

MAE 320 - Chapter 3

Properties of Pure Substances

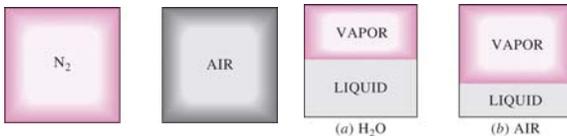
The content and the pictures are from the text book: Çengel, Y. A. and Boles, M. A., "Thermodynamics: An Engineering Approach," McGraw-Hill, New York, 6th Ed., 2008

Objectives

- Introduce the **concept of a pure substance**.
- Discuss the physics of **phase-change processes**.
- Illustrate the P - v , T - v , and P - T property diagrams and P - v - T surfaces of pure substances.
- Demonstrate the procedures for determining thermodynamic properties of pure substances from **tables of property data**.
- Describe the hypothetical substance "**ideal gas**" and the ideal-gas equation of state.
- Introduce the **compressibility factor**, which accounts for the deviation of real gases from ideal-gas behavior.
- Present some of the best-known equations of state.

Pure Substance

- **Pure substance:** A substance that has a **fixed chemical composition throughout**.
- Air is a **mixture** of several gases, but it is considered to be a pure substance, because this mixture is **homogeneous**.
- In a multi-phase system, if the chemical composition of all the phases is the **same**, it is a pure substance.



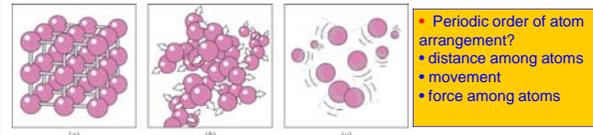
Nitrogen and gaseous air are pure substances.

A mixture of liquid and gaseous water is a pure substance, but a mixture of liquid and gaseous air is not.

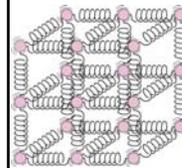
Phases of a Pure Substance

SOLID - LIQUID - GAS

The arrangement of atoms in different phases: (a) molecules are at relatively fixed positions in a **solid**, (b) groups of molecules move about each other in the **liquid** phase, and (c) molecules move about at random in the **gas** phase.



- Periodic order of atom arrangement?
- distance among atoms
- movement
- force among atoms



The molecules in a solid are kept at their positions by the large spring-like inter-molecular forces.

In a solid, the attractive and repulsive forces between the molecules tend to maintain them at relatively constant distances from each other.

Phase-change Processes of water

- **Compressed liquid (subcooled liquid):** A substance that it is *not about to vaporize*. At 1 atm, 20° to <100 °C, water exists in the liquid phase
- **Saturated liquid:** A liquid that is *about to vaporize*. At 1 atm pressure and 100°C, water exists as a liquid that is ready to vaporize.

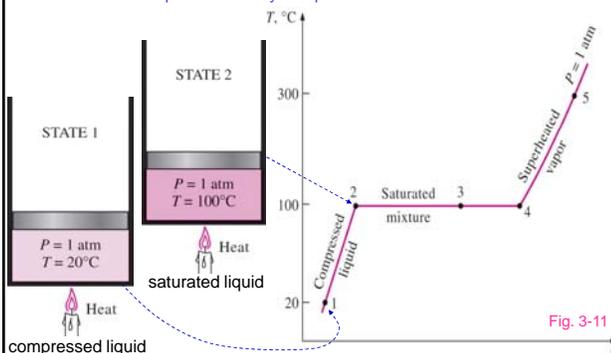


Fig. 3-11

Phase-change Processes of water

Saturated liquid–vapor mixture: The state at which the *liquid and vapor phases coexist in equilibrium*. At 1 atm pressure, the temperature remains constant at 100°C until the last drop of liquid is vaporized.

Superheated vapor: A vapor that is *not about to condense* (i.e., not a saturated vapor). As more heat is transferred, the temperature of the vapor starts to rise.

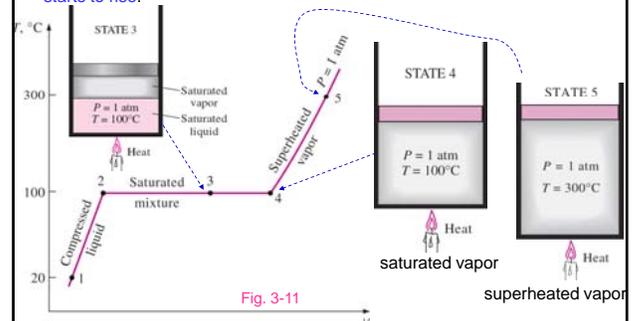


Fig. 3-11

Phase-change Processes of water

If the entire process between state 1 and 5 described in the figure is reversed by cooling the water while maintaining the pressure at the same value, the water will go back to state 1, retracing the same path.

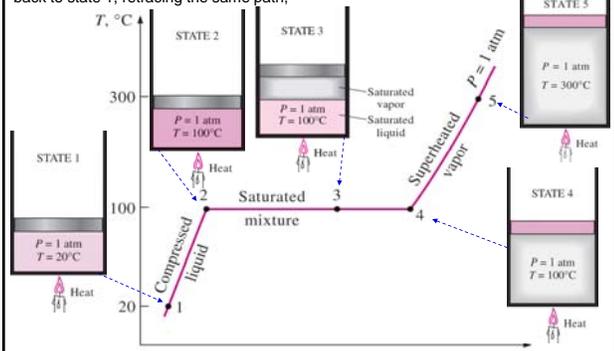


Fig. 3-11 T-v diagram for the heating process of water at constant pressure.

Saturation Temperature and Saturation Pressure

- Latent heat:** The amount of energy absorbed or released during a phase-change process.
- Latent heat of fusion:** The amount of energy absorbed during melting. It is equivalent to the amount of energy released during freezing.
- Latent heat of vaporization:** The amount of energy absorbed during vaporization and it is equivalent to the energy released during condensation.
- The magnitudes of the latent heats depend on the temperature or pressure at which the phase change occurs.

At 1 atm pressure, the latent heat of fusion of water is 333.7 kJ/kg and the latent heat of vaporization is 2256.5 kJ/kg=kg_{h₂O}.

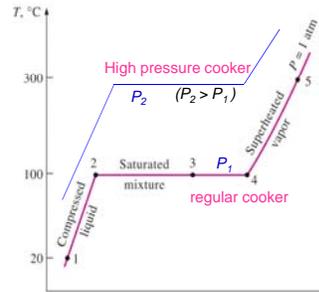
Saturation Temperature and Saturation Pressure

- The temperature at which water starts boiling depends on the pressure; therefore, if the pressure is fixed, so is the boiling temperature.
- Water boils at 100°C at 1 atm pressure.
- Saturation temperature T_{sat} :** The temperature at which a pure substance changes phase at a given pressure.
- Saturation pressure P_{sat} :** The pressure at which a pure substance changes phase at a given temperature.

