# Plasma turbulence in the tokamak scrape-off layer

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Theory of Fusion Plasmas
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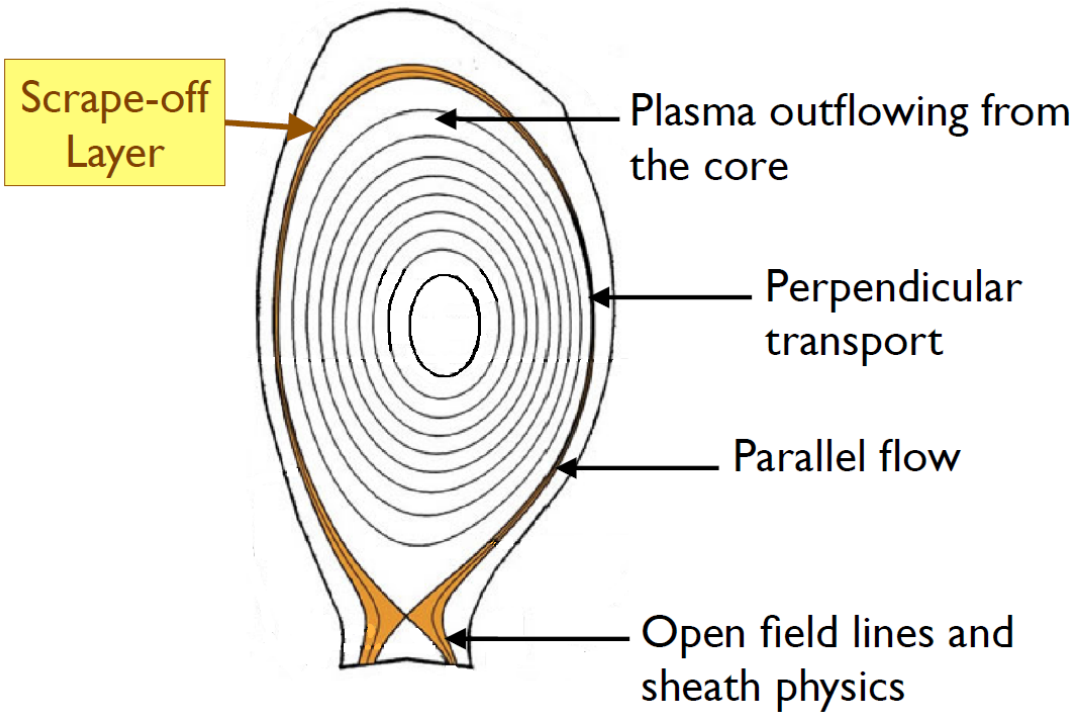
