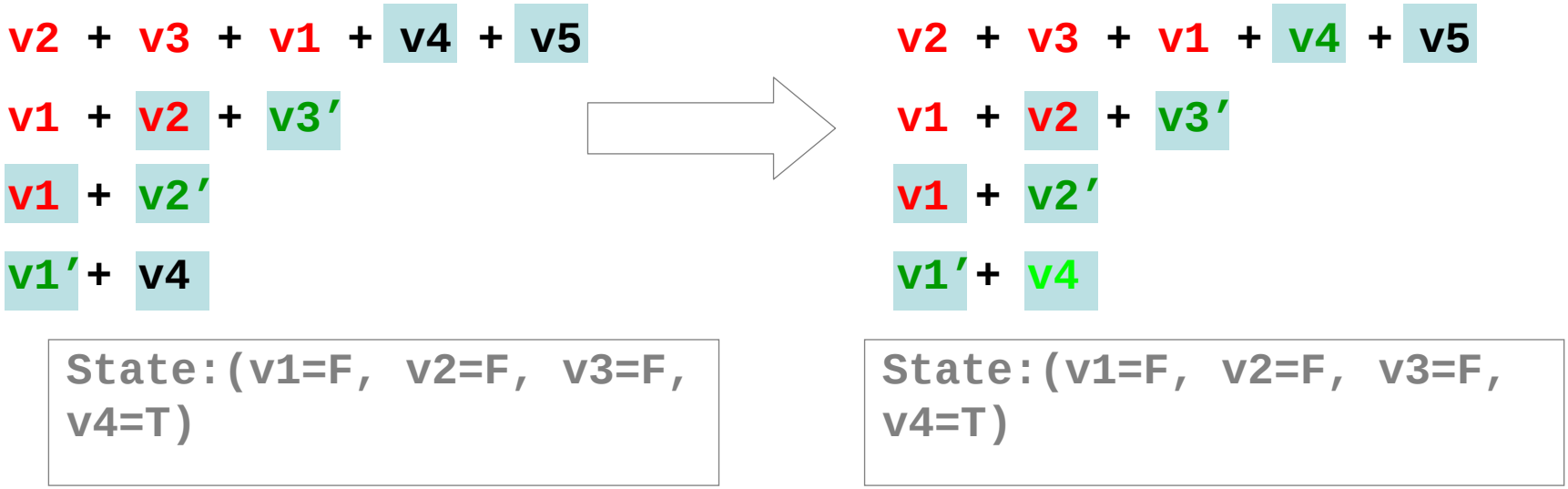
- Next, we process v3'. We only examine the first clause.



- For the first clause, we replace v3 with v5 as a new watched literal. Since v5 is not assigned to F, this maintains our invariants.
- Since there are no pending assignments, and no conflict, BCP terminates and we make a decision. Both v4 and v5 are unassigned. Let's say we decide to assign v4=T and proceed.