TABLE 3-2

Variation of the standard atmospheric pressure and the boiling (saturation) temperature of water with altitude

Elevation, m	Atmospheric pressure, kPa	Boiling temperature, °C
0	101.33	100.0
1,000	89.55	96.5
2,000	79.50	93.3
5,000	54.05	83.3
10,000	26.50	66.3
20,000	5.53	34.7



Saturation Temperature and Saturation Pressure

- During a phase-change process, pressure and temperature dependent on each other. A plot of T_{sat} versus P_{sat} is called a **liquid-vapor saturation curve**.

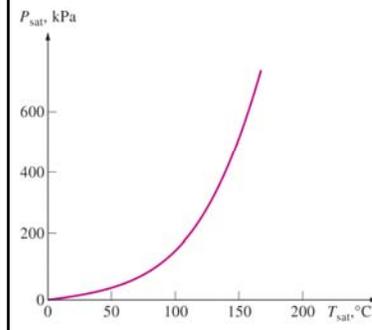


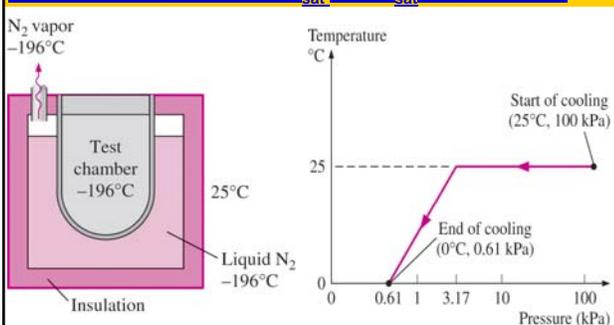
TABLE 3-1

Saturation (boiling) pressure of water at various temperatures

Temperature, T , °C	Saturation pressure, P_{sat} , kPa
-10	0.26
-5	0.40
0	0.61
5	0.87
10	1.23
15	1.71
20	2.34
25	3.17
30	4.25
40	7.39
50	12.35
100	101.4
150	476.2
200	1555
250	3976
300	8588

Fig.3-12 The liquid-vapor saturation curve of a pure substance (numerical values are for water).

Some Consequences of T_{sat} and P_{sat} Dependence



The temperature of liquid nitrogen exposed to the atmosphere remains constant at 196°C, and thus it maintains the test chamber at 196°C.

The variation of the temperature of fruits and vegetables with pressure during vacuum cooling from 25°C to 0°C.

Property Diagrams for Phase-change Processes

- The variations of properties during phase-change processes are described with the property diagrams such as the T-v, P-v, and P-T diagrams for pure substances.

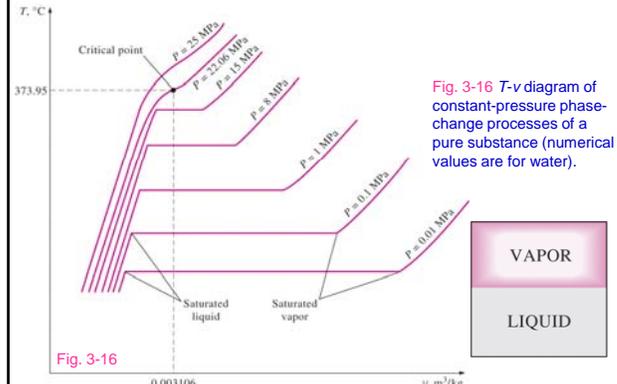


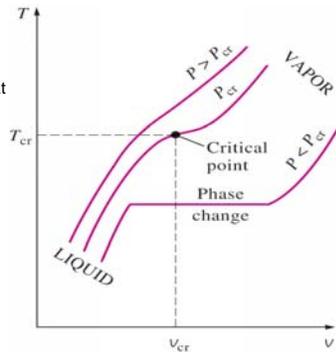
Fig. 3-16 T-v diagram of constant-pressure phase-change processes of a pure substance (numerical values are for water).

Fig. 3-16

Critical Point

- **Critical point:** The point at which the saturated liquid and saturated vapor states are identical.
- At supercritical pressures ($P > P_{cr}$), there is no distinct phase-change (boiling) process.

The temperature, pressure and specific volume of a substance at the critical point are called the **critical temperature**, **critical pressure** and **critical specific volume**, respectively



T-v Diagram of a Pure Substance

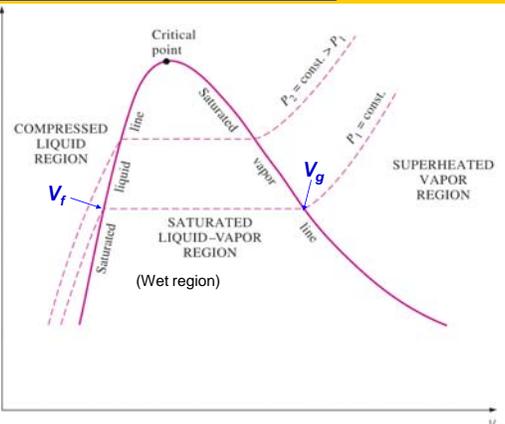


Fig.3-18

P-v Diagram of a Pure Substance

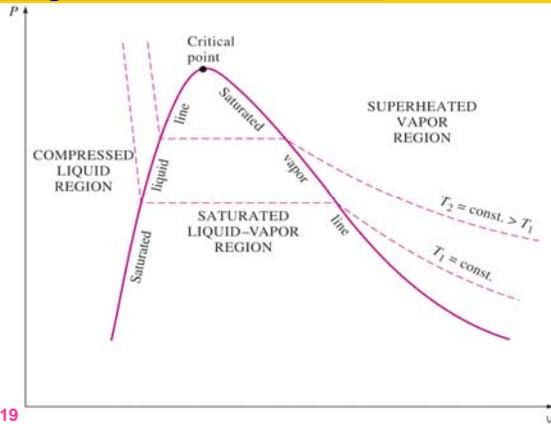


Fig.3-19

Extending the Diagrams to Include the Solid Phase

For water,
 $T_{tp} = 0.01^\circ\text{C}$
 $P_{tp} = 0.6117 \text{ kPa}$

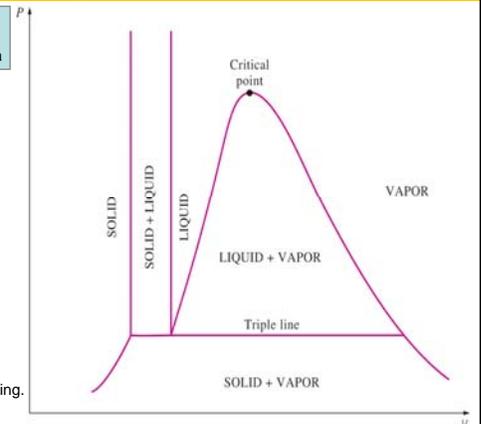


Fig. 3-21

P-v diagram of a substance that contracts on freezing.

Extending the Diagrams to Include the Solid Phase

At triple-point pressure and temperature, a substance exists in three phases in equilibrium.

