

Eppur Si Muove – Stellar Parallax?

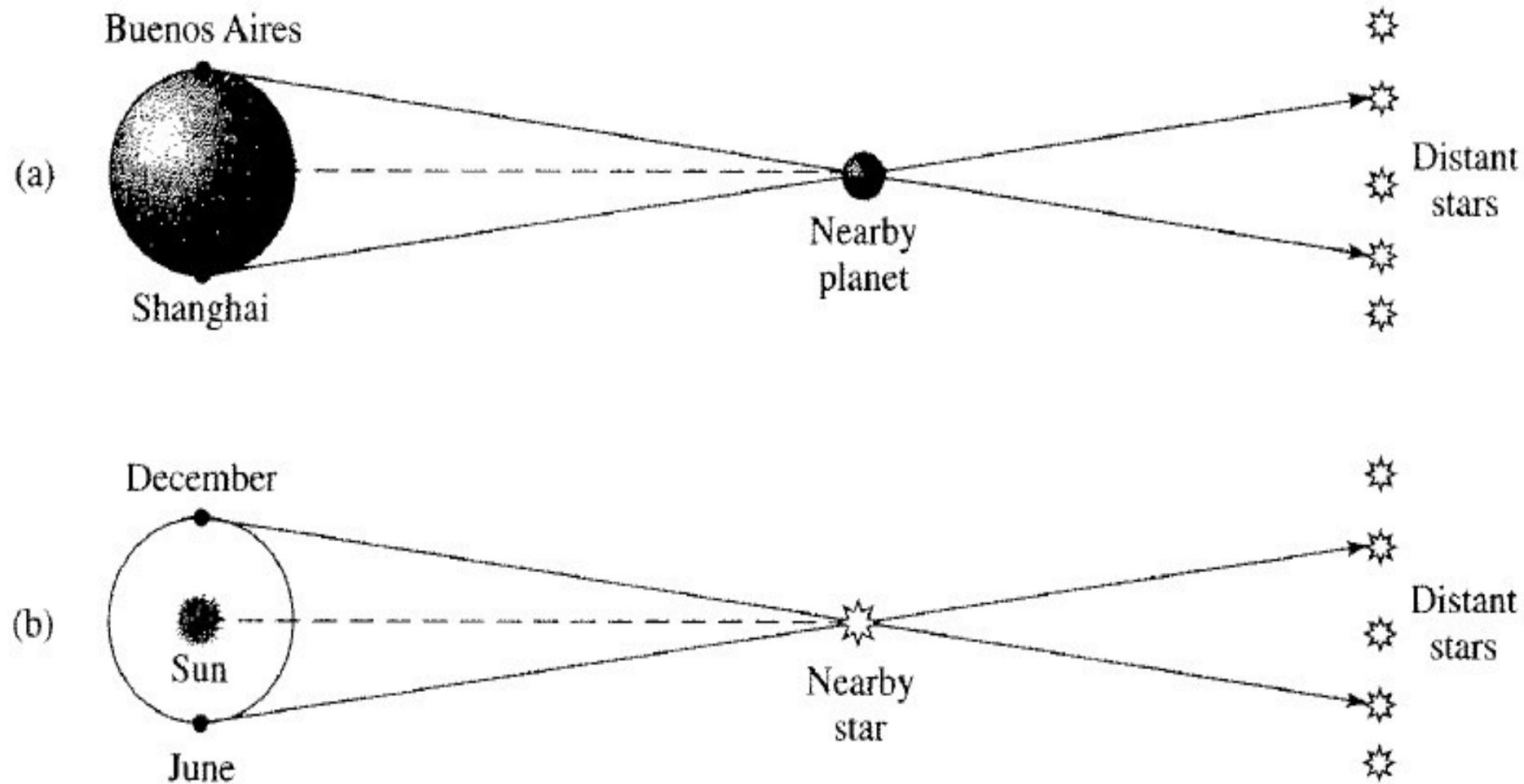
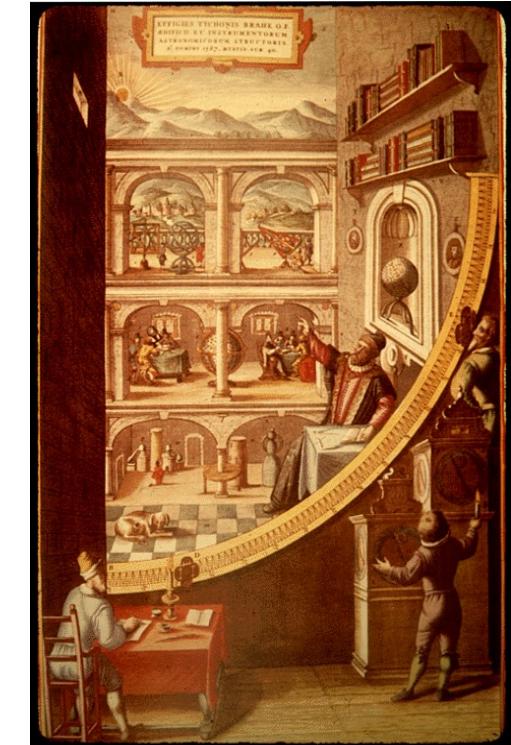


FIGURE 2.5 (a) Geocentric, or diurnal, parallax due to a change in position relative to the Earth's center. (b) Heliocentric, or annual, parallax due to a change in position relative to the Sun.

Eppur Si Muove – Stellar Parallax?

- As early as the time of Copernicus observational astronomers began an earnest search for parallax.
- Tycho Brahe – one of the best and most precise observers of the pre-telescopic era (correctly) dismissed claims of observation of parallax as purely observational error.
 - the best non-telescopic precision being about 1 arcminute (60 arcseconds)
 - the Sun (and Moon) have angular diameters of $\frac{1}{2}$ degree (30 arcminutes)



Eppur Si Muove – Stellar Parallax?

- Even with the introduction of the telescope, true parallax remained elusive.
- Beginning in about 1680, with the observations of Jean Picard (really!... actually Jean Felix Picard) telescopic observers began to notice something fishy was going on associated with Earth's orbit around the Sun.
 - Polaris seemed to be displaced in different directions at different times of year by tens of arcseconds.

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Just how big is an arcsecond??

The Small Angle Approximation

An exact expression for $\sin(\theta)$ via Taylor Series is...

$$\sin \theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} \dots$$

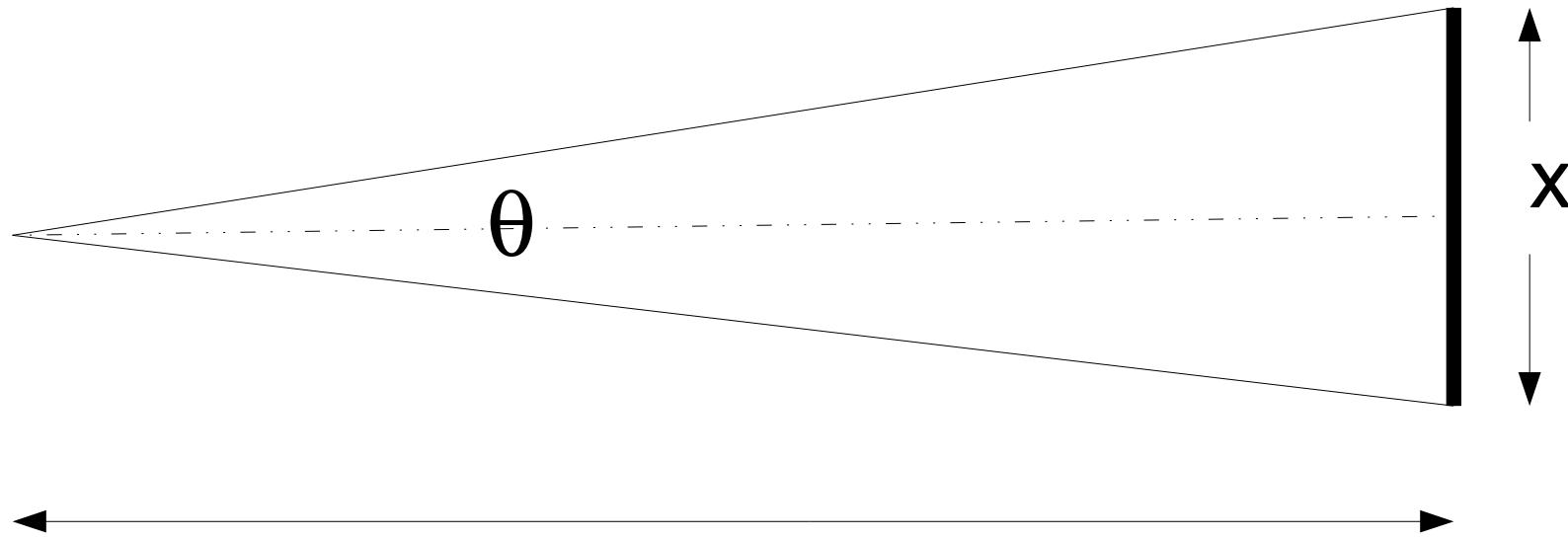
For small values of theta (expressed in radians)

$$\sin \theta \simeq \theta \text{ with error of order } \theta^3$$

- If θ is 0.1 radian (5.7 degrees) the error is already <0.1%
- At these small angles $\tan \theta \simeq \sin \theta$

The Small Angle Approximation

$\sin \theta \simeq \theta \simeq \tan \theta$ with error of order θ^3



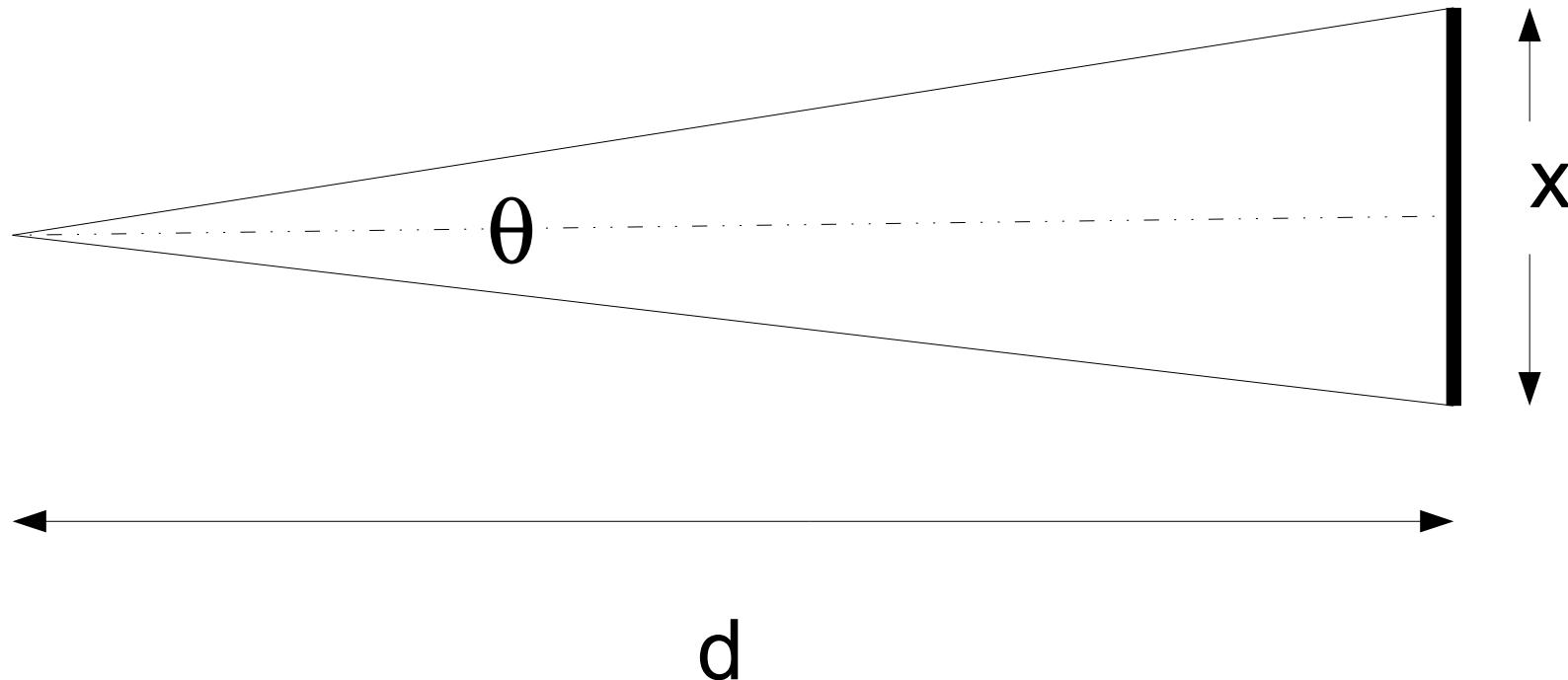
d

Observing an object of physical dimension, x, at distance, d

$$\tan \frac{\theta}{2} = \frac{x/2}{d}$$

The Small Angle Approximation

$$\sin \theta \simeq \theta \simeq \tan \theta \text{ with error of order } \theta^3$$



