Modeling and Analysis of Simple Market Strategies

Alexander R. Block

July 17, 2017

Introduction

- This project aims to observe what happens when simple market strategies are used in a particular market place.
- Modeling based on real-valued functions.
- Simple software developed in SageMath [S+17] (arbitrary precision computing) to track statistics and plot data for analysis.

Model

- We analyze the following model.
  - There is a market place for one product, with n sellers and a market price mp.
  - Each seller has a pricing strategy which depends on at most $(n-1)$ other sellers and/or the current or starting market price.
  - Each seller has an initial price for the market.
  - The market price also has a strategy.
- This model can be thought of as a *seller's only market*, where the pricing strategies of the sellers is the sole determinant of the market as a whole.

Model

- Experiments in this model follow the following outline.
  - The market spans over t rounds or time instances.
  - During a round t_i, we determine which seller(s) apply their pricing strategy(-ies) and update their price(s).
  - At the end of a round t_i, the market price is updated.

Model

- Model is broad in order to allow a wider range of possible results and observations, but at the same time is limited.
  - No classical supply and demand for example.
- Many parameters are fixed to allow comparable results to be generated.
- General assumptions
  - Sellers cannot price below \$0.
  - Sellers may not update their price more than once per round.
  - Each seller strategy is *round independent*.
- Fixed Market Price Strategy
  - the average of all seller prices at the end of round t_i.

Seller Strategies

- Guideline for seller strategies
  - Deterministic vs. Nondeterministic
  - Unbounded vs. Bounded
- Experiments are performed with each of the above combinations, in addition to a mix of bounded and unbounded constraints.
  - For deterministic strategy experiments, all sellers have deterministic strategies.
  - For nondeterministic, at least one seller must have a nondeterministic strategy.

Seller Strategies and Market Skew

- What is Market Skew?
  - When the strategies of the seller pull the market upwards or downwards (in terms of price).
- Three possible: Low-Skew, No-Skew, and High-Skew
  - Low-Skew inherently floored at \$0 by our assumptions, but is otherwise symmetric to High-Skew.
  - No-Skew means *roughly on average*, the market doesn't skew up or down "too much".
  - High-Skew not considered, as it is symmetric to Low-Skew (especially when a market cap is provided).
- Broad categorization, does not distinguish between the rate at which a market skews in a particular direction.

Experiment Parameters

- How to choose a seller to update their price in a round?
  - One Random Seller: one seller chosen uniformly at random out of n.
  - Two Random Sellers: two distinct sellers chosen uniformly at random.
  - All Sellers: each seller updates their price
- More parameter fixing
  - Number of rounds: $t = 500$.
  - Number of seller: $n = 5$ and $n = 10$.
  - Starting market price: $smp = \$2500$.
- Still a great degree of freedom left in the model because of seller strategies.
- We run each experiment for 20 trials; i.e., at the end of 500 rounds, statistics are collected, the market is reset, and the experiment is run again.

Results from a Low-Skew Market

	Nondeterministic Unbounded	Nondeterministic Bounded	Nondeterministic Mix
Seller 1	$f_1(\mathbf{x}_{[n]}) = \underset{i}{\text{ave}}\{x_i\} - \mathbf{rr}(5,16)$	$g_1(\mathbf{x}_{[n]}) = \max\left\{100, f_1(\mathbf{x}_{[n]})\right\}$	$h_1(\mathbf{x}_{[n]}) = g_1(\mathbf{x}_{[n]})$
Seller 2	$f_2(\mathbf{x}_{[n]}) = \frac{\mathbf{rr}(145,161)}{100}\underset{i}{\text{ave}}\{x_i\}$	$g_2(\mathbf{x}_{[n]}) = \min\left\{5(mp), f_2(\mathbf{x}_{[n]})\right\}$	$h_2(\mathbf{x}_{[n]}) = f_2(\mathbf{x}_{[n]})$
Seller 3	$f_3(\mathbf{x}_{[n]}) = \frac{\mathbf{rr}(67,74)}{100}\underset{i}{\text{ave}}\{x_i\}$	$g_3(\mathbf{x}_{[n]}) = \max\left\{0.15(mp), f_3(\mathbf{x}_{[n]})\right\}$	$h_3(\mathbf{x}_{[n]}) = f_3(\mathbf{x}_{[n]})$
Seller 4	$f_4(\mathbf{x}_{[n]}) = \frac{\mathbf{rr}(75,86)}{100}\min_i\{x_i\}$	$g_4(\mathbf{x}_{[n]}) = \max\left\{75, f_4(\mathbf{x}_{[n]})\right\}$	$h_4(\mathbf{x}_{[n]}) = g_4(\mathbf{x}_{[n]})$
Seller 5	$f_5(\mathbf{x}_{[n]}) = \underset{x_i<mp}{\text{ave}}\{x_i\} - \mathbf{rr}(20,31)$	$g_5(\mathbf{x}_{[n]}) = \max\left\{0.1(smp), f_5(\mathbf{x}_{[n]})\right\}$	$h_5(\mathbf{x}_{[n]}) = f_5(\mathbf{x}_{[n]})$
Seller 6	$f_6(\mathbf{x}_{[n]}) = \frac{\mathbf{rr}(107,116)}{100}\max_i\{x_i\}$	$g_6(\mathbf{x}_{[n]}) = \min\left\{3(smp), f_6(\mathbf{x}_{[n]})\right\}$	$h_6(\mathbf{x}_{[n]}) = f_6(\mathbf{x}_{[n]})$
Seller 7	$f_7(\mathbf{x}_{[n]}) = \min_{x_i \geqslant 0.75(mp)}\{x_i\} - \mathbf{rr}(9,21)$	$g_7(\mathbf{x}_{[n]}) = \max\left\{50, f_7(\mathbf{x}_{[n]})\right\}$	$h_7(\mathbf{x}_{[n]}) = g_7(\mathbf{x}_{[n]})$
Seller 8	$f_8(\mathbf{x}_{[n]}) = \frac{75}{100}\underset{i \text{ even}}{\text{ave}}\{x_i\}$	$g_8(\mathbf{x}_{[n]}) = \max\left\{100, f_8(\mathbf{x}_{[n]})\right\}$	$h_8(\mathbf{x}_{[n]}) = g_8(\mathbf{x}_{[n]})$
Seller 9	$f_9(\mathbf{x}_{[n]}) = \frac{\mathbf{rr}(120,136)}{100}\underset{i \text{ odd}}{\text{ave}}\{x_i\}$	$g_9(\mathbf{x}_{[n]}) = \min\left\{500 + (mp), f_9(\mathbf{x}_{[n]})\right\}$	$h_9(\mathbf{x}_{[n]}) = f_9(\mathbf{x}_{[n]})$
Seller 10	$f_{10}(\mathbf{x}_{[n]}) = \underset{i}{\text{ave}}\{x_i\}$	$g_{10}(\mathbf{x}_{[n]}) = \max\left\{60, f_{10}(x_{[n]})\right\}$	$h_{10}(\mathbf{x}_{[n]}) = g_{10}(\mathbf{x}_{[n]})$

