Stochastic Event-triggered Sensor Schedule for Remote State Estimation

Ling Shi

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香港科技大學 THE HONG KONG UNIVERSITY OF SCIENCE AND TECHNOLOGY

Brief Introduction

- Wireless sensor network widely used for monitoring the process state, e.g., transportation, environmental monitoring, industrial process, smart grid, etc.
- Remote state estimation, e.g., humidity in forest, water temperature in ocean.
- Good: mobility, scalability, etc.
- Bad: limited power and bandwidth, less reliability, etc.
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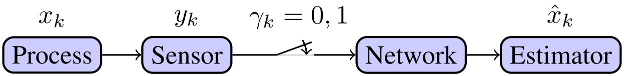

$$x_{k+1} = Ax_k + w_k,$$
$$y_k = Cx_k + v_k.$$

- $x_k \in \mathbb{R}^n$ is the state at time k.
- $y_k \in \mathbb{R}^m$ is the measurement from the sensor.
- w_k, v_k, x_0 are independent Gaussian r.v., and $x_0 \sim \mathcal{N}(\bar{x},\ \Sigma)$, $w_k \sim \mathcal{N}(0,\ Q)$ and $v_k \sim \mathcal{N}(0,\ R)$. (A, Q) controllable and (A, C) observable.
- Based on the current and the past available observations, the estimator computes the MMSE estimate of the current state x_k.

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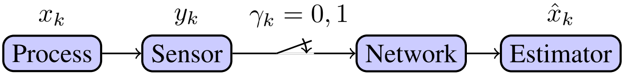

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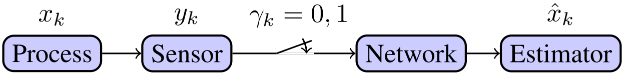

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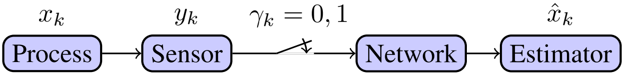

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Kalman Filtering

Time update:

$$\hat{x}_{k+1}^{-} = A\hat{x}_k,$$
$$P_{k+1}^{-} = AP_kA' + Q,$$

Measurement update:

$$\hat{x}_k = \hat{x}_k^{-} + K_k(y_k - C\hat{x}_k^{-}) \,,$$
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with initial condition

$$\hat{x}_0^{-} = \bar{x}, \quad P_0^{-} = \Sigma,$$

where

$\hat{x}_k \triangleq \mathbb{E}[x_k|y_k, \ldots, y_0]$, $e_k \triangleq x_k - \hat{x}_k$, $P_k \triangleq \mathrm{Cov}(e_k)$.
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Kalman Filtering Preliminaries

- KF is the MMSE estimator to obtain $\hat{x}_k$.
- For KF, we need to keep track on both the state estimate $\hat{x}_k$ and the "accuracy" of the estimate P_k.
- P_k is deterministic, which does not depends on y_k.
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Sensor Scheduling

- Communication costs too much energy.
- Sensors could deliberately and smartly drop some packets to save energy. Certainly, optimal filter derivation should be revisited.
- **Tradeoff**: Reduce the communication frequency to save energy and bandwidth. Inevitably sacrifice the estimation performance.
- *To send, or not to send* y_k: that's a question for the sensor.
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What does a *sensor schedule* mean here?

- **Definition**: basically, a sequence of transmission decisions based on some decision rule. Decision variable $\gamma_k \in \{0, 1\}$: $\gamma_k = 1$ indicates that y_k is sent and $\gamma_k = 0$ otherwise. A sensor schedule $S : \{\gamma_0 \gamma_1 \dots \gamma_k \dots\}$.
- Off-line schedule $\begin{cases} \text{Deterministic} \\ \text{Stochastic} \end{cases}$.
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The schedule is designed based on the communication rate requirement and the statistics of the system, e.g.:

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- Stochastic schedule: $\gamma_k = \begin{cases} 0 & 0 \leq r < 0.5 \\ 1 & 0.5 \leq r \leq 1 \end{cases}$, $r \sim B(1,\ 0.5)$, i.i.d., send with 50% probability.

