

Lecture slides for  
*Automated Planning: Theory and Practice*

**Chapter 10**  
**Control Rules in Planning**

Dana S. Nau

University of Maryland  
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# Motivation

- Often, planning can be done much more efficiently if we have domain-specific information
- Example:
  - ◆ classical planning is EXPSPACE-complete
  - ◆ block-stacking can be done in time  $O(n^3)$
- But we don't want to have to write a new domain-specific planning system for each problem!
- *Domain-configurable* planning algorithm
  - ◆ Domain-independent search engine (usually a forward state-space search)
  - ◆ Input includes domain-specific information that allows us to avoid a brute-force search
    - » Prevent the planner from visiting unpromising states

# Motivation (Continued)

- If we're at some state  $s$  in a state space, sometimes a domain-specific test can tell us that
  - ◆  $s$  doesn't lead to a solution, or
  - ◆ for any solution below  $s$ , there's a better solution along some other path
- In such cases we can to prune  $s$  immediately
- Rather than writing the domain-dependent test as low-level computer code, we'd prefer to talk directly about the planning domain
- One approach:
  - ◆ Write logical formulas giving conditions that states must satisfy; prune states that don't satisfy the formulas
- Presentation similar to the chapter, but not identical
  - ◆ Based partly on TLPlan [Bacchus & Kabanza 2000]

```
Abstract-search( $u$ )
  if Terminal( $u$ ) then return( $u$ )
   $u \leftarrow$  Refine( $u$ )           ;; refinement step
   $B \leftarrow$  Branch( $u$ )        ;; branching step
   $B' \leftarrow$  Prune( $B$ )        ;; pruning step
  if  $B' = \emptyset$  then return(failure)
  nondeterministically choose  $v \in B'$ 
  return(Abstract-search( $v$ ))
end
```

# Quick Review of First Order Logic

- First Order Logic (FOL):
  - ◆ constant symbols, function symbols, predicate symbols
  - ◆ logical connectives ( $\vee, \wedge, \neg, \Rightarrow, \Leftrightarrow$ ), quantifiers ( $\forall, \exists$ ), punctuation
  - ◆ Syntax for formulas and sentences
    - $on(A,B) \wedge on(B,C)$
    - $\exists x on(x,A)$
    - $\forall x (ontable(x) \Rightarrow clear(x))$
- First Order Theory  $T$ :
  - ◆ “Logical” axioms and inference rules – encode logical reasoning in general
  - ◆ Additional “nonlogical” axioms – talk about a particular domain
  - ◆ Theorems: produced by applying the axioms and rules of inference
- Model: set of objects, functions, relations that the symbols refer to
  - ◆ For our purposes, a model is some state of the world  $s$
  - ◆ In order for  $s$  to be a model, all theorems of  $T$  must be true in  $s$
  - ◆  $s \models on(A,B)$  read “ $s$  satisfies  $on(A,B)$ ” or “ $s$  models  $on(A,B)$ ”
    - » means that  $on(A,B)$  is true in the state  $s$

# Linear Temporal Logic

- Modal logic: FOL plus *modal operators*  
to express concepts that would be difficult to express within FOL
- Linear Temporal Logic (LTL):
  - ◆ Purpose: to express a limited notion of time
    - » An infinite sequence  $\langle 0, 1, 2, \dots \rangle$  of time instants
    - » An infinite sequence  $M = \langle s_0, s_1, \dots \rangle$  of states of the world
  - ◆ Modal operators to refer to the states in which formulas are true:
    - $\bigcirc f$  - *next f* -  $f$  holds in the next state, e.g.,  $\bigcirc on(A,B)$
    - $\Diamond f$  - *eventually f* -  $f$  either holds now or in some future state
    - $\Box f$  - *always f* -  $f$  holds now and in all future states
    - $f_1 \cup f_2$  -  $f_1$  *until*  $f_2$  -  $f_2$  either holds now or in some future state, and  $f_1$  holds until then
  - ◆ Propositional constant symbols TRUE and FALSE