Results from a Low-Skew Market

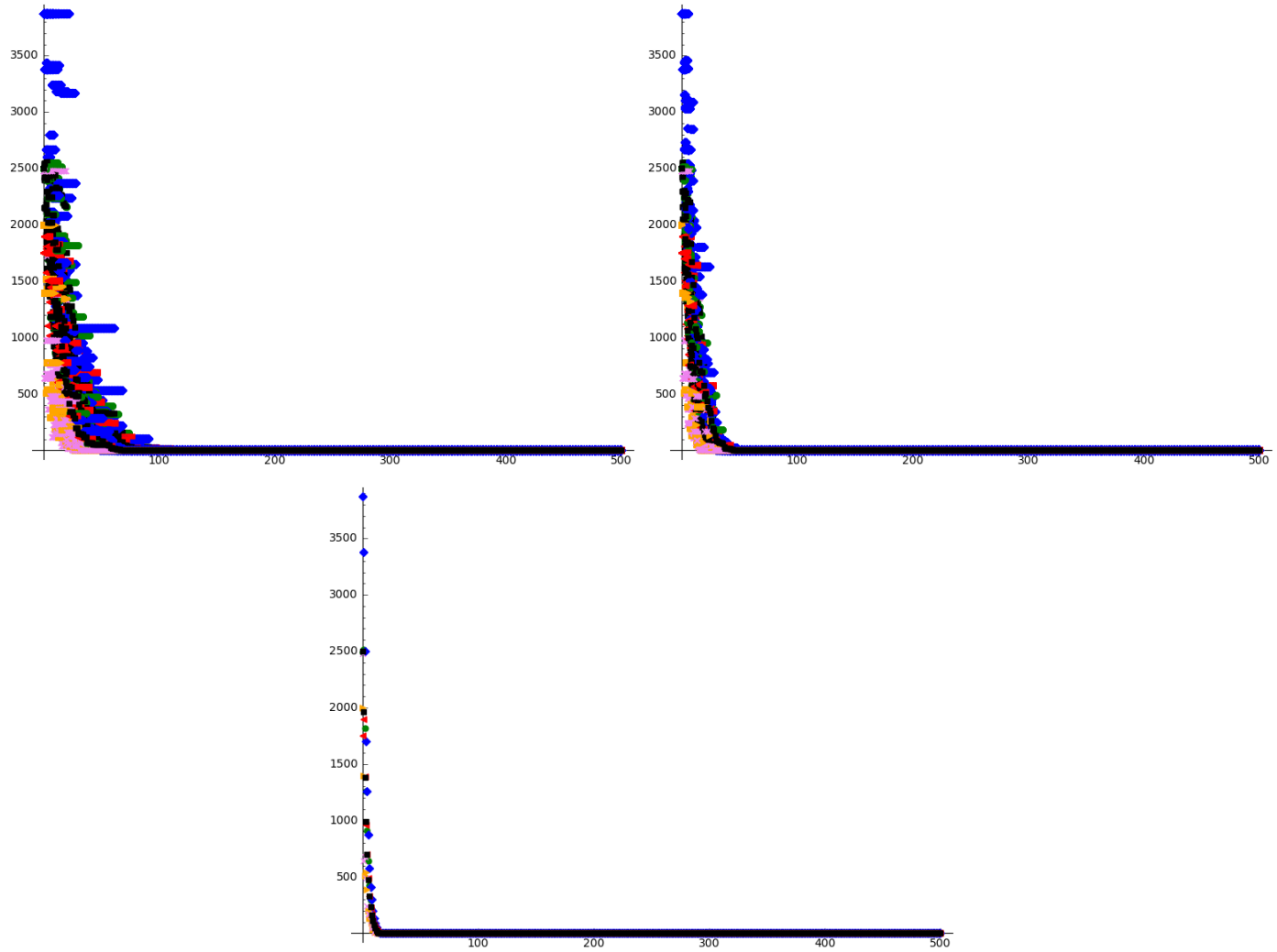


Figure 1: Data plots for Low-Skew, 5 Sellers, Deterministic Unbounded. Clockwise from the top left: One Update, Two Updates, All Update. The x-axis is number of rounds t; the y-axis is price in dollars. Market Price in black.

