Contextual Abductive Reasoning with Side-Effects

Luís Moniz Pereira[1] **Emmanuelle-Anna Dietz**[1,2] **Steffen Hölldobler**[2]

[1]Centro de Inteligência Artificial (CENTRIA)
Universidade Nova de Lisboa, Portugal

[2]International Center for Computational Logic (ICCL)
TU Dresden, Germany

How to explain the forest fire?


we observe a forest fire

How to explain the forest fire?

we observe a forest fire

$\xleftarrow{\text{forest fire occurred because of lightning}}$

lightning

$\xleftarrow{\text{forest fire occurred because of barbecue}}$

barbecue

How to explain the forest fire?

we observe a storm

forest fire occurred because of lightning

only if a storm was observed

lightning

we observe a forest fire

forest fire occurred because of barbecue

barbecue

How to explain the forest fire?

we observe a storm

forest fire occurred because of lightning

only if a storm was observed

lightning

we observe a forest fire

forest fire occurred because of barbecue

barbecue

We want to express that when observing a storm, the forest fire occurred because of a lightning. Otherwise, it occurred because of a barbecue.

Motivation

Evans, Barston, and Pollard carried out an experiment where participants had to evaluate whether certain syllogisms are valid.

- Their answers were strongly influenced by the contextual setting and whether the syllogisms' conclusions confirmed or contradicted the participants' beliefs.

Motivation

Evans, Barston, and Pollard carried out an experiment where participants had to evaluate whether certain syllogisms are valid.

- Their answers were strongly influenced by the contextual setting and whether the syllogisms' conclusions confirmed or contradicted the participants' beliefs.

Classical logic cannot adequately represent this syllogistic reasoning task.

Hölldobler and Kencana Ramli [2009] propose to model human reasoning by

- logic programs
- under weak completion semantics
- based on the three-valued Łukasiewicz (1920) logic.

It seems to adequately model Byrne's suppression task and Wason's selection task.

Motivation

Evans, Barston, and Pollard carried out an experiment where participants had to evaluate whether certain syllogisms are valid.

- Their answers were strongly influenced by the contextual setting and whether the syllogisms' conclusions confirmed or contradicted the participants' beliefs.

Classical logic cannot adequately represent this syllogistic reasoning task.

Hölldobler and Kencana Ramli [2009] propose to model human reasoning by

- logic programs
- under weak completion semantics
- based on the three-valued Łukasiewicz (1920) logic.

It seems to adequately model Byrne's suppression task and Wason's selection task.

While modeling this syllogistic reasoning task under weak completion semantics we needed to define a contextual abductive framework.

Logic Programs

We restrict ourselves to datalog programs. A logic program $\mathcal{P}$ is a finite set of clauses

$$A \leftarrow A_1 \wedge \ldots \wedge A_n \wedge \neg B_1 \wedge \ldots \wedge \neg B_m, \quad (1)$$

$$A \leftarrow \perp, \quad (2)$$

- where A and A_i, $0 \leq i \leq n$, are atoms and $\neg B_j$, $1 \leq j \leq m$, are negated atoms.
- If $i = 0$, then we write $A \leftarrow \top$, which is called a positive fact.
- A clause of the form (2) is called a negative fact.
- A is undefined if it is not the head of any clause.
- $\mathrm{g}\,\mathcal{P}$ denotes ground $\mathcal{P}$, that is, it consists of all the ground instances of its clauses.
- $\mathrm{undef}(\mathcal{P})$ is the set of all undefined atoms in $\mathrm{g}\,\mathcal{P}$.

Logic Programs

We restrict ourselves to datalog programs. A logic program $\mathcal{P}$ is a finite set of clauses

$$A \leftarrow A_1 \wedge \ldots \wedge A_n \wedge \neg B_1 \wedge \ldots \wedge \neg B_m, \quad (1)$$
$$A \leftarrow \bot, \quad (2)$$

- where A and A_i, $0 \leq i \leq n$, are atoms and $\neg B_j$, $1 \leq j \leq m$, are negated atoms.
- If $i = 0$, then we write $A \leftarrow \top$, which is called a positive fact.
- A clause of the form (2) is called a negative fact.
- A is undefined if it is not the head of any clause.
- $\mathrm{g}\,\mathcal{P}$ denotes ground $\mathcal{P}$, that is, it consists of all the ground instances of its clauses.
- $\mathrm{undef}(\mathcal{P})$ is the set of all undefined atoms in $\mathrm{g}\,\mathcal{P}$.

The following transformation is the weak completion of $\mathcal{P}$

1. Replace all clauses in $\mathrm{g}\,\mathcal{P}$ with the same head $A \leftarrow body_1, \ldots, A \leftarrow body_n$ by the single expression $A \leftarrow body_1 \vee \ldots \vee body_n$.
2. Replace all occurrences of $\leftarrow$ by $\leftrightarrow$.

Three-Valued Łukasiewicz [1920] Logic

	$\neg$
$\top$	$\bot$
$\bot$	$\top$
U	U

$\wedge$	$\top$	U	$\bot$
$\top$	$\top$	U	$\bot$
U	U	U	$\bot$
$\bot$	$\bot$	$\bot$	$\bot$

$\vee$	$\top$	U	$\bot$
$\top$	$\top$	$\top$	$\top$
U	$\top$	U	U
$\bot$	$\top$	U	$\bot$

$\leftarrow_L$	$\top$	U	$\bot$
$\top$	$\top$	$\top$	$\top$
U	U	$\top$	$\top$
$\bot$	$\bot$	U	$\top$

$\leftrightarrow_L$	$\top$	U	$\bot$
$\top$	$\top$	U	$\bot$
U	U	$\top$	U
$\bot$	$\bot$	U	$\top$

Table: $\top$, $\bot$, and U denote *true*, *false*, and *unknown*, respectively.

Three-Valued Łukasiewicz [1920] Logic

	$\neg$
$\top$	$\bot$
$\bot$	$\top$
U	U

$\wedge$	$\top$	U	$\bot$
$\top$	$\top$	U	$\bot$
U	U	U	$\bot$
$\bot$	$\bot$	$\bot$	$\bot$

$\vee$	$\top$	U	$\bot$
$\top$	$\top$	$\top$	$\top$
U	$\top$	U	U
$\bot$	$\top$	U	$\bot$

$\leftarrow_L$	$\top$	U	$\bot$
$\top$	$\top$	$\top$	$\top$
U	U	$\top$	$\top$
$\bot$	$\bot$	U	$\top$

$\leftrightarrow_L$	$\top$	U	$\bot$
$\top$	$\top$	U	$\bot$
U	U	$\top$	U
$\bot$	$\bot$	U	$\top$

Table: $\top$, $\bot$, and U denote *true*, *false*, and *unknown*, respectively.

An interpretation I of g$\mathcal{P}$ is a mapping of the Herbrand base $\mathcal{B}_\mathcal{P}$ to $\{\top, \bot, \mathrm{U}\}$ and is represented by an unique pair, $\langle I^\top, I^\bot \rangle$, where

$$I^\top = \{A \in \mathcal{B}_\mathcal{P} \mid A \text{ is mapped to } \top\} \text{ and } I^\bot = \{A \in \mathcal{B}_\mathcal{P} \mid A \text{ is mapped to } \bot\}.$$

Three-Valued Łukasiewicz [1920] Logic

	$\neg$
$\top$	$\bot$
$\bot$	$\top$
U	U

$\wedge$	$\top$	U	$\bot$
$\top$	$\top$	U	$\bot$
U	U	U	$\bot$
$\bot$	$\bot$	$\bot$	$\bot$

$\vee$	$\top$	U	$\bot$
$\top$	$\top$	$\top$	$\top$
U	$\top$	U	U
$\bot$	$\top$	U	$\bot$

$\leftarrow_L$	$\top$	U	$\bot$
$\top$	$\top$	$\top$	$\top$
U	U	$\top$	$\top$
$\bot$	$\bot$	U	$\top$

$\leftrightarrow_L$	$\top$	U	$\bot$
$\top$	$\top$	U	$\bot$
U	U	$\top$	U
$\bot$	$\bot$	U	$\top$

Table: $\top$, $\bot$, and U denote *true*, *false*, and *unknown*, respectively.