Observing an object of physical dimension, x , at distance, d

$$\frac{\theta}{2} \simeq \frac{x/2}{d} \quad \text{or} \quad \theta \simeq \frac{x}{d}$$

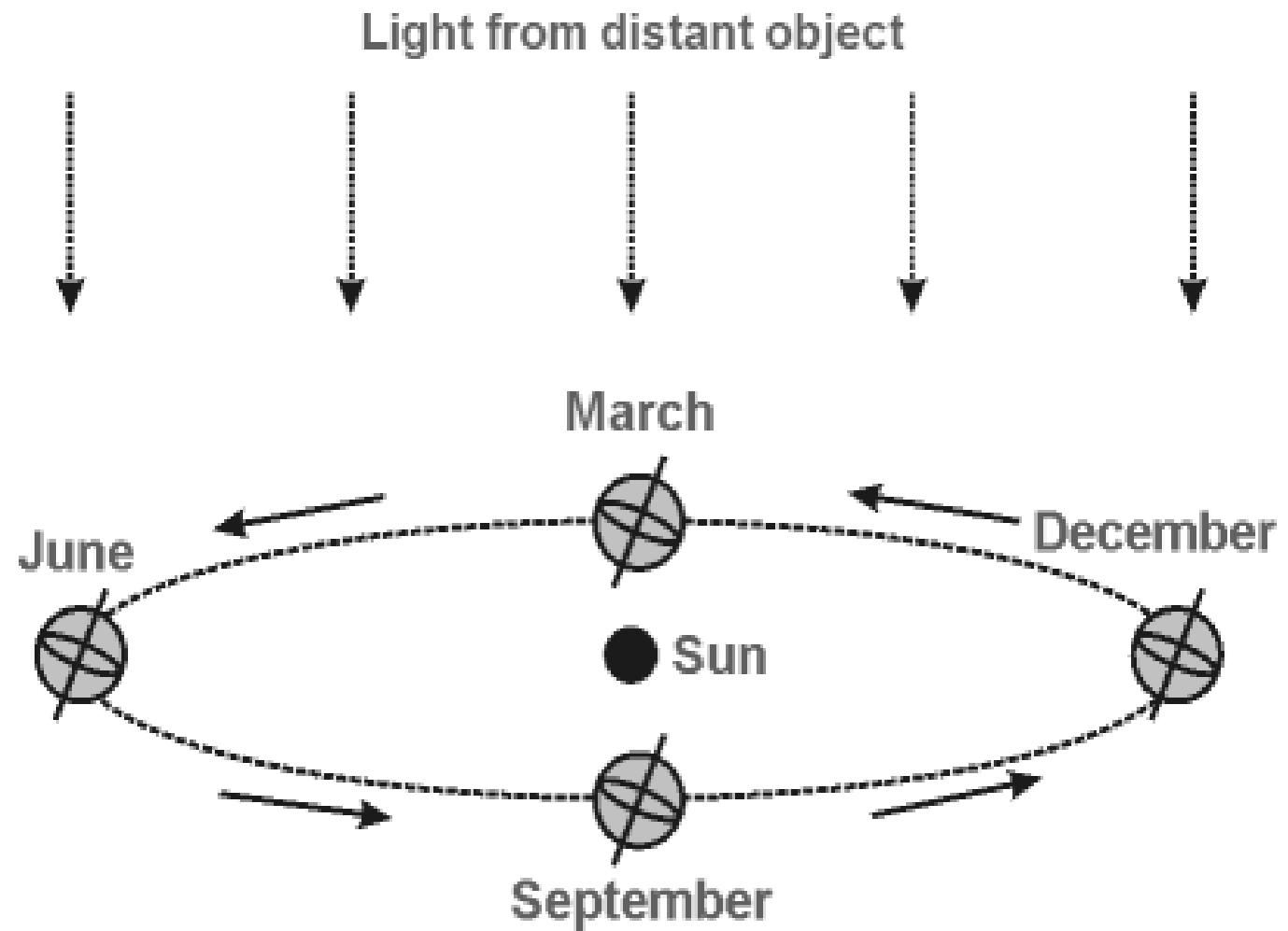
Small Angles

- Angular relationships
 - 57.293... degrees in a radian
 - 60 arcminutes in a degree
 - 60 arcseconds in an arcminute
 - 3600 arcseconds in a degree
 - 206265 arcseconds in a radian (one of the most important numbers)
- Angular references
 - Your fist at arms length is ~10 degrees. Your index finger is a little more than a degree (measure it yourself – try covering the Moon with your finger)
 - The Moon and Sun have angular sizes of $\frac{1}{2}$ degree
 - The average angular size of Jupiter is 40 arcseconds.
 - The average angular size of Pluto is 0.1 arcsecond.

Eppur Si Muove – Aberration of Starlight



James Bradley
(1693-1762)



James Bradley recognized the relatively large changes in stellar positions resulted from the speed of Earth's motion relative to the speed of light – like raindrops seen falling at an angle due to the observer's velocity.

Eppur Si Muove – Aberration of Starlight



James Bradley
(1693-1762)

$$\theta_{ab} = \frac{v}{c}$$

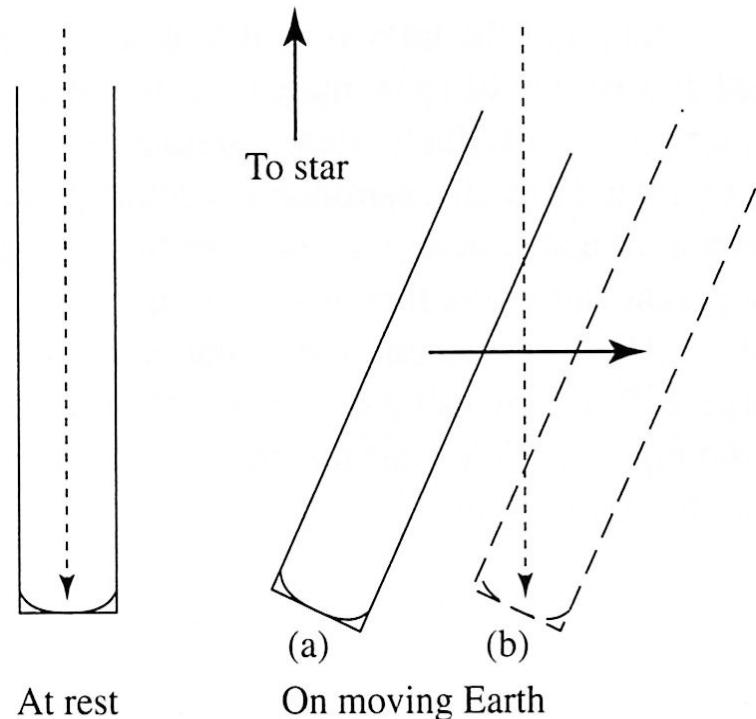


FIGURE 2.22 Telescopes must be tilted in the direction of the Earth's motion by an angle $\theta \approx v/c$ to assure that photons arrive at point P at the same time as the bottom of the telescope.

He strapped a telescope to a chimney and made precise measurements of Gamma Draconis (declination +51°) from London (latitude +51°)

Stellar Parallax

- Actual parallax was finally observed by Fredrich Bessel in 1838. The target was the obscure star 61 Cygni.
 - 61 Cygni has a huge **proper motion** – 5 arcseconds per year – making it a likely candidate for being nearby
 - The observed parallax was 0.3". 61 Cygni was three **parsecs** (600,000 AU) away.

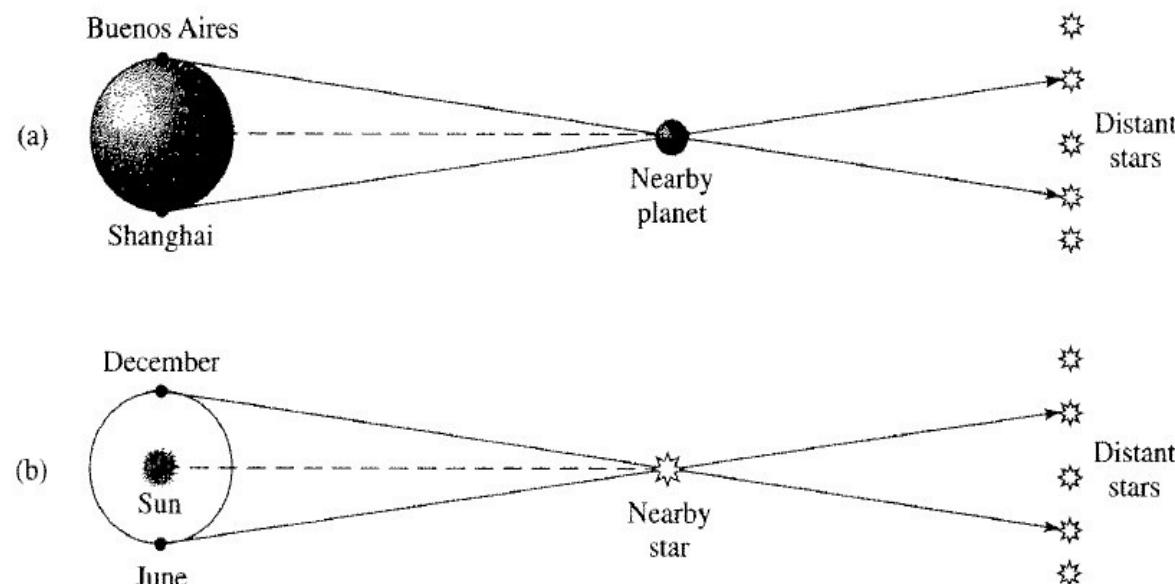


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What's a Parsec?

- In terms of other units one parsec is 3.26 light years.
- More importantly, a parsec is a unit of measure defined by the Earth's orbit and an arcsecond.
 - Consider a star's parallax motion observed as the Earth shifts in position by 1 AU (half of the maximum shift that can be observed across the diameter of the Earth's orbit)
 - If that star is placed at a distance where the shift due to 1AU displacement of the Earth is one arcsecond the star is, by definition, one parsec away.
 - One parsec = 206265 AU – there's that number again, the number of arcseconds in a radian....
- Some distances:
 - It is 1.3 parsecs, more than 250,000 AU to the nearest star system – Alpha Centauri. The dozen or so nearest stars are a few parsecs away.
 - The Orion nebula star forming region is 500 pc away. The center of the Galaxy is 8,500 pc (~30,000 light years) away.
 - The Andromeda galaxy is 600,00 parsecs away. The Virgo Cluster is 10 million parsecs (10 megaparsecs) away.

