Analysis of incomplete data due to double truncation

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Outline

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2. The NPMLE revisited
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6. Conclusions

Motivation examples

- Astronomy
- Economy
- Epidemiology
- Survival Analysis

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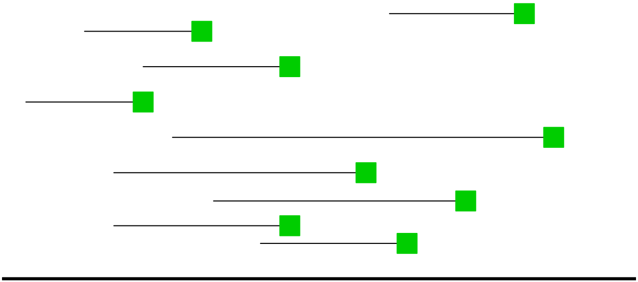
Related with Epidemiology and/or Survival Analysis:

- Time from HIV infection to diagnosis of AIDS (Bilker and Wang, 1996)
- Time from birth to diagnosis in childhood cancer (Moreira and De Uña-Álvarez, 2007)

Truncation Scheme



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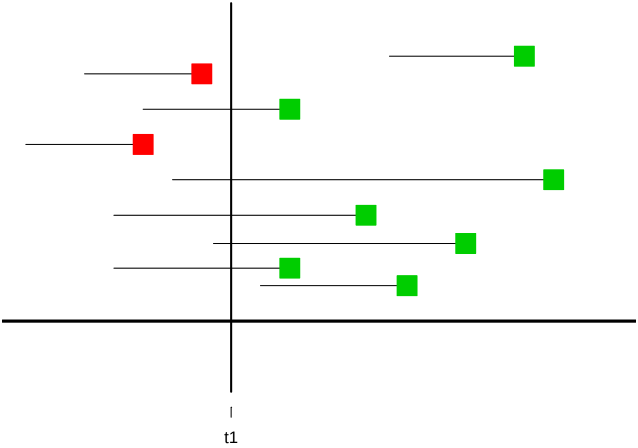

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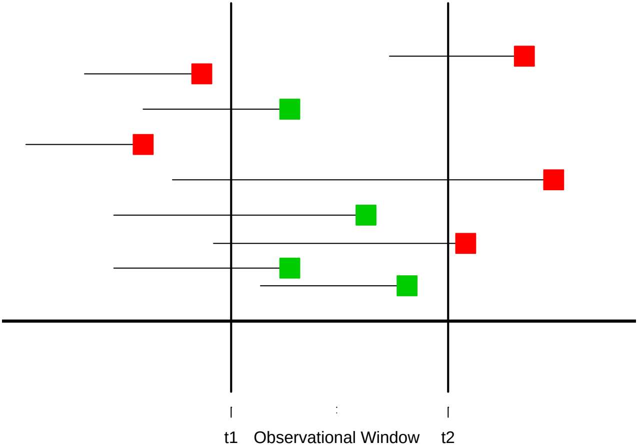

Truncation Scheme

- Let X^* be the ultimate time of interest with df F
- (U^*, V^*) the pair of truncation times, with joint df K
- We observe (U^*, X^*, V^*) if and only if $U^* \leq X^* \leq V^*$
- Let $(U_i, X_i, V_i), i = 1, ..., n$ be the observed data.

Under the assumption of independence between X^* and (U^*, V^*):

The full likelihood is given by:

$$L_n(f,k) = \prod_{j=1}^{n} \frac{f_j k_j}{\sum_{i=1}^{n} F_i k_i}$$

Truncation Scheme

Where:

- $f = (f_1, f_2, ..., f_n)$
- $k = (k_1, k_2, ..., k_n)$
- $F_i = \sum_{m=1}^{n} f_m J_{i_m}$

and

$$J_{i_m} = I_{[U_i \leq X_m \leq V_i]} = 1 \quad \text{if} \quad U_i \leq X_m \leq V_i,$$

or zero otherwise.