An interpretation I of $g\,\mathcal{P}$ is a mapping of the Herbrand base $\mathcal{B}_\mathcal{P}$ to $\{\top, \bot, \mathsf{U}\}$ and is represented by an unique pair, $\langle I^\top, I^\bot \rangle$, where

$$I^\top = \{A \in \mathcal{B}_\mathcal{P} \mid A \text{ is mapped to } \top\} \text{ and } I^\bot = \{A \in \mathcal{B}_\mathcal{P} \mid A \text{ is mapped to } \bot\}.$$

- For every I it holds that $I^\top \cap I^\bot = \emptyset$.
- A model of a formula F is an interpretation I such that F is true under I.
- A model of $g\,\mathcal{P}$ is an interpretation that is a model of each clause in $g\,\mathcal{P}$.

Computing Least Models

Hölldobler and Kencana Ramli [2009] propose to compute the least model of the weak completion of $\mathcal{P}$ ($\mathsf{lm}_{Ł}\mathsf{wc}\,\mathcal{P}$) which is identical to the least fixed point of $\Phi_{\mathcal{P}}$, by an operator defined by Stenning and van Lambalgen [2008].

Computing Least Models

Hölldobler and Kencana Ramli [2009] propose to compute the least model of the weak completion of $\mathcal{P}$ ($\mathrm{lm}_{\text{Ł}}\mathrm{wc}\,\mathcal{P}$) which is identical to the least fixed point of $\Phi_{\mathcal{P}}$, by an operator defined by Stenning and van Lambalgen [2008].

Let I be an interpretation in $\Phi_{\mathcal{P}}(I) = \langle J^{\top}, J^{\perp} \rangle$, where

$$\begin{aligned} J^{\top} &= \{A \mid \text{there exists } A \leftarrow body \in \mathrm{g}\,\mathcal{P} \text{ with } I(body) = \top\}, \\ J^{\perp} &= \{A \mid \text{there exists } A \leftarrow body \in \mathrm{g}\,\mathcal{P} \text{ and} \\ &\qquad \text{for all } A \leftarrow body \in \mathrm{g}\,\mathcal{P} \text{ we find } I(body) = \perp\}. \end{aligned}$$

Computing Least Models

Hölldobler and Kencana Ramli [2009] propose to compute the least model of the weak completion of $\mathcal{P}$ ($\mathrm{lm}_{\text{Ł}}\mathrm{wc}\,\mathcal{P}$) which is identical to the least fixed point of $\Phi_{\mathcal{P}}$, by an operator defined by Stenning and van Lambalgen [2008].

Let I be an interpretation in $\Phi_{\mathcal{P}}(I) = \langle J^{\top}, J^{\perp}\rangle$, where

$$\begin{aligned} J^{\top} &= \{A \mid \text{there exists } A \leftarrow \mathit{body} \in \mathrm{g}\,\mathcal{P} \text{ with } I(\mathit{body}) = \top\}, \\ J^{\perp} &= \{A \mid \text{there exists } A \leftarrow \mathit{body} \in \mathrm{g}\,\mathcal{P} \text{ and} \\ &\qquad \text{for all } A \leftarrow \mathit{body} \in \mathrm{g}\,\mathcal{P} \text{ we find } I(\mathit{body}) = \perp\}. \end{aligned}$$

Hölldobler and Kencana Ramli showed that the model intersection property holds for weakly completed programs. This guarantees the existence of least models for every $\mathcal{P}$.

Computing Least Models

Hölldobler and Kencana Ramli [2009] propose to compute the least model of the weak completion of $\mathcal{P}$ ($\mathrm{lm}_{Ł}\mathrm{wc}\,\mathcal{P}$) which is identical to the least fixed point of $\Phi_{\mathcal{P}}$, by an operator defined by Stenning and van Lambalgen [2008].

Let I be an interpretation in $\Phi_{\mathcal{P}}(I) = \langle J^{\top}, J^{\perp}\rangle$, where

$$\begin{aligned} J^{\top} &= \{A \mid \text{there exists } A \leftarrow \mathit{body} \in \mathrm{g}\,\mathcal{P} \text{ with } I(\mathit{body}) = \top\}, \\ J^{\perp} &= \{A \mid \text{there exists } A \leftarrow \mathit{body} \in \mathrm{g}\,\mathcal{P} \text{ and} \\ &\qquad\quad \text{for all } A \leftarrow \mathit{body} \in \mathrm{g}\,\mathcal{P} \text{ we find } I(\mathit{body}) = \perp\}. \end{aligned}$$

Hölldobler and Kencana Ramli showed that the model intersection property holds for weakly completed programs. This guarantees the existence of least models for every $\mathcal{P}$.

In (Dietz, Hölldobler, and Wernhard [2013]) we show that weak completion semantics corresponds to well-founded semantics for modified tight logic programs.

Computing Least Models

Hölldobler and Kencana Ramli [2009] propose to compute the least model of the weak completion of $\mathcal{P}$ ($\mathrm{lm}_{Ł}\mathrm{wc}\,\mathcal{P}$) which is identical to the least fixed point of $\Phi_{\mathcal{P}}$, by an operator defined by Stenning and van Lambalgen [2008].

Let I be an interpretation in $\Phi_{\mathcal{P}}(I) = \langle J^{\top}, J^{\perp} \rangle$, where

$$
\begin{aligned}
J^{\top} &= \{A \mid \text{there exists } A \leftarrow \mathit{body} \in \mathrm{g}\,\mathcal{P} \text{ with } I(\mathit{body}) = \top\}, \\
J^{\perp} &= \{A \mid \text{there exists } A \leftarrow \mathit{body} \in \mathrm{g}\,\mathcal{P} \text{ and} \\
&\qquad\quad \text{for all } A \leftarrow \mathit{body} \in \mathrm{g}\,\mathcal{P} \text{ we find } I(\mathit{body}) = \perp\}.
\end{aligned}
$$

Hölldobler and Kencana Ramli showed that the model intersection property holds for weakly completed programs. This guarantees the existence of least models for every $\mathcal{P}$.

In (Dietz, Hölldobler, and Wernhard [2013]) we show that weak completion semantics corresponds to well-founded semantics for modified tight logic programs.

- $\mathcal{P}$ is tight, that is, it does not contain positive cycles.
- Modified $\mathcal{P}$ is $\mathcal{P} \cup \{A \leftarrow \neg n\_A,\ n\_A \leftarrow \neg A \mid \text{ for all undefined atoms } A \in \mathcal{P}\}$.

Abduction (Kakas, Kowalski, and Toni [1993])

Given an abductive framework $\langle \mathcal{P}, \mathcal{A}, \models_{Ł}^{\mathsf{lmwc}} \rangle$ where

Abduction (Kakas, Kowalski, and Toni [1993])

Given an abductive framework $\langle \mathcal{P}, \mathcal{A}, \models_{Ł}^{lmwc} \rangle$ where

- set of abducibles $\mathcal{A}$ contains all positive and negative facts of each $A \in \text{undef}(\mathcal{P})$,
- $\mathcal{E}$ is an explanation and a consistent subset of $\mathcal{A}$,
- logical consequence relation $\models_{Ł}^{lmwc}$, where $\mathcal{P} \models_{Ł}^{lmwc} F$ iff $\text{lm}_{Ł}\text{wc}\,\mathcal{P}(F) = \top$, and
- $\mathcal{O}$ is an observation which is a set of (at least one) literals.

