Locally Optimal Takagi-Sugeno Fuzzy Controllers

Amir massoud Farahmand
amir@cs.ualberta.ca

Mohammad javad Yazdanpanah
yazdan@ut.ac.ir

Department of Electrical and Computer Engineering
University of Tehran
Tehran, Iran

Fuzzy Control

- Successful in many applications
- Ease of use
  - Intuitive and interpretable
- Powerful nonlinear controller

Takagi-Sugeno Plant Model

$$\mathrm{R_i}: \text{IF } x_1(t) \text{ is } \mathrm{M_1^i} \text{ and} \ldots \text{and } x_n(t) \text{ is } \mathrm{M_n^i} \text{ THEN } \dot{\mathrm{x}}(\mathrm{t}) = \mathrm{A_i} x(t) + B_i u(t)$$

$$\dot{x}(t) = \sum_{i=1}^{r} h_i\big(x(t)\big)\big(A_i x(t) + B_i u(t)\big)$$

,

$$h_i(x) = \frac{w_i(x)}{\sum_{i=1}^{r} w_i(x)} \qquad w_k(x) = \prod_{i=1}^{r} \mu_{M_i^k}(x_i(t))$$

Theorem 1. *The continuous uncontrolled T-S fuzzy system is globally quadratically stable if there exists a common positive definite matrix P such that*

$$A_i^T P + P A_i < 0, \quad \mathrm{i} = 1, \ldots, \mathrm{r}$$

Parallel Distributed Compensation

Control rule$_i$: IF $x_1(t)$ is M_1^i and ... and $x_n(t)$ is M_n^i THEN u(t) = -K_ix(t)

$$u = -\sum_{i=1}^{r} h_i(x) K_i x(t)$$

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(x) h_j(x) \left[A_i - B_i K_j\right] x(t)$$

$$= \sum_{i=1}^{r} h_i^2(x) G_{ii} x(t) + \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(x) h_j(x) . \left[\frac{G_{ij} + G_{ji}}{2}\right] x(t) \qquad G_{ij} = A_i - B_i K_j$$

Stability condition: $$\left(\frac{G_{ij} + G_{ji}}{2}\right)^T P + P\left(\frac{G_{ij} + G_{ji}}{2}\right) < 0, \quad \mathrm{i, j = 1,...,r}$$

Locally Optimal Design

Linearization

$$\dot{x} = f(x,u) \quad \Rightarrow \quad A_i, B_i$$

Locally optimal design

$$\dot{x} = A_i x + B_i u \quad \Rightarrow \quad K_i = R^{-1} B^T P$$

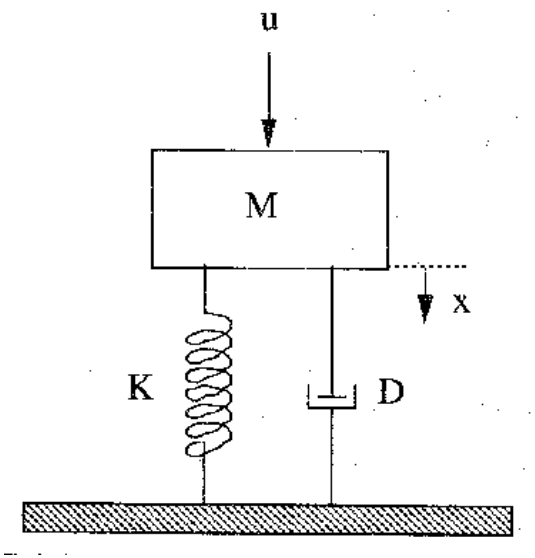
$$J(x(t),u(t)) = \int_0^\infty \left[x^T Q x + u^T R u \right] dt$$