Results from a Low-Skew Market

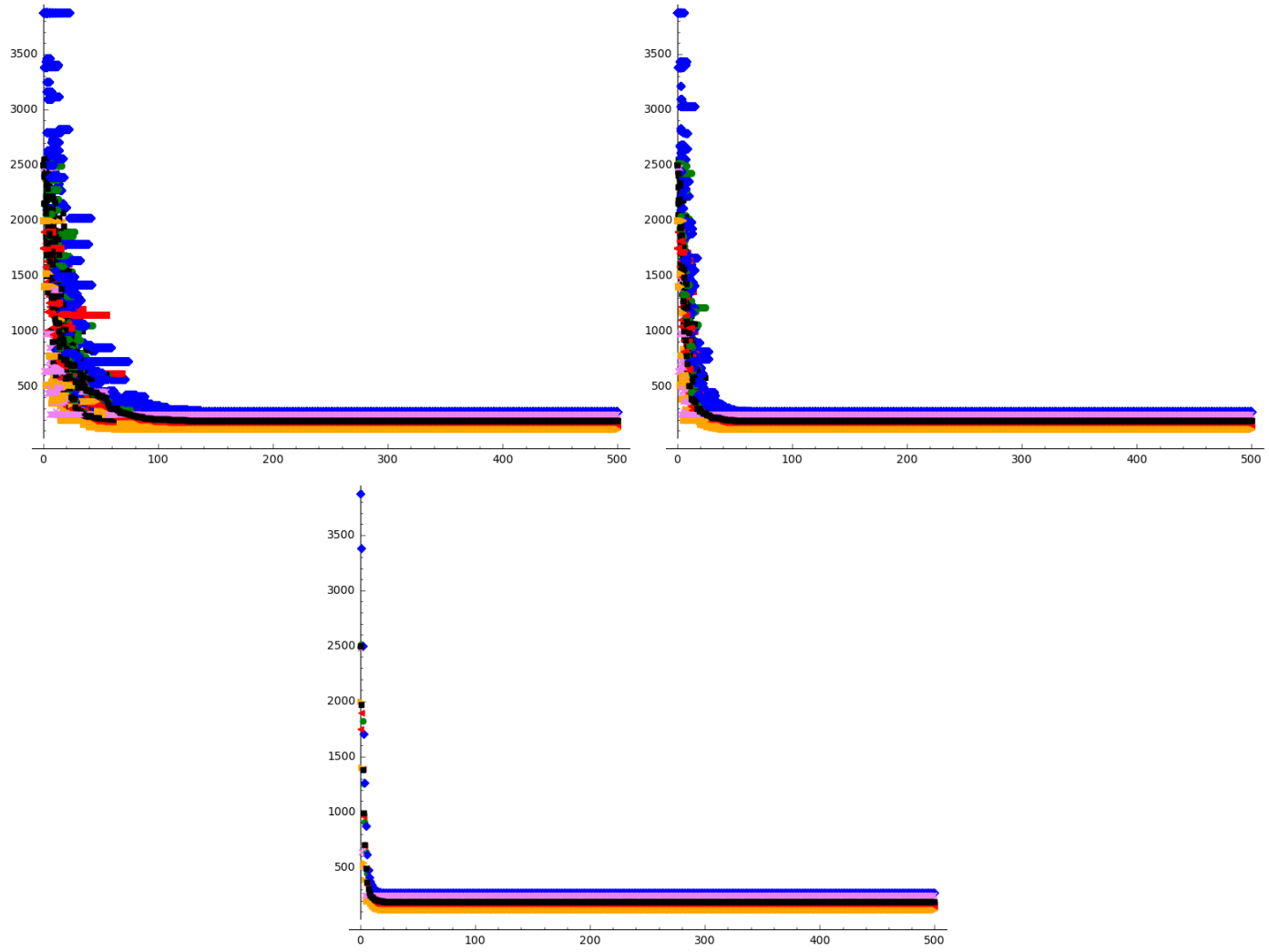


Figure 2: Data plots for Low-Skew, 5 Sellers, Deterministic Bounded. Clockwise from the top left: One Update, Two Updates, All Update. The x-axis is number of rounds t; the y-axis is price in dollars. Market Price in black.

Results from a Low-Skew Market

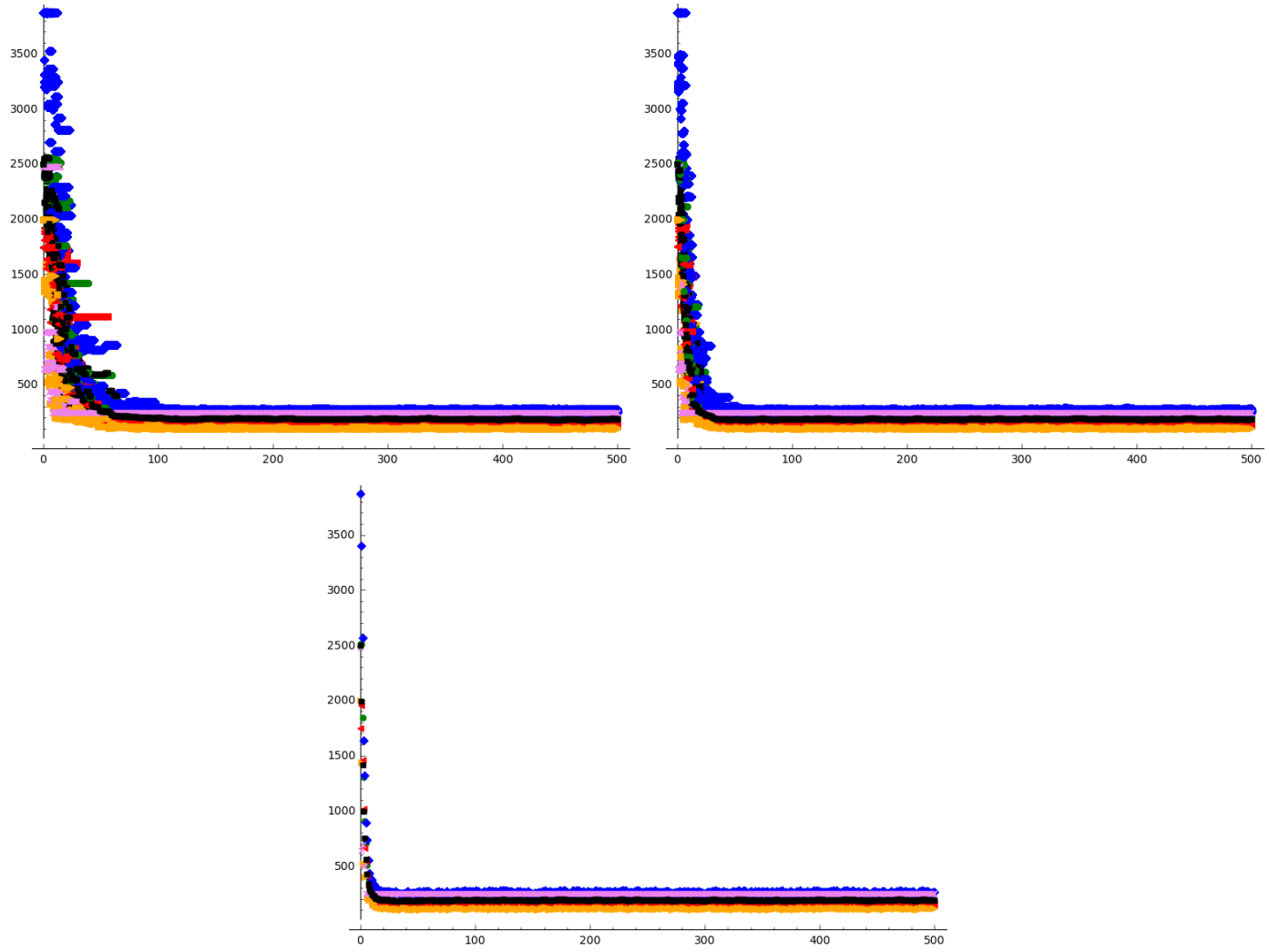


Figure 3: Data plots for Low-Skew, 5 Sellers, Nondeterministic Bounded. Clockwise from the top left: One Update, Two Updates, All Update. The x-axis is number of rounds t; the y-axis is price in dollars. Market Price in black.