Abduction (Kakas, Kowalski, and Toni [1993])

Given an abductive framework $\langle \mathcal{P}, \mathcal{A}, \models_{Ł}^{\text{lmwc}} \rangle$ where

- set of abducibles $\mathcal{A}$ contains all positive and negative facts of each $A \in \text{undef}(\mathcal{P})$,
- $\mathcal{E}$ is an explanation and a consistent subset of $\mathcal{A}$,
- logical consequence relation $\models_{Ł}^{\text{lmwc}}$, where $\mathcal{P} \models_{Ł}^{\text{lmwc}} F$ iff $\text{lm}_{Ł}\text{wc}\,\mathcal{P}(F) = \top$, and
- $\mathcal{O}$ is an observation which is a set of (at least one) literals.

$\mathcal{O}$ is **explained by** $\mathcal{E}$ **given** $\mathcal{P}$ iff $\mathcal{P} \cup \mathcal{E} \models_{Ł}^{\text{lmwc}} O$, where $\mathcal{P} \not\models_{Ł}^{\text{lmwc}} \mathcal{O}$.

$\mathcal{O}$ is **explained given** $\mathcal{P}$ iff there exists an $\mathcal{E}$ such that $\mathcal{O}$ is explained by $\mathcal{E}$ given $\mathcal{P}$.

Abduction (Kakas, Kowalski, and Toni [1993])

Given an abductive framework $\langle \mathcal{P}, \mathcal{A}, \models_{Ł}^{lmwc} \rangle$ where

- set of abducibles $\mathcal{A}$ contains all positive and negative facts of each $A \in \text{undef}(\mathcal{P})$,
- $\mathcal{E}$ is an explanation and a consistent subset of $\mathcal{A}$,
- logical consequence relation $\models_{Ł}^{lmwc}$, where $\mathcal{P} \models_{Ł}^{lmwc} F$ iff $\text{lm}_{Ł}\text{wc}\,\mathcal{P}(F) = \top$, and
- $\mathcal{O}$ is an observation which is a set of (at least one) literals.

$\mathcal{O}$ is **explained by** $\mathcal{E}$ **given** $\mathcal{P}$ iff $\mathcal{P} \cup \mathcal{E} \models_{Ł}^{lmwc} O$, where $\mathcal{P} \not\models_{Ł}^{lmwc} \mathcal{O}$.

$\mathcal{O}$ is **explained given** $\mathcal{P}$ iff there exists an $\mathcal{E}$ such that $\mathcal{O}$ is explained by $\mathcal{E}$ given $\mathcal{P}$.

F **follows skeptically from** $\mathcal{P}$**,and** $\mathcal{O}$ iff $\mathcal{O}$ can be explained given $\mathcal{P}$, and for all minimal explanations $\mathcal{E}$ we find that $\mathcal{P} \cup \mathcal{E} \models_{Ł}^{lmwc} \mathcal{O}$.

F **follows credulously from** $\mathcal{P}$**, and** $\mathcal{O}$ iff there exists a minimal explanation $\mathcal{E}$ such that $\mathcal{P} \cup \mathcal{E} \models_{Ł}^{lmwc} \mathcal{O}$.

Explaining the Forest Fire

Consider $\mathcal{P}_{fire,dry}$

$$
\begin{array}{llllll}
storm & \leftarrow & lightning \wedge tempest, & forest\_fire & \leftarrow & lightning \wedge smoke \wedge \neg ab, \\
ab & \leftarrow & \neg dry\_leaves, & forest\_fire & \leftarrow & barbecue \wedge smoke \wedge \neg ab, \\
smoke & \leftarrow & \top, & dry\_leaves & \leftarrow & \top.
\end{array}
$$

Explaining the Forest Fire

Consider $\mathcal{P}_{fire,dry}$

$$\begin{array}{lllllll} storm & \leftarrow & lightning \wedge tempest, & forest\_fire & \leftarrow & lightning \wedge smoke \wedge \neg ab, \\ ab & \leftarrow & \neg dry\_leaves, & forest\_fire & \leftarrow & barbecue \wedge smoke \wedge \neg ab, \\ smoke & \leftarrow & \top, & dry\_leaves & \leftarrow & \top. \end{array}$$

Let us assume that we observe a forest fire

$$\mathcal{O}_{forest\_fire} = \{forest\_fire\}$$

Explaining the Forest Fire

Consider $\mathcal{P}_{fire,dry}$

$$
\begin{array}{lcllcl}
storm & \leftarrow & lightning \wedge tempest, & forest\_fire & \leftarrow & lightning \wedge smoke \wedge \neg ab, \\
ab & \leftarrow & \neg dry\_leaves, & forest\_fire & \leftarrow & barbecue \wedge smoke \wedge \neg ab, \\
smoke & \leftarrow & \top, & dry\_leaves & \leftarrow & \top.
\end{array}
$$

Let us assume that we observe a forest fire

$$\mathcal{O}_{forest\_fire} = \{forest\_fire\}$$

The set of abducibles, $\mathcal{A}_{\mathcal{P}_{fire,dry}}$, is

$$
\begin{array}{lcllcl}
barbecue & \leftarrow & \top, & barbecue & \leftarrow & \bot, \\
lightning & \leftarrow & \top, & lightning & \leftarrow & \bot, \\
tempest & \leftarrow & \top, & tempest & \leftarrow & \bot.
\end{array}
$$

Explaining the Forest Fire

Consider $\mathcal{P}_{fire,dry}$

$$
\begin{array}{lclclcl}
storm & \leftarrow & lightning \wedge tempest, & & forest\_fire & \leftarrow & lightning \wedge smoke \wedge \neg ab, \\
ab & \leftarrow & \neg dry\_leaves, & & forest\_fire & \leftarrow & barbecue \wedge smoke \wedge \neg ab, \\
smoke & \leftarrow & \top, & & dry\_leaves & \leftarrow & \top.
\end{array}
$$

There are two minimal explanations for $\mathcal{O}_{forest\_fire}$

$$
\begin{array}{lcl}
\mathcal{E}_{barbecue} & = & \{barbecue \leftarrow \top\}, \\
\mathcal{E}_{lightning} & = & \{lightning \leftarrow \top\}.
\end{array}
$$

Explaining the Forest Fire

Consider $\mathcal{P}_{fire,dry}$

$$
\begin{array}{lclclcl}
storm & \leftarrow & lightning \wedge tempest, & & forest\_fire & \leftarrow & lightning \wedge smoke \wedge \neg ab, \\
ab & \leftarrow & \neg dry\_leaves, & & forest\_fire & \leftarrow & barbecue \wedge smoke \wedge \neg ab, \\
smoke & \leftarrow & \top, & & dry\_leaves & \leftarrow & \top.
\end{array}
$$

There are two minimal explanations for $\mathcal{O}_{forest\_fire}$

$$
\begin{array}{lcl}
\mathcal{E}_{barbecue} & = & \{barbecue \leftarrow \top\}, \\
\mathcal{E}_{lightning} & = & \{lightning \leftarrow \top\}.
\end{array}
$$

The least model of the weak completion of $\mathcal{P}_{fire,dry}$ with each of them is

$$
\begin{array}{lcl}
\mathrm{lm}_{Ł}\mathrm{wc}\,(\mathcal{P}_{fire,dry} \cup \mathcal{E}_{barbecue}) & = & \langle\{forest\_fire, barbecue, smoke, dry\_leaves\}, \{ab\}\rangle, \\
\mathrm{lm}_{Ł}\mathrm{wc}\,(\mathcal{P}_{fire,dry} \cup \mathcal{E}_{lightning}) & = & \langle\{forest\_fire, lightning, smoke, dry\_leaves\}, \{ab\}\rangle.
\end{array}
$$

Contextual Abductive Reasoning

How to express that *barbecue* describes the usual and *lightning* the exceptional case, only explaining the *forest fire* in the context of a *storm*?

Contextual Abductive Reasoning

How to express that *barbecue* describes the usual and *lightning* the exceptional case, only explaining the *forest fire* in the context of a *storm*?

Introduce for every A, two reserved (meta-) predicates (Pereira and Pinto [2011]).

$\text{inspect}(A)$ and $\text{inspect}_{\text{not}}(A)$

These are special abducibles only to be abduced
if A or $\neg A$ are abduced somewhere else.