# Linear Temporal Logic (continued)

- Quantifiers cause problems with computability
  - ◆ Suppose  $f(x)$  is true for infinitely many values of  $x$
  - ◆ Problem evaluating truth of  $\forall x f(x)$  and  $\exists x f(x)$
- Bounded quantifiers
  - ◆ Let  $g(x)$  be such that  $\{x : g(x)\}$  is finite and easily computed
$$\forall[x:g(x)] f(x)$$
    - means  $\forall x (g(x) \Rightarrow f(x))$
    - expands into  $f(x_1) \wedge f(x_2) \wedge \dots \wedge f(x_n)$
$$\exists[x:g(x)] f(x)$$
    - means  $\exists x (g(x) \wedge f(x))$
    - expands into  $f(x_1) \vee f(x_2) \vee \dots \vee f(x_n)$

# Models for LTL

- A model is a triple  $(M, s_i, v)$ 
  - ◆  $M = \langle s_0, s_1, \dots \rangle$  is a sequence of states
  - ◆  $s_i$  is the  $i$ 'th state in  $M$ ,
  - ◆  $v$  is a *variable assignment* function
    - » a substitution that maps all variables into constants
- Write  $(M, s_i, v) \models f$   
to mean that  $v(f)$  is true in  $s_i$
- Always require that
$$(M, s_i, v) \models \text{TRUE}$$
$$(M, s_i, v) \models \neg \text{FALSE}$$

# Examples

- Suppose  $M = \langle s_0, s_1, \dots \rangle$

$(M, s_0, v) \models \bigcirc\bigcirc on(A, B)$  means  $A$  is on  $B$  in  $s_2$

- Abbreviations:

$(M, s_0) \models \bigcirc\bigcirc on(A, B)$  no free variables, so  $v$  is irrelevant:  
 $M \models \bigcirc\bigcirc on(A, B)$  if we omit the state, it defaults to  $s_0$

- Equivalently,

$(M, s_2, v) \models on(A, B)$  same meaning with no modal operators  
 $s_2 \models on(A, B)$  same thing in ordinary FOL

- $M \models \Box \neg holding(C)$

◆ in every state in  $M$ , we aren't holding  $C$

- $M \models \Box (on(B, C) \Rightarrow (on(B, C) \cup on(A, B)))$

◆ whenever we enter a state in which  $B$  is on  $C$ ,  $B$  remains on  $C$  until  $A$  is on  $B$ .



# Where We're Going

- Basic idea:
  - ◆ TLPLan does a forward search, using LTL to do pruning tests
  - ◆ Input includes a current state  $s$ , and a **control formula**  $f$  written in LTL
    - » If  $f$  isn't satisfied, then  $s$  is unacceptable  $\Rightarrow$  backtrack
    - » Else keep going
- We'll need to augment LTL to include a way to refer to goal states
  - ◆ Include a GOAL operator such that  $\text{GOAL}(f)$  means  $f$  is true in every goal state
  - ◆  $((M, s_i, V), g) \models \text{GOAL}(f)$  iff  $(M, s_i, V) \models f$  for every  $s_i \in g$
- Next, some examples of control formulas

# Example: Blocks World

**unstack(x,y)**

Precond:  $\text{on}(x,y)$ ,  $\text{clear}(x)$ ,  $\text{handempty}$

Effects:  $\neg\text{on}(x,y)$ ,  $\neg\text{clear}(x)$ ,  $\neg\text{handempty}$ ,  
 $\text{holding}(x)$ ,  $\text{clear}(y)$

**stack(x,y)**

Precond:  $\text{holding}(x)$ ,  $\text{clear}(y)$

Effects:  $\neg\text{holding}(x)$ ,  $\neg\text{clear}(y)$ ,  
 $\text{on}(x,y)$ ,  $\text{clear}(x)$ ,  $\text{handempty}$