Results from a Low-Skew Market

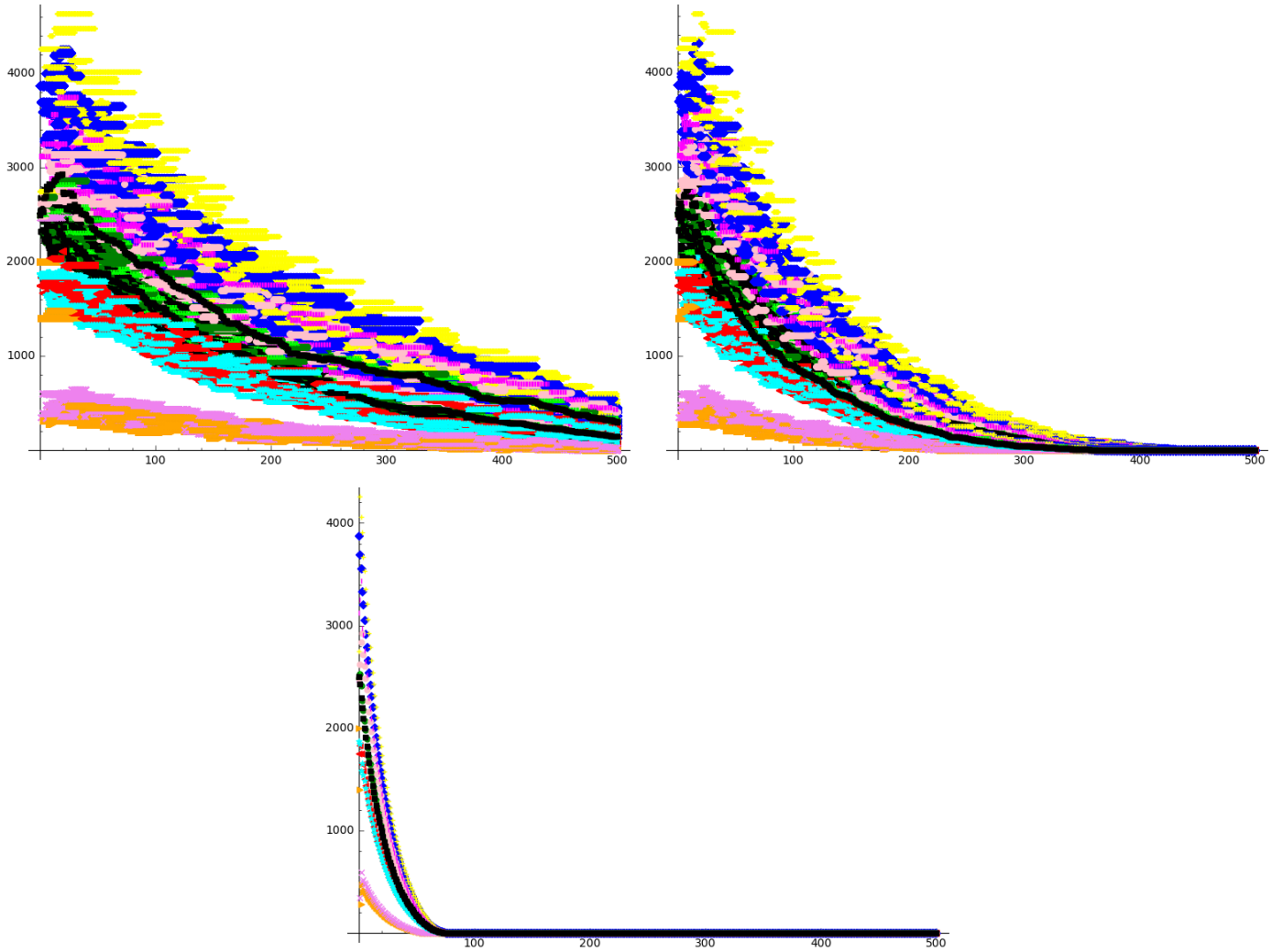


Figure 4: Data plots for Low-Skew, 10 Sellers, Deterministic Unbounded. Clockwise from the top left: One Update, Two Updates, All Update. The x-axis is number of rounds t; the y-axis is price in dollars. Market Price in black.