Carl Freidrich Gauss

Friedrich Bessel

Heinrich Scherk Berlin

Josef Stefan Vienna (1858)

Ernst Kummer Wittneberg

Ludwig Boltzmann Vienna (1866)

Carl Runge Berlin (1880)

Paul Ehrenfest Universitat Wein (1904)

Max Born Gottingen (1906)

Samuel Goudsmit Leiden (1927)

J. Robert Oppenheimer Gottingen (1927)

Robert Bacher Michigan (1930)

Philip Morrison Berkeley (1940)

Boyce McDaniel Cornell (1943)

Robert Walker Cornell (1948)

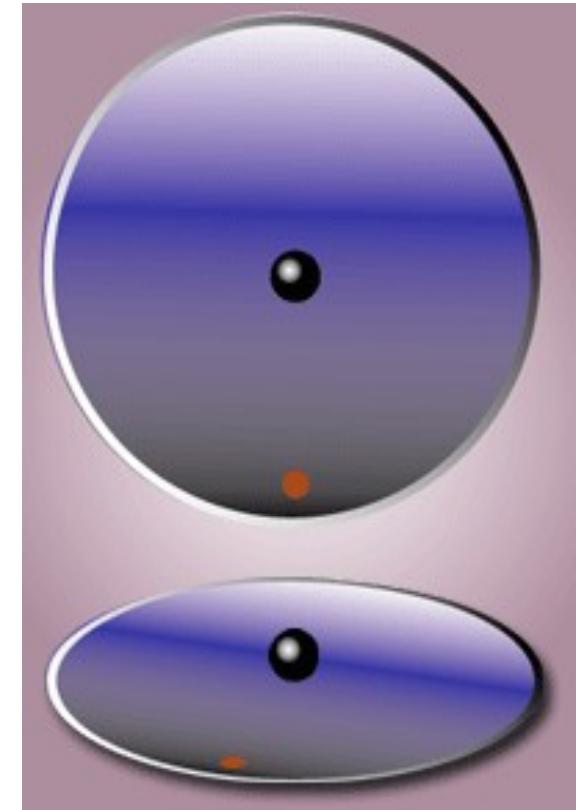
Gerry Neugebauer Caltech (1960)

Steven Beckwith Caltech (1981)

Michael Skrutskie Cornell (1987)

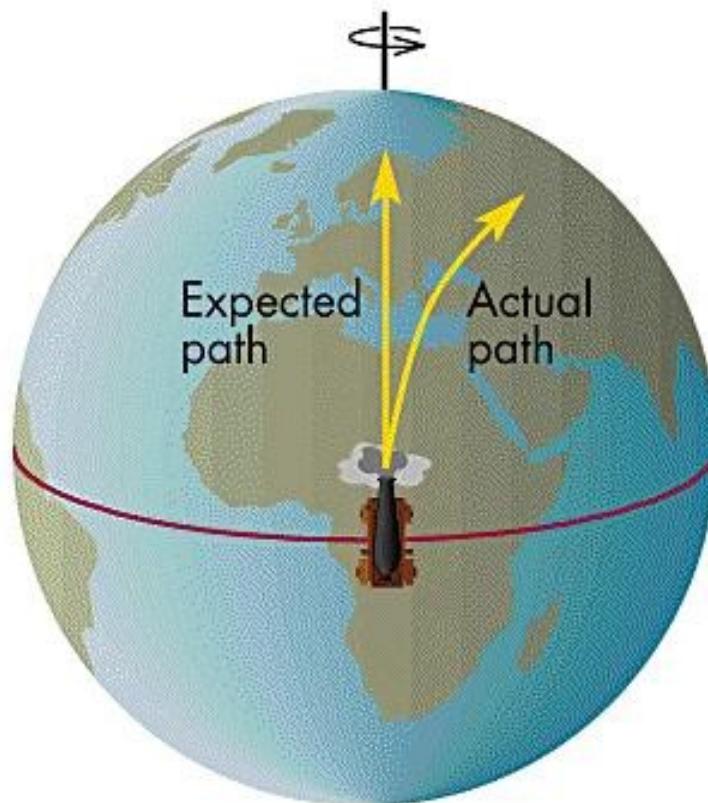
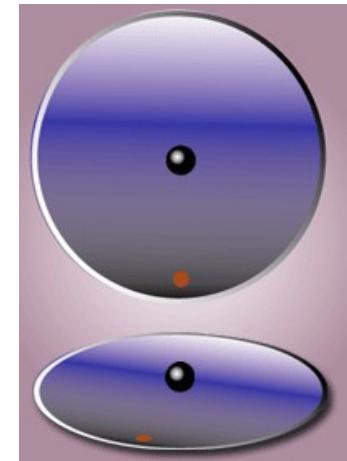
Eppur Si Muove – Coriolis Force

- Rotation of the Earth is evident from effects due to observation in a rotating frame of reference (although the effects are subtle).
- Gaspard-Gustav Coriolis first characterized this rotating frame effect in 1835.
- A resident of Earth near the equator is carried along at about 450 meters/s by the rotational motion of the Earth.
 - Rockets launch to the East from equatorial latitudes (French Guyana, for example) to get a boost from Earth rotation.
- Heading north (or south) from the equator an individual moves to a latitude where the rotation velocity of the Earth is $\cos(\text{latitude})$ smaller but the individual retains their original velocity.
 - They end up being “ahead” of where they thought they should go
 - Half of this motion is due to reference frame shift and half is due to conservation of angular momentum.

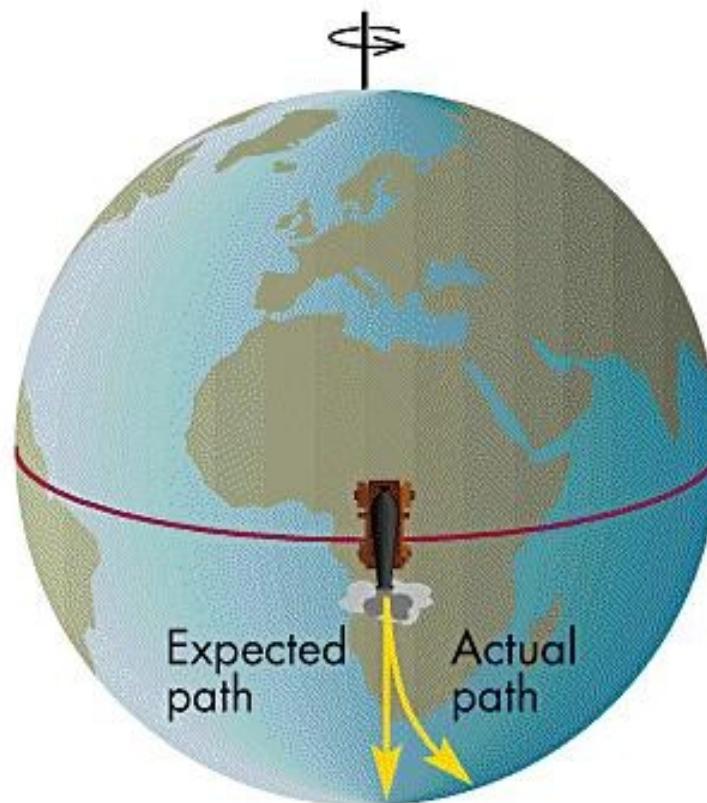


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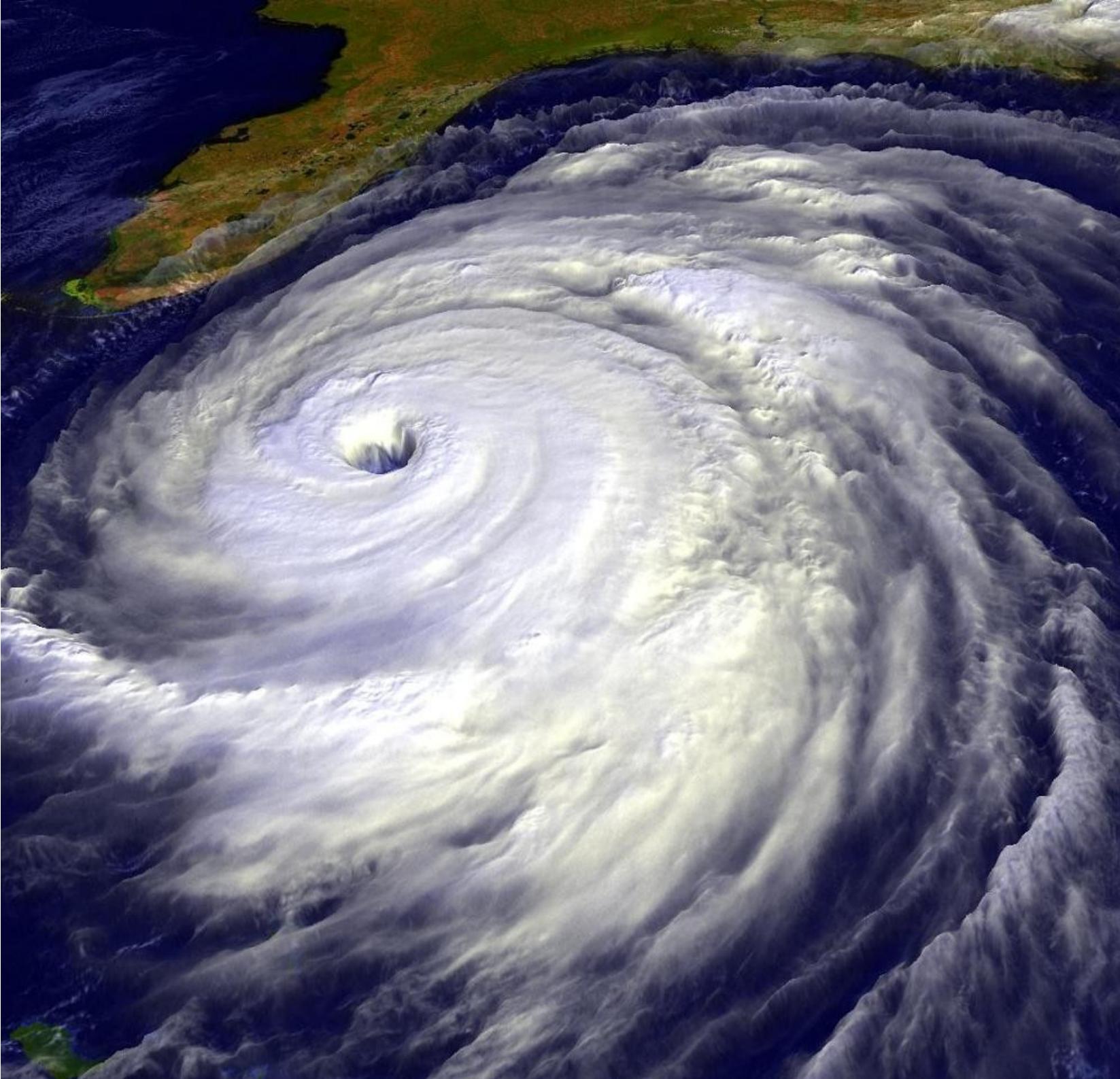
$$F_{rot} = m \vec{a} - m \vec{\omega} \times (\vec{\omega} \times \vec{r}) - 2m \vec{\omega} \times \vec{v}$$