Contextual Side-effects

Consider $\mathcal{P}_{fire,dry}$

$$\begin{array}{llll} storm \leftarrow lightning \wedge tempest, & & forest\_fire \leftarrow \text{inspect}(lightning) \wedge smoke \wedge \neg ab, \\ ab \leftarrow \neg dry\_leaves, & & forest\_fire \leftarrow barbecue \wedge smoke \wedge \neg ab, \\ smoke \leftarrow \top, & & dry\_leaves \leftarrow \top. \end{array}$$

Contextual Side-effects

Consider $\mathcal{P}_{fire,dry}$

$$
\begin{array}{lll}
storm \leftarrow lightning \wedge tempest, & & forest\_fire \leftarrow \text{inspect}(lightning) \wedge smoke \wedge \neg ab, \\
ab \leftarrow \neg dry\_leaves, & & forest\_fire \leftarrow barbecue \wedge smoke \wedge \neg ab, \\
smoke \leftarrow \top, & & dry\_leaves \leftarrow \top.
\end{array}
$$

lightning

forest fire

$$\xLeftarrow{\mathcal{E}_{\text{insp}(lightning)} = \{\text{inspect}(lightning) \leftarrow \top\}}$$

$$\xLeftarrow{\mathcal{E}_{barbecue} = \{barbecue \leftarrow \top\}}$$

barbecue

Contextual Side-effects

Consider $\mathcal{P}_{fire,dry}$

$$
\begin{array}{llllll}
storm & \leftarrow & lightning \wedge tempest, & forest\_fire & \leftarrow & \mathrm{inspect}(lightning) \wedge smoke \wedge \neg ab, \\
ab & \leftarrow & \neg dry\_leaves, & forest\_fire & \leftarrow & barbecue \wedge smoke \wedge \neg ab, \\
smoke & \leftarrow & \top, & dry\_leaves & \leftarrow & \top.
\end{array}
$$

lightning

$$\xleftarrow{\mathcal{E}_{\mathrm{insp}(lightning)} = \{\mathrm{inspect}(lightning) \leftarrow \top\}}$$

forest fire

$$\xleftarrow{\mathcal{E}_{barbecue} = \{barbecue \leftarrow \top\}}$$

barbecue

Contextual Side-effects

Consider $\mathcal{P}_{fire,dry}$

$$
\begin{array}{llll}
storm & \leftarrow lightning \wedge tempest, & forest\_fire & \leftarrow inspect(lightning) \wedge smoke \wedge \neg ab, \\
ab & \leftarrow \neg dry\_leaves, & forest\_fire & \leftarrow barbecue \wedge smoke \wedge \neg ab, \\
smoke & \leftarrow \top, & dry\_leaves & \leftarrow \top.
\end{array}
$$

storm

$$\xLeftarrow{\mathcal{E}_{lightning,tempest} = \{lightning \leftarrow \top, tempest \leftarrow \top\}}$$

$$\xLeftarrow{\mathcal{E}_{insp(lightning)} = \{inspect(lightning) \leftarrow \top\}}$$

lightning

forest fire

$$\cancel{\xLeftarrow{\mathcal{E}_{barbecue} = \{barbecue \leftarrow \top\}}}$$

barbecue

$inspect(lightning) \leftarrow \top$ can only be abduced by $\mathcal{O}_{forest\_fire} = \{forest\_fire\}$ if $lightning \leftarrow \top$ is explained somewhere else, e.g. by observing a $storm$.

Contextual Side-effects

Consider $\mathcal{P}_{\mathit{fire},\mathit{dry}}$

$$
\begin{aligned}
&\mathit{storm} \leftarrow \mathit{lightning} \wedge \mathit{tempest}, && \mathit{forest\_fire} \leftarrow \mathrm{inspect}(\mathit{lightning}) \wedge \mathit{smoke} \wedge \neg \mathit{ab}, \\
&\mathit{ab} \leftarrow \neg \mathit{dry\_leaves}, && \mathit{forest\_fire} \leftarrow \mathit{barbecue} \wedge \mathit{smoke} \wedge \neg \mathit{ab}, \\
&\mathit{smoke} \leftarrow \top, && \mathit{dry\_leaves} \leftarrow \top.
\end{aligned}
$$


storm

$$\xLeftarrow{\mathcal{E}_{\mathit{lightning},\mathit{tempest}} = \{\mathit{lightning} \leftarrow \top, \mathit{tempest} \leftarrow \top\}}$$

$$\xLeftarrow{\mathcal{E}_{\mathrm{insp}(\mathit{lightning})} = \{\mathrm{inspect}(\mathit{lightning}) \leftarrow \top\}}$$


lightning


forest fire

$$\cancel{\xLeftarrow{\mathcal{E}_{\mathit{barbecue}} = \{\mathit{barbecue} \leftarrow \top\}}}$$


barbecue

$\mathcal{O}_{\mathit{forest\_fire}}$ is a contextual side-effect of $\mathcal{O}_{\mathit{storm}}$ given $\mathcal{P}_{\mathit{fire},\mathit{dry}}$.

Contestable Contextual Side-effects

Consider $\mathcal{P}_{fire,rained}$

$$
\begin{array}{llll}
storm & \leftarrow lightning \wedge tempest, & forest\_fire & \leftarrow \mathrm{inspect}(lightning) \wedge smoke \wedge \neg ab, \\
ab & \leftarrow \neg dry\_leaves, & forest\_fire & \leftarrow barbecue \wedge smoke \wedge \neg ab, \\
smoke & \leftarrow \top, & rained & \leftarrow \mathrm{inspect}_{\mathrm{not}}(dry\_leaves).
\end{array}
$$

Contestable Contextual Side-effects

Consider $\mathcal{P}_{fire,rained}$

$$
\begin{array}{llll}
storm \leftarrow lightning \wedge tempest, & forest\_fire & \leftarrow & \text{inspect}(lightning) \wedge smoke \wedge \neg ab, \\
ab \leftarrow \neg dry\_leaves, & forest\_fire & \leftarrow & barbecue \wedge smoke \wedge \neg ab, \\
smoke \leftarrow \top, & rained & \leftarrow & \text{inspect}_{\text{not}}(dry\_leaves).
\end{array}
$$

- The *rained*-clause states that if for some other observation, we explained that the leaves are not dry, then it rained.

Contestable Contextual Side-effects

Consider $\mathcal{P}_{fire,rained}$

$$
\begin{array}{llll}
storm \leftarrow lightning \wedge tempest, & forest\_fire \leftarrow \text{inspect}(lightning) \wedge smoke \wedge \neg ab, \\
ab \leftarrow \neg dry\_leaves, & forest\_fire \leftarrow barbecue \wedge smoke \wedge \neg ab, \\
smoke \leftarrow \top, & rained \leftarrow \text{inspect}_{\text{not}}(dry\_leaves).
\end{array}
$$

- The *rained*-clause states that if for some other observation, we explained that the leaves are not dry, then it rained.
- When we observe $\mathcal{O}_{forest\_fire}$, we need to abduce $dry\_leaves \leftarrow \top$.

Contestable Contextual Side-effects

Consider $\mathcal{P}_{fire,rained}$

$$
\begin{array}{lll}
storm \leftarrow lightning \wedge tempest, & forest\_fire \leftarrow \mathrm{inspect}(lightning) \wedge smoke \wedge \neg ab, \\
ab \leftarrow \neg dry\_leaves, & forest\_fire \leftarrow barbecue \wedge smoke \wedge \neg ab, \\
smoke \leftarrow \top, & rained \leftarrow \mathrm{inspect}_{\mathrm{not}}(dry\_leaves).
\end{array}
$$

- The *rained*-clause states that if for some other observation, we explained that the leaves are not dry, then it rained.
- When we observe $\mathcal{O}_{forest\_fire}$, we need to abduce $dry\_leaves \leftarrow \top$.
- This will make $\mathrm{inspect}_{\mathrm{not}}(dry\_leaves)$ false and we conclude that it did not rain.