The optimal filter is

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Event-triggered Schedule

- The schedule depends on both the statistics and the realization of the system.
- For example, if the temperature of the room is normal distributed around 72.5, we could design the schedule to be: send y_k if the temperature is outside $[70, 75]$, e.g., $\gamma_k = \begin{cases} 1, & |y_k - 72.5| > 2.5; \\ 0, & otherwise. \end{cases}$
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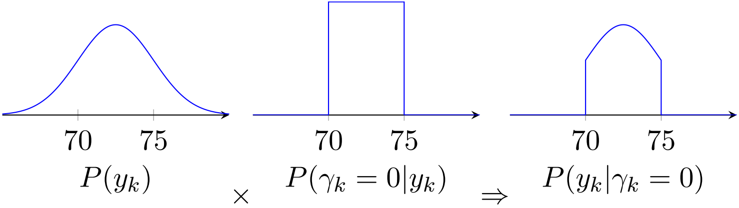

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- Not linear in general, difficult to analyze the estimation performance.

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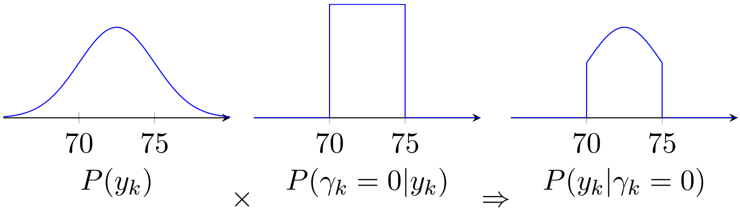

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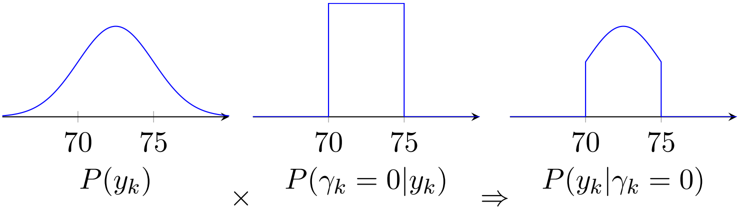

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General Decision Rule

$$\gamma_k = \begin{cases} 0 & \zeta_k \leq \Phi(y_k, \hat{y}_k^-) \\ 1 & \zeta_k > \Phi(y_k, \hat{y}_k^-) \end{cases}, \tag{1}$$

where ζ_k is an i.i.d. random variable with sample space $[a, b]$, $\Phi(y_k, \hat{y}_k^-) : \mathbb{R}^m \times \mathbb{R}^m \to [a, b]$.

- Special case: deterministic event-based rule, Wu et al. (TAC13),

$$\gamma_k = \begin{cases} 0 & \zeta_k \leq -\|\hat{z}_k\|_\infty \\ 1 & \zeta_k > -\|\hat{z}_k\|_\infty \end{cases}, \tag{2}$$

$a = b = -\delta$, $\Phi(y_k, \hat{y}_k^-)$ is the negative H_∞ norm of $\hat{z}_k$ which normalizes the innovation $z_k = y_k - \hat{y}_k^-$.

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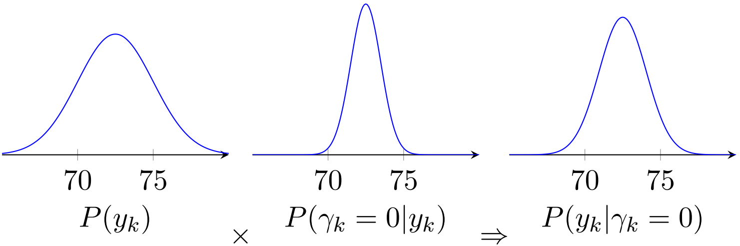
Proposed Stochastic Event-triggered Schedule



- At each time k, the sensor generates a random variable $\zeta_k \sim U[0,1]$.
- Decide whether to send or not based on the following rule:

$$\gamma_k = \begin{cases} 0 & \zeta_k \le \Phi(y_k) \\ 1 & \zeta_k > \Phi(y_k) \end{cases},$$

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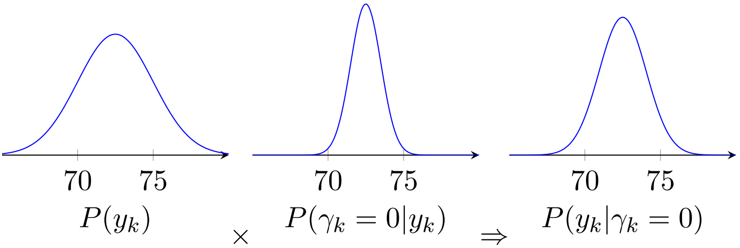

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Optimal Filter

We can derive the exact MMSE estimator without any approximation.