Experiments: *Problem description*

- Nonlinear Mass-Spring-Damper system

$$M\ddot{x}(t) + g\left(x(t), \dot{x}(t)\right) + f\left(x(t)\right) = \phi\left(\dot{x}(t)\right)u(t)$$

$$\underline{\dot{x}}(t) = \begin{pmatrix} x_2 \\ -x_2 - 0.01x_1 - 0.1x_1^3 + \left(1.4387 - 0.13x_2^2\right)u \end{pmatrix}$$



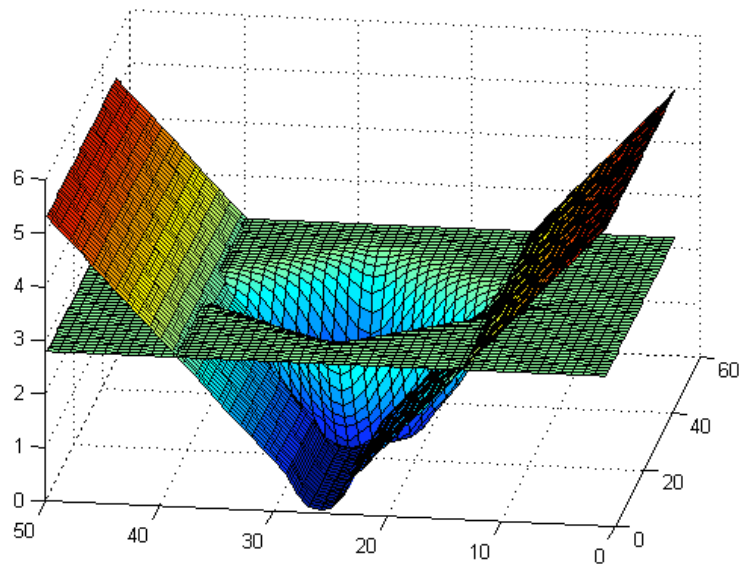
$$A = \left.\frac{\partial f(x,u)}{\partial x}\right|_{\substack{x=x_0 \\ u=0}} = \left.\begin{pmatrix} 0 & 1 \\ -0.01 - 0.3x_1^2 & -1 \end{pmatrix}\right|_{x_0} \qquad B = \left.\frac{\partial f(x,u)}{\partial u}\right|_{x=x_0} = \left.\begin{pmatrix} 0 \\ 1.4387 - 0.13x_2^2 \end{pmatrix}\right|_{x_0}$$

Experiments: *Fuzzy Settings*

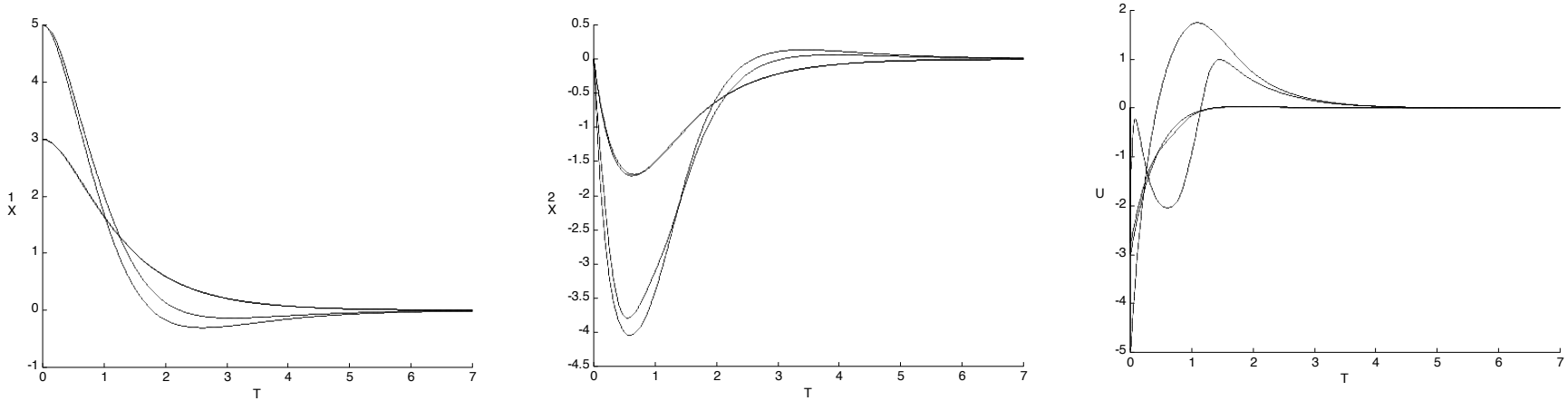
The dynamics of the plant is approximated using Gaussian membership function