Contestable Contextual Side-effects

Consider $\mathcal{P}_{fire,rained}$

$$\begin{array}{lll} storm \leftarrow lightning \wedge tempest, & forest\_fire \leftarrow \mathrm{inspect}(lightning) \wedge smoke \wedge \neg ab, \\ ab \leftarrow \neg dry\_leaves, & forest\_fire \leftarrow barbecue \wedge smoke \wedge \neg ab, \\ smoke \leftarrow \top, & rained \leftarrow \mathrm{inspect}_{\mathrm{not}}(dry\_leaves). \end{array}$$

- The *rained*-clause states that if for some other observation, we explained that the leaves are not dry, then it rained.
- When we observe $\mathcal{O}_{forest\_fire}$, we need to abduce $dry\_leaves \leftarrow \top$.
- This will make $\mathrm{inspect}_{\mathrm{not}}(dry\_leaves)$ false and we conclude that it did not rain.

$\mathcal{O}_{\neg rained}$ is a contested contextual side-effect of $\mathcal{O}_{forest\_fire}$ given $\mathcal{P}_{fire,rained}$.

Jointly Supported Contextual Relevant Consequences

Consider $\mathcal{P}_{\mathit{ffight}}$

$$
\begin{array}{lcllcl}
\mathit{smoke} & \leftarrow & \mathit{fire} & \wedge & \mathrm{inspect}(\mathit{fire\_fighters}), \\
\mathit{sirens} & \leftarrow & \mathrm{inspect}(\mathit{fire}) & \wedge & \mathit{fire\_fighters}.
\end{array}
$$

Jointly Supported Contextual Relevant Consequences

Consider $\mathcal{P}_{\mathit{ffight}}$

$$\begin{array}{lllll} \mathit{smoke} & \leftarrow & \mathit{fire} & \wedge & \mathrm{inspect}(\mathit{fire\_fighters}), \\ \mathit{sirens} & \leftarrow & \mathrm{inspect}(\mathit{fire}) & \wedge & \mathit{fire\_fighters}. \end{array}$$

- Let's observe $\mathcal{O}_{\mathit{smoke}} = \{\mathit{smoke}\}$. We abduce $\mathit{fire} \leftarrow \top$.

Jointly Supported Contextual Relevant Consequences

Consider $\mathcal{P}_{\mathit{ffight}}$

$$
\begin{array}{lcllcl}
\mathit{smoke} & \leftarrow & \mathit{fire} & & \wedge & \mathrm{inspect}(\mathit{fire\_fighters}), \\
\mathit{sirens} & \leftarrow & \mathrm{inspect}(\mathit{fire}) & & \wedge & \mathit{fire\_fighters}.
\end{array}
$$

- Let's observe $\mathcal{O}_{\mathit{smoke}} = \{\mathit{smoke}\}$. We abduce $\mathit{fire} \leftarrow \top$.
- $\mathrm{inspect}(\mathit{fire\_fighters}) \leftarrow \top$ is only abducible if $\mathit{fire\_fighters} \leftarrow \top$ is abduced.

Jointly Supported Contextual Relevant Consequences

Consider $\mathcal{P}_{\mathit{ffight}}$

$$\begin{array}{lllll} \mathit{smoke} & \leftarrow & \mathit{fire} & \wedge & \mathrm{inspect}(\mathit{fire\_fighters}), \\ \mathit{sirens} & \leftarrow & \mathrm{inspect}(\mathit{fire}) & \wedge & \mathit{fire\_fighters}. \end{array}$$

- Let's observe $\mathcal{O}_{\mathit{smoke}} = \{\mathit{smoke}\}$. We abduce $\mathit{fire} \leftarrow \top$.
- $\mathrm{inspect}(\mathit{fire\_fighters}) \leftarrow \top$ is only abducible if $\mathit{fire\_fighters} \leftarrow \top$ is abduced.
- Given $\mathcal{O}_{\mathit{sirens}}$, we can abduce $\mathit{fire\_fighters} \leftarrow \top$ but not $\mathrm{inspect}(\mathit{fire}) \leftarrow \top$.

Jointly Supported Contextual Relevant Consequences

Consider $\mathcal{P}_{ffight}$

$$\begin{array}{lcccl} smoke & \leftarrow & fire & \wedge & \text{inspect}(fire\_fighters), \\ sirens & \leftarrow & \text{inspect}(fire) & \wedge & fire\_fighters. \end{array}$$

- Let's observe $\mathcal{O}_{smoke} = \{smoke\}$. We abduce $fire \leftarrow \top$.
- $\text{inspect}(fire\_fighters) \leftarrow \top$ is only abducible if $fire\_fighters \leftarrow \top$ is abduced.
- Given $\mathcal{O}_{sirens}$, we can abduce $fire\_fighters \leftarrow \top$ but not $\text{inspect}(fire) \leftarrow \top$.

If we observe both, $\mathcal{O}_{smoke}$ and $\mathcal{O}_{sirens}$ then the minimal explanation is

$$\begin{array}{lclclcl} fire & \leftarrow & \top, & \quad & fire\_fighters & \leftarrow & \top, \\ \text{inspect}(fire) & \leftarrow & \top, & \quad & \text{inspect}(fire\_fighters) & \leftarrow & \top. \end{array}$$

Jointly Supported Contextual Relevant Consequences

Consider $\mathcal{P}_{\mathit{ffight}}$

$$\begin{array}{lclcl} \mathit{smoke} & \leftarrow & \mathit{fire} & \wedge & \mathrm{inspect}(\mathit{fire\_fighters}), \\ \mathit{sirens} & \leftarrow & \mathrm{inspect}(\mathit{fire}) & \wedge & \mathit{fire\_fighters}. \end{array}$$

- Let's observe $\mathcal{O}_{\mathit{smoke}} = \{\mathit{smoke}\}$. We abduce $\mathit{fire} \leftarrow \top$.
- $\mathrm{inspect}(\mathit{fire\_fighters}) \leftarrow \top$ is only abducible if $\mathit{fire\_fighters} \leftarrow \top$ is abduced.
- Given $\mathcal{O}_{\mathit{sirens}}$, we can abduce $\mathit{fire\_fighters} \leftarrow \top$ but not $\mathrm{inspect}(\mathit{fire}) \leftarrow \top$.

If we observe both, $\mathcal{O}_{\mathit{smoke}}$ and $\mathcal{O}_{\mathit{sirens}}$ then the minimal explanation is

$$\begin{array}{lclclcl} \mathit{fire} & \leftarrow & \top, & \quad & \mathit{fire\_fighters} & \leftarrow & \top, \\ \mathrm{inspect}(\mathit{fire}) & \leftarrow & \top, & \quad & \mathrm{inspect}(\mathit{fire\_fighters}) & \leftarrow & \top. \end{array}$$

$\mathcal{O}_{\mathit{smoke}}$ and $\mathcal{O}_{\mathit{sirens}}$ are jointly supported contextual relevant consequences given $\mathcal{P}_{\mathit{ffight}}$.

Conclusion

- Weak completion semantics is based on a previously proposed approach that seems to adequately model
  - Wason's selection task, and
  - Byrne's suppression task.

Conclusion

- Weak completion semantics is based on a previously proposed approach that seems to adequately model
  - Wason's selection task, and
  - Byrne's suppression task.
- While modeling other human reasoning tasks, we identified the need to express contextual abductive reasoning.

Conclusion

- Weak completion semantics is based on a previously proposed approach that seems to adequately model
  - Wason's selection task, and
  - Byrne's suppression task.
- While modeling other human reasoning tasks, we identified the need to express contextual abductive reasoning.
- By modeling contextual abduction with inspection points, more specific relations between observations can be defined as
  - contextual side-effects,
  - (jointly supported) contextual relevant consequences, and
  - contestable contextual side-effects.

Thank you very much for your attention!

References I

Emmanuelle-Anna Dietz, Steffen Hölldobler, and Christoph Wernhard. Modeling the suppression task under weak completion and well-founded semantics. Journal of Applied Non-Classical Logics, 2013.