Theorem 1

Time update:

$$\hat{x}_k^- = A\hat{x}_{k-1},$$
$$P_k^- = AP_{k-1}A' + Q.$$

Measurement update:

$$\hat{x}_k = \begin{cases} \hat{x}_k^- + P_k^- C'(CP_k^- C' + R)^{-1}(y_k - C\hat{x}_k^-), & \gamma_k = 1 \\ (I - P_k^- C'(CP_k^- C' + R + Y^{-1})^{-1}C)\hat{x}_k^-, & \gamma_k = 0 \end{cases}$$

$$P_k = \begin{cases} \left[\left(P_k^-\right)^{-1} + C'R^{-1}C\right]^{-1}, & \gamma_k = 1 \\ \left[\left(P_k^-\right)^{-1} + C'\left(R + Y^{-1}\right)^{-1}C\right]^{-1}, & \gamma_k = 0 \end{cases}$$

with initial condition

$$\hat{x}_0^- = 0,\ P_0^- = \Sigma_0.$$

Short Summary

- Scheduling the transmission based on the importance of the measurement.
- The similarity of the event-triggering function and the pdf of a Gaussian random variable plays a key role in the derivation of the MMSE estimator.
- Facilitate performance analysis.

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- Average communication rate, e.g.,

$$\gamma \triangleq \limsup_{T\to\infty} \frac{1}{T} \sum_{k=0}^{T} \mathbb{E}[\gamma_k]$$

- Stability,
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Average Communication Rate

Theorem 2

If the system is stable, e.g., $\rho(A) < 1$, then the communication rate γ can be computed as

$$\gamma = 1 - \frac{1}{\sqrt{\det(I + \Pi Y)}}, \tag{3}$$

where Π satisfies $\Pi = C\Sigma C' + R$, Σ is given by $\Sigma = A\Sigma A' + Q$.

Stability

- The estimator is stable with bounded estimation error covariance.
- Trivial bound for the estimation error covariance is the solution to the following discrete algebraic Riccati equation:

$$P_{k+1}^{-} = AP_k^{-}A' + Q - AP_k^{-}C'(CP_k^{-}C' + R + Y^{-1})^{-1}CP_k^{-}A'.$$

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Asymptotic Lower Bound of $\mathbb{E}[P_k^-]$

Define $g(W, X) \triangleq AXA' + Q - AXC'(CXC' + W)^{-1}CXA'$.

Theorem 3

$E[P_k^-]$ is asymptotically bounded by

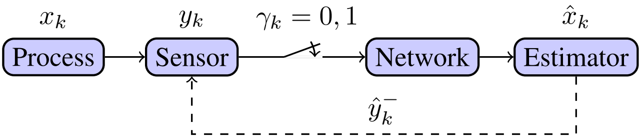
$$\underline{X}_{open} \leq \lim_{k\to\infty} \mathbb{E}[P_k^-] \leq \overline{X}_{open}, \tag{4}$$

where $\underline{X}_{open}$ is the unique positive-definite solution to $g(R_1, X) = X$ with

$$R_1 = \left(\gamma R^{-1} + (1-\gamma)(R + Y^{-1})^{-1}\right)^{-1}, \tag{5}$$

and $\overline{X}_{open}$ is the unique positive-definite solution to $g(R_2, X) = X$ with $R_2 = R + Y^{-1}$.

Closed-loop Case



$$\gamma_k = \begin{cases} 0 & \zeta_k \le \exp\left(-\frac{1}{2} z_k' Z z_k\right) \\ 1 & \zeta_k > \exp\left(-\frac{1}{2} z_k' Z z_k\right) \end{cases},$$

where $z_k = y_k - C\hat{x}_k^-, Z > 0$, $\zeta_k \sim Uniform[0, 1]$.

- If we close the loop, the estimator feeds $\hat{x}_k^-$ back to the sensor, we can define a similar event to the previous one.

Optimal Filter

We can derive the exact MMSE estimator for the closed-loop case.

Theorem 4

Time update:

$$\begin{aligned}\hat{x}_k^- &= A\hat{x}_{k-1},\\ P_k^- &= AP_{k-1}A' + Q.\end{aligned}$$

Measurement update:

$$\hat{x}_k = \begin{cases} \hat{x}_k^- + P_k^- C'(CP_k^- C' + R)^{-1}(y_k - C\hat{x}_k^-), & \gamma_k = 1 \\ \hat{x}_k^-, & \gamma_k = 0 \end{cases}$$

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Closed-loop case (Continued)

- Better than open-loop case.
- Since $\mathbb{E}[y_k y_k^T]$ is larger than $\mathbb{E}[z_k z_k^T]$, with the same communication rate, the matrix Z for the closed-loop schedule is larger than Y for the open-loop schedule. Recall how Y^{-1} and Z^{-1} enlarge P_k.
- Closed-loop schedule can be used in unstable systems.
- Similar analytical results.