**pickup(x)**

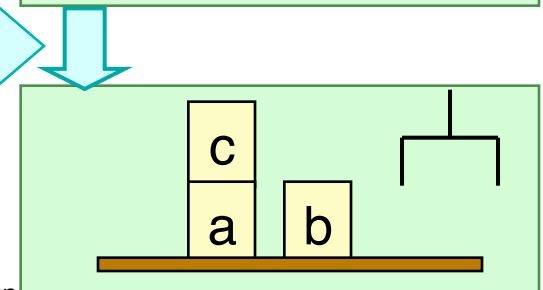
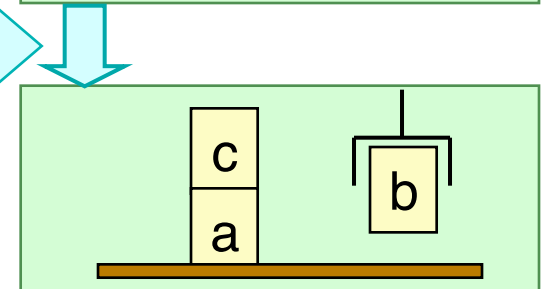
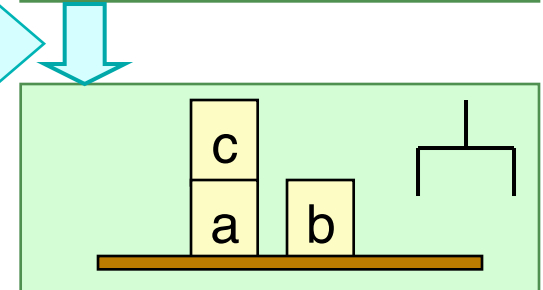
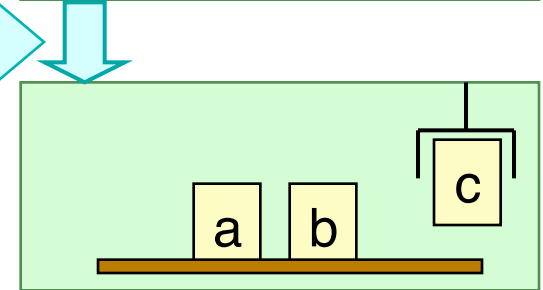
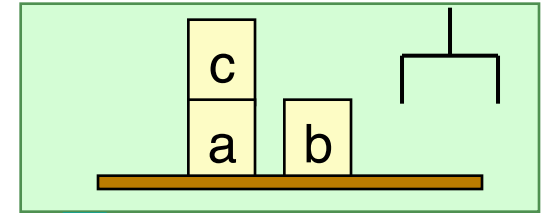
Precond:  $\text{ontable}(x)$ ,  $\text{clear}(x)$ ,  $\text{handempty}$

Effects:  $\neg\text{ontable}(x)$ ,  $\neg\text{clear}(x)$ ,  
 $\neg\text{handempty}$ ,  $\text{holding}(x)$

**putdown(x)**

Precond:  $\text{holding}(x)$

Effects:  $\neg\text{holding}(x)$ ,  $\text{ontable}(x)$ ,  
 $\text{clear}(x)$ ,  $\text{handempty}$



# Supporting Axioms

- Want to define conditions under which a stack of blocks will never need to be moved
- If  $x$  is the top of a stack of blocks, then we want  $goodtower(x)$  to hold if
  - ◆  $x$  doesn't need to be anywhere else
  - ◆ None of the blocks below  $x$  need to be anywhere else
- Definitions to support this:
  - ◆  $goodtower(x) \Leftrightarrow clear(x) \wedge \neg GOAL(holding(x)) \wedge goodtowerbelow(x)$
  - ◆  $goodtowerbelow(x) \Leftrightarrow$ 

$$[ontable(x) \wedge \neg \exists[y:GOAL(on(x,y))]]$$

$$\vee \exists[y:on(x,y)] \{ \neg GOAL(ontable(x)) \wedge \neg GOAL(holding(y))$$

$$\wedge \neg GOAL(clear(y)) \wedge \forall[z:GOAL(on(x,z))] (z = y)$$

$$\wedge \forall[z:GOAL(on(z,y))] (z = x) \wedge goodtowerbelow(y) \}$$
  - ◆  $badtower(x) \Leftrightarrow clear(x) \wedge \neg goodtower(x)$

# Blocks World Example (continued)

Three different control formulas:

(1) Every goodtower must always remain a goodtower:

$$\Box \left( \forall [x:clear(x)] goodtower(x) \Rightarrow \bigcirc (clear(x) \vee \exists [y:on(y, x)] goodtower(y)) \right)$$