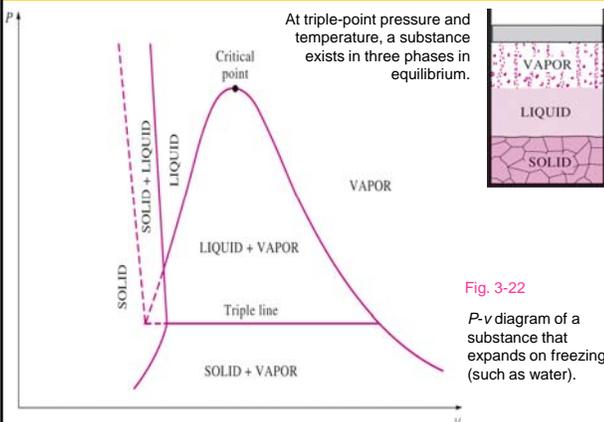


Fig. 3-22

P-v diagram of a substance that expands on freezing (such as water).

P-T Diagram: Phase Diagram

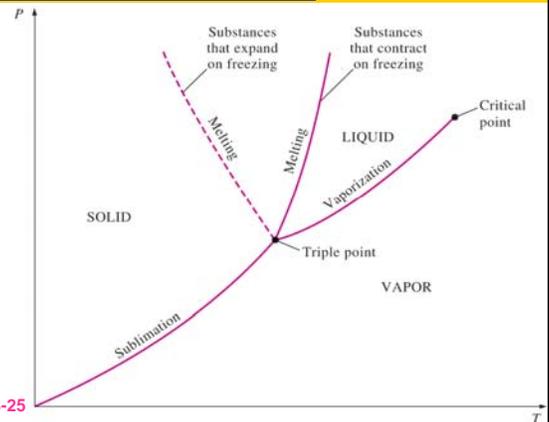


Fig.3-25

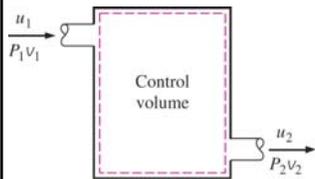
Property Tables

- For most substances, the relationships among thermodynamic properties are too complex to be expressed by simple equations. Therefore, properties are frequently presented in the form of tables (for example, Table A-6 in Appendix 1 in the text book).

Enthalpy — A Combination Property

$$h = u + Pv \quad (\text{kJ/kg})$$

$$H = U + PV \quad (\text{kJ})$$



The combination $u + Pv$ is frequently encountered in the analysis of control volumes

$\text{kPa} \cdot \text{m}^3 \equiv \text{kJ}$
$\text{kPa} \cdot \text{m}^3/\text{kg} \equiv \text{kJ/kg}$
$\text{bar} \cdot \text{m}^3 \equiv 100 \text{ kJ}$
$\text{MPa} \cdot \text{m}^3 \equiv 1000 \text{ kJ}$
$\text{psi} \cdot \text{ft}^3 \equiv 0.18505 \text{ Btu}$

The product *pressure × volume* has energy units.

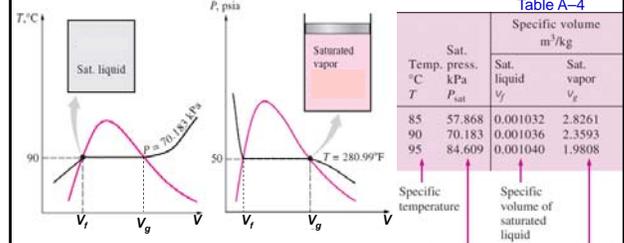
Saturated Liquid and Saturated Vapor States

- Table A-4:** Saturation properties of water under temperature.
- Table A-5:** Saturation properties of water under pressure.

V_f = specific volume of saturated liquid

V_g = specific volume of saturated vapor

$$V_{fg} = V_g - V_f$$



Enthalpy of vaporization, h_{fg} (Latent heat of vaporization): The amount of energy needed to vaporize a unit mass of saturated liquid at a given temperature or pressure.

Corresponding saturation pressure
Specific volume of saturated liquid
Specific volume of saturated vapor

Property Tables

Example 3-1 A sealed rigid tank contains 50 kg of saturated liquid water at 90 °C. Determine the pressure in the tank and the volume of liquid water.

Analysis: Since the saturation condition exist in a sealed tank, the pressure in the tank is the saturation pressure:

$$P = P_{sat @ 90^\circ\text{C}} = \mathbf{70.183 \text{ kPa}} \quad (\text{Table A-4})$$

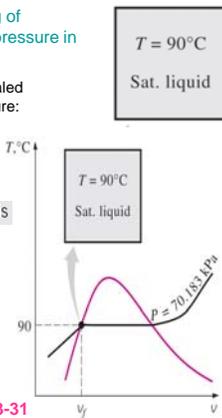
The specific volume of the saturated liquid at 90 °C is

$$v = v_f @ 90^\circ\text{C} = 0.001036 \text{ m}^3/\text{kg} \quad (\text{Table A-4})$$

The total volume of liquid water:

$$V = mv = (50 \text{ kg})(0.001036 \text{ m}^3/\text{kg}) = \mathbf{0.0518 \text{ m}^3}$$

Fig.3-31



Property Tables – Interpolation Method

A piston-cylinder contains 3 m³ of saturated water vapor at 340 kPa pressure, determine the temperature and the mass of the vapor inside the cylinder.

Analysis: Since the cylinder contains the saturated vapor at 340 kPa pressure, the temperature inside must be T_{sat} at 340 kPa.

In Table A-5, there is no T_{sat} data for 340 kPa

However, there are

$$P_{sat} = 325 \text{ kPa}, T_{sat} = 136.27^\circ\text{C}$$

$$P_{sat} = 350 \text{ kPa}, T_{sat} = 138.86^\circ\text{C}$$

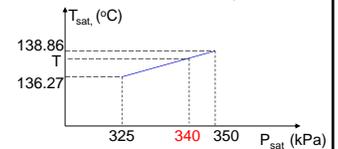
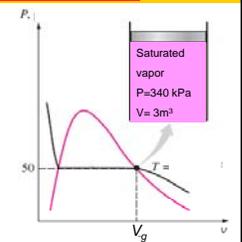
Use the **interpolation method** to get the Unknown temperature

$$T = \mathbf{137.54^\circ\text{C}}$$

Use the similar approach to get:

$$v_g = 0.539328 \text{ (m}^3/\text{kg)}$$

$$m = V/v_g = 3 \text{ (m}^3)/0.539328 \text{ (m}^3/\text{kg}) = \mathbf{5.56 \text{ kg}}$$



Interpolation method

Property Tables

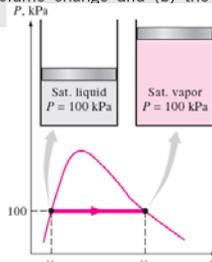
Example 3-3

A mass of 200 g of saturated liquid water is completely vaporized at a constant pressure of 100 kPa. Determine (a) the volume change and (b) the amount of energy transferred to the water.