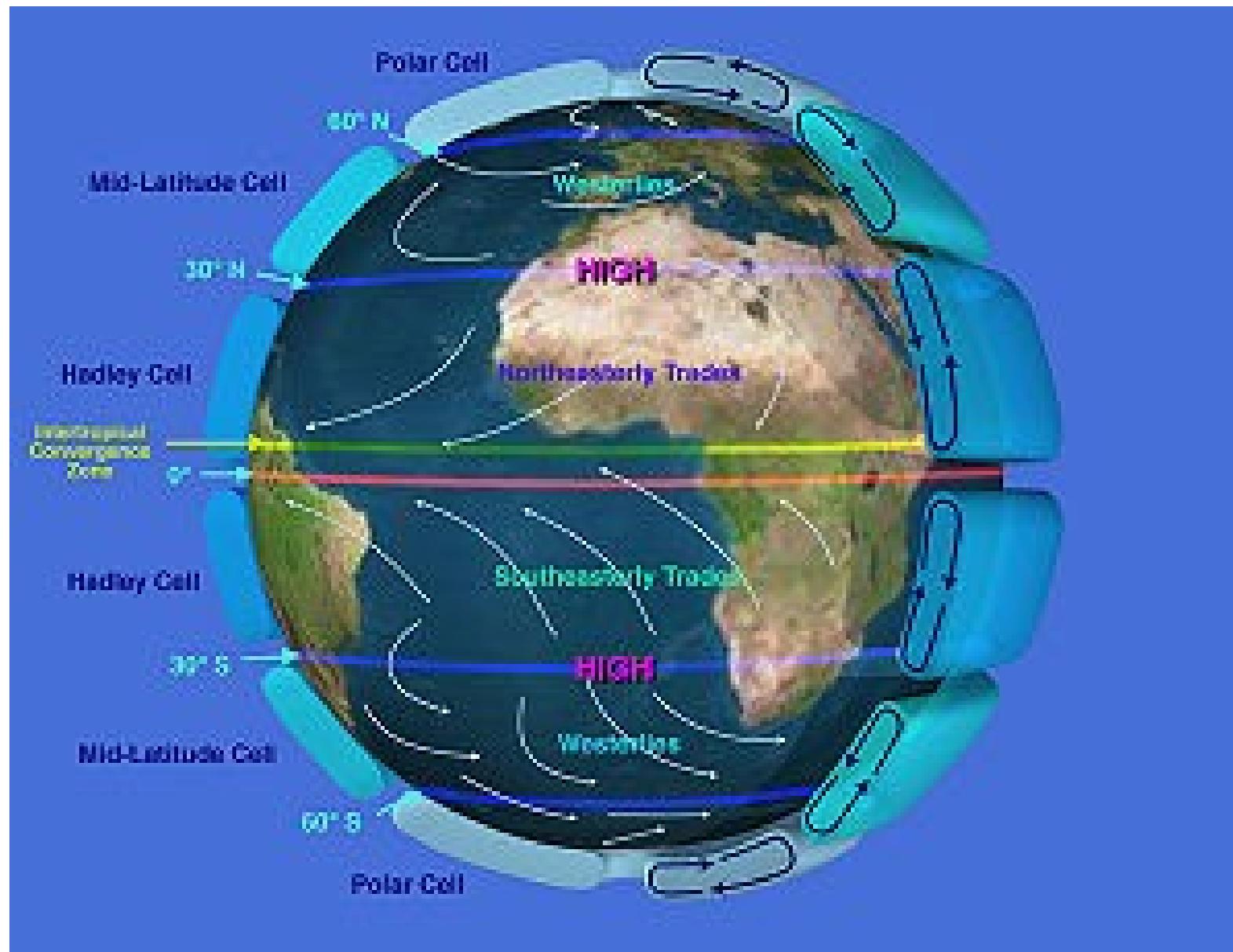
A Projectile fired northward

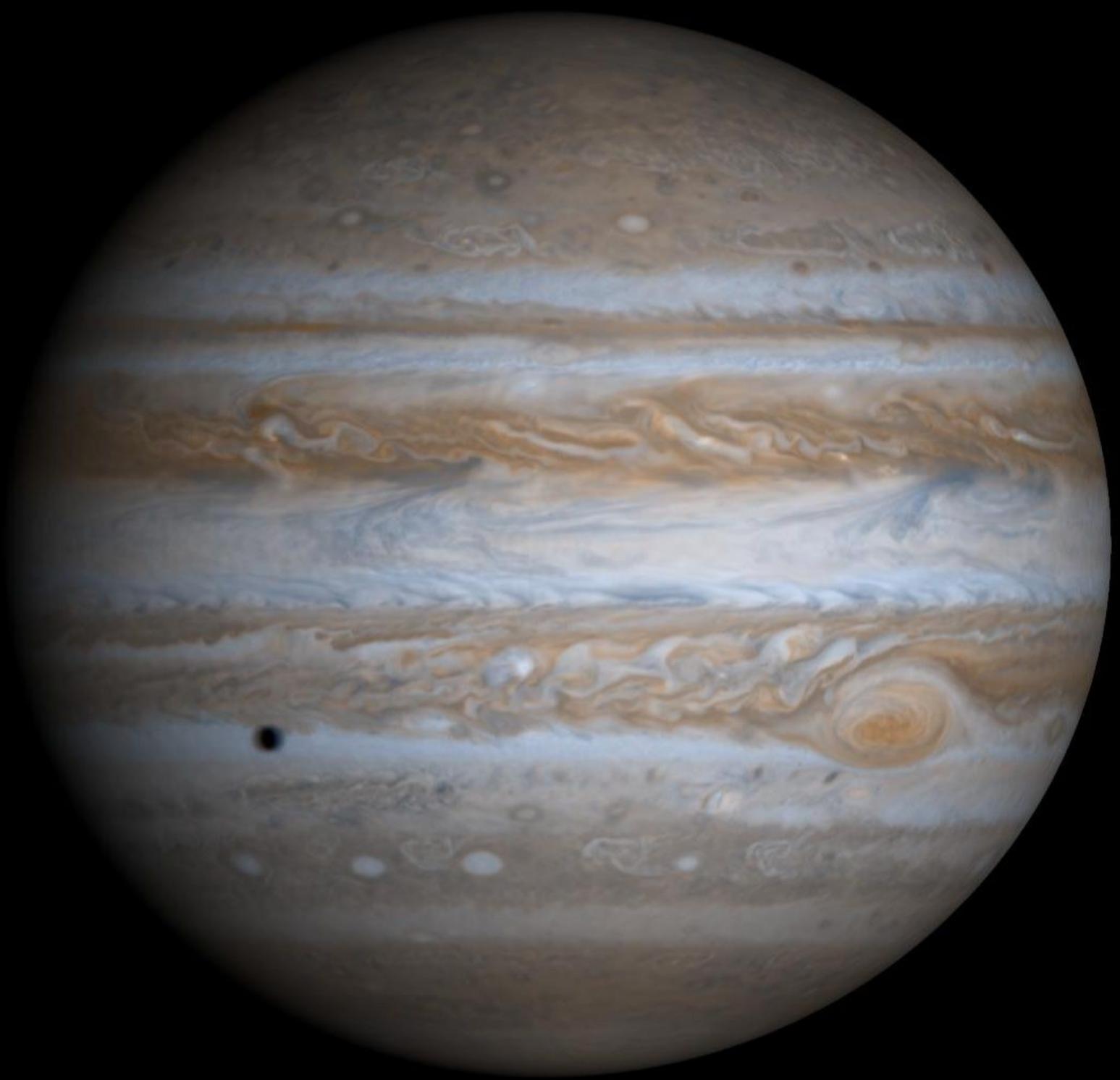


B Projectile fired southward

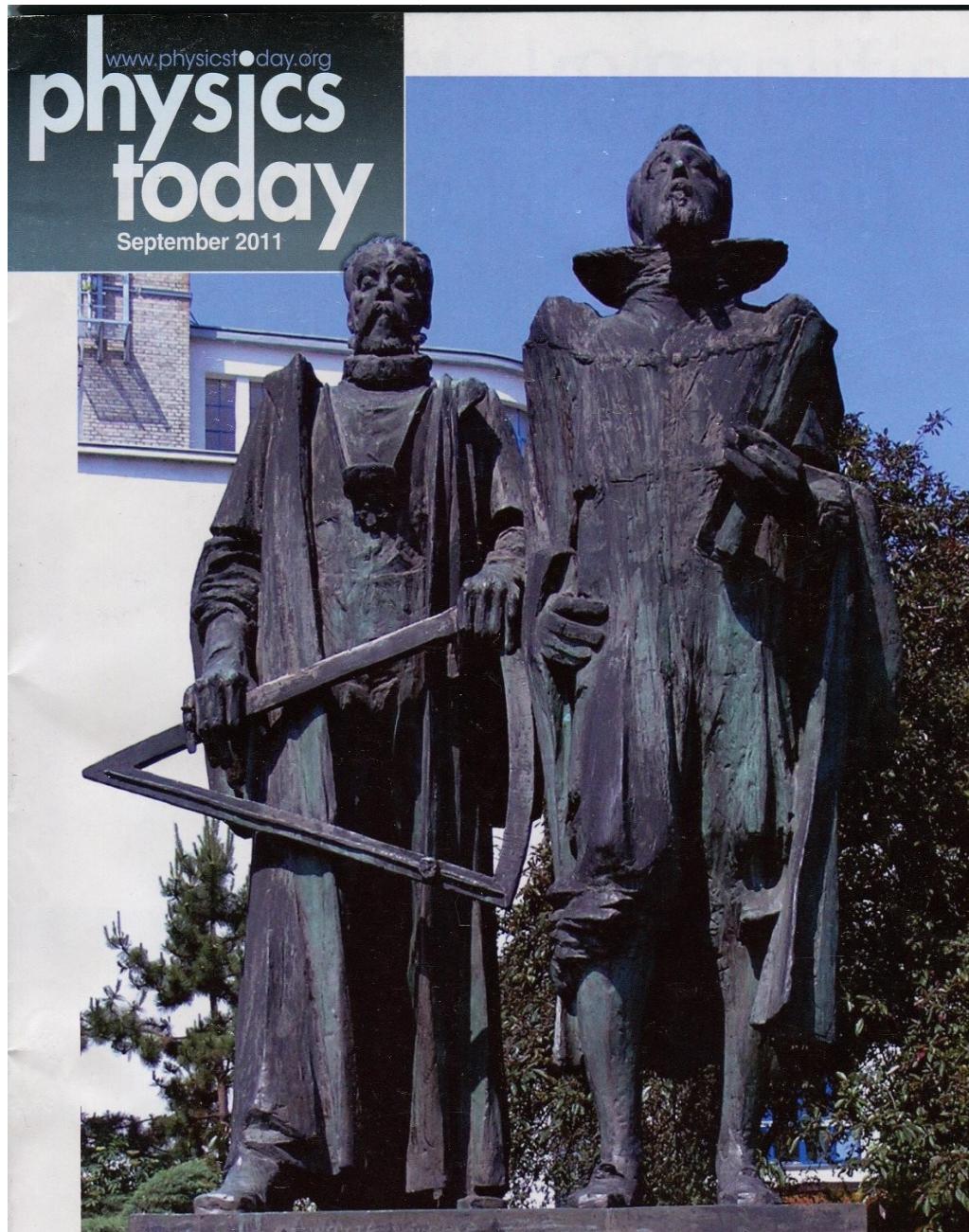


Hadley Circulation





Tycho and Kepler



Orbital eccentrics

Tycho and Kepler

- Ironically, Tycho Brahe was an adherent to the Earth-centered universe. His assistant, mathematical genius, and mystic Johannes Kepler, eagerly waited for the opportunity to analyze Tycho's data (which Tycho kept close and secret).



Kepler discovered remarkable properties of planetary orbits:

1. All planetary orbits were ellipses with the Sun at one focus.
2. Planets moved faster when closer to the Sun in a way that a line between the Sun and planet swept out equal area in equal time.
3. The orbital period of a planet was related to its average distance from the Sun. $P^2=a^3$

Kepler's First Law – Elliptical Orbits

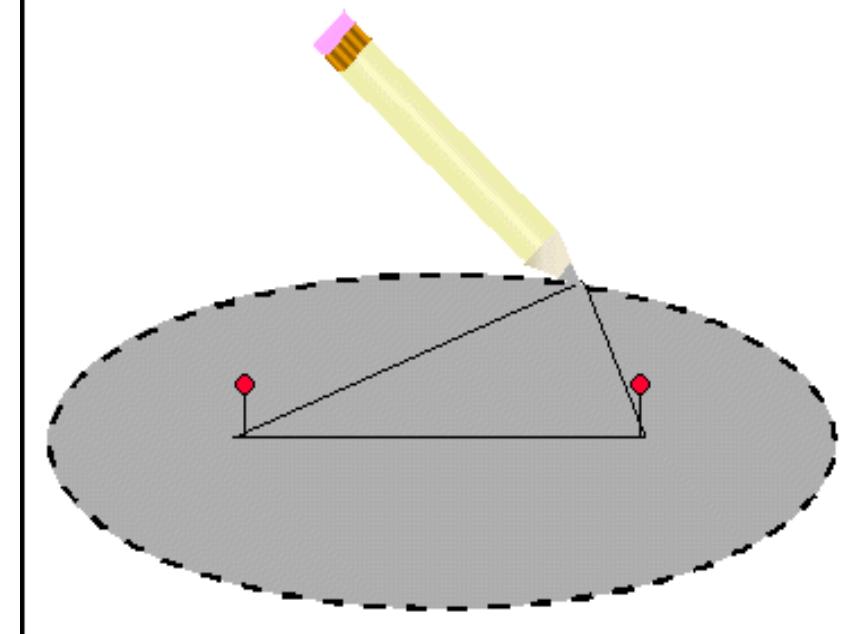
- An astounding discovery – Planets followed detailed mathematical relationships.

All planetary orbits were *ellipses* with the Sun at one focus.