(2) Like (1), but also says never to put anything onto a badtower:

$$\Box \left( \forall [x:clear(x)] goodtower(x) \Rightarrow \bigcirc (clear(x) \vee \exists [y:on(y, x)] goodtower(y) \wedge badtower(x) \Rightarrow \bigcirc (\neg \exists [y:on(y, x)])) \right)$$

(3) Like (2), but also says never to pick up a block from the table unless you can put it onto a goodtower:

$$\Box \left( \forall [x:clear(x)] goodtower(x) \Rightarrow \bigcirc (clear(x) \vee \exists [y:on(y, x)] goodtower(y)) \wedge badtower(x) \Rightarrow \bigcirc (\neg \exists [y:on(y, x)]) \wedge (ontable(x) \wedge \exists [y:GOAL(on(x, y))] \neg goodtower(y)) \Rightarrow \bigcirc (\neg holding(x)) \right)$$

# Outline of How TLPlan Works

- Recall that TLPlan's input includes a current state  $s$ , and a control formula  $f$  written in LTL
  - ◆ How can TLPlan determine whether there exists a sequence of states  $M$  beginning with  $s$ , such that  $M \models f$ ?
- We can compute a formula  $f^+$  such that for every sequence  $M = \langle s, s^+, s^{++}, \dots \rangle$ ,
  - ◆  $M \models f^+$  iff  $M^+ = \langle s^+, s^{++}, \dots \rangle$  satisfies  $f^+$
  - ◆  $f^+$  is called the **progression** of  $f$  through  $s$
- If  $f^+ = \text{FALSE}$  then no  $M^+$  can satisfy  $f^+$ 
  - ◆ Thus no  $M$  can satisfy  $f$ , so TLPlan can backtrack
- Otherwise, need to determine whether there is an  $M^+$  that satisfies  $f^+$ 
  - ◆ For every child  $s^+$  of  $s$ , call TLPlan recursively on  $s^+$  and  $f^+$
- How to compute the progression of  $f$  through  $s$ ?

## Procedure Progress( $f, s$ )

### Case

1.  $f$  contains no temporal operators:  
 $f^+ := \text{TRUE if } s \models f, \text{ FALSE otherwise.}$
2.  $f = f_1 \wedge f_2$ :  
 $f^+ := \text{Progress}(f_1, s) \wedge \text{Progress}(f_2, s)$
3.  $f = \neg f_1$ :  
 $f^+ := \neg \text{Progress}(f_1, s)$
4.  $f = \bigcirc f_1$ :  
 $f^+ := f_1$
5.  $f = f_1 \cup f_2$ :  
 $f^+ := \text{Progress}(f_2, s) \vee (\text{Progress}(f_1, s) \wedge f)$
6.  $f = \Diamond f_1$ :  
 $f^+ := \text{Progress}(f_1, s) \vee f$
7.  $f = \Box f_1$ :  
 $f^+ := \text{Progress}(f_1, s) \wedge f$
8.  $f = \forall[x:\gamma(x)] f_1$ :  
 $f^+ := \bigwedge_{i=1, \dots, n} \text{Progress}(f_i, s)$
9.  $f = \exists[x:\gamma(x)] f_1$ :  
 $f^+ := \bigvee_{i=1, \dots, n} \text{Progress}(f_i, s)$

where  $\{c_1, \dots, c_n\} = \{x : s \models \gamma(x)\}$ , and  $f_i = f$  with  $x$  replaced by  $c_i$

### Boolean simplification rules:

1.  $[\text{FALSE} \wedge \phi | \phi \wedge \text{FALSE}] \mapsto \text{FALSE},$
2.  $[\text{TRUE} \wedge \phi | \phi \wedge \text{TRUE}] \mapsto \phi,$
3.  $\neg \text{TRUE} \mapsto \text{FALSE},$
4.  $\neg \text{FALSE} \mapsto \text{TRUE}.$