As noted by Shen (2008):

$$L_n(f,k) = \prod_{j=1}^{n} \frac{f_j}{F_j} \times \prod_{j=1}^{n} \frac{F_j k_j}{\sum_{i=1}^{n} F_i k_i} = L_1(f) \times L_2(f,k)$$

Efron-Petrosian estimator

The conditional NPMLE of F (Efron-Petrosian, 1999) is defined as the maximizer of $L_1(f)$.

$$\frac{1}{\hat{f}_j} = \sum_{i=1}^{n} J_{ij} \times \frac{1}{\hat{F}_i}, \quad j = 1, ..., n$$

where $\hat{F}_i = \sum_{m=1}^{n} \hat{f}_m J_{im}$.

This equation was used by Efron and Petrosian (1999) to introduce the EM algorithm to compute $\hat{f}$.

EM algorithm from Efron and Petrosian (1999)

EP1. Compute the initial estimate $\hat{F}_{(0)}$ corresponding to $\hat{f}_{(0)} = (1/n, ..., 1/n)$;

EP2. Apply (1) to get an improved estimator $\hat{f}_{(1)}$ to compute the $\hat{F}_{(1)}$ pertaining to $\hat{f}_{(1)}$;

EP3. Repeat Step EP2 until convergence criterion is reached.

Shen estimator

Interchanging the roles of X's and (U_i, V_i):

$$L_n(f,k) = \prod_{j=1}^{n} \frac{k_j}{K_j} \times \prod_{j=1}^{n} \frac{K_j f_j}{\sum_{i=1}^{n} K_i f_i} = L_1(k) \times L_2(k,f)$$

where

$$K_i = \sum_{m=1}^{n} k_m I_{[U_m \leq X_i \leq V_m]} = \sum_{m=1}^{n} k_m J_{im}$$

and maximizing $L_1(k)$:

$$\frac{1}{\hat{k}_j} = \sum_{i=1}^{n} J_{ji} \frac{1}{\hat{K}_i}, \quad j = 1, ..., n$$

with $\hat{K}_i = \sum_{m=1}^{n} \hat{k}_m J_{im}$.

Shen Estimator

Shen (2008) showed that the solutions are the unconditional NPMLE of F

and K, respectively, and both estimators can be obtained by:

$$\hat{f}_j = \left[\sum_{i=1}^{n} \frac{1}{\hat{K}_j} \right]^{-1} \frac{1}{\hat{K}_j}, \quad j = 1, ..., n$$

$$\hat{k}_j = \left[\sum_{i=1}^{n} \frac{1}{\hat{F}_j} \right]^{-1} \frac{1}{\hat{F}_j}, \quad j = 1, ..., n$$

EM algorithm from Shen (2008)

S1. Compute the initial estimate $\hat{F}_{(0)}$ corresponding to $\hat{f}_{(0)} = (1/n, ..., 1/n)$;

S2. Apply (4) to get the first step estimator $\hat{k}_{(1)}$ and compute the $\hat{K}_{(1)}$ pertaining to $\hat{k}_{(1)}$;

S3. Apply (3) to get the first step estimator $\hat{f}_{(1)}$ and its corresponding $\hat{F}_{(1)}$;

S4. Repeat Steps S2 and S3 until convergence criterion is reached.