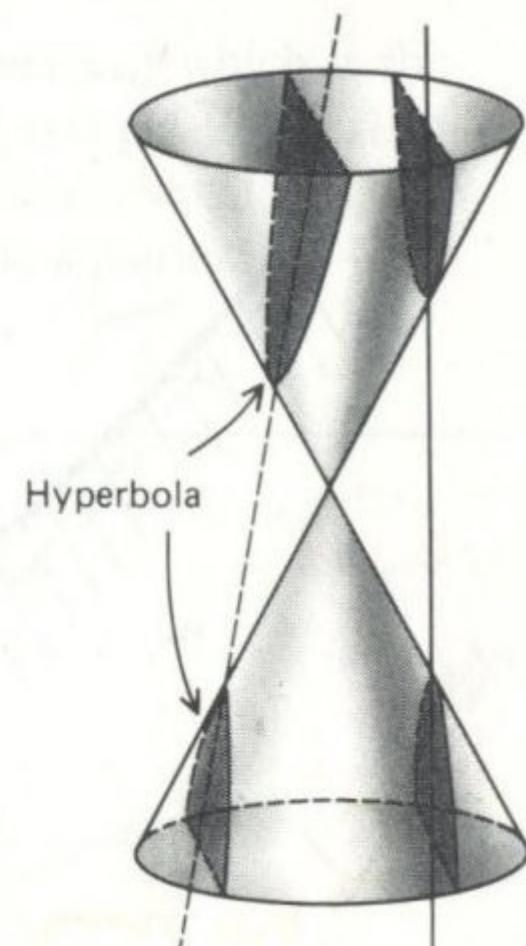
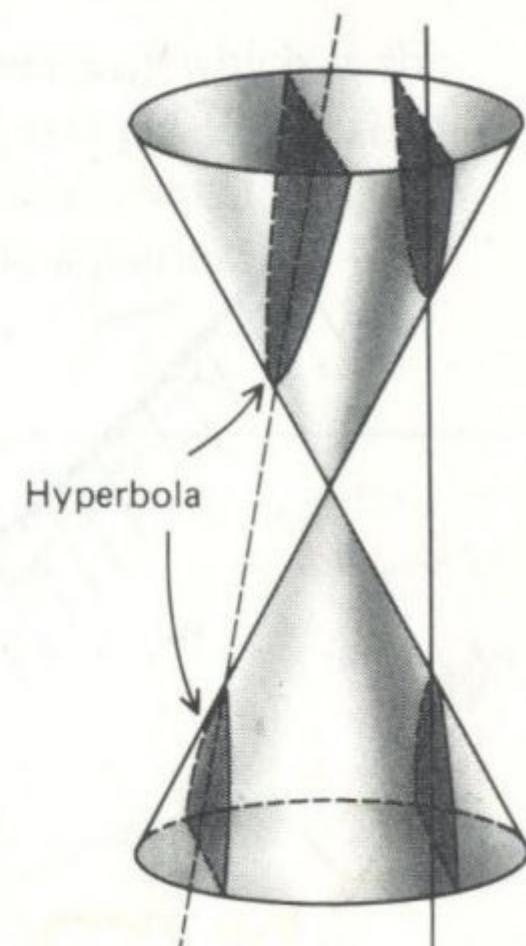
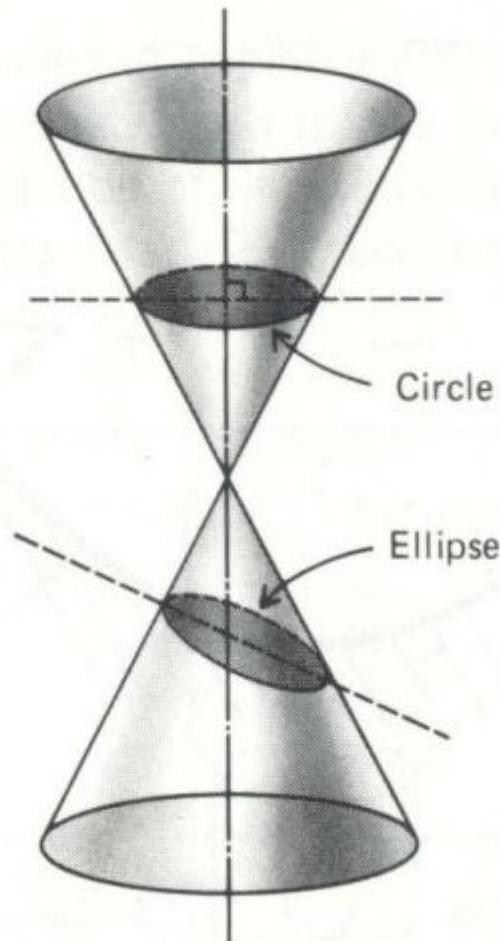
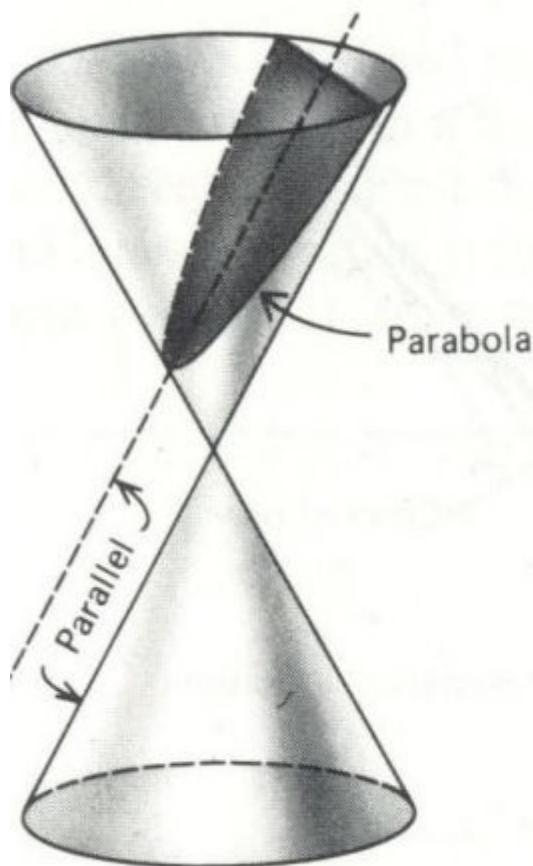
A circle is a special case of an ellipse where the foci are on top of each other.

The more separated the foci the more *eccentric* the ellipse.

How to draw an ellipse



Conic Sections

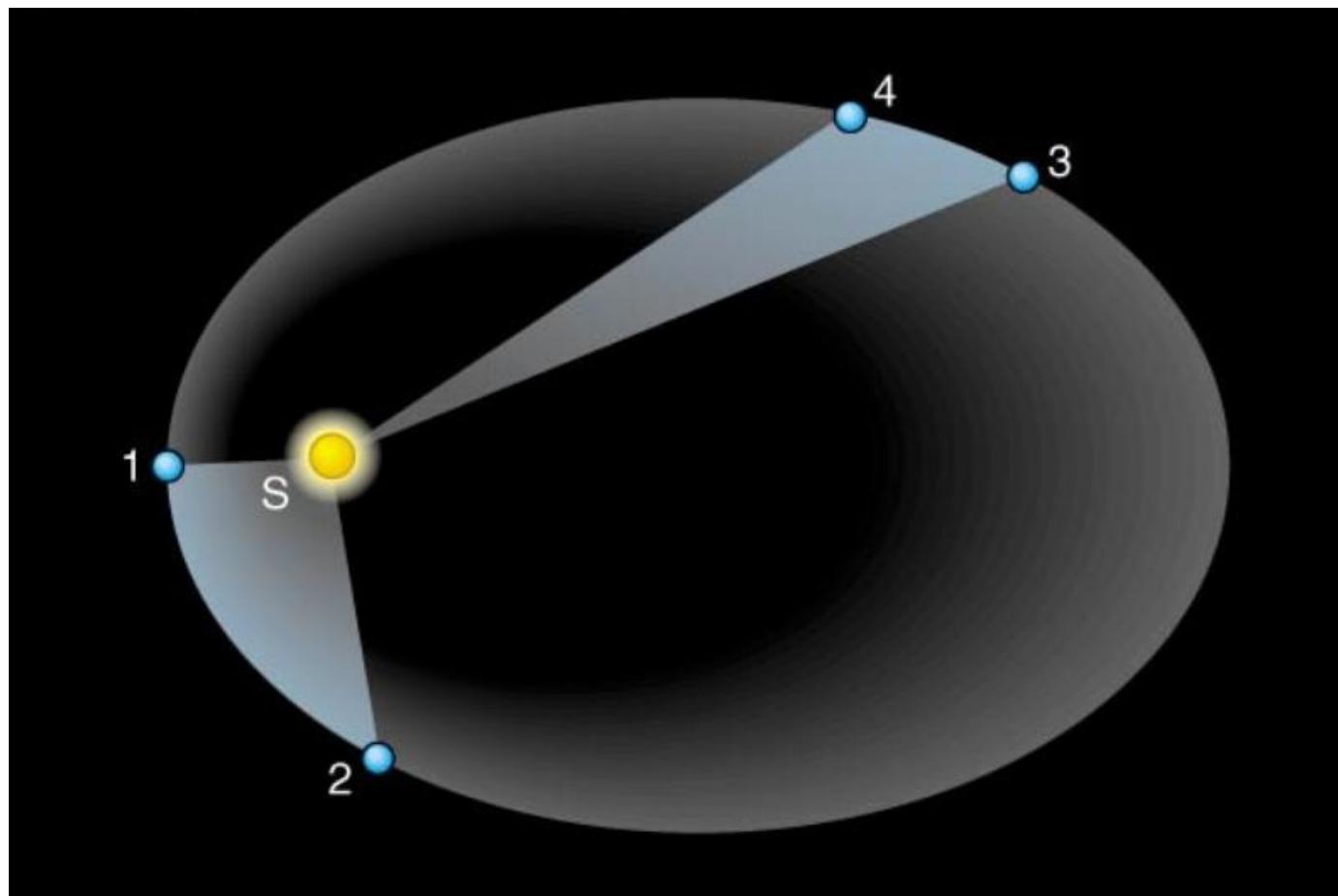


Kepler's Second Law – Equal Areas

- Another quantitative relationship....

Planets move faster when they are closer to the Sun in such a way that a line between the planet and the Sun sweeps out an equal area in the same time interval.

The Law of Equal Areas



Kepler's Third “Harmonic” Law

- Relates

A planet's average distance from the sun - “a”

specifically $\frac{1}{2}$ of the long axis of the elliptical orbit

for a circular orbit this “semi-major axis” is just the radius of the circle

The time it takes the planet to orbit the Sun – the orbital period, “P”

$$P^2 = a^3$$

This equation works as written if P, the orbital period, is expressed in years, and a, the semi-major axis, is expressed in astronomical units.

The Earth's orbital period is 1 year and it is 1 A.U. from the Sun.

$1^2 = 1^3$ it works.

An Example Using Kepler's Third Law

- Consider an asteroid discovered in a circular orbit 4 A.U. from the Sun. How long does it take this asteroid to orbit the Sun?
 $a = 4 \text{ A.U.}$

$$P^2 = a^3$$

$$?^2 = 4^3 = 4 \times 4 \times 4 = 64$$

$$8^2 = 64$$

This asteroid takes 8 years to complete an orbit around the Sun.

An Example Using Kepler's Third Law

- Jupiter's orbit has a semimajor axis of $a=5.2$ A.U.

$$P^2 = a^3 = (5.2)^3 = 140.6$$

$$P = \sqrt{140.6} = 11.9 \textit{ years}$$

Kepler's Laws in Motion

Deriving Kepler's Third Law

- Recall the derivation of circular orbital velocity

$$F_c = \frac{mv^2}{R} \quad F_g = \frac{GMm}{R^2}$$

$$v_{orbit} = \sqrt{\frac{GM}{R}}$$

Deriving Kepler's Third Law

- Express velocity on the left side in terms of orbital period, P

$$v = \frac{2\pi R}{P} = \sqrt{\frac{GM}{R}}$$

$$MP^2 = \frac{4\pi^2}{G} R^3$$

This result generalizes to the elliptical orbit with semi-major axis, a , in place of the radius, R .