# Examples

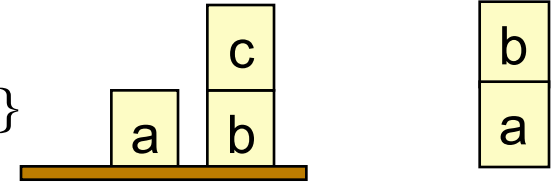
- Suppose  $f = \Box \text{on}(a,b)$ 
  - ◆  $f^+ = \text{Progress}(\text{on}(a,b), s) \wedge \Box \text{on}(a,b)$
  - ◆ If  $\text{on}(a,b)$  is true in  $s$  then
    - »  $f^+ = \text{TRUE} \wedge \Box \text{on}(a,b)$
    - » simplifies to  $\Box \text{on}(a,b)$
  - ◆ If  $\text{on}(a,b)$  is false in  $s$  then
    - »  $f^+ = \text{FALSE} \wedge \Box \text{on}(a,b)$
    - » simplifies to FALSE
- Summary:
  - ◆  $\Box$  generates a test on the current state
  - ◆ If the test succeeds,  $\Box$  propagates it to the next state

## Examples (continued)

- Suppose  $f = \Box(on(a,b) \Rightarrow \bigcirc clear(a))$ 
  - ◆  $f^+ = \text{Progress}[\Box(on(a,b) \Rightarrow \bigcirc clear(a)), s]$
  - ◆  $= \text{Progress}[on(a,b) \Rightarrow \bigcirc clear(a), s] \wedge \Box(on(a,b) \Rightarrow \bigcirc clear(a))$
  - ◆ If  $on(a,b)$  is true in  $s$ , then
    - »  $f^+ = clear(a) \wedge \Box(on(a,b) \Rightarrow \bigcirc clear(a))$ 
      - Since  $on(a,b)$  is true in  $s$ ,  
 $s^+$  must satisfy  $clear(a)$
      - The “always” constraint is propagated to  $s^+$
  - ◆ If  $on(a,b)$  is false in  $s$ , then
    - »  $f^+ = \Box(on(a,b) \Rightarrow \bigcirc clear(a))$ 
      - The “always” constraint is propagated to  $s^+$