BCP Algorithm

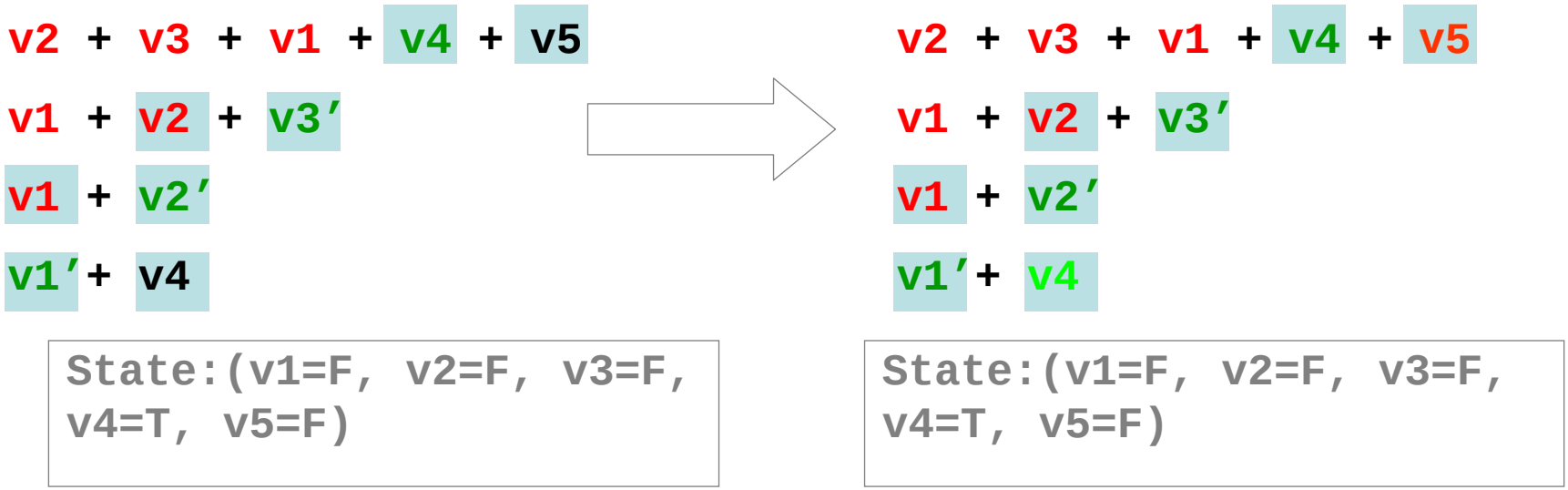
- Next, we process v4. We do nothing at all.



- Since there are no pending assignments, and no conflict, BCP terminates and we make a decision. Only v5 is unassigned. Let's say we decide to assign v5=F and proceed.

BCP Algorithm

- Next, we process v5=F. We examine the first clause.



- The first clause is implied. However, the implication is v4=T, which is a duplicate (since v4=T already) so we ignore it
- Since there are no pending assignments, and no conflict, BCP terminates and we make a decision. No variables are unassigned, so the problem is sat, and we are done

BCP Algorithm Summary

- During forward progress: Decisions and Implications
  - Only need to examine clauses where watched literal is set to F
    - Can ignore any assignments of literals to T
    - Can ignore any assignments to non-watched literals
- During backtrack: Unwind Assignment Stack
  - Any sequence of chronological unassignments will maintain our invariants
    - So no action is required at all to unassign variables
- Overall
  - Minimize clause access
  - Better memory locality

Decision Heuristics – Conventional Wisdom

- DLIS is a relatively simple dynamic decision heuristic
  - Simple and intuitive: At each decision simply choose the assignment that satisfies the most unsatisfied clauses
  - However, considerable work is required to maintain the statistics necessary for this heuristic – for one implementation:
    - Must touch *every* clause that contains a literal that has been set to true. Often restricted to initial (not learned) clauses
    - Maintain “sat” counters for each clause
    - When counters transition 0→1, update rankings
    - Need to reverse the process for unassignment
  - The total effort required for this and similar decision heuristics is *much more* than for our BCP algorithm.
- Look ahead algorithms even more compute intensive

  C. Li, Anbulagan, “Look-ahead versus look-back for satisfiability problems” *Proc. of CP*, 1997.

Chaff Decision Heuristic - VSIDS

- Variable State Independent Decaying Sum
  - Rank variables by literal count in the initial clause database
  - Periodically, divide all counts by a constant
  - Only increment counts as new clauses are added
- Quasi-static:
  - Static because it doesn't depend on var state
  - Not static because it gradually changes as new clauses are added
    - Decay causes bias toward *recent* conflicts.
- Use heap to find unassigned var with the highest ranking
    - Even single linear pass though variables on each decision would dominate run-time!
- Seems to work fairly well in terms of # decisions
    - hard to compare with other heuristics because they have too much overhead

Interplay of BCP and Decision

- This is only an intuitive description …
  - Reality depends heavily on specific instance
- Take some variable ranking (from the decision engine)
  - Assume several decisions are made
    - Say v2=T, v7=F, v9=T, v1=T (and any implications thereof)
  - Then a conflict is encountered that forces v2=F
    - The next decisions may still be v7=F, v9=T, v1=T !
    - But the BCP engine has recently processed these assignments … so these variables are unlikely to still be watched.
    - Thus, the BCP engine *inherently does a differential update.*
  - And the Decision heuristic makes differential changes more likely to occur in practice.
- In a more general sense, the more “active” a variable is, the more likely it is to *not* be watched.