Results from a Low-Skew Market

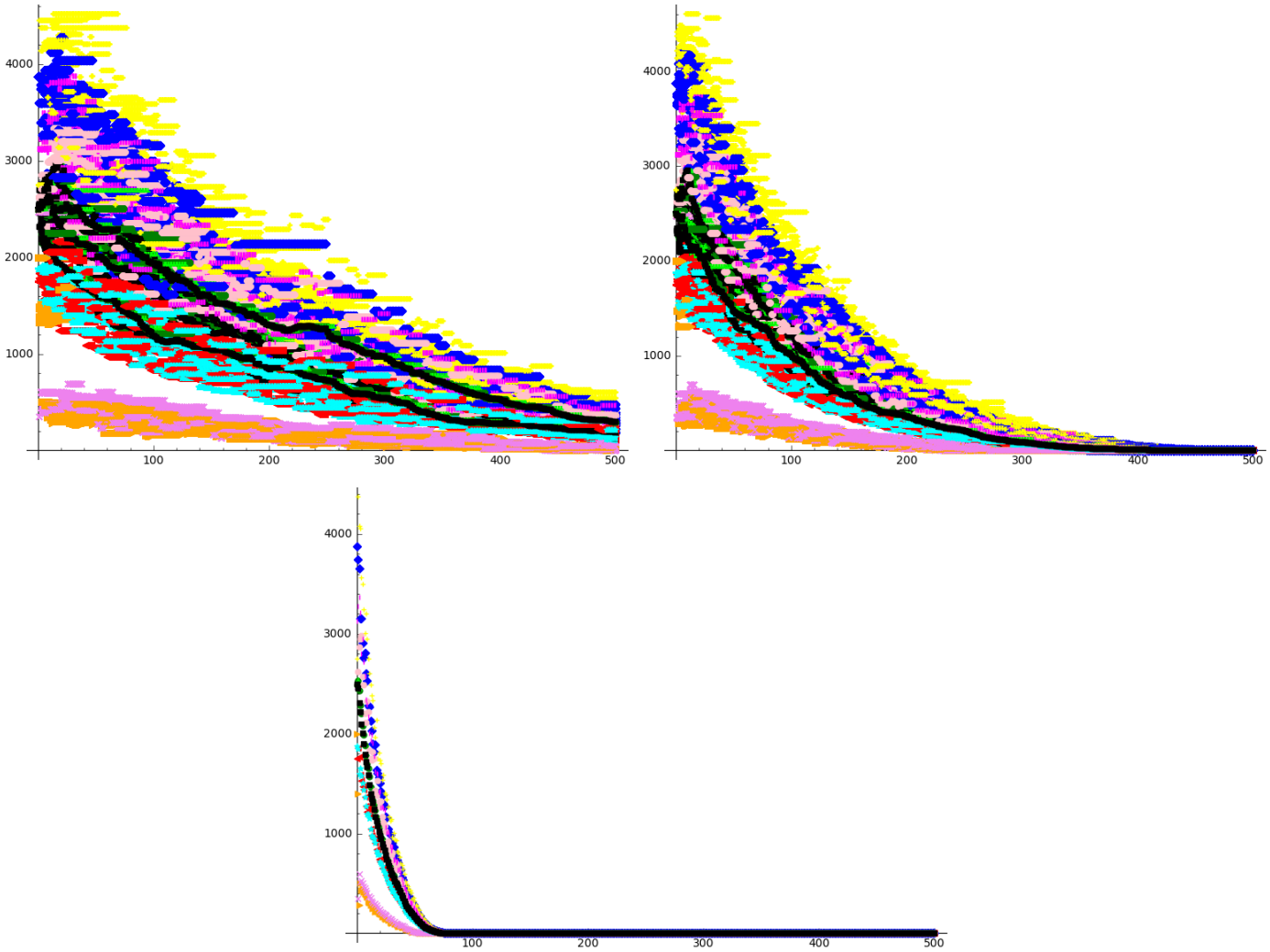
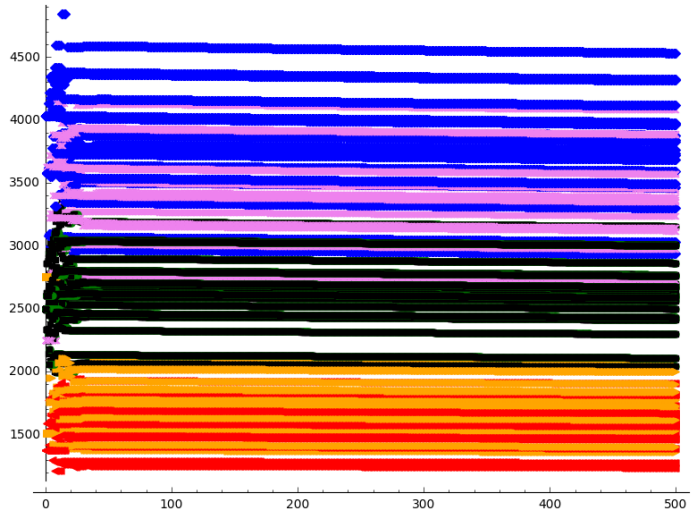
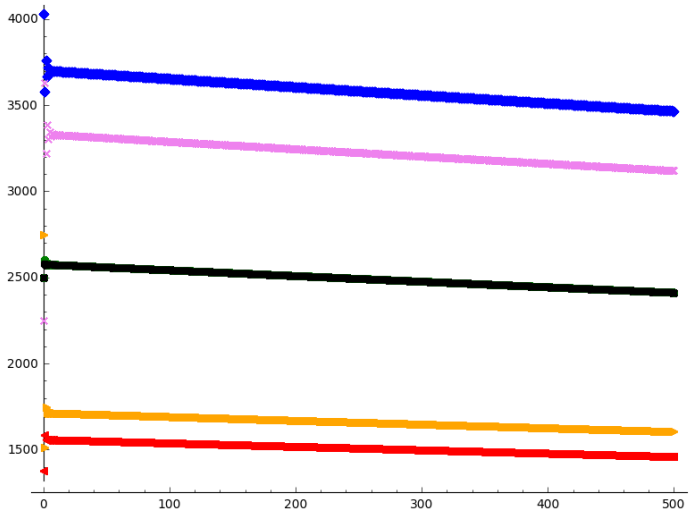


Figure 5: Data plots for Low-Skew, 10 Sellers, Nondeterministic Unbounded. Clockwise from the top left: One Update, Two Updates, All Update. The x-axis is number of rounds t; the y-axis is price in dollars. Market Price in black.

Results from a No-Skew Market

	Deterministic Unbounded	Deterministic Bounded	Deterministic Mix
Seller 1	$f_1(\mathbf{x}_{[n]}) = \underset{i}{\text{ave}}\{x_i\}$	$g_1(\mathbf{x}_{[n]}) = \max\{0.3(smp), f_1(\mathbf{x}_{[n]})\}$	$h_1(\mathbf{x}_{[n]}) = g_1(\mathbf{x}_{[n]})$
Seller 2	$f_2(\mathbf{x}_{[n]}) = 1.613\left(\underset{i}{\text{ave}}\{x_i\}\right)$	$g_2(\mathbf{x}_{[n]}) = \min\{4000, f_2(\mathbf{x}_{[n]})\}$	$h_2(\mathbf{x}_{[n]}) = f_2(\mathbf{x}_{[n]})$
Seller 3	$f_3(\mathbf{x}_{[n]}) = 0.55\left(\underset{i}{\text{ave}}\{x_i\}\right)$	$g_3(\mathbf{x}_{[n]}) = \max\{500, f_3(\mathbf{x}_{[n]})\}$	$h_3(\mathbf{x}_{[n]}) = f_3(\mathbf{x}_{[n]})$
Seller 4	$f_4(\mathbf{x}_{[n]}) = 1.1\,(\min_i\{x_i\})$	$g_4(\mathbf{x}_{[n]}) = \max\{0.5(mp), f_4(\mathbf{x}_{[n]})\}$	$h_4(\mathbf{x}_{[n]}) = g_4(\mathbf{x}_{[n]})$
Seller 5	$f_5(\mathbf{x}_{[n]}) = 0.9\,(\max_i\{x_i\})$	$g_5(\mathbf{x}_{[n]}) = \min\{mp + 1500, f_5(\mathbf{x}_{[n]})\}$	$h_5(\mathbf{x}_{[n]}) = f_5(\mathbf{x}_{[n]})$