(a) At 100 kPa (See Table A-5):

$$v_{fg} = v_g - v_f = 1.6941 - 0.001043 = \mathbf{1.6931 \text{ m}^3/\text{kg}}$$

$$\Delta U = mv_{fg} = (0.2 \text{ kg})(1.6931 \text{ m}^3/\text{kg}) = \mathbf{0.3386 \text{ m}^3}$$



(b) The amount of energy needed to vaporize a unit mass of a substance at a given pressure is the enthalpy of vaporization at that pressure, which is $h_{fg} = 2257.5 \text{ kJ/kg}$ for water at 100 kPa. Thus, the amount of energy transferred is $mh_{fg} = (0.2 \text{ kg})(2257.5 \text{ kJ/kg}) = \mathbf{451.5 \text{ kJ}}$

Saturated Liquid–Vapor Mixture

Quality, x : The ratio of the mass of vapor to the total mass of the mixture.

The properties of the saturated liquid are the same whether it exists alone or in a mixture with saturated vapor.

$$x = \frac{m_{\text{vapor}}}{m_{\text{total}}}$$

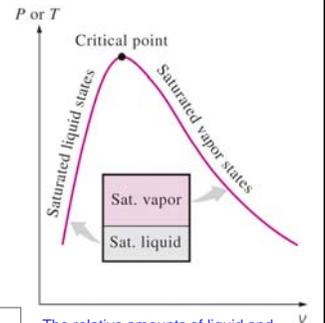
$$m_{\text{total}} = m_{\text{liquid}} + m_{\text{vapor}}$$

$$m_t = m_f + m_g$$

Quality, x , is between 0 and 1 for a mixture:

- 0 for saturated liquid,
- 1 for saturated vapor.

Temperature and pressure are dependent properties for a mixture.



The relative amounts of liquid and vapor phases in a saturated mixture are specified by the **quality x** .

Saturated Liquid–Vapor Mixture

Consider a tank that contains a saturated liquid–vapor mixture. The volume occupied by saturated liquid is V_f and the volume occupied by saturated vapor is V_g , the total volume V is the sum of the two:

$$V = V_f + V_g$$

$$\therefore V = mv$$

$$\therefore m_t v_{avg} = m_f v_f + m_g v_g$$

$$\therefore m_f = m_t - m_g$$

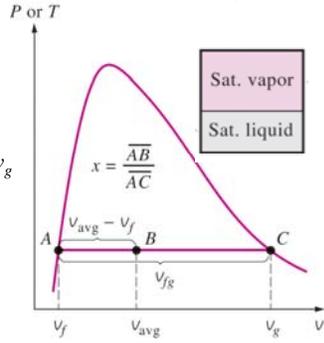
$$\therefore m_t v_{avg} = (m_t - m_g) v_f + m_g v_g$$

$$x = m_g / m_t \Rightarrow$$

$$v_{avg} = (1 - x) v_f + x v_g$$

$$v_{avg} = v_f + x v_{fg}$$

$$x = \frac{v_{avg} - v_f}{v_{fg}}$$



Quality is related to the horizontal distances on P-v and T-v diagrams.

Saturated Liquid–Vapor Mixture

$$v_{avg} = v_f + x v_{fg} \quad (m^3/kg)$$

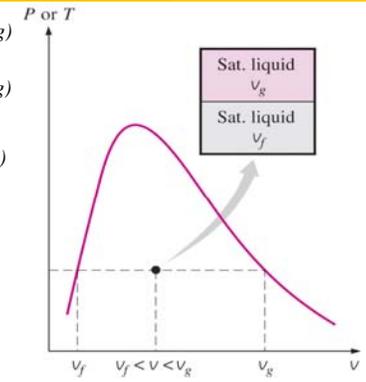
$$u_{avg} = u_f + x u_{fg} \quad (kJ/kg)$$

$$h_{avg} = h_f + x h_{fg} \quad (kJ/kg)$$

$y \rightarrow v, u, \text{ or } h$

$$y_{avg} = y_f + x y_{fg}$$

$$y_f \leq y_{avg} \leq y_g$$



The v value of a saturated liquid–vapor mixture lies between the v_f and v_g values at the specified T or P .

Saturated Liquid–Vapor Mixture

Example 3-4

A rigid tank contains 10 kg of water at 90°C . If 8 kg of the water is in the liquid form and the rest is in the vapor form, determine (a) the pressure in the tank and (b) the volume of the tank.

Analysis: (a) the state of the saturated liquid–vapor mixture is shown in Fig. 3-38. Since the two phases coexist in equilibrium, we have a saturated mixture, and the pressure must be the saturation pressure at the given temperature:

$$P = P_{\text{sat}} @ 90^\circ\text{C} = 70.183 \text{ kPa} \quad (\text{Table A-4})$$

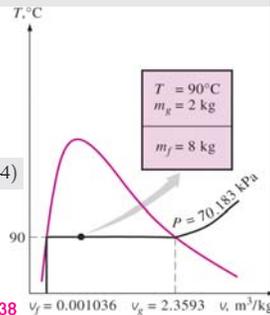


FIGURE 3-38

Saturated Liquid–Vapor Mixture

Example 3-4

(b) At 90°C , we have $v_f = 0.001036 \text{ m}^3/\text{kg}$ and $v_g = 2.3593 \text{ m}^3/\text{kg}$ (Table A-4). One way of finding the volume of the tank is to determine the volume occupied by each phase and then add them:

$$V = V_f + V_g = m_f v_f + m_g v_g$$

$$= (8 \text{ kg})(0.001036 \text{ m}^3/\text{kg}) + (2 \text{ kg})(2.3593 \text{ m}^3/\text{kg})$$

$$= 4.73 \text{ m}^3$$

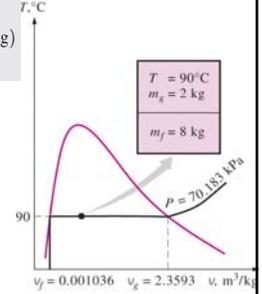


FIGURE 3-38

Saturated Liquid–Vapor Mixture

Example 3-5

An 80-L vessel contains 4 kg of refrigerant-134a at a pressure of 160 kPa. Determine (a) the temperature, (b) the quality, (c) the enthalpy of the refrigerant, and (d) the volume occupied by the vapor phase.