Steffen Hölldobler and Carroline Dewi Kencana Ramli. Logic programs under three-valued Łukasiewicz semantics. In Patricia M. Hill and David Scott Warren, editors, Logic Programming, 25th International Conference, ICLP 2009, volume 5649 of Lecture Notes in Computer Science, pages 464–478, Heidelberg, 2009. Springer.

A. C. Kakas, R. A. Kowalski, and F. Toni. Abductive logic programming. Journal of Logic and Computation, 2(6):719–770, 1993.

Jan Łukasiewicz. O logice trójwartościowej. Ruch Filozoficzny, 5:169–171, 1920. English translation: On three-valued logic. In: Łukasiewicz J. and Borkowski L. (ed.). (1990). *Selected Works*, Amsterdam: North Holland, pp. 87–88.

Luís Moniz Pereira and Alexandre Miguel Pinto. Inspecting side-effects of abduction in logic programs. In M. Balduccini and Tran Cao Son, editors, Logic Programming, Knowledge Representation, and Nonmonotonic Reasoning: Essays in honour of Michael Gelfond, volume 6565 of LNAI, pages 148–163. Springer, 2011.

Keith Stenning and Michiel van Lambalgen. Human Reasoning and Cognitive Science. A Bradford Book. MIT Press, Cambridge, MA, 2008. ISBN 9780262195836.

Background Knowledge & Abnormality Predicates

It is commonly known that

Cigarettes are addictive. (1)

Background Knowledge & Abnormality Predicates

It is commonly known that

Cigarettes are addictive. (1)

After reading PREMISE 2 participants seem to assume that

Cigarettes are inexpensive (compared to other addictive things). (2)

which biases the reasoning towards a representation.

Background Knowledge & Abnormality Predicates

It is commonly known that

Cigarettes are addictive. (1)

After reading PREMISE 2 participants seem to assume that

Cigarettes are inexpensive (compared to other addictive things). (2)

which biases the reasoning towards a representation.

Stenning & van Lambalgen (2008) propose to model conditionals with *ab* predicates.

PREMISE 1 of S_{add} can be represented as

If something is inexpensive and not abnormal, then it is not addictive. (3)
Nothing (as a rule) is abnormal (wrt (3)).

Background Knowledge & Abnormality Predicates

It is commonly known that

Cigarettes are addictive. (1)

After reading PREMISE 2 participants seem to assume that

Cigarettes are inexpensive (compared to other addictive things). (2)

which biases the reasoning towards a representation.

Stenning & van Lambalgen (2008) propose to model conditionals with *ab* predicates.

PREMISE 1 of S_{add} can be represented as

If something is inexpensive and not abnormal, then it is not addictive. (3)
Nothing (as a rule) is abnormal (wrt (3)).

The belief in (1) and (2) generates an exception for cigarettes

If something is a cigarette, then it is abnormal (wrt (3)). (4)

Modeling S_{add} of the Syllogistic Reasoning Task

$\mathcal{P}_{add}$ represents the first two premises of S_{add}

$$add'(X) \quad \leftarrow \quad inex(X) \wedge \neg ab_{add'}(X), \qquad add(X) \quad \leftarrow \quad \neg add'(X), \quad \textsc{Premise 1}$$

Modeling S_{add} of the Syllogistic Reasoning Task

$\mathcal{P}_{add}$ represents the first two premises of S_{add}

$$
\begin{array}{rclrcll}
add'(X) & \leftarrow & inex(X) \wedge \neg ab_{add'}(X), & add(X) & \leftarrow & \neg add'(X), & \textsc{Premise 1} \\
cig(a) & \leftarrow & \top, & inex(a) & \leftarrow & \top, & \textsc{Premise 2}
\end{array}
$$

Modeling S_{add} of the Syllogistic Reasoning Task

$\mathcal{P}_{add}$ represents the first two premises of S_{add}

$$
\begin{array}{rclrcll}
add'(X) & \leftarrow & inex(X) \wedge \neg ab_{add'}(X), & add(X) & \leftarrow & \neg add'(X), & \text{Premise 1} \\
cig(a) & \leftarrow & \top, & inex(a) & \leftarrow & \top, & \text{Premise 2} \\
inex(X) & \leftarrow & cig(X) \wedge \neg ab_{inex}(X), & ab_{add'}(X) & \leftarrow & cig(X), & \text{Bias (2, 4)} \\
ab_{add'}(X) & \leftarrow & \bot, & ab_{inex}(X) & \leftarrow & \bot. & \text{Abnormality}
\end{array}
$$

Modeling S_{add} of the Syllogistic Reasoning Task

$\mathcal{P}_{add}$ represents the first two premises of S_{add}

$$
\begin{array}{rclrcll}
add'(X) & \leftarrow & inex(X) \wedge \neg ab_{add'}(X), & add(X) & \leftarrow & \neg add'(X), & \textsc{Premise 1} \\
cig(a) & \leftarrow & \top, & inex(a) & \leftarrow & \top, & \textsc{Premise 2} \\
inex(X) & \leftarrow & cig(X) \wedge \neg ab_{inex}(X), & ab_{add'}(X) & \leftarrow & cig(X), & \text{Bias (2, 4)} \\
ab_{add'}(X) & \leftarrow & \bot, & ab_{inex}(X) & \leftarrow & \bot. & \text{Abnormality}
\end{array}
$$

The least model of the weak completion of $\mathcal{P}_{add}$ is

$$
\langle\ \{cig(a), inex(a), add(a), ab_{add'}(a)\}, \{add'(a), ab_{inex}(a)\}\rangle.
$$

Modeling S_{add} of the Syllogistic Reasoning Task

$\mathcal{P}_{add}$ represents the first two premises of S_{add}

$$
\begin{array}{rcllcll}
add'(X) & \leftarrow & inex(X) \wedge \neg ab_{add'}(X), & add(X) & \leftarrow & \neg add'(X), & \text{PREMISE 1} \\
cig(a) & \leftarrow & \top, & inex(a) & \leftarrow & \top, & \text{PREMISE 2} \\
inex(X) & \leftarrow & cig(X) \wedge \neg ab_{inex}(X), & ab_{add'}(X) & \leftarrow & cig(X), & \text{Bias (2, 4)} \\
ab_{add'}(X) & \leftarrow & \bot, & ab_{inex}(X) & \leftarrow & \bot. & \text{Abnormality}
\end{array}
$$

The least model of the weak completion of $\mathcal{P}_{add}$ is

$$\langle \{cig(a), inex(a), add(a), ab_{add'}(a)\}, \{add'(a), ab_{inex}(a)\}\rangle.$$

The CONCLUSION, some addictive things are not cigarettes, is sth. that cannot be a.

Modeling S_{add} of the Syllogistic Reasoning Task

$\mathcal{P}_{add}$ represents the first two premises of S_{add}

$$
\begin{array}{rcllcll}
add'(X) & \leftarrow & inex(X) \wedge \neg ab_{add'}(X), & add(X) & \leftarrow & \neg add'(X), & \text{PREMISE 1} \\
cig(a) & \leftarrow & \top, & inex(a) & \leftarrow & \top, & \text{PREMISE 2} \\
inex(X) & \leftarrow & cig(X) \wedge \neg ab_{inex}(X), & ab_{add'}(X) & \leftarrow & cig(X), & \text{Bias (2, 4)} \\
ab_{add'}(X) & \leftarrow & \bot, & ab_{inex}(X) & \leftarrow & \bot. & \text{Abnormality}
\end{array}
$$

The least model of the weak completion of $\mathcal{P}_{add}$ is

$$\langle \{cig(a), inex(a), add(a), ab_{add'}(a)\}, \{add'(a), ab_{inex}(a)\}\rangle.$$

The CONCLUSION, some addictive things are not cigarettes, is sth. that cannot be a.

To explain why people validate S_{add} we need to show that they reason abductively.