Results from a No-Skew Market

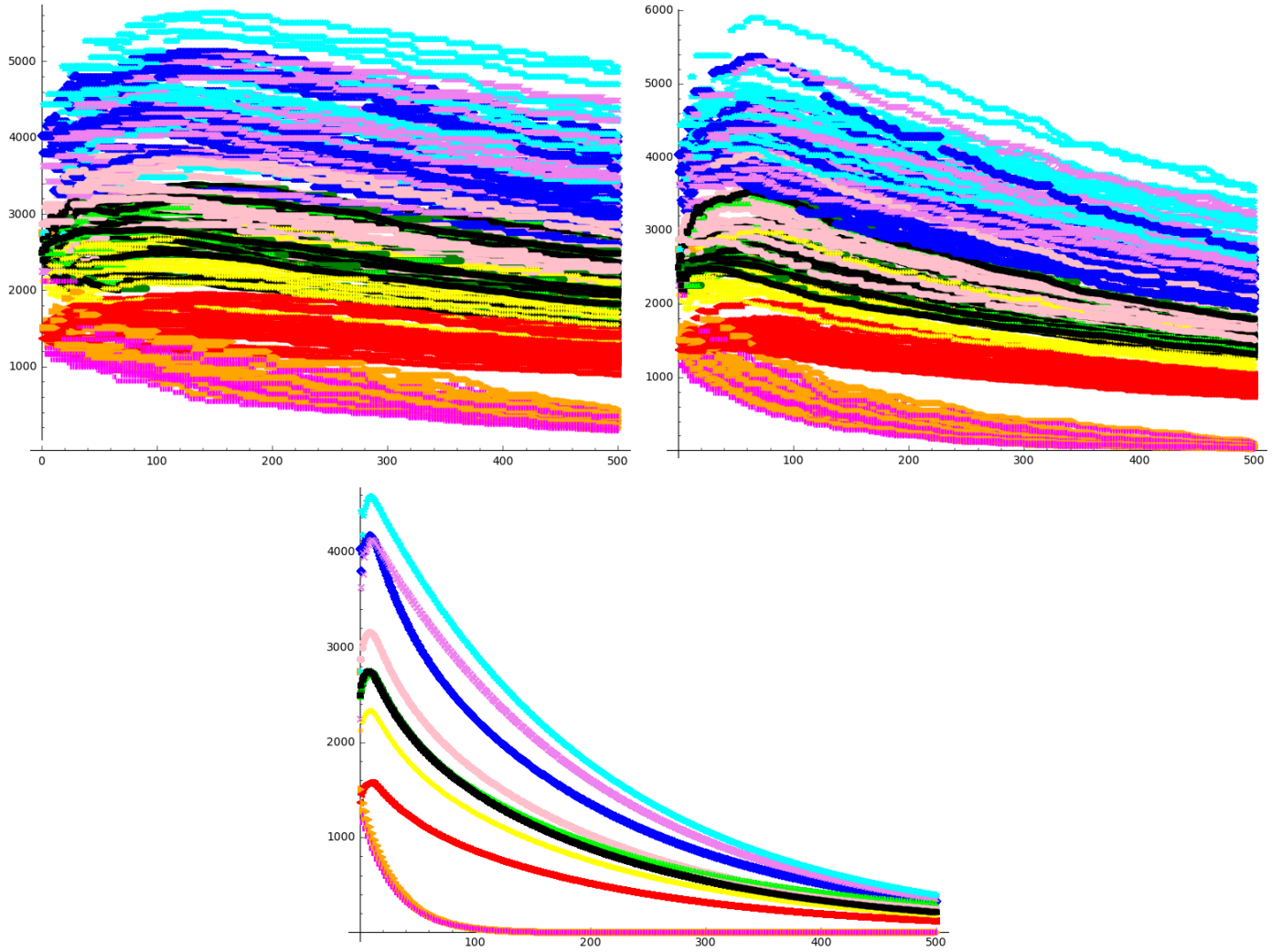


Figure 6: Data plots for No-Skew, 10 Sellers, Deterministic Unbounded. Clockwise from the top left: One Update, Two Updates, All Update. The x-axis is number of rounds t; the y-axis is price in dollars. Market Price in black.