Analysis: (a) The state of the saturated liquid–vapor mixture is shown in Fig. 3-39. At this point we do not know whether the refrigerant is in the compressed liquid, superheated vapor, or saturated mixture region. This can be determined by comparing a suitable property to the saturated liquid and saturated vapor values. From the information given, we can determine the specific volume:

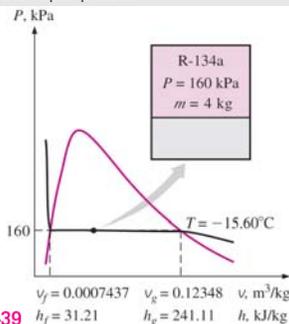


FIGURE 3-39

Saturated Liquid–Vapor Mixture

Example 3-5

$$v = \frac{V}{m} = \frac{0.080 \text{ m}^3}{4 \text{ kg}} = 0.02 \text{ m}^3/\text{kg}$$

At 160 kPa, we read Table A-12

$$v_f = 0.0007437 \text{ m}^3/\text{kg}$$

$$v_g = 0.12348 \text{ m}^3/\text{kg}$$

Obviously, $v_f < v < v_g$.

Therefore, the refrigerant is in the wet region, the temperature must be the saturation temperature:

$$T = T_{\text{sat}} @ 160 \text{ kPa} = -15.60^\circ\text{C}$$

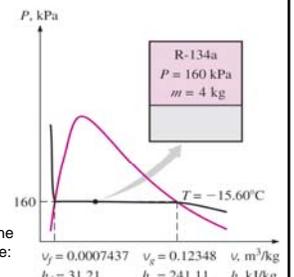


FIGURE 3-39

Saturated Liquid-Vapor Mixture

(b) Quality can be determined from

$$x = \frac{v - v_f}{v_{fg}} = \frac{0.02 - 0.0007437}{0.12348 - 0.0007437} = 0.157$$

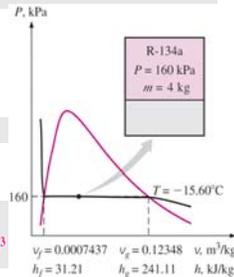
(c) At 160 kPa, we also read from Table A-12 that $h_f = 31.21$ kJ/kg and $h_{fg} = 209.90$ kJ/kg. Then,

$$\begin{aligned} h &= h_f + xh_{fg} \\ &= 31.21 \text{ kJ/kg} + (0.157)(209.90 \text{ kJ/kg}) \\ &= 64.2 \text{ kJ/kg} \end{aligned}$$

(d) The mass of the vapor is $m_g = xm_t = (0.157)(4 \text{ kg}) = 0.628 \text{ kg}$

the volume occupied by the vapor phase is

$$V_g = m_g v_g = (0.628 \text{ kg})(0.12348 \text{ m}^3/\text{kg}) = 0.0775 \text{ m}^3$$



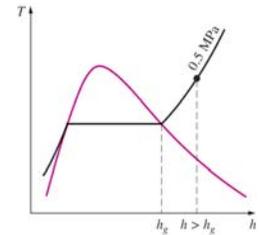
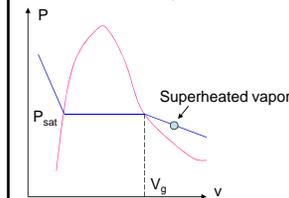
Superheated Vapor

In the region to the right of the saturated vapor line and at temperatures above the critical point temperature, a substance exists as superheated vapor.

In this region, temperature and pressure are independent properties.

Compared to the saturated vapor, superheated vapor is characterized by:

- Lower pressures ($P < P_{sat}$ at given T)
- higher temperatures ($T > T_{sat}$ at given P)
- higher specific volumes ($v > v_g$ at given P or T)
- higher internal energies ($u > u_g$ at given P or T)
- higher enthalpies ($h > h_g$ at given P or T)



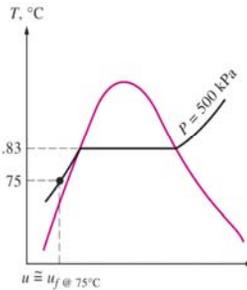
At a specified P , superheated vapor exists at a higher h than the saturated vapor.

Compressed Liquid

The compressed liquid properties depend on temperature much more strongly than they do on pressure.

Compressed liquid is characterized by:

- higher pressures ($P > P_{sat}$ at given T)
- lower temperatures ($T < T_{sat}$ at given P)
- lower specific volumes ($v < v_f$ at given P or T)
- lower internal energies ($u < u_f$ at given P or T)
- Lower enthalpies ($h < h_f$ at given P or T)



At a given P and T , a pure substance will exist as a compressed liquid if $T < T_{sat @ P}$

Compressed Liquid

The compressed liquid properties depend on temperature much more strongly than they do on pressure.

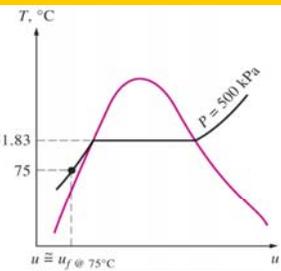
A compressed liquid may be approximated as a saturated liquid at the given temperature.

$$y \cong y_f @ T$$

$$y \rightarrow v, u, \text{ or } h$$

A more accurate relation for h

$$h \cong h_f @ T + v_f @ T (P - P_{sat @ T})$$



Given: P and T

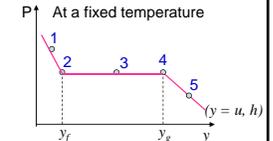
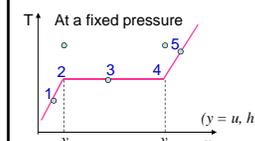
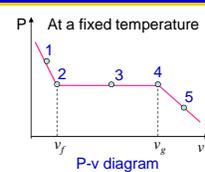
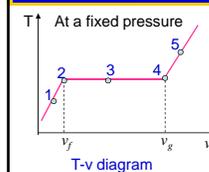
$$\begin{aligned} v &\cong v_f @ T \\ u &\cong u_f @ T \\ h &\cong h_f @ T \end{aligned}$$

Summary of Properties of Pure Substance

property	state				
	compressed liquid	saturated liquid	saturated liquid-vapor mixture	saturated vapor	superheated vapor
T	$T < T_{sat}$	$T_{sat @ P}$	$T_{sat @ P}$	$T_{sat @ P}$	$T > T_{sat}$
P	$P > P_{sat}$	$P_{sat @ T}$	$P_{sat @ T}$	$P_{sat @ T}$	$P < P_{sat}$
x		0	$0 < x < 1$	1	
v	$v \approx v_f @ T$	v_f	$v_f < v < v_g$	v_g	$v > v_g$
u	$u \approx u_f @ T$	u_f	$u_f < u < u_g$	u_g	$u > u_g$
h	$h \approx h_f @ T$	h_f	$h_f < h < h_g$	h_g	$h > h_g$

More accurate $h \cong h_f @ T + v_f @ T (P - P_{sat @ T})$

Summary of Diagrams of Pure Substances



Point 1: compressed liquid;
Point 3: liquid-vapor mixture;
Point 5: superheated vapor

Point 2: saturated liquid;
Point 4: saturated vapor;

Reference State and Reference Values

- The values of u , h , and s cannot be measured directly, and they are calculated from measurable properties using the relations between properties.
- However, those relations give the *changes* in properties, not the values of properties at specified states.
- Therefore, we need to choose a convenient *reference state* and assign a value of zero for a convenient property or properties at that state.
- Some properties may have negative values as a result of the reference state chosen.
- Sometimes different tables list different values for some properties at the same state as a result of using a different reference state.
- However, in thermodynamics we are concerned with the *changes* in properties, and the reference state chosen is of no consequence in calculations.