Abducing the CONCLUSION

Given our background knowledge we know, there are addictive things, let's say about b

$$\mathcal{O}_{add(b)} = \{add(b)\}$$

We have two minimal explanations for $\mathcal{O}_{add(b)}$

$$\begin{aligned}
\mathrm{lm}_{\L}\mathrm{wc}\,(\mathcal{P}_{add} \cup \mathcal{E}_{cig(b)}) &= \langle\{add(b), cig(b), inex(b), \ldots\}, \quad \{\ldots\}\rangle \\
\mathrm{lm}_{\L}\mathrm{wc}\,(\mathcal{P}_{add} \cup \mathcal{E}_{\neg cig(b)}) &= \langle\{add(b), \ldots\}, \quad \{cig(b), inex(b), \ldots\}\rangle
\end{aligned}$$

Abducing the Conclusion

Given our background knowledge we know, there are addictive things, let's say about b

$$\mathcal{O}_{add(b)} = \{add(b)\}$$

We have two minimal explanations for $\mathcal{O}_{add(b)}$

$$\begin{array}{lcll}
\mathrm{Im}_{Ł}\mathrm{wc}\,(\mathcal{P}_{add} \cup \mathcal{E}_{cig(b)}) & = & \langle\{add(b), cig(b), inex(b), \ldots\}, & \{\ldots\}\rangle \\
\mathrm{Im}_{Ł}\mathrm{wc}\,(\mathcal{P}_{add} \cup \mathcal{E}_{\neg cig(b)}) & = & \langle\{add(b), \ldots\}, & \{cig(b), inex(b), \ldots\}\rangle
\end{array}$$

Recall $\mathcal{P}_{add}$. Together with $\mathcal{E}_{cig(b)}$ it contains

$$\begin{array}{rcllrcl}
add'(X) & \leftarrow & inex(X) \wedge \neg ab_{add'}(X), & & add(X) & \leftarrow & \neg add'(X), \\
inex(X) & \leftarrow & cig(X) \wedge \neg ab_{inex}(X), & & ab_{add'}(X) & \leftarrow & cig(X), \\
ab_{add'}(X) & \leftarrow & \bot, & & ab_{inex}(X) & \leftarrow & \bot, \\
cig(b) & \leftarrow & \top. & & & &
\end{array}$$

Abducing the CONCLUSION

Given our background knowledge we know, there are addictive things, let's say about b

$$\mathcal{O}_{add(b)} = \{add(b)\}$$

We have two minimal explanations for $\mathcal{O}_{add(b)}$

$$\begin{array}{lcll}
\mathrm{Im}_{Ł}\mathrm{wc}\,(\mathcal{P}_{add} \cup \mathcal{E}_{cig(b)}) & = & \langle\{add(b), cig(b), inex(b), ...\}, & \{...\}\rangle \\
\mathrm{Im}_{Ł}\mathrm{wc}\,(\mathcal{P}_{add} \cup \mathcal{E}_{\neg cig(b)}) & = & \langle\{add(b), ...\}, & \{cig(b), inex(b), ...\}\rangle
\end{array}$$

Recall $\mathcal{P}_{add}$. Together with $\mathcal{E}_{\neg cig(b)}$ it contains

$$\begin{array}{rcl@{\qquad}rcl}
add'(X) & \leftarrow & inex(X) \wedge \neg ab_{add'}(X), & add(X) & \leftarrow & \neg add'(X), \\
inex(X) & \leftarrow & cig(X) \wedge \neg ab_{inex}(X), & ab_{add'}(X) & \leftarrow & cig(X), \\
ab_{add'}(X) & \leftarrow & \bot, & ab_{inex}(X) & \leftarrow & \bot, \\
cig(b) & \leftarrow & \bot. & & &
\end{array}$$

Abducing the CONCLUSION

Given our background knowledge we know, there are addictive things, let's say about b

$$\mathcal{O}_{add(b)} = \{add(b)\}$$

We have two minimal explanations for $\mathcal{O}_{add(b)}$

$$\begin{array}{lcll}
\mathrm{lm}_{Ł}\mathrm{wc}\,(\mathcal{P}_{add} \cup \mathcal{E}_{cig(b)}) & = & \langle\{add(b), cig(b), inex(b), \ldots\}, & \{\ldots\}\rangle \\
\mathrm{lm}_{Ł}\mathrm{wc}\,(\mathcal{P}_{add} \cup \mathcal{E}_{\neg cig(b)}) & = & \langle\{add(b), \ldots\}, & \{cig(b), inex(b), \ldots\}\rangle
\end{array}$$

Recall $\mathcal{P}_{add}$. Together with $\mathcal{E}_{\neg cig(b)}$ it contains

$$\begin{array}{rcl@{\qquad}rcl}
add'(X) & \leftarrow & inex(X) \wedge \neg ab_{add'}(X), & add(X) & \leftarrow & \neg add'(X), \\
inex(X) & \leftarrow & cig(X) \wedge \neg ab_{inex}(X), & ab_{add'}(X) & \leftarrow & cig(X), \\
ab_{add'}(X) & \leftarrow & \bot, & ab_{inex}(X) & \leftarrow & \bot, \\
cig(b) & \leftarrow & \bot. & & &
\end{array}$$

Credulously, we validate some addictive things are not cigarettes.

Modeling S_{rich}

If we model S_{rich} analogously to S_{add} we have the following background knowledge

Rich people are millionaires. (1)

Modeling S_{rich}

If we model S_{rich} analogously to S_{add} we have the following background knowledge

Rich people are millionaires. (1)

additionally, given PREMISE 2

Rich people are hard workers (compared to other millionaires). (2)

Modeling S_{rich}

If we model S_{rich} analogously to S_{add} we have the following background knowledge

Rich people are millionaires. (1)

additionally, given PREMISE 2

Rich people are hard workers (compared to other millionaires). (2)

PREMISE 1 of S_{rich} would then be represented as

If someone is a hard worker and not abnormal, then this person is not a millionaire. (3)
Nobody is abnormal (wrt (3)).

The belief in (1) and (2) would generate the exception for rich people

If someone is rich, then this person is abnormal (wrt (3)). (4)

Modeling S_{rich}

If we model S_{rich} analogously to S_{add} we have the following background knowledge

Rich people are millionaires. (1)

additionally, given PREMISE 2

Rich people are hard workers (compared to other millionaires). (2)

PREMISE 1 of S_{rich} would then be represented as

If someone is a hard worker and not abnormal, then this person is not a millionaire. (3)
Nobody is abnormal (wrt (3)).

The belief in (1) and (2) would generate the exception for rich people

If someone is rich, then this person is abnormal (wrt (3)). (4)

Even though not tested yet, our hypothesis is, while checking S_{rich}, participants did not make these assumptions and thus, had not been influenced by the belief-bias effect.

Contextual Abductive Reasoning

How to express that PREMISE 1 describes the usual and PREMISE 2 the exceptional case? Inexpensive cigarette should be the exception in the context of addictive things.

Contextual Abductive Reasoning

How to express that PREMISE 1 describes the usual and PREMISE 2 the exceptional case? Inexpensive cigarette should be the exception in the context of addictive things.

Introduce for every A, two reserved (meta-) predicates (Pereira and Pinto [2011]).

$$\mathsf{inspect}(A) \qquad \text{and} \qquad \mathsf{inspect}_{\mathsf{not}}(A)$$

These are special abducibles only to be abduced if A or $\neg A$ are abduced somewhere else.

Contextual Abductive Reasoning

How to express that PREMISE 1 describes the usual and PREMISE 2 the exceptional case? Inexpensive cigarette should be the exception in the context of addictive things.

Introduce for every A, two reserved (meta-) predicates (Pereira and Pinto [2011]).

$$\text{inspect}(A) \qquad \text{and} \qquad \text{inspect}_{\text{not}}(A)$$

These are special abducibles only to be abduced if A or $\neg A$ are abduced somewhere else. We replace the $ab_{add'}$-clause in $\mathcal{P}_{add}$. $\mathcal{P}_{add_{\text{insp}}}$ contains

$$\begin{array}{rcl rcl}
add'(X) & \leftarrow & inex(X) \wedge \neg ab_{add'}(X), & add(X) & \leftarrow & \neg add'(X), \\
inex(X) & \leftarrow & cig(X) \wedge \neg ab_{inex}(X), & ab_{add'}(X) & \leftarrow & \text{inspect}(cig(X)), \\
ab_{add'}(X) & \leftarrow & \bot, & ab_{inex}(X) & \leftarrow & \bot,
\end{array}$$

Contextual Abductive Reasoning

How to express that PREMISE 1 describes the usual and PREMISE 2 the exceptional case? Inexpensive cigarette should be the exception in the context of addictive things.