# Example

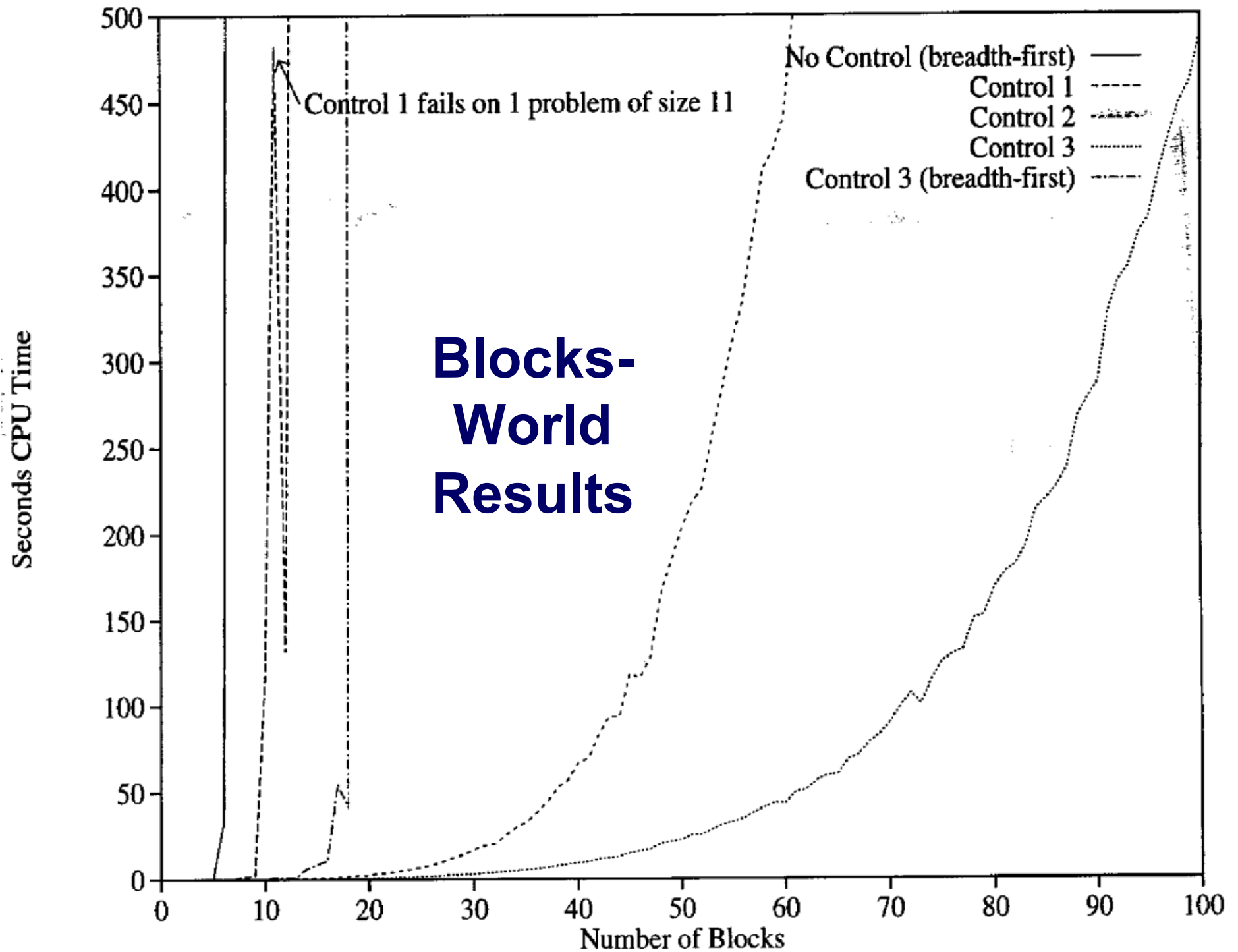


- $s = \{ontable(a), ontable(b), clear(a), clear(c), on(c,b)\}$
- $g = \{on(b, a)\}$
- $f = \Box \forall [x:clear(x)] \{ (ontable(x) \wedge \neg \exists [y:GOAL(on(x,y))]) \Rightarrow O \neg holding(x) \}$ 
  - ◆ never pick up a block  $x$  if  $x$  is not required to be on another block  $y$
- $f^+ = Progress(f,s) \wedge f$
- $Progress(f,s)$ 
  - $= Progress(\forall [x:clear(x)] \{ (ontable(x) \wedge \neg \exists [y:GOAL(on(x,y))]) \Rightarrow O \neg holding(x) \}, s)$
  - $= Progress((ontable(a) \wedge \neg \exists [y:GOAL(on(a,y))]) \Rightarrow O \neg holding(a)), s)$
  - $\quad \wedge Progress((ontable(b) \wedge \neg \exists [y:GOAL(on(b,y))]) \Rightarrow O \neg holding(b)), s)$
  - $= \neg holding(a) \wedge TRUE$
- $f^+ = \neg holding(a) \wedge TRUE \wedge f$ 
  - $= \neg holding(a) \wedge$
  - $\quad \Box \forall [x:clear(x)] \{ (ontable(x) \wedge \neg \exists [y:GOAL(on(x,y))]) \Rightarrow O \neg holding(x) \}$