Deriving Kepler's Third Law

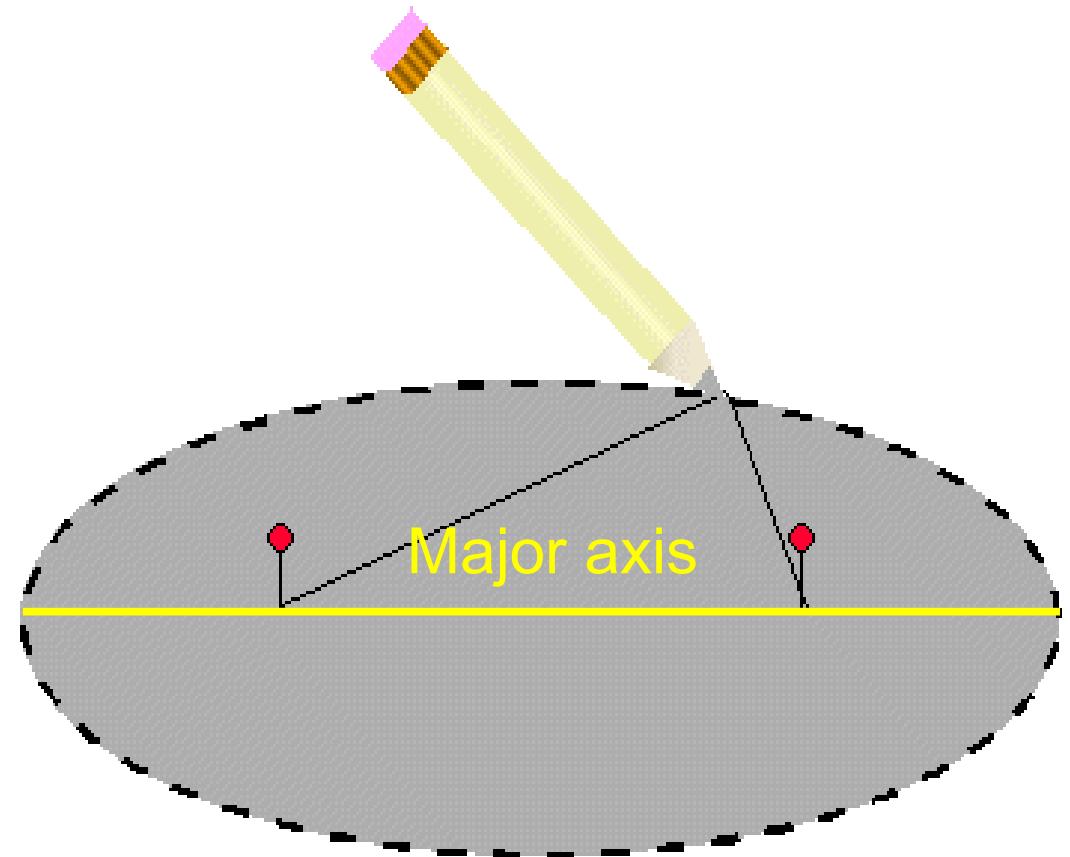
- If the equation is calibrated using units of “years” and “AU” then all of the constants multiply out to 1 and you are left with the elementary expression of Kepler's Third Law – $P^2=a^3$.
- This way of expressing Kepler's equation implicitly assumes the mass of the star being orbited is 1 solar mass. The general simplified expression for any given star is.

$$MP^2 = a^3$$

Ellipse Basics

- Geometrically, an ellipse is the set of all points that are equidistant from the two foci.
 - Fixing the focus locations but changing this distance leads to ellipses of different eccentricity.
- The “semimajor axis” (a) is one half the long axis of the ellipse
 - the “semiminor axis” (b) is one half the short axis.
- In cartesian coordinates the formula for an ellipse is:

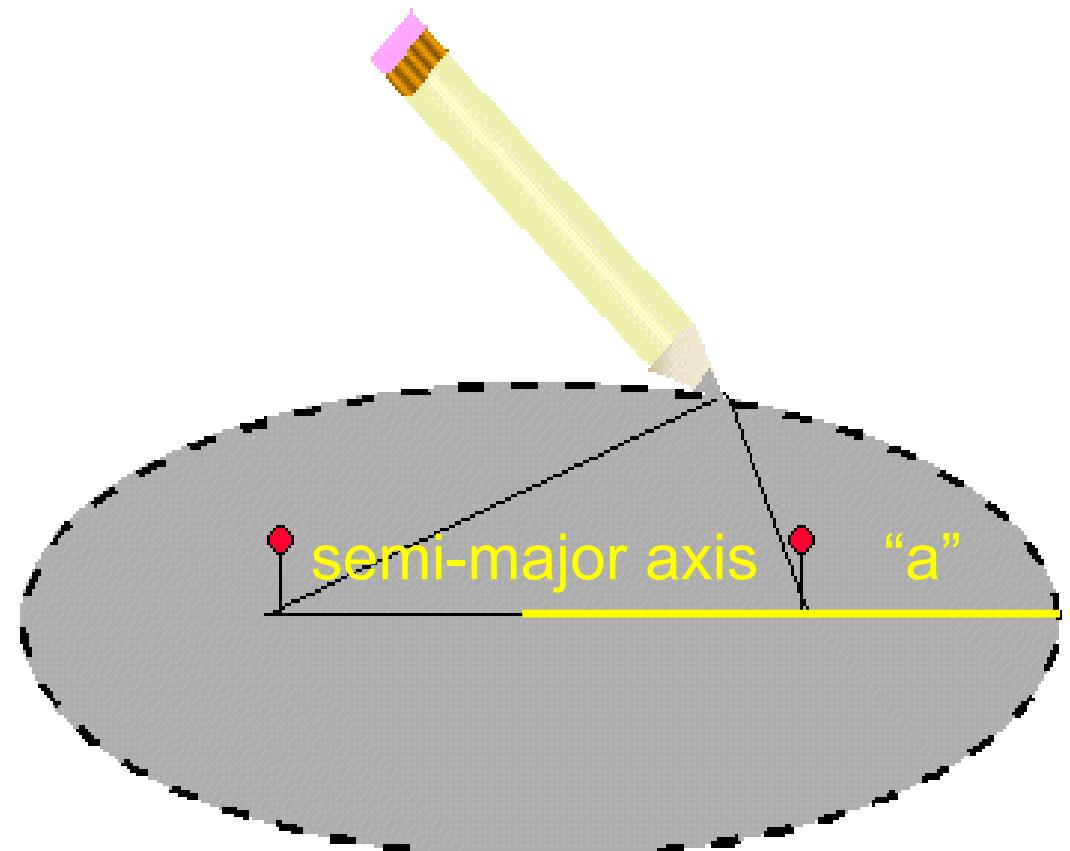
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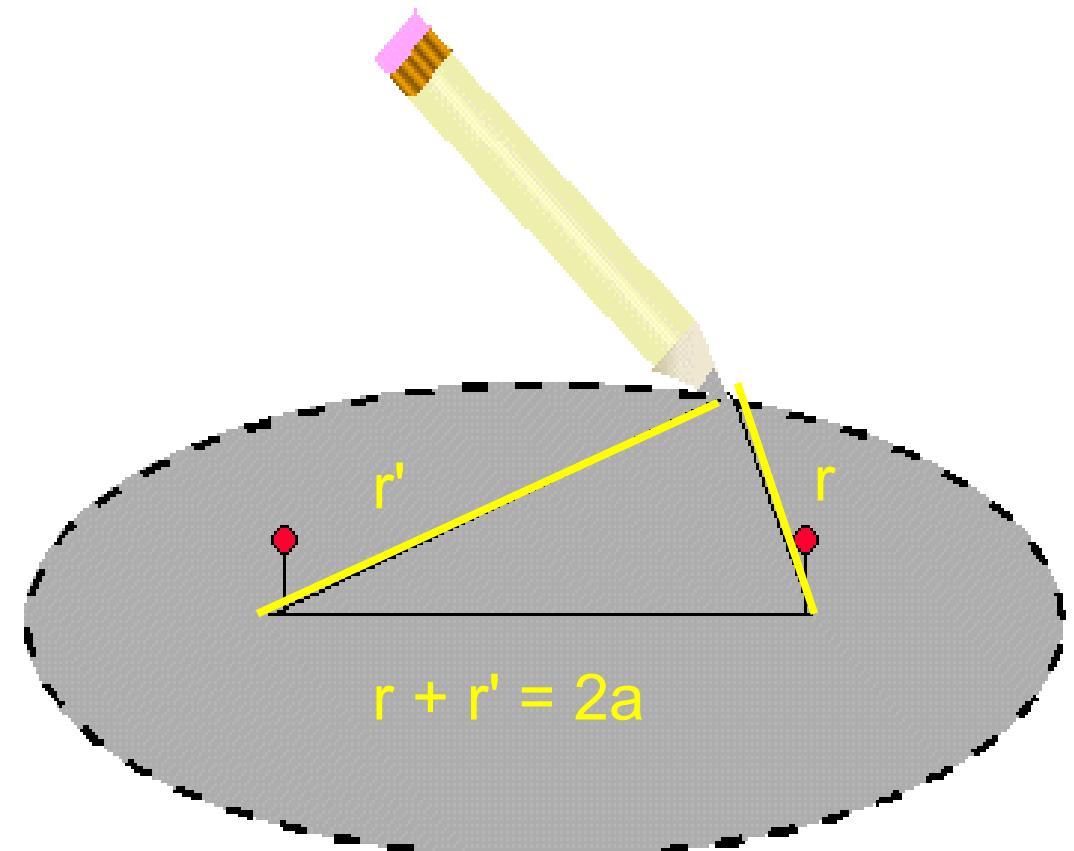
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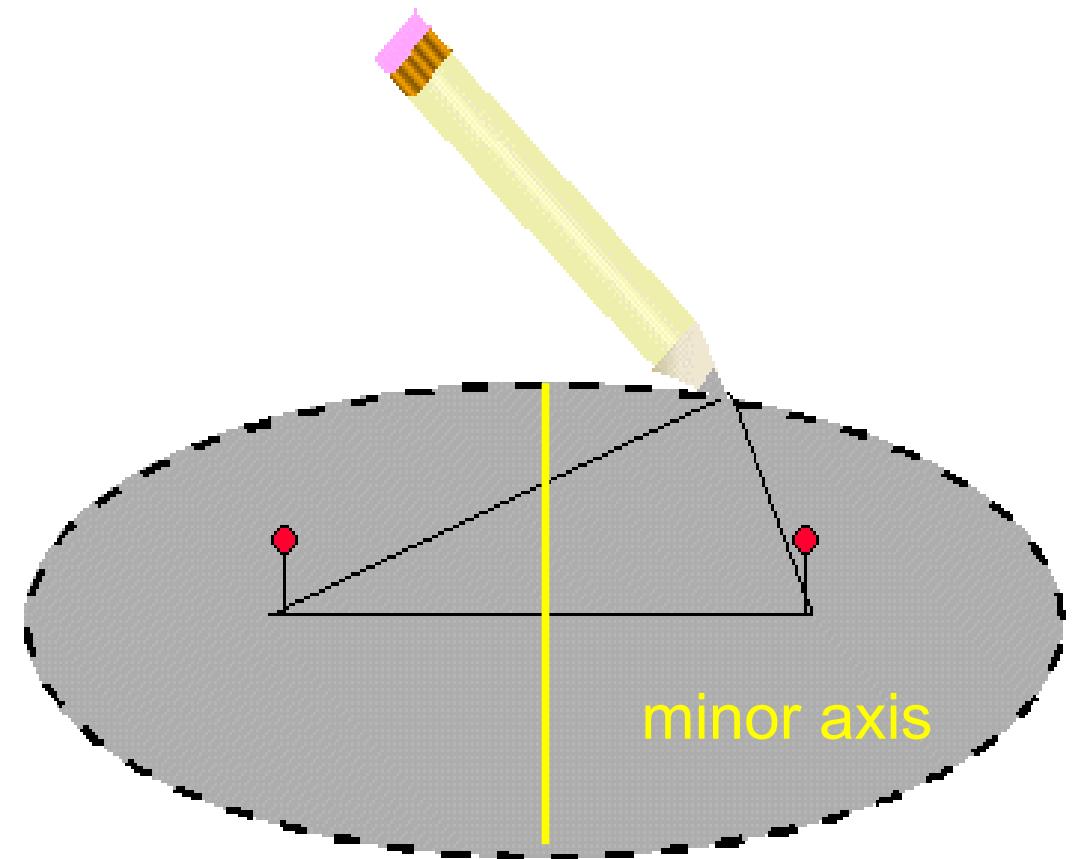
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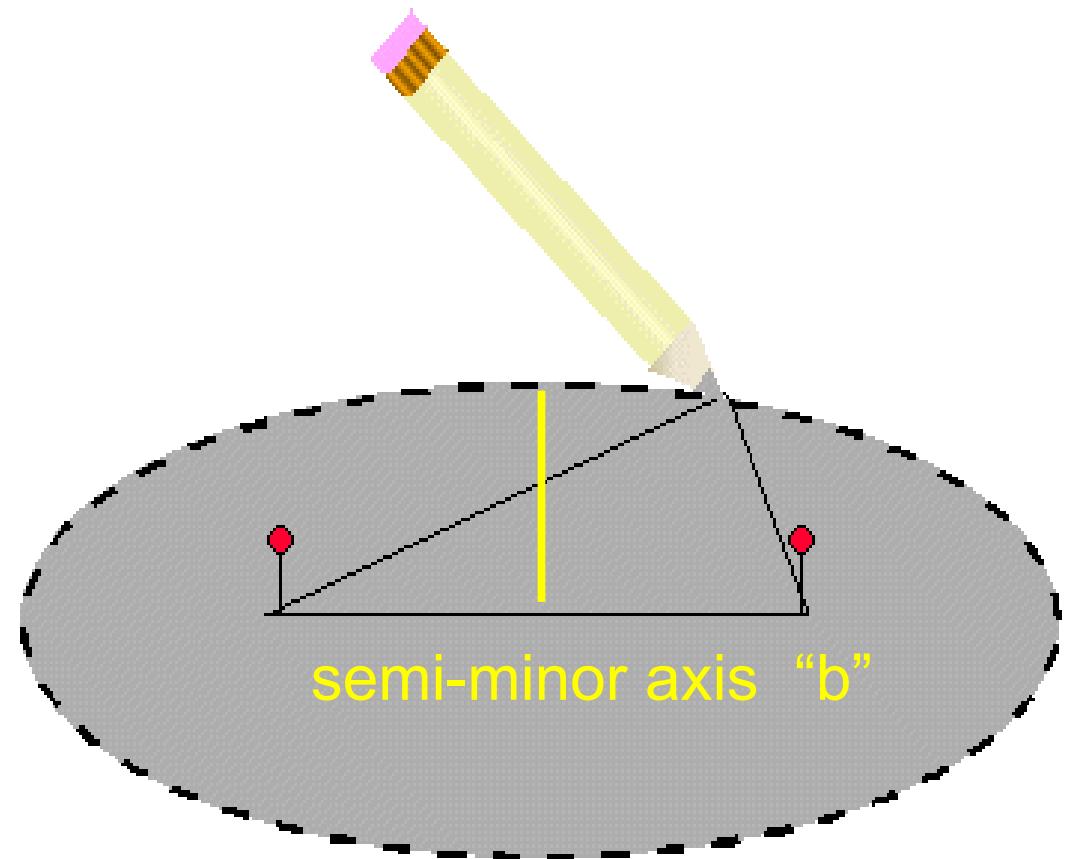
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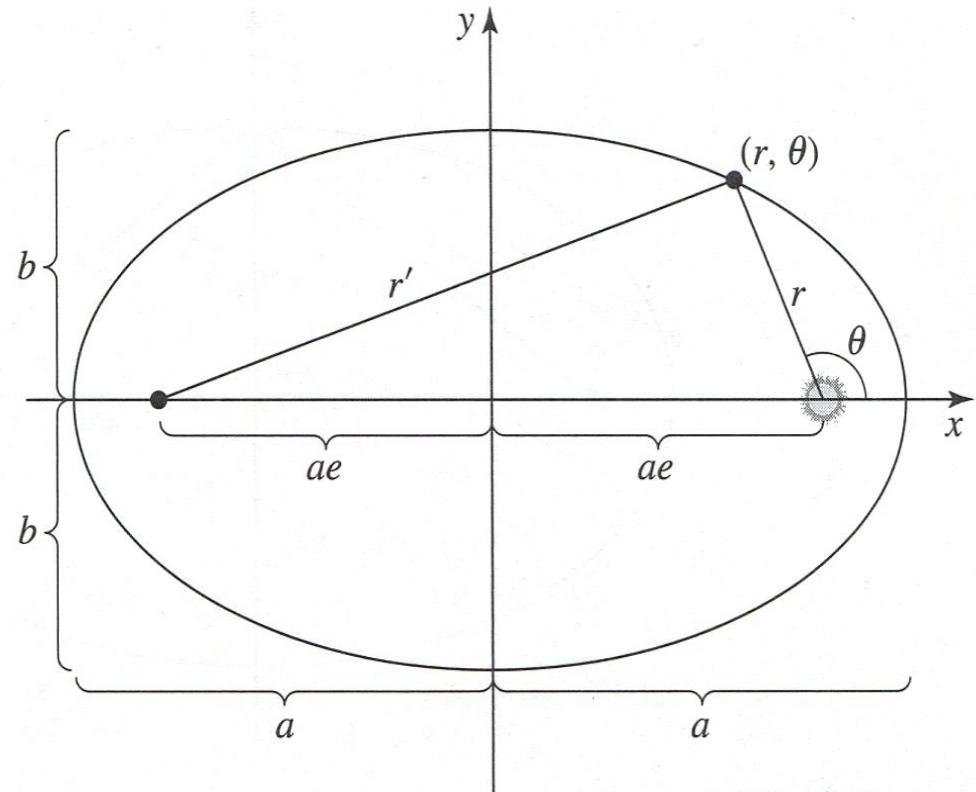


