

Anisotropic representations for superresolution of hyperspectral data

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Goals of Talk

- ➊ Introduce the notion of *superresolution*.
- ➋ Briefly survey the mathematical discipline of anisotropic harmonic analysis.
- ➌ Describe a superresolution algorithm based on *shearlets*.
- ➍ Evaluate the results of superresolution experiments on synthetic and hyperspectral data.

Background on Superresolution (1/2)

- The goal of superresolution is to increase the resolution of an image I , while preserving detail and without producing artifacts.
- The outcome of a superresolution algorithm is an image \tilde{I} , which is of the same scene as I , but at a higher resolution.
- Let I be an $M \times N$ matrix and \tilde{I} an $\tilde{M} \times \tilde{N}$ matrix, with $M < \tilde{M}$, $N < \tilde{N}$. We consider the common case where $\tilde{M} = 2M$ and $\tilde{N} = 2N$, which corresponds to doubling the resolution of the original image.
- Images with multiple channels, such as hyperspectral images, can be superresolved by superresolving each channel separately.

Background on Superresolution (2/2)

- Superresolution can be implemented by using information in addition to I , such as low resolution images at sub-pixel shifts of the scene or images of the scene with different modalities.
- There are several standard approaches to superresolving I without using additional information. Among the most common are *nearest neighbor interpolation* and *bicubic interpolation*.
- In the case of nearest neighbor interpolation, new pixel values are computed by replicating current pixel values. This method is simple and computationally efficient, but leads to extremely jagged superresolved images.
- Other methods involve convolving the image with an interpolation kernel, which amounts to taking a weighted average of pixel values within some neighborhood. For example, bicubic interpolation determines \tilde{I} by computing each new pixel as a weighted average of the 16 nearest neighbors in I .

- Harmonic analysis decomposes signals into simpler elements.
- Classical methods include Fourier series and wavelets. These have proven extremely influential and quite effective for many applications¹.
- However, they are fundamentally *isotropic*, meaning they decompose signals without considering how the signal varies *directionally*.
- In a broad sense, wavelets decompose an image signal with respect to *translation* and *scale*. Since the early 2000s, there have been several attempts to incorporate directionality into the wavelet construction.

¹I. Daubechies. *Ten lectures on wavelets*. Society for industrial and applied mathematics, 1992.

Shearlets and Anisotropic Harmonic Analysis (2/3)

- These constructions incorporate directionality in a variety of ways.
- Some of the major constructions include:
 - Curvelets (Donoho and Candès)².
 - Contourlets (Do and Vetterli)³.
 - Shearlets (Guo, Labate, Kutyniok, Weiss, et al.)⁴.
- We consider shearlets for superresolution, since they have several efficient numerical implementations in MATLAB⁵.

²E. J. Candès and D. L. Donoho. *New tight frames of curvelets and optimal representations of objects with piecewise \mathcal{C}^2 singularities*. Communications on pure and applied mathematics, 57(2):219–266, 2004.

³M. N. Do and M. Vetterli. Contourlets: a directional multiresolution image representation. In Proceedings of 2002 IEEE International Conference on Image Processing, 2002.

⁴D. Labate, W. Q. Lim, G. Kutyniok, and G. Weiss. Sparse multidimensional representation using shearlets. In Proceedings of International Society for Optics and Phototonics: Optics and Phototonics, 2005.

⁵S. Häuser. Fast finite shearlet transform. arXiv preprint, arXiv:1202.1773, 2012.

Shearlets and Anisotropic Harmonic Analysis (3/3)

- Mathematically, given a signal $f \in L^2([0, 1]^2)$ and an appropriate shearlet function ψ , we may decompose f as

$$f = \sum_{i \in \mathbb{Z}} \sum_{j \in \mathbb{Z}} \sum_{k \in \mathbb{Z}^2} \langle f, \psi_{i,j,k} \rangle \psi_{i,j,k}, \quad (1)$$

where: $\psi_{i,j,k}(x) := 2^{\frac{3i}{4}} \psi(B^j A^i x - k)$, $A = \begin{pmatrix} 2 & 0 \\ 0 & 2^{\frac{1}{2}} \end{pmatrix}$, $B = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$.