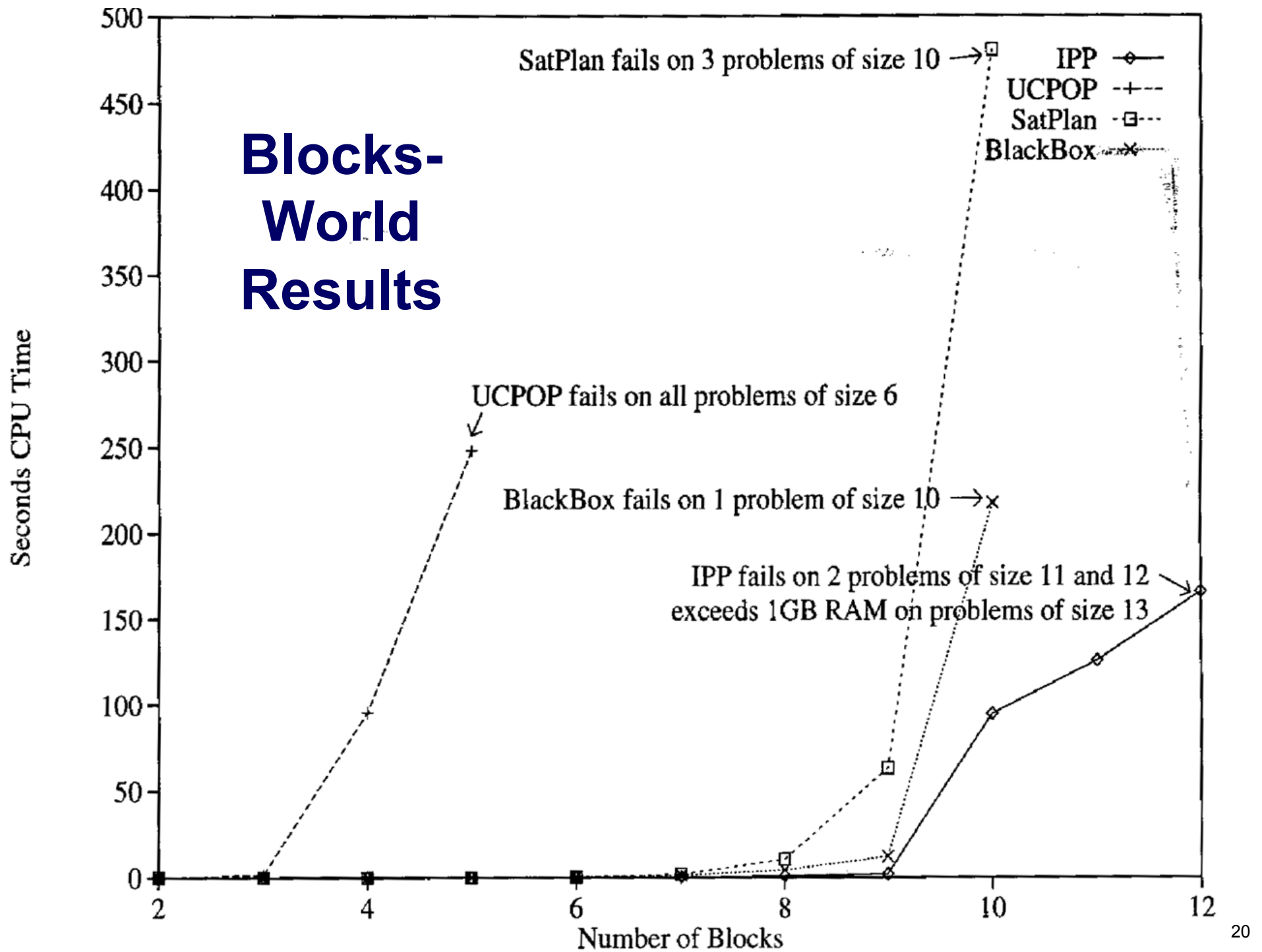
# Pseudocode for TLPlan

- Nondeterministic forward search
  - ◆ Input includes a control formula  $f$  for the current state  $s$
  - ◆ When we expand a state  $s$ , we progress its formula  $f$  through  $s$
  - ◆ If the progressed formula is false,  $s$  is a dead-end
  - ◆ Otherwise the progressed formula is the control formula for  $s$ 's children