FIGURE 3.6 An ellipse of semimajor axis a and semiminor axis b .

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Ellipse Basics

- In polar coordinates the equation defining an ellipse is

$$r = \frac{a(1-e^2)}{1 + e \cos \theta}$$

- where θ is called the “true anomaly” and defined to be zero in the direction of positive x.
- e is the “eccentricity”
- Things to note:
 - The focus is ae from the center
 - $r + r'$ is a constant equal to $2a$ (the length of the major axis)

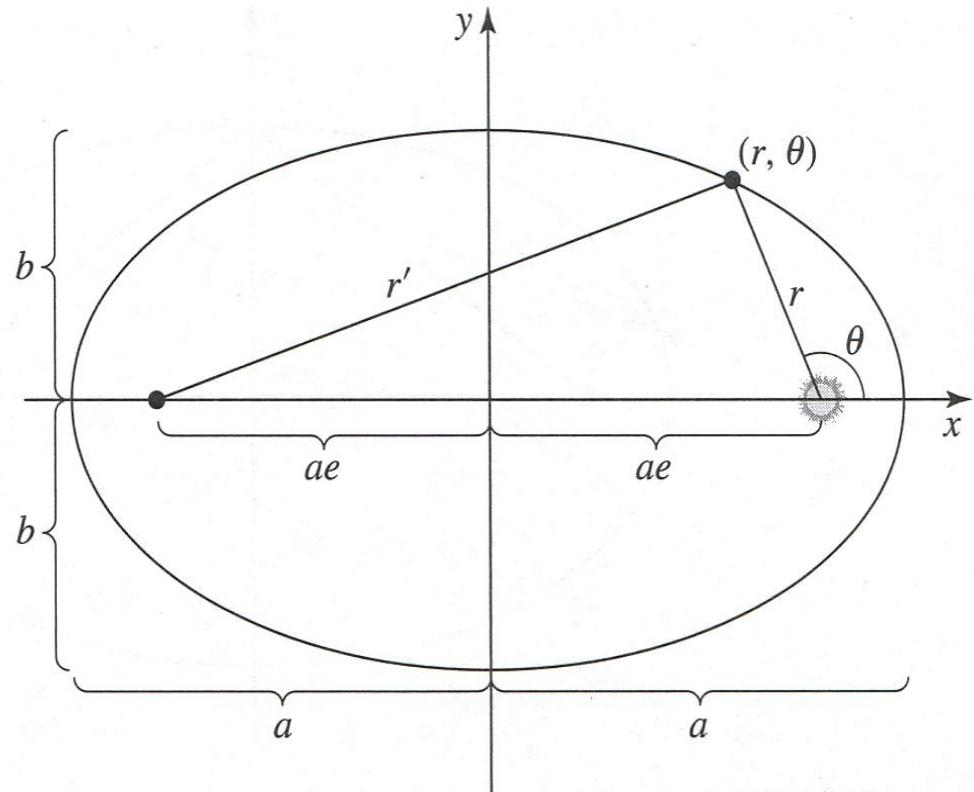


FIGURE 3.6 An ellipse of semimajor axis a and semiminor axis b .

Getting from Cartesian to Polar Coordinates (for the interested)

38

- Note the triangle with sides r' , and $2ae$.

- One interior angle of this triangle is $\pi - \theta$
- Apply the Law of Cosines

$$r'^2 = 4a^2e^2 + r^2 - 2aer \cos(\pi - \theta)$$

- note $\cos(\pi - \theta) = -\cos(\theta)$ and use $r + r' = 2a$ to substitute for r' above.

- rearrange to recover

$$r = \frac{a(1 - e^2)}{1 + e \cos \theta}$$

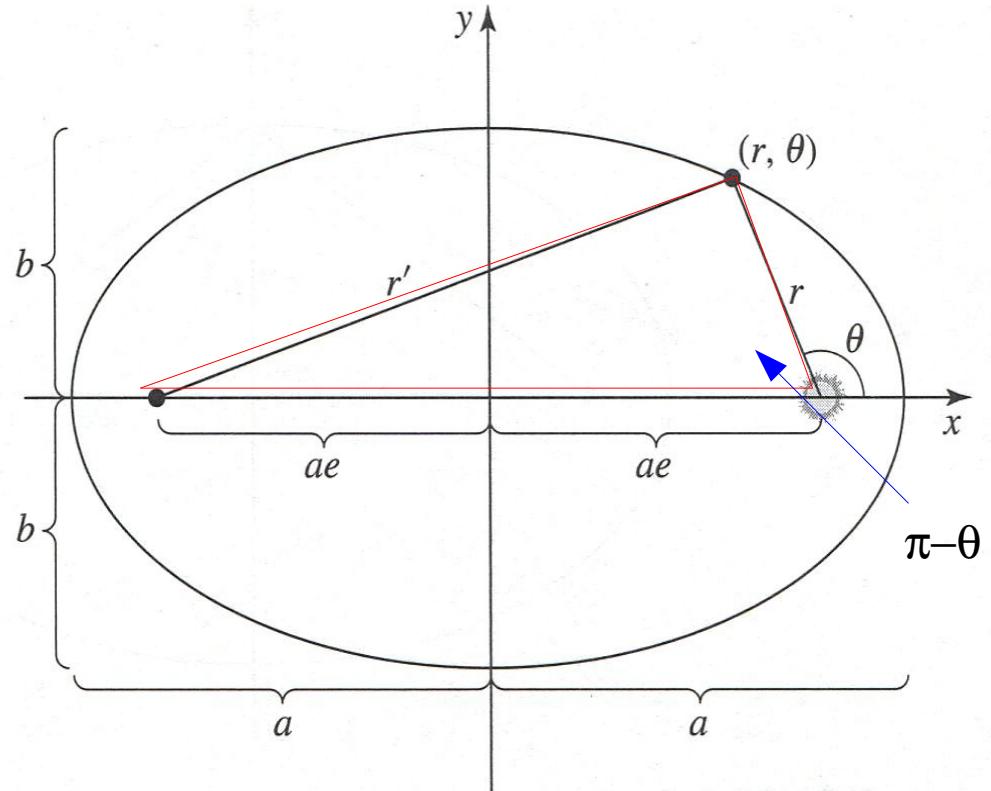


FIGURE 3.6 An ellipse of semimajor axis a and semiminor axis b .