- Note that A is anisotropic, hence it will allow our new analyzing functions to be more pronounced in a particular direction. The new matrix B , a shearing matrix, lets us select the direction.
- The anisotropic character of shearlets has proven useful for a variety of problems in image processing, including image denoising⁶ and image registration⁷.

⁶G.R. Easley, D. Labate, and F. Colonna. *Shearlet-based total variation diffusion for denoising*. Image Processing, IEEE Transactions on 18.2 (2009): 260-268.

⁷J.M. Murphy and J. Le Moigne. *Anisotropic Features for Registration of Multitemporal Data*. IEEE Geoscience and Remote Sensing Society, the International Geoscience and Remote Sensing Symposium. 2015.

Superresolution and Harmonic Analysis

- Harmonic analysis has been applied to image superresolution, for its ability to capture essential aspects of images.
- Particular instances include:
 - Wavelets ⁸.
 - TV-regularization ⁹.
 - Circulant matrices and frame theory ¹⁰.
- The method of circulant matrices is anisotropic; it deduces directional content of an image by adaptively constructing tight frames.

⁸G. Anbarjafari and H. Demirel. *Image super resolution based on interpolation of wavelet domain high frequency subbands and the spatial domain input image*. ETRI journal 32.3 (2010): 390-394.

⁹A. Marquina and S.J. Osher. *Image super-resolution by TV-regularization and Bregman iteration*. Journal of Scientific Computing 37.3 (2008): 367-382.

¹⁰E.H. Bosch, A. Castrodad, J.S. Cooper, W. Czaja, and J. Dobrosotskaya. *Tight frames for multiscale and multidirectional image analysis*. In SPIE Defense, Security, and Sensing, pp. 875004-875004. International Society for Optics and Photonics, 2013.

Algorithm Overview

- The approach taken in our new algorithm is to use anisotropic shearlets to determine the directional content present in an image.
- - 1 An image is up-sampled with a conventional, isotropic superresolution method, for example nearest neighbor or bicubic interpolation.
 - 2 The shearlet transform is then applied to the image, and dominant directions are computed based on these coefficients.
 - 3 This information is then used to smoothly blur the image in certain directions locally, to provide a smoother superresolved image.
- Our algorithm for shearlet-based superresolution is coded in MATLAB.

Overview of Synthetic Experiments

- To test our algorithms, we first considered synthetic experiments.
- Since our algorithms incorporate anisotropic information, we wanted to study their efficacy on images that have very prominent directional content.
- We first constructed 1024×1024 half planes in MATLAB, at various slopes. Two such half planes appear below.

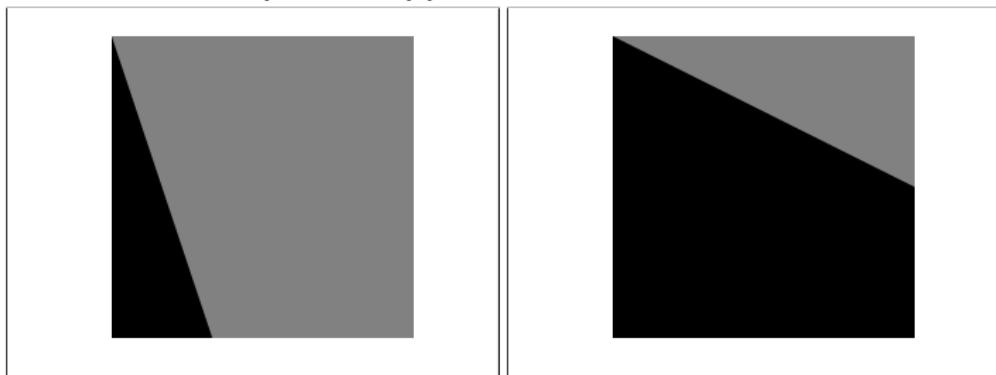


Figure: Half planes of slope -3 and $-.5$, respectively.

Results of Synthetic Experiments

- We evaluate our algorithm by computing the PSNR for each plane, superresolved with each of four techniques: our shearlets algorithm with 16 and 32 directions, the circulant matrices method of Bosch et al., and bicubic interpolation.