Saturated water—Temperature table

Temp., T , °C	Sat. press., P_{sat} , kPa	Specific volume, m^3/kg		Internal energy, kJ/kg			Enthalpy, kJ/kg			Entropy, kJ/kg · K		
		Sat. liquid, v_f	Sat. vapor, v_g	Sat. liquid, u_f	Evap., u_{fg}	Sat. vapor, u_g	Sat. liquid, h_f	Evap., h_{fg}	Sat. vapor, h_g	Sat. liquid, s_f	Evap., s_{fg}	Sat. vapor, s_g
0.01	0.6117	0.001000	206.00	0.000	2374.9	2374.9	0.001	2500.9	2500.9	0.0000	9.1556	9.1556
5	0.8725	0.001000	147.03	21.019	2360.8	2381.8	21.020	2489.1	2510.1	0.0763	8.9487	9.0249

Properties of Water

Example: Determine the internal energy of water at 200 kPa and 400 °C

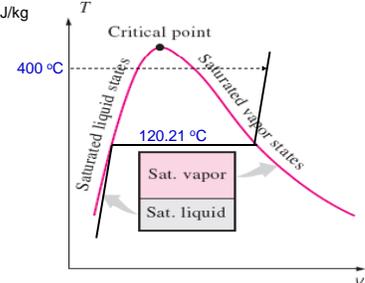
Analysis: we do not know whether water is in compressed liquid, saturated mixture, or superheated region. The key is to determine the status of water.

Solution:

From Table A-5, the $T_{\text{sat}@ 200 \text{ kPa}} = 120.21$ °C

Since $T = 400$ °C $>$ $T_{\text{sat}@ 200 \text{ kPa}} = 120.21$ °C, it is superheated vapor

From Table A-6, $u = 2967.2$ kJ/kg



Property of Water

Example 3-7

Determine the temperature of water at a state of $P = 0.5$ MPa and $h = 2890$ kJ/kg.

Analysis: At 0.5 MPa, the enthalpy of saturated water vapor is $h_g = 2748.1$ kJ/kg. Since $h > h_g$, as shown in Fig. 3–41, we again have superheated vapor. Under 0.5 MPa in Table A–6 we read

T , °C	h , kJ/kg
200	2855.8
250	2961.0

The temperature is between 200 and 250 °C, by linear interpolation:

$$T = 216.3$$
 °C

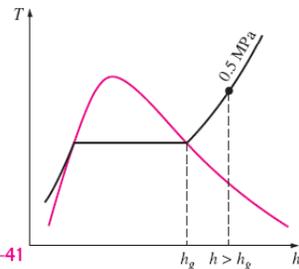


FIGURE 3–41

The Use of Steam table

Example 3-9:

Determine the missing properties and the phase descriptions in the following table for water:

	T , °C	P , kPa	u , kJ/kg	x	Phase description
(a)		200		0.6	
(b)	125		1600		
(c)		1000	2950		
(d)	75	500			
(e)		850		0.0	

The Use of Steam table

Example 3-9:

Analysis (a) The quality is given to be $x = 0.6$, which implies that 60 percent of the mass is in the vapor phase and the remaining 40 percent is in the liquid phase. Therefore, we have saturated liquid–vapor mixture at a pressure of 200 kPa. Then the temperature must be the saturation temperature at the given pressure:

$$T = T_{\text{sat}@ 200 \text{ kPa}} = 120.21$$
 °C (Table A–5)

At 200 kPa, we also read from Table A–5 that $u_f = 504.50$ kJ/kg and $u_{fg} = 2024.6$ kJ/kg. Then the average internal energy of the mixture is

$$\begin{aligned} u &= u_f + xu_{fg} \\ &= 504.50 \text{ kJ/kg} + (0.6)(2024.6 \text{ kJ/kg}) \\ &= 1719.26 \text{ kJ/kg} \end{aligned}$$

Property of Water

Example 3-9:

- (b) if $u < u_f$ we have *compressed liquid*
 if $u_f \leq u \leq u_g$ we have *saturated mixture*
 if $u > u_g$ we have *superheated vapor*

In our case the given u value is 1600, which falls between the u_f and u_g values at 125°C. Therefore, we have saturated liquid–vapor mixture. Then the pressure must be the saturation pressure at the given temperature:

$$P = P_{\text{sat}@ 125^\circ\text{C}} = 232.23 \text{ kPa} \quad (\text{Table A–4})$$

The quality is determined from

$$x = \frac{u - u_f}{u_{fg}} = \frac{1600 - 524.83}{2009.5} = 0.535$$

Property of Water

Example 3-9:

(c) This is similar to case (b), except pressure is given instead of temperature. Following the argument given above, we read the u_f and u_g values at the specified pressure. At 1 MPa, we have $u_f = 761.39$ kJ/kg and $u_g = 2582.8$ kJ/kg. The specified u value is 2950 kJ/kg, which is greater than the u_g value at 1 MPa. Therefore, we have superheated vapor, and the temperature at this state is determined from the superheated vapor table by interpolation to be

$$T = 395.2^\circ\text{C} \quad (\text{Table A-6})$$

We would leave the quality column blank in this case since quality has no meaning for a superheated vapor.

Property of Water

Example 3-9:

- (d)
- | | | |
|----|---------------------------------|----------------------------------|
| if | $T < T_{\text{sat @ given } P}$ | we have <i>compressed liquid</i> |
| if | $T = T_{\text{sat @ given } P}$ | we have <i>saturated mixture</i> |
| if | $T > T_{\text{sat @ given } P}$ | we have <i>superheated vapor</i> |

In our case, the given T value is 75°C , which is less than the T_{sat} value at the specified pressure. Therefore, we have compressed liquid (Fig. 3-44), and normally we would determine the internal energy value from the compressed liquid table. But in this case the given pressure is much lower than the lowest pressure value in the compressed liquid table (which is 5 MPa), and therefore we are justified to treat the compressed liquid as saturated liquid at the given temperature (*not* pressure):

$$u \cong u_f @ 75^\circ\text{C} = 313.99 \text{ kJ/kg} \quad (\text{Table A-4})$$

Property of Water

Example 3-9:

(e) The quality is given to be $x = 0$, and thus we have saturated liquid at the specified pressure of 850 kPa. Then the temperature must be the saturation temperature at the given pressure, and the internal energy must have the saturated liquid value:

$$T = T_{\text{sat @ 850 kPa}} = 172.94^\circ\text{C}$$

$$u = u_f @ 850 \text{ kPa} = 731.00 \text{ kJ/kg} \quad (\text{Table A-5})$$

The Ideal-gas Equation of State

- Equation of state:** Any equation that relates the pressure, temperature, and specific volume of a substance.
- The simplest and best-known equation of state for substances in the gas phase is the **ideal-gas equation of state**. This equation predicts the P - v - T behavior of a gas quite accurately within some properly selected region.