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Effectiveness

- Consider a scalar stable system with parameters $A = 0.8, C = 1, Q = 1, R = 1$. We use four strategies to reduce the communication rate.

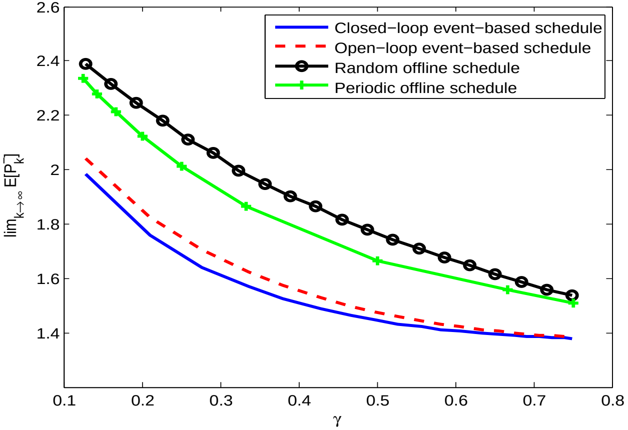

Figure: $\lim_{k\to\infty} \mathbb{E}[P_k^-]$ under four scheduling strategies versus communication rate γ

Comparison with Wu et al. (TAC13)

- Consider a target tracking problem where a sensor is deployed to track the state x_k which consists of the position, speed and acceleration of the target.
- $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, Q = I_{3\times 3}, R = I_{3\times 3}.$
- Communication rate limit: 0.65
- The closed-loop stochastic event-triggered schedule (CLSET-KF) used with $Z = 0.52 \times I_{3\times 3}$ and Wu's deterministic event-triggered schedule (DET-KF) used with the threshold being 1.60, where the parameters are carefully designed to satisfy the communication rate limitation.
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Comparison with Wu's Work

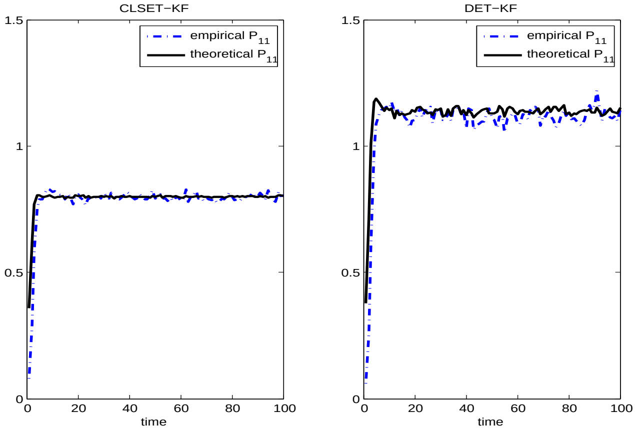
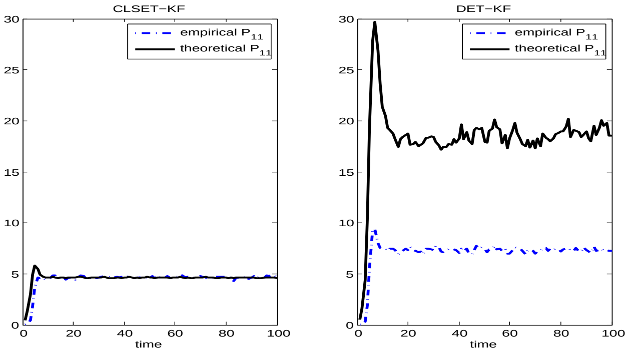


Figure: Variance of the target position error. The target is tracked by the CLSET-KF (left) and DET-KF (right) with the average communication rate being 0.65.

- Communication rate limit: 0.25
- CLSET-KF with $Z = 0.047 \times I_{3\times 3}$ and Wu's DET-KF used with the threshold being 4.3



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Conclusion

1. We propose a stochastic event-trigger and derive the corresponding optimal filter.
2. The most crucial feature is reserving the Gaussian properties of Kalman filtering while introducing the nonlinear event-triggered mechanism.
3. Some analytical results on stability, average communication rate and average estimation error covariance are given.
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Thank You !