Ellipse Basics – Relating axis ratio to eccentricity

- In polar coordinates the equation defining an ellipse is

$$r = \frac{a(1-e^2)}{1 + e \cos \theta}$$

- where θ is called the “true anomaly” and defined to be zero in the direction of positive x.
- e is the “eccentricity”
- Things to note:
 - The focus is ae from the center
 - $r + r'$ is a constant equal to $2a$ (the length of the major axis)
- consider the right triangle formed by a focus and a point on the y-axis

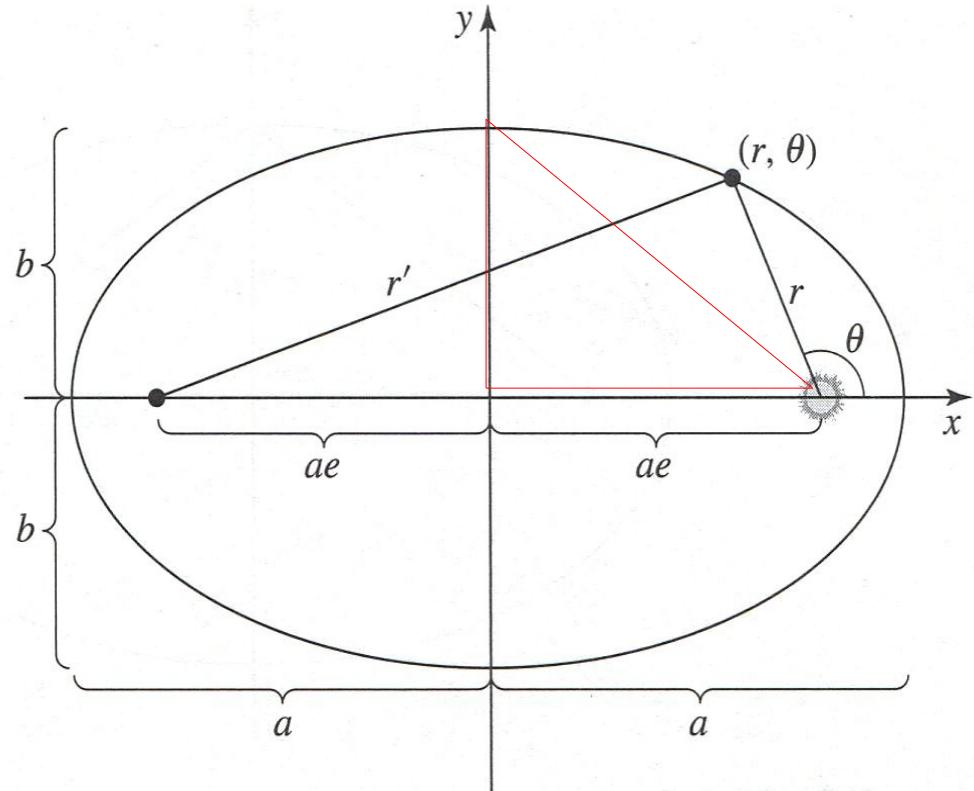


FIGURE 3.6 An ellipse of semimajor axis a and semiminor axis b .

$$b^2 + (ae)^2 = r^2 = a^2$$

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

Periapse and Apoapse

- In polar coordinates the equation defining an ellipse is

$$r = \frac{a(1-e^2)}{1 + e \cos \theta}$$

- where θ is called the “true anomaly” and defined to be zero in the direction of positive x.
- e is the “eccentricity”
- Things to note:

- a particle is closest (periapse/perihelion/perigee) to the focus when $\cos(\theta)=0$, farthest when $\cos(\theta)=\pi$ (apoapse)

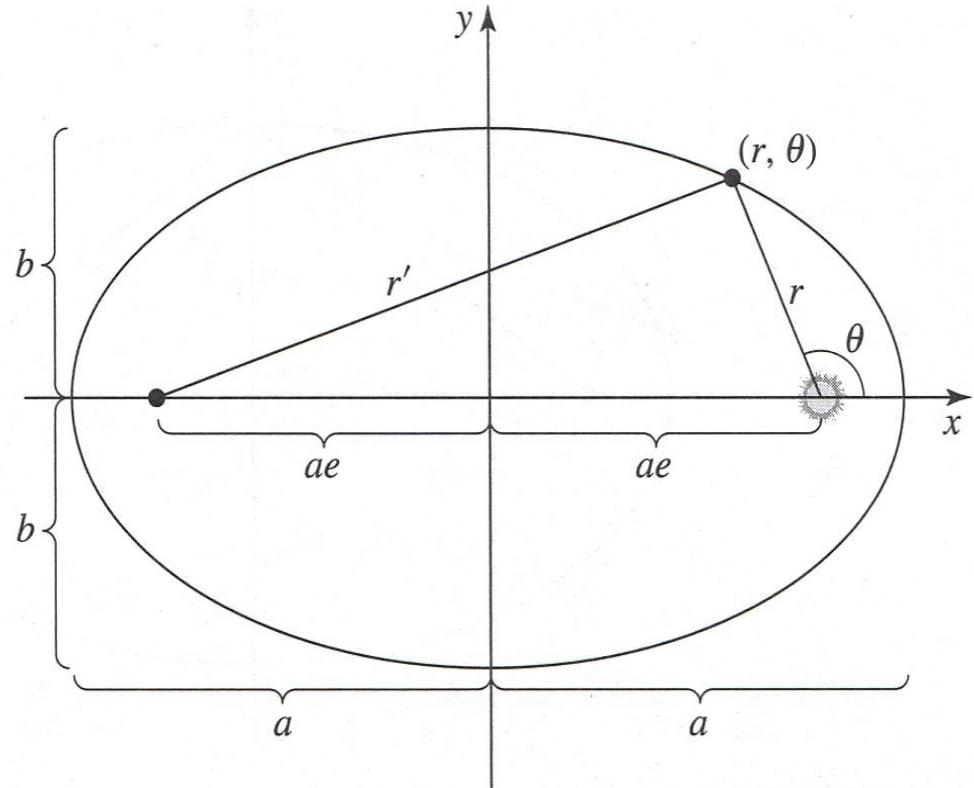


FIGURE 3.6 An ellipse of semimajor axis a and semiminor axis b .

$$\text{periapse} = a(1 - e)$$

$$\text{apoapse} = a(1 + e)$$

Orbital Velocity

- A complete derivation of Kepler's first law (section 3.1.2) yields

$$r = \frac{L^2}{GMm^2(1 + e \cos \theta)} \quad \text{but} \quad r = \frac{a(1 - e^2)}{1 + e \cos \theta}$$

- so

$$\frac{L^2}{m^2} = GMa(1 - e^2) = r^2 v_t^2 \quad \text{since} \quad L = mr v_{tangential}$$

- velocity is fully tangential at periapse and apoapse, so

$$v_{peri} = \left[\frac{GM}{a} \frac{1+e}{1-e} \right]^{1/2} \quad v_{apo} = \left[\frac{GM}{a} \frac{1-e}{1+e} \right]^{1/2}$$

note these equations yield circular orbital velocity when $e=0$.

Orbital Velocity

In general, the orbital velocity is given by

$$v^2 = GM \left(\frac{2}{r} - \frac{1}{a} \right)$$

note this equation yields circular orbital velocity when $a=r$ (i.e zero eccentricity)

Interplanetary Travel

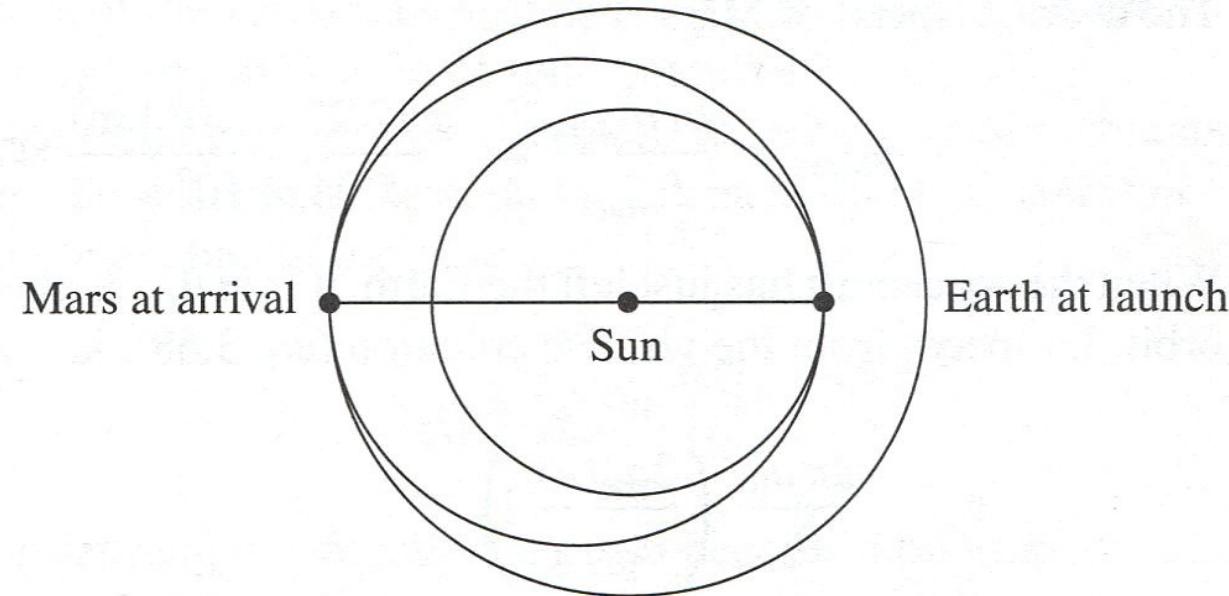
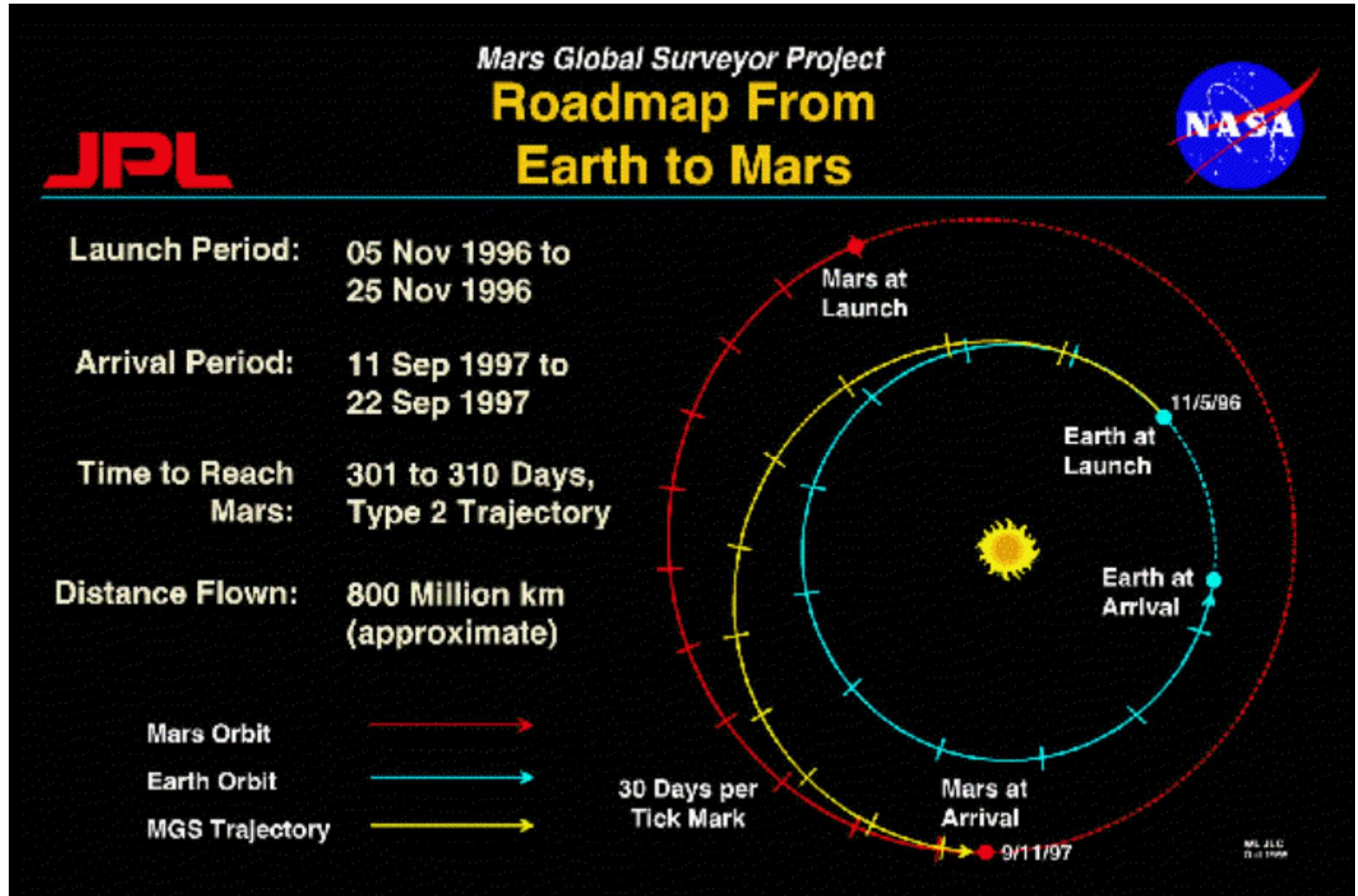


FIGURE 3.10 A Hohmann transfer orbit for interplanetary travel (here from Earth to Mars). The transfer orbit is an ellipse with its perihelion at Earth and its aphelion at the orbit of Mars.



Orbital Energy

- Bound orbits have negative energy

$$E_{\text{total}} = K + U = \frac{1}{2} m v^2 - \frac{GMm}{r}$$

- Negative total energy corresponds to a bound orbit.
- Zero energy corresponds to a parabolic trajectory (velocity reaches zero at infinity)
- Positive total energy corresponds to a hyperbolic (unbound) trajectory.