$$Pv = RT \quad \text{Ideal gas equation of state}$$

- P: Pressure (kPa)
 V: Specific volume (m^3/kg)
 R: gas constant ($\text{kJ}/\text{kg} \cdot \text{K}$)
 T: Thermodynamic temperature (K)

Note:

In the SI unit system, the unit of pressure is Pa, and the unit of R is $\text{J}/\text{kg} \cdot \text{K}$
 In the text book, the unit of pressure is kPa, and the unit of R is $\text{kJ}/\text{kg} \cdot \text{K}$

The Ideal-gas Equation of State

$$R = \frac{R_u}{M}$$

R: gas constant ($\text{kJ}/\text{kg} \cdot \text{K}$) or ($\text{kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K}$) $J = N \cdot m = Pa \cdot m^2 \cdot m = Pa \cdot m^3$

R_u : universal gas constant ($\text{kJ}/\text{kmol} \cdot \text{K}$)

M: molar mass (kg/kmol) or (g/mol)

$$R_u = \begin{cases} 8.31447 \text{ kJ}/\text{kmol} \cdot \text{K} \\ 8.31447 \text{ kPa} \cdot \text{m}^3/\text{kmol} \cdot \text{K} \\ 0.0831447 \text{ bar} \cdot \text{m}^3/\text{kmol} \cdot \text{K} \\ 1.98588 \text{ Btu}/\text{lbmol} \cdot \text{R} \\ 10.7316 \text{ psia} \cdot \text{ft}^3/\text{lbmol} \cdot \text{R} \\ 1545.37 \text{ ft} \cdot \text{lb}/\text{lbmol} \cdot \text{R} \end{cases}$$

$$m = MN \quad (\text{kg}) \quad \text{Mass} = \text{Molar mass} \times \text{Mole number}$$

The Ideal-gas Equation of State

Various expressions of ideal gas equation

$$\begin{aligned} U &= mU \longrightarrow PV = mRT \\ mR &= (MN)R = NR_u \longrightarrow PV = NR_uT \\ V &= N\bar{v} \longrightarrow P\bar{v} = R_uT \end{aligned}$$

where \bar{v} is the molar specific volume, that is, the volume per unit mole

Real gases behave as an ideal gas at low densities (i.e., low pressure, high temperature).

The ideal-gas relation often is not applicable to real gases; thus, care should be exercised when using it.

Ideal gas equation at two states for a fixed mass:

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

The Ideal-gas Equation of State

Example 3-10:

Determine the mass of the air in a room whose dimensions are 4 m × 5 m × 6 m at 100 kPa and 25°C.

Analysis A sketch of the room is given in Fig. 3–48. Air at specified conditions can be treated as an ideal gas. From Table A–1, the gas constant of air is $R = 0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K}$, and the absolute temperature is $T = 25^\circ\text{C} + 273 = 298 \text{ K}$. The volume of the room is

$$V = (4 \text{ m})(5 \text{ m})(6 \text{ m}) = 120 \text{ m}^3$$

The mass of air in the room is determined from the ideal-gas relation to be

$$m = \frac{PV}{RT} = \frac{(100 \text{ kPa})(120 \text{ m}^3)}{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(298 \text{ K})} = 140.3 \text{ kg}$$

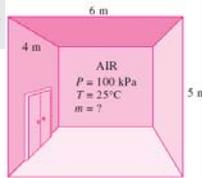


FIGURE 3–48

Is Water Vapor an Ideal Gas?

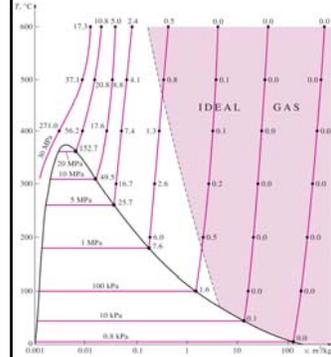


Fig. 3-49 Percentage of error $([v_{\text{table}} - v_{\text{ideal}}]/v_{\text{table}}) \times 100$ involved in assuming steam to be an ideal gas, and the region where steam can be treated as an ideal gas with less than 1 percent error.

- At pressures below 10 kPa, water vapor can be treated as an ideal gas, regardless of its temperature, with negligible error (less than 0.1 percent).
- At higher pressures, however, the ideal gas assumption yields unacceptable errors, particularly in the vicinity of the critical point and the saturated vapor line.
- In air-conditioning applications, the water vapor in the air can be treated as an ideal gas. Why?
- In steam power plant applications, however, the pressures involved are usually very high; therefore, ideal-gas relations should not be used.

Compressibility Factor

Compressibility factor Z: A factor that accounts for the deviation of real gases from ideal-gas behavior at a given temperature and pressure.

- The farther away Z is from unity, the more the gas deviates from ideal-gas behavior.
- If $z = 0.95-1$, the gas can be treated as the idea gas
- Gases behave as an ideal gas at low densities (i.e., low pressure, high temperature).

$$Pv = zRT$$

$$Z = \frac{PV}{RT}$$

$$Z = \frac{v_{\text{actual}}}{v_{\text{ideal}}}$$



The compressibility factor is unity for ideal gases.

At very low pressures, all gases approach ideal-gas behavior (regardless of their temperature).

Compressibility Factor

The pressure or temperature of a gas is high or low relative to its critical temperature or pressure.

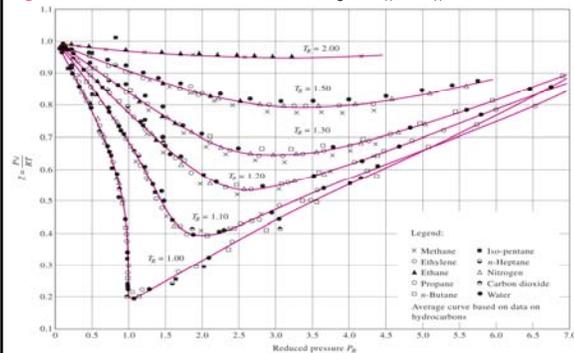
Reduced pressure:
$$P_R = \frac{P}{P_{cr}}$$

Reduced temperature:
$$T_R = \frac{T}{T_{cr}}$$

Pseudo-reduced specific volume:
$$v_R = \frac{v_{\text{actual}}}{RT_{cr}/P_{cr}}$$

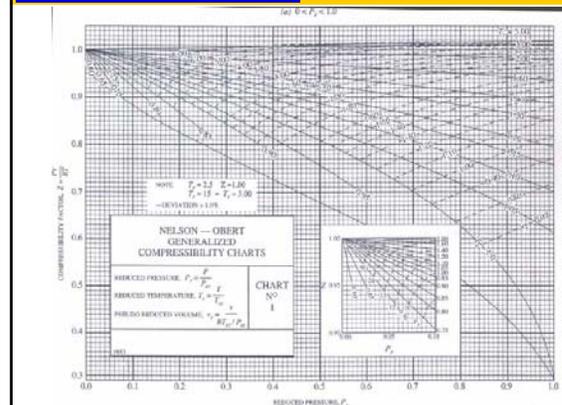
Compressibility Factor

Fig. 3-51 Z can be determined from a knowledge of P_R and v_R .