Simple bootstrap procedure

- From the original data, we take a bootstrap resample (U_{ib}, V_{ib}, X_{ib}), $i = 1, ..., n$ putting weight $1/n$ at each of the observations (U_i, V_i, X_i), $i = 1, ..., n$
- Repeat this procedure a large number B of times
- Put $\hat{F}_b$ for the estimator $\hat{F}$ computed from the b^{th} bootstrap resample, $b = 1, ..., B$
- The values of $\hat{F}_1(t), ..., \hat{F}_b(t)$ can be used to empirically approximate the finite sample distribution of $\hat{F}(t)$ for a given t

Simulated model

- X^* is independent of (U^*, V^*) but $U^* = V^* - \delta$
- $X^* \sim Unif(0, 15)$, $U^* \sim Unif(-5, 15)$ and $V^* = U^* + 5$

Simulated model

PT	n	Deciles	Coverage	Mean Length CI	Length sd. CI
37,5%	50	1	0.926	0.3019516	0.033585412
		2	0.951	0.4139273	0.027835971
		3	0.958	0.4704103	0.018342575
		4	0.971	0.4981912	0.011472745
		5	0.957	0.5042559	0.008720808
		6	0.960	0.4942161	0.010959723
		7	0.955	0.4624988	0.017726775
		8	0.940	0.3994099	0.026072178
		9	0.917	0.2852907	0.032080445
37,5%	250	1	0.950	0.09643203	0.0005328409
		2	0.941	0.13397596	0.0007621275
		3	0.950	0.15348897	0.0007692665
		4	0.950	0.16222925	0.0006908011
		5	0.956	0.16459729	0.0006598123
		6	0.959	0.16209428	0.0006474786
		7	0.958	0.15367155	0.0006482495
		8	0.951	0.13382406	0.0006319017
		9	0.975	0.09619246	0.0004300545

Table: Coverages of the 95% bootstrap confidence intervals for the NPMLE of F along 1000 trials for sample sizes 50 and 250. $X^* \sim Unif(0, 15)$, $U^* \sim Unif(-5, 15)$ were independently simulated and $V^* = U^* + 5$. Means and standard deviations of the interval lengths are also reported. Simple bootstrap method was considered.

Childhood cancer data description

- Includes all the cases diagnosed in Northern region of Portugal between January 1st, 1999 and December 31st, 2003;
- Follow-up until April 30th, 2006;
- Variables included: birth date, date of death, censoring status, source of diagnosis, residence, sex, age at diagnosis, date of first symptom, date of first examination, date of diagnosis and type of cancer; according to paediatric classification tumours whose based according the International Childhood Cancer Classification, 3rd Edition;

Childhood cancer data description

- Data correspond to 409 children, with age below 15 years old (180 female and 229 male);
- Birth date varying between May 13th, 1984 and July 2nd, 2003;
- In the five years of recruitment, the number of cases ranged almost uniformly (63 in 2002 to 90 in 2003);
- The more frequent diagnosis are the precocious: 50% of the cases correspond to children below six years old, and 75% of the cases correspond to children below ten years old.

Data Formulation

- Let X^* be the age (in years) at diagnosis and U^* the age of the individual at January 1 st, 1999;
- (U^*, V^*) is observed only when $U^* \leq X^* \leq U^* + 5$;
- X^* is doubly truncated by (U^*, V^*) where $V^* = U^* + 5$;
- V^* is doubly truncated by $(X^*, X^* + 5)$.

NPMLE of the df of X^*

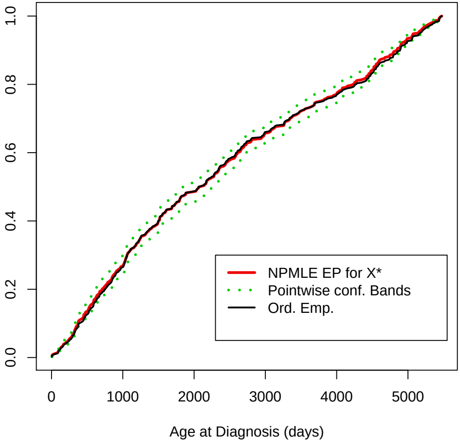

Figure: NPMLE of the distribution of the age at diagnosis for the childhood cancer data, and 95% pointwise confidence band based on the simple boostrap. The ordinary empirical distribution of the age at diagnosis is included for comparison.