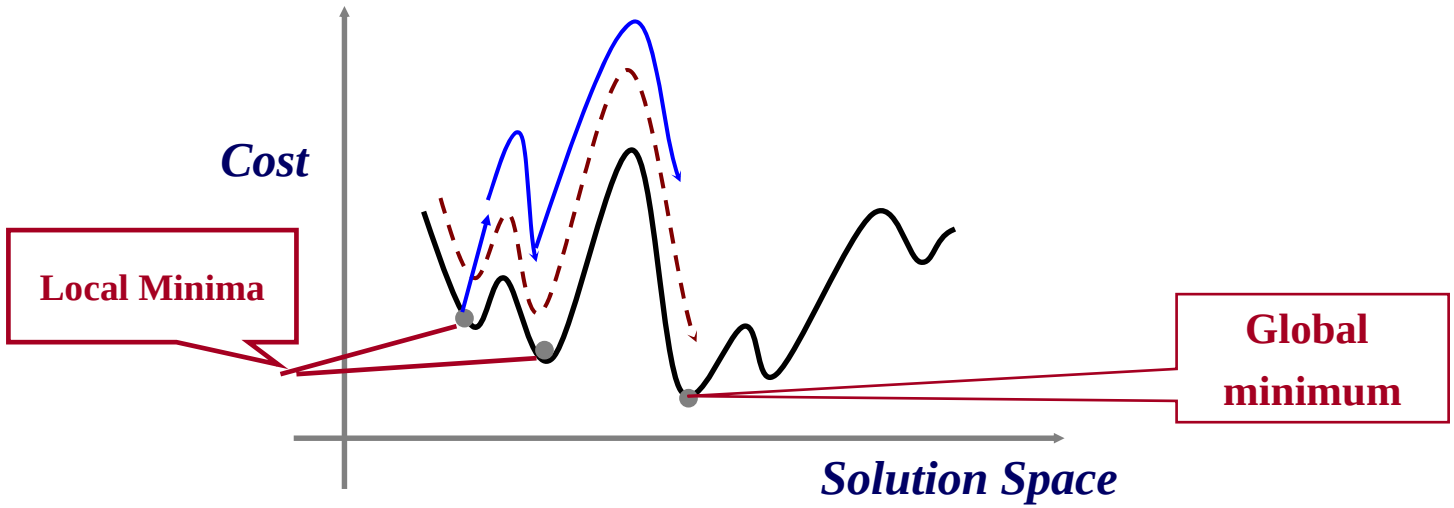
Missing

- Post Chaff SAT solvers
  - BerkMin
  - Seige
  - miniSat
  - HaifaSAT
  - JeruSAT (Alex Nadel)
- The Stålmarck's algorithm
- Hyperresolution
- Local Search

Local Search (GSAT, WSAT)

B. Selman, H. Levesque, and D. Mitchell. “A new method for solving hard satisfiability problems”. *Proc. AAAI*, 1992.

- Incomplete SAT solvers
  - Geared towards satisfiable instances, cannot prove unsatisfiability
- Hill climbing algorithm for local search
- Make short local moves
- Probabilistically accept moves that worsen the cost function to enable exits from local minima



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- **Efficient Conflict Driven Learning in a Boolean Satisfiability Solver** Lintao Zhang, Conor F. Madigan, Matthew H. Moskewicz (IJCAD'01)
- **A New Method for Solving Hard Satisfiability Problems**
  Bart Selman, Hector Levesque, David Mitchell
  (AAI'92)

Open Question

- Is there a subset of propositional logic beyond Horn clauses which:
  - Allows polynomial SAT
  - Includes many of the practical instances

Summary

- Rich history of emphasis on practical efficiency
- Need to account for computation cost in search space pruning
- Need to match algorithms with underlying processing system architectures
- Specific problem classes can benefit from specialized algorithms
  - Identification of problem classes?
  - Dynamically adapting heuristics?
- We barely understand the tip of the iceberg here
  - much room to learn and improve