Principle of corresponding states: The Z factor for all gases is almost the same at the same reduced pressure and temperature

Compressibility Chart

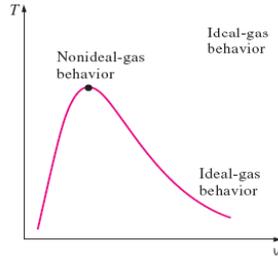


P934

Compressibility Factor

The following observation can be made from the generalized compressibility chart:

- At very low pressure ($P_R \ll 1$), gas behavior as an ideal gas regardless of temperature
- At high temperature ($T_R > 2$), ideal gas behavior can be assumed regardless of pressure ($P_R \gg 1$)
- The deviation of a gas from ideal-gas behavior is greatest in the vicinity of the critical point.



Compressibility Factor

Example 3-11

Determine the specific volume of refrigerant-134a at 1 MPa and 50°C, using (a) the ideal-gas equation of state and (b) the generalized compressibility chart. Compare the values obtained to the actual value of 0.021796 m³/kg and determine the error involved in each case.

Solution The specific volume of refrigerant-134a is to be determined assuming ideal- and nonideal-gas behavior.

Analysis The gas constant, the critical pressure, and the critical temperature of refrigerant-134a are determined from Table A-1 to be

$$R = 0.0815 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K}$$

$$P_{cr} = 4.059 \text{ MPa}$$

$$T_{cr} = 374.2 \text{ K}$$

Compressibility Factor

(a) The specific volume of refrigerant-134a under the ideal-gas assumption is

$$v = \frac{RT}{P} = \frac{(0.0815 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(323 \text{ K})}{1000 \text{ kPa}} = \mathbf{0.026325 \text{ m}^3/\text{kg}}$$

Therefore, treating the refrigerant-134a vapor as an ideal gas would result in an error of $(0.026325 - 0.021796)/0.021796 = \mathbf{0.208}$, or 20.8 percent in this case.

(b) To determine the correction factor Z from the compressibility chart, we first need to calculate the reduced pressure and temperature:

$$\left. \begin{aligned} P_R &= \frac{P}{P_{cr}} = \frac{1 \text{ MPa}}{4.059 \text{ MPa}} = 0.246 \\ T_R &= \frac{T}{T_{cr}} = \frac{323 \text{ K}}{374.2 \text{ K}} = 0.863 \end{aligned} \right\} Z = 0.84 \quad (\text{from Fig. A-15 in pg.934})$$

Thus

$$v = Zv_{ideal} = (0.84)(0.026325 \text{ m}^3/\text{kg}) = \mathbf{0.022113 \text{ m}^3/\text{kg}}$$

Other Equations of State

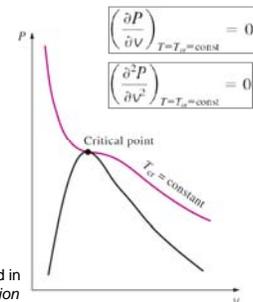
Several equations have been proposed to represent the P - v - T behavior of substances accurately over a larger region with no limitations.

Van der Waals Equation of State:

$$\left(P + \frac{a}{v^2}\right)(v - b) = RT$$

$$a = \frac{27R^2 T_{cr}^2}{64P_{cr}}$$

$$b = \frac{RT_{cr}}{8P_{cr}}$$



This model includes two effects not considered in the ideal-gas model: the *intermolecular attraction forces* and the *volume occupied by the molecules themselves*. The accuracy of the van der Waals equation of state is often inadequate.

Critical isotherm of a pure substance has an inflection point at the critical state.

Other Equations of State

Beattie-Bridgeman Equation of State:

$$P = \frac{R_u T}{\bar{v}^2} \left(1 - \frac{c}{T^3}\right) (\bar{v} + B) - \frac{A}{\bar{v}^2}$$

The constants are given in Table 3-4 for various substances. It is known to be reasonably accurate for densities up to about $0.8\rho_{cr}$.

$$A = A_0 \left(1 - \frac{a}{\bar{v}}\right) \quad B = B_0 \left(1 - \frac{b}{\bar{v}}\right)$$

Benedict-Webb-Rubin Equation of State:

$$P = \frac{R_u T}{\bar{v}} + \left(B_0 R_u T - A_0 - \frac{C_0}{T^2}\right) \frac{1}{\bar{v}^2} + \frac{b R_u T - a}{\bar{v}^3} + \frac{aa'}{\bar{v}^6} + \frac{c}{\bar{v}^3 T^2} \left(1 + \frac{\gamma}{\bar{v}^2}\right) e^{-\gamma/\bar{v}^2}$$

The constants are given in Table 3-4. This equation can handle substances at densities up to about $2.5\rho_{cr}$.

Virial Equation of State:

$$P = \frac{RT}{v} + \frac{a(T)}{v^2} + \frac{b(T)}{v^3} + \frac{c(T)}{v^4} + \frac{d(T)}{v^5} + \dots$$

The coefficients $a(T)$, $b(T)$, $c(T)$, and so on, that are functions of temperature alone are called *virial coefficients*.

Other Equations of State

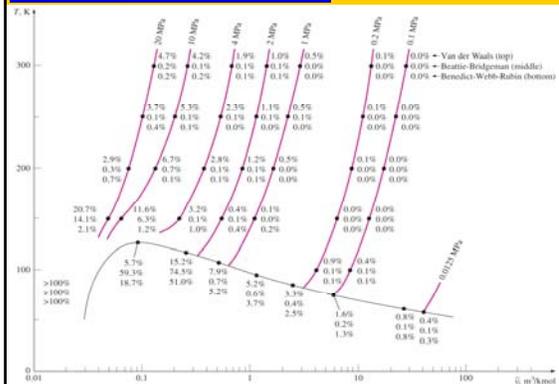


Fig. 3-60 Percentage of error involved in various equations of state for nitrogen (% error = $[(v_{table} - v_{equation})/v_{table}] \times 100$).

Summary

- Pure substance
- Phases of a pure substance
- Phase-change processes of pure substances
 - ✓ Compressed liquid, Saturated liquid, Saturated vapor, Superheated vapor
 - ✓ Saturation temperature and Saturation pressure
- Property diagrams for phase change processes
 - ✓ The T - v diagram, The P - v diagram, The P - T diagram
 - ✓ Property tables
 - ✓ Enthalpy
 - ✓ Saturated liquid, saturated vapor, Saturated liquid vapor mixture, Superheated vapor, compressed liquid
 - ✓ Reference state and reference values
- The ideal gas equation of state
- Compressibility factor
- Other equations of state