Modeling using Variation

Objectives: To find equations of direct, inverse, and joint variation, and to solve applied problems involving variation.

Variation – An Introduction

- Variation formulas show how one quantity changes in relation to other quantities.
- Quantities can vary three ways:

1. _____

2. _____

3. _____

Steps for Solving Variation Problems

1. Write an _____
2. that describes the given statement.
3. _____ the given pair of values into the equation from step 1 and solve for *k*, the constant of variation.
4. Substitute the _____ into the equation in step 1.
5. Use the equation from step 3 to _____ the problem's question.

Direct Variation

- If a situation gives rise to a linear function

$$f(x) = kx, \text{ or } y = kx,$$

where *k* is a positive constant, we say that we have **direct variation**, or that ***y* varies directly as *x*,** or that ***y* is directly proportional to *x*.**

- The number *k* is called the _____ _____, or **constant of proportionality**.
- The graph of this type of variation is a _____.

Direct Variation Practice

Find the variation constant and an equation of variation in which *y* varies directly as *x*, and *y* = 42 when *x* = 3.

Example

The number of gallons of water, *w*, used when taking a shower varies directly as the time, *t*, in minutes, in the shower. A shower lasting 5 minutes uses 30 gallons of water. How much water is used in a shower lasting 11 minutes?

Direct Variation with Powers

Y varies directly as the *n*th power of x if there exists some nonzero constant *k* such that

_____

The graph of this type of variation is a _____ located in quadrant one.

Example

- The distance required to stop a car varies directly as the square of its speed. If 200 feet are required to stop a car traveling 60 miles per hour, how many feet are required to stop a car traveling 100 miles per hour?

Inverse Variation

- If a situation gives rise to a function

_____ **or** _____

where *k* is a positive constant, we say that we have **inverse variation**, or that ***y* varies inversely as *x*,** or that ***y* is inversely proportional to *x*.**

The number *k* is called the **variation constant,** or **constant of proportionality**.

Example

The length of a violin string varies inversely as the frequency of its vibrations. A violin string 8 inches long vibrates at a frequency of 640 cycles per second. What is the frequency of a 10-inch string?

Another type of variation

- Combined Variation -- when **direct** and **inverse** variation occur at the same time.

Example

The number of minutes needed to solve an exercise set of variation problems varies directly as the number of problems and inversely as the number of people working to solve the problems. It takes 4 people 32 minutes to solve 16 problems. How many minutes will it take 8 people to solve 24 problems?

Joint Variation

- Joint Variation is a type of variation in which a variable varies directly as the _____ of two or more other variables.
- For example: *y varies* ***jointly as x and z*** if there is some positive constant k such that $y = kxz$.

Example

- The volume of a cone, V, varies jointly as its height, h, and the square of its radius, r. A cone with a radius measuring 6 feet and a height measuring 10 feet has a volume of 120π cubic feet. Find the volume of a cone having a radius of 12 feet and a height of 2 feet.