Planar Slope	PSNR Shearlet (16 directions)	PSNR Shearlet (32 directions)	PSNR Circulant Matrices	PSNR Bicubic
0.20	34.3692	34.2751	34.5594	34.2194
0.25	34.3305	34.2185	34.1892	34.2100
0.33	35.4151	35.4039	35.3457	35.0192
0.50	34.8050	35.5906	33.4600	34.3771
1.00	35.6006	35.7117	36.8685	35.3984
2.00	36.6007	35.3213	36.6829	35.9840
3.00	34.4332	34.5253	35.5237	34.2145
4.00	35.6012	34.9848	34.8555	34.7286
5.00	35.3447	34.8927	34.6148	34.6299

Table: The PSNR values for various methods on angled half planes.

- We notice that the anisotropic harmonic analysis methods outperform bicubic interpolation in all cases.

Overview of Hyperspectral Experiments

- We also tested our algorithm on real data. We considered a hyperspectral data set of the University of Houston¹¹, which consists of 144 bands of size 349×1905 .



- This dataset has a spatial resolution of 2.5 m, and spectral resolution of between 380 nm and 1050 nm. For convenience, we extracted a 256×256 subset from band 70.

¹¹ IEEE Data Fusion Contest. <http://www.grss-ieee.org/community/technical-committees/data-fusion/>. 2013

Results of Hyperspectral Experiments (1/3)

- As before, we ran our algorithm for superresolution and compared it to the circulant matrix method and bicubic interpolation. We only considered the shearlet algorithm with 16 directions, due to the size of the image.

PSNR Shearlet (16 directions)	PSNR Circulant Matrices	PSNR Bicubic
29.2720	29.2561	29.2125

Table: The PSNR values for the University of Houston scene.

- The shearlet method produces optimal PSNR for this hyperspectral image.

Results of Hyperspectral Experiments (2/3)



Figure: Image of Houston, superresolved with bicubic interpolation (left) and shearlet algorithm (right). Notice the smoother edges in the case of shearlet superresolution.

Results of Hyperspectral Experiments (3/3)



Figure: Subset from image of Houston, superresolved with bicubic interpolation (left) and shearlet algorithm (right). Notice the smoother edges around circular and rectangular buildings.

Experiment Conclusions

- We described a shearlet-based algorithm for the superresolution of images.
- We performed the algorithm on simple synthetic examples of half planes. We compared our algorithm at two different scales with bicubic interpolation and a recent circulant matrix method.
- Our shearlet algorithm tested the best in 5 out of 9 of the half-plane examples, while the circulant matrix method performed the best in the remaining 4 examples. This indicates that algorithms based on anisotropic harmonic analysis outperform bicubic interpolation in all iterations of our half plane experiments.
- We also applied our algorithm to one channel of a hyperspectral image. Compared to bicubic interpolation, our algorithm produced an image having fewer jagged edges and in fact also has improved PSNR.

Summary and Future Directions (1/2)

- In general, anisotropic harmonic provides a powerful set of techniques for superresolution, both in terms of PSNR and visual quality.
- One of the greatest challenges for our algorithm is superresolving images with many textures without degrading the PSNR.
- In future work, we would like to find a method for filtering out the textures so as to only smooth the edges.
- In addition, we would like to consider more sophisticated ways of improving edges beyond motion blurring, which tends to decrease image sharpness.
- State-of-the-art methods, incorporating directional interpolation and statistical methods, have been proposed by Mallat et al.¹², and shall be studied.

¹²S. Mallat and G. Yu. *Super-resolution with sparse mixing estimators*. IEEE Transactions on Image Processing 19.11 (2010): 2889-2900.

Summary and Future Directions (2/2)

- It is also of interest to study *data cubes*, such as full hyperspectral or lidar data, with anisotropic harmonic analysis.
- These would require $3D$ transform methods that are more general than the ones described in this talk.
- Implementations of $3D$ shearlets have been recently developed, and are being implemented for the problem of $3D$ superresolution of hyperspectral and lidar data.