Results from a No-Skew Market

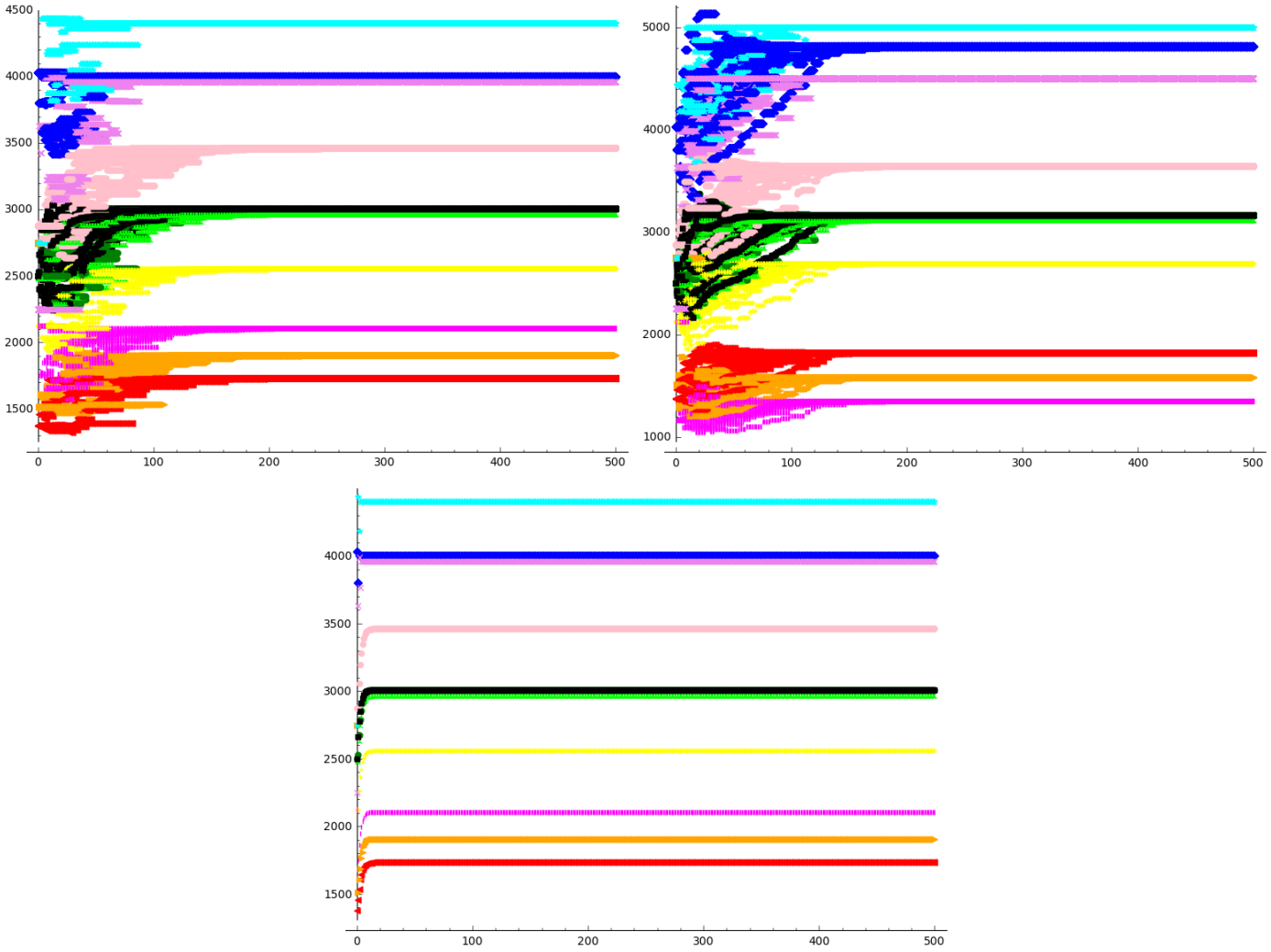


Figure 7: Data plots for No-Skew, 10 Sellers, Deterministic Bounded. Clockwise from the top left: One Update, Two Updates, All Update. The x-axis is number of rounds t; the y-axis is price in dollars. Market Price in black.

Results from a No-Skew Market

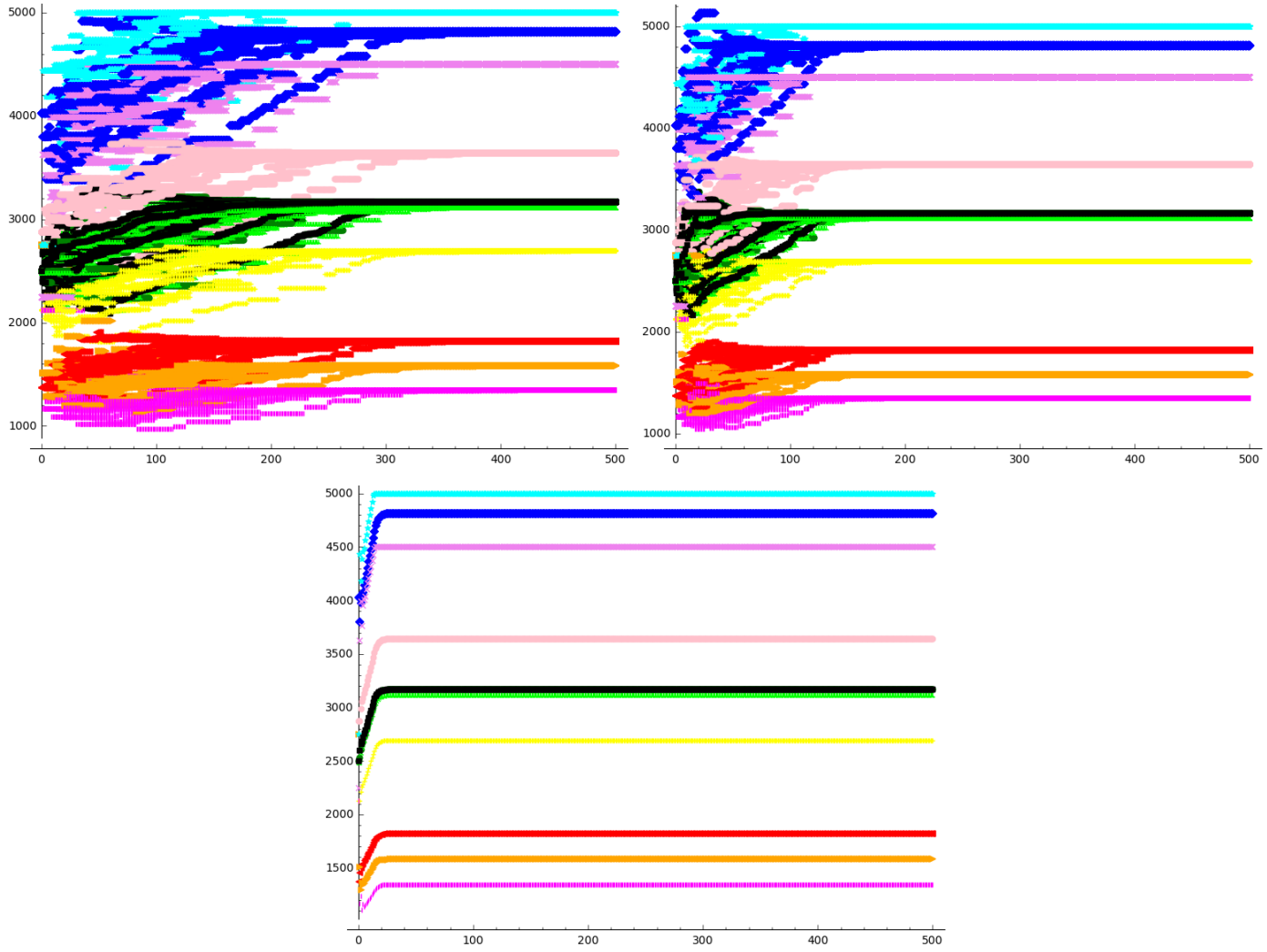


Figure 8: Data plots for No-Skew, 10 Sellers, Deterministic Mix. Clockwise from the top left: One Update, Two Updates, All Update. The x-axis is number of rounds t; the y-axis is price in dollars.