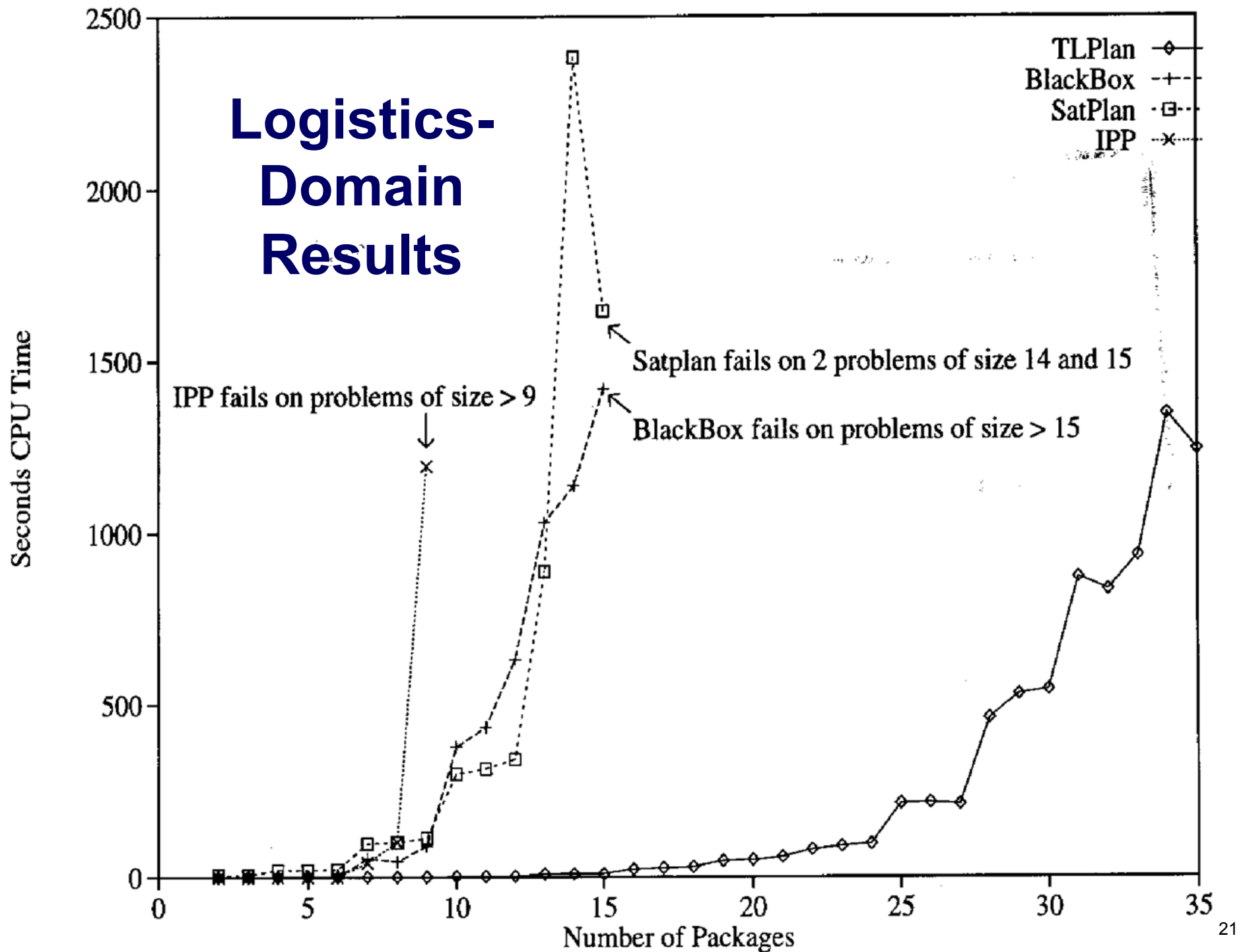
```
Procedure TLPlan ( $s, f, g, \pi$ )  
     $f^+ \leftarrow \text{Progress}(f, s)$   
    if  $f^+ = \text{FALSE}$  then return failure  
    if  $s$  satisfies  $g$  then return  $\pi$   
     $A \leftarrow \{\text{actions applicable to } s\}$   
    if  $A = \text{empty}$  then return failure  
    nondeterministically choose  $a \in A$   
     $s^+ \leftarrow \gamma(s, a)$   
    return TLPlan ( $s^+, f^+, g, \pi.a$ )
```



# Blocks-World Results



# Logistics- Domain Results



# Discussion

- 2000 International Planning Competition
  - ◆ TALplanner: same kind of algorithm, different temporal logic
    - » received the top award for a “hand-tailored” (i.e., domain-configurable) planner
- TLPlan won the same award in the 2002 International Planning Competition
- Both of them:
  - ◆ Ran several orders of magnitude faster than the “fully automated” (i.e., domain-independent) planners
    - » especially on large problems
  - ◆ Solved problems on which the domain-independent planners ran out of time/memory