$$\mu_{M^k}(x(t)) = \exp\left(-\frac{\|(x - x_c)\|^2}{2\sigma^2}\right)$$

Approximation error



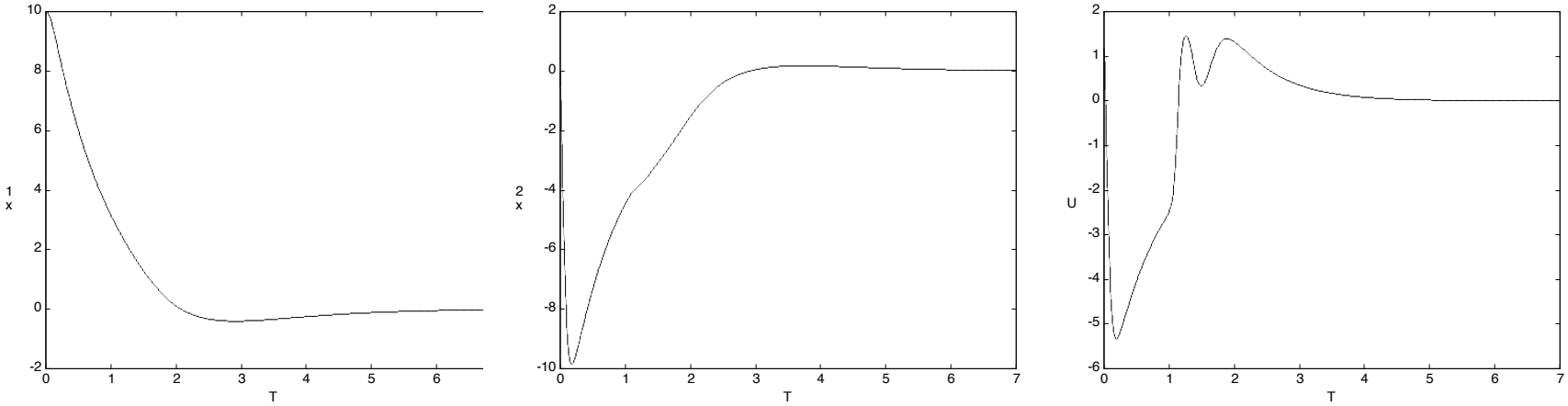
Experiments: *Stabilization (I)*



Comparison of T-S controller (bold) and linear controller (dotted) with different initial conditions

Both TS and linear controller are stable in this case. However, the behavior of fuzzy controller is smoother and with lower overshoot.

Experiments: *Stabilization (II)*



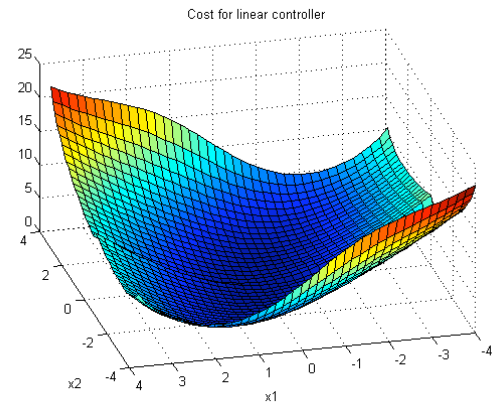
The linear controller is not stable in this case, but the fuzzy controller can handle it easily.