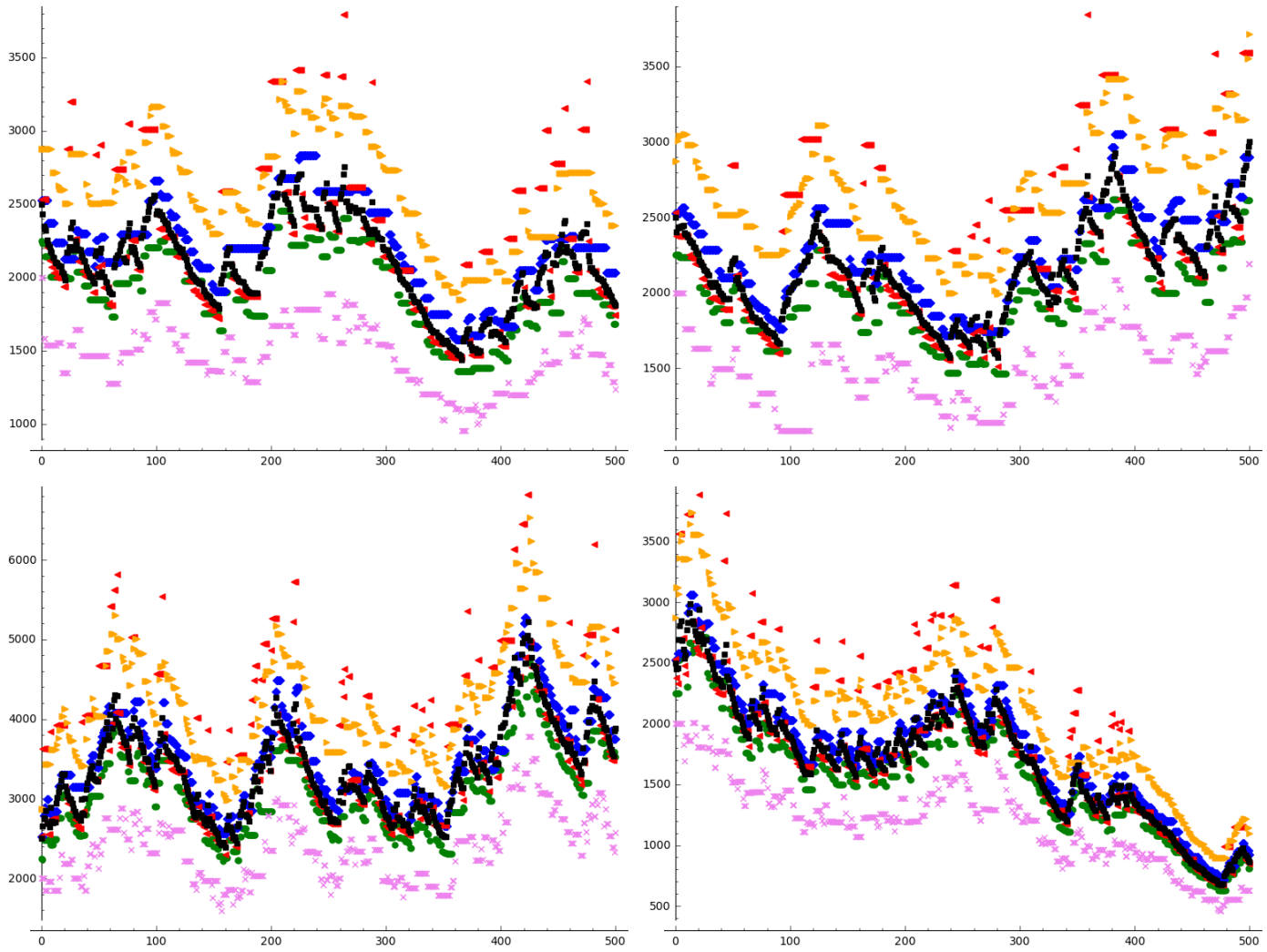
Results from Program Testing

- These results weren't formally part of the overall experiments, but are interesting to mention and put in the report.
- More or less achieved what I thought of as a "realistic" looking market.
- Has 5 sellers, mix of bounded and unbounded functions. 2 of 5 sellers have nondeterministic strategies.

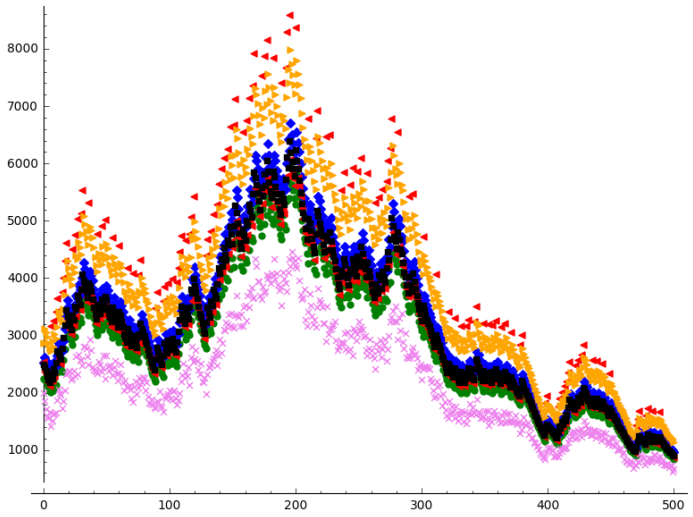
Results from Program Testing

- The functions from this testing
  - $f_1(\mathbf{x}_{[n]}) = 0.9(\text{ave}\{x_i\})$
  - $f_2(\mathbf{x}_{[n]}) = \min\{1.05(mp), 1.1(\text{ave}\{x_i\})\}$
  - $$\begin{aligned} f_3(\mathbf{x}_{[n]}) = \{ & \\ & b = \mathbf{rr}(0,4) \\ & \text{if } b == 0: \text{ return } 1.25(mp) \\ & \text{else: return } 0.95(mp) \\ \} & \end{aligned}$$
  - $f_4(\mathbf{x}_{[n]}) =$
    - do not exceed $1.2(mp)$
    - do not go below $0.85(mp)$
    - otherwise, return $\sum_i \frac{i}{n} x_i$
  - $f_5(\mathbf{x}_{[n]}) = \max\{(\mathbf{rr}(65,76)/100)(mp), 80\}$

Results from Program Testing



Results from Program Testing



Conclusions

- Pricing purely based on other sellers and market price can quickly fall prey to groups who undercut/overcut the market price.
- Even randomness in determining which sellers update their prices doesn't help much (think expectation over a large number of trials).
- Randomness in seller strategies contributes more than the randomness tested in determining seller updates (in these experiments).
- Difficult with this model to get a relatively "stable" no-skew market when trying more intuitive functions (i.e., having two seller strategies designed to cancel each other).
- Much room for other testing, different functions, different market price update strategies, different seller choice strategies, assumptions, ...

Thank You!

Any questions?

[S+17] W. A. Stein et al.
Sage Mathematics Software (Version 7.4.)*.
The Sage Development Team, 2017.
http://www.sagemath.org.