NPMLE of the df of V^*

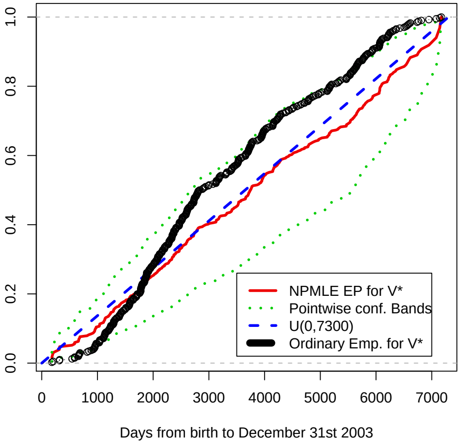

Figure: NPMLE of the distribution of NPMLE of the distribution of the time from birth to December 31st, 2003 for the childhood cancer data, and 95% pointwise confidence band based on the bootstrap. The uniform distribution and the ordinary empirical df of V^* are included for comparison.

DT vs LTRC

- Doubly truncated(DT) data are not the same as left-truncated-right-censored (LTRC) data as considered in Wang (1991) or Gross and Lai (1996).

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- In LTRC setup, one would have observed those cases with $X^* > U^* + 5$, with the information on the lifetime X^* limited to $U^* + 5$ (right censored information).
- In our DT scenario, we have no information on these subjects, and hence inference procedures are expected to be less efficient than those corresponding to LTRC data.

Simulated model

- X^* is independent of (U^*, V^*) but $U^* = V^* - \delta$
- $X^* \sim Unif(0, 15)$, $U^* \sim Unif(-5, 15)$ and $V^* = U^* + 5$
- Let $(U_i, X_i, V_i), i = 1, ..., n$ be the simulated data
- Accept the pairs that verified $U_i \leq X_i$
- If $V_i < X_i, i = 1, ..., n$, the case is censored, otherwise is doubly truncated.

DT vs LTRC

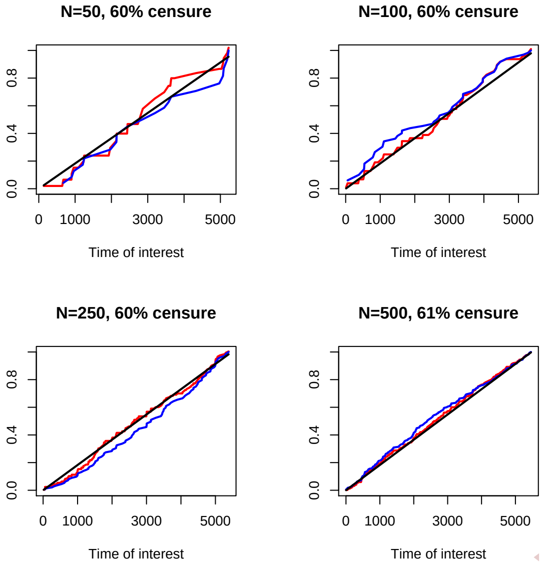

Figure 3: NPMLE of the distribution function of the time of interest for doubly truncated data (blue line), Kaplan-Meier estimator for LTRC data (red line). The interest distribution function (black line).

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- NPMLE for doubly truncated data has been revisited;
- Existing algorithms for the numerical approximation of the NPMLE has been reviewed;
- Both the estimation of the doubly truncated distribution and of the (joint) distribution of the truncation times were considered;
- We suggest using the first algorithm in Efron and Petrosian (1999) or the alternative method in Shen (2008) for the computation of the NPMLE;

Summary

- The bootstrap has been introduced as a method to approximate the sampling distribution of the NPMLE;
- The behaviour of the simple bootstrap was tested in a simulation study;
- Ignoring the double truncation issue may introduce a severe bias in estimation;
- All methods were implemented in R language and included in DTDA R package.

Future Research

- Semiparametric estimator for doubly truncated data
- Regression with doubly truncated responses
- Application of the NPMLE to kernel estimation of the density and the hazard rate under double truncation

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