Introduce for every A, two reserved (meta-) predicates (Pereira and Pinto [2011]).

$$\text{inspect}(A) \qquad \text{and} \qquad \text{inspect}_{\text{not}}(A)$$

These are special abducibles only to be abduced if A or $\neg A$ are abduced somewhere else. We replace the $ab_{add'}$-clause in $\mathcal{P}_{add}$. $\mathcal{P}_{add_{\text{insp}}}$ contains

$$\begin{array}{rcl@{\qquad}rcl}
add'(X) & \leftarrow & inex(X) \wedge \neg ab_{add'}(X), & add(X) & \leftarrow & \neg add'(X), \\
inex(X) & \leftarrow & cig(X) \wedge \neg ab_{inex}(X), & ab_{add'}(X) & \leftarrow & \text{inspect}(cig(X)), \\
ab_{add'}(X) & \leftarrow & \bot, & ab_{inex}(X) & \leftarrow & \bot,
\end{array}$$

Suppose again b is addictive , i.e. $\mathcal{O}_{add(b)} = \{add(b)\}$. $\mathcal{E}_{cig} = \{cig(b) \leftarrow \top\}$ cannot be abduced anymore to explain $\mathcal{O}_{add(b)}$. Its only minimal explanation is $\mathcal{E}_{\neg cig(b)}$.

Contextual Abductive Reasoning

How to express that PREMISE 1 describes the usual and PREMISE 2 the exceptional case? Inexpensive cigarette should be the exception in the context of addictive things.

Introduce for every A, two reserved (meta-) predicates (Pereira and Pinto [2011]).

$$\text{inspect}(A) \qquad \text{and} \qquad \text{inspect}_{\text{not}}(A)$$

These are special abducibles only to be abduced if A or $\neg A$ are abduced somewhere else. We replace the $ab_{add'}$-clause in $\mathcal{P}_{add}$. $\mathcal{P}_{add_{\text{insp}}}$ contains

$$\begin{array}{rclrcl}
add'(X) & \leftarrow & inex(X) \wedge \neg ab_{add'}(X), & add(X) & \leftarrow & \neg add'(X), \\
inex(X) & \leftarrow & cig(X) \wedge \neg ab_{inex}(X), & ab_{add'}(X) & \leftarrow & \text{inspect}(cig(X)), \\
ab_{add'}(X) & \leftarrow & \bot, & ab_{inex}(X) & \leftarrow & \bot,
\end{array}$$

Suppose again b is addictive , i.e. $\mathcal{O}_{add(b)} = \{add(b)\}$. $\mathcal{E}_{cig} = \{cig(b) \leftarrow \top\}$ cannot be abduced anymore to explain $\mathcal{O}_{add(b)}$. Its only minimal explanation is $\mathcal{E}_{\neg cig(b)}$.

Skeptically, we validate some addictive things are not cigarettes.

Contextual Relevant Consequences

Consider $\mathcal{P}_{fire}$

$$
\begin{array}{llllll}
storm & \leftarrow & lightning, & forest\_fire & \leftarrow & \text{inspect}(lightning) \wedge smoke \wedge \neg ab, \\
ab & \leftarrow & \neg dry\_leaves, & forest\_fire & \leftarrow & barbecue \wedge smoke \wedge \neg ab, \\
smoke & \leftarrow & \top. & & &
\end{array}
$$

Contextual Relevant Consequences

Consider $\mathcal{P}_{fire}$

$$\begin{array}{lllllll} storm & \leftarrow & lightning, & forest\_fire & \leftarrow & \mathrm{inspect}(lightning) \wedge smoke \wedge \neg ab, \\ ab & \leftarrow & \neg dry\_leaves, & forest\_fire & \leftarrow & barbecue \wedge smoke \wedge \neg ab, \\ smoke & \leftarrow & \top. & & & \end{array}$$

- $\mathrm{inspect}(lightning) \leftarrow \top$ is only partly explaining $\mathcal{O}_{forest\_fire} = \{forest\_fire\}$ if $lightning \leftarrow \top$ is abduced somewhere else.

Contextual Relevant Consequences

Consider $\mathcal{P}_{fire}$

$$\begin{array}{lllllll} storm & \leftarrow & lightning, & forest\_fire & \leftarrow & \text{inspect}(lightning) \wedge smoke \wedge \neg ab, \\ ab & \leftarrow & \neg dry\_leaves, & forest\_fire & \leftarrow & barbecue \wedge smoke \wedge \neg ab, \\ smoke & \leftarrow & \top. & & & \end{array}$$

- inspect($lightning$) $\leftarrow \top$ is only partly explaining $\mathcal{O}_{forest\_fire} = \{forest\_fire\}$ if $lightning \leftarrow \top$ is abduced somewhere else.
- Otherwise, only explanation is $\mathcal{E}_{barbecue,dry} = \{barbecue \leftarrow \top, dry\_leaves \leftarrow \top\}$.

Contextual Relevant Consequences

Consider $\mathcal{P}_{fire}$

$$
\begin{array}{lllllll}
storm & \leftarrow & lightning, & forest\_fire & \leftarrow & \text{inspect}(lightning) \wedge smoke \wedge \neg ab, \\
ab & \leftarrow & \neg dry\_leaves, & forest\_fire & \leftarrow & barbecue \wedge smoke \wedge \neg ab, \\
smoke & \leftarrow & \top. & & &
\end{array}
$$

- inspect($lightning$) $\leftarrow \top$ is only partly explaining $\mathcal{O}_{forest\_fire} = \{forest\_fire\}$ if $lightning \leftarrow \top$ is abduced somewhere else.
- Otherwise, only explanation is $\mathcal{E}_{barbecue,dry} = \{barbecue \leftarrow \top, dry\_leaves \leftarrow \top\}$.

Given $\mathcal{O}_{storm}$ explained by $\mathcal{E}_{lightning}$, $\mathcal{O}_{forest\_fire}$'s minimal explanation is

$$\mathcal{E}_{\text{inspect}(lightning),dry} = \{\text{inspect}(lightning) \leftarrow \top, dry\_leaves \leftarrow \top\}.$$

Contextual Relevant Consequences

Consider $\mathcal{P}_{fire}$

$$
\begin{array}{lllllll}
storm & \leftarrow & lightning, & forest\_fire & \leftarrow & \mathrm{inspect}(lightning) \wedge smoke \wedge \neg ab, \\
ab & \leftarrow & \neg dry\_leaves, & forest\_fire & \leftarrow & barbecue \wedge smoke \wedge \neg ab, \\
smoke & \leftarrow & \top. & & &
\end{array}
$$

- $\mathrm{inspect}(lightning) \leftarrow \top$ is only partly explaining $\mathcal{O}_{forest\_fire} = \{forest\_fire\}$ if $lightning \leftarrow \top$ is abduced somewhere else.
- Otherwise, only explanation is $\mathcal{E}_{barbecue,dry} = \{barbecue \leftarrow \top, dry\_leaves \leftarrow \top\}$.

Given $\mathcal{O}_{storm}$ explained by $\mathcal{E}_{lightning}$, $\mathcal{O}_{forest\_fire}$'s minimal explanation is

$$\mathcal{E}_{\mathrm{inspect}(lightning),dry} \quad = \quad \{\mathrm{inspect}(lightning) \leftarrow \top, dry\_leaves \leftarrow \top\}.$$

$\mathcal{O}_{forest\_fire}$ is a contextual relevant consequence of $\mathcal{O}_{storm}$ given $\mathcal{P}_{fire}$.