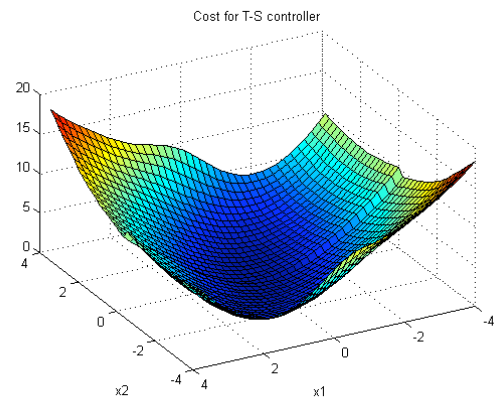
Response of T-S controller to (10 0)'

Experiments: *Performance Comparison*

”

	Linear	TS
Q=I, R=1	5.80	**5.47**
Q=10I,R=1	9.06	**8.44**
Q=I, R=10	**5.58**	5.62





Experiments: Performance Comparison

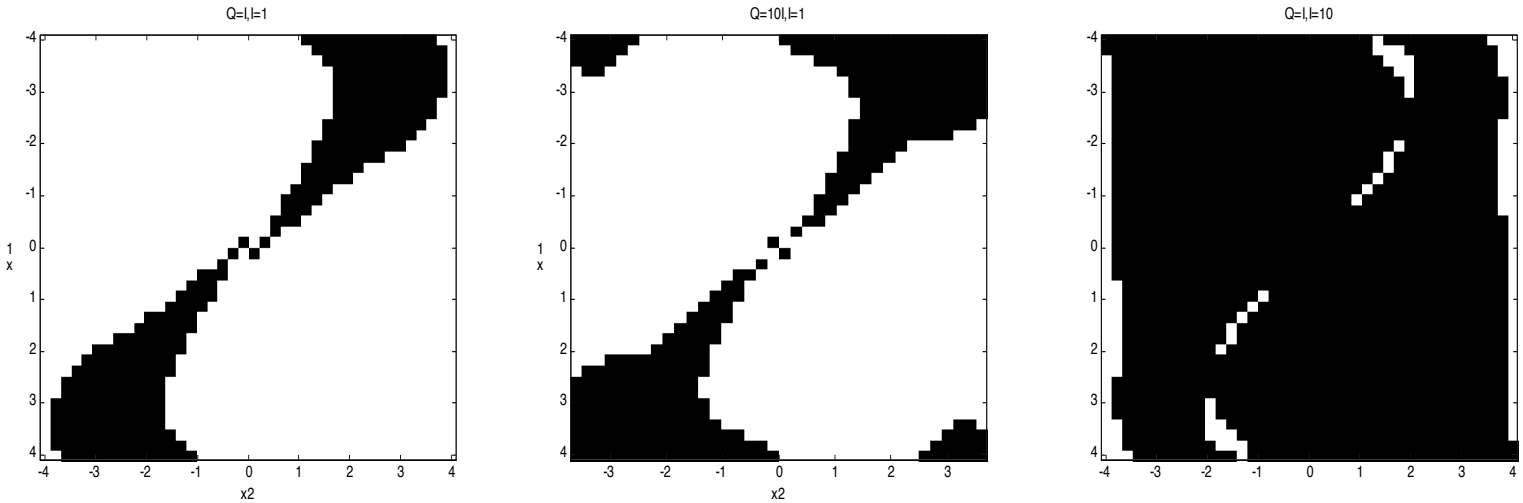


Fig. 3. Performance region comparison for different performance indices: (Q=1, R=1), (Q=10, R=1), and (Q=1, R=10), from left to right, respectively (dark region means linear one has better performance).

Conclusions and Suggestions

- Conclusions
  - Stable Fuzzy Controller
  - Local Optimality
    - How close is it to the global optimal solution?!
- Suggestions
  - Comparison with other T-S controllers
  - Modeling error and stability (polytopic systems)
  - Considering the effect of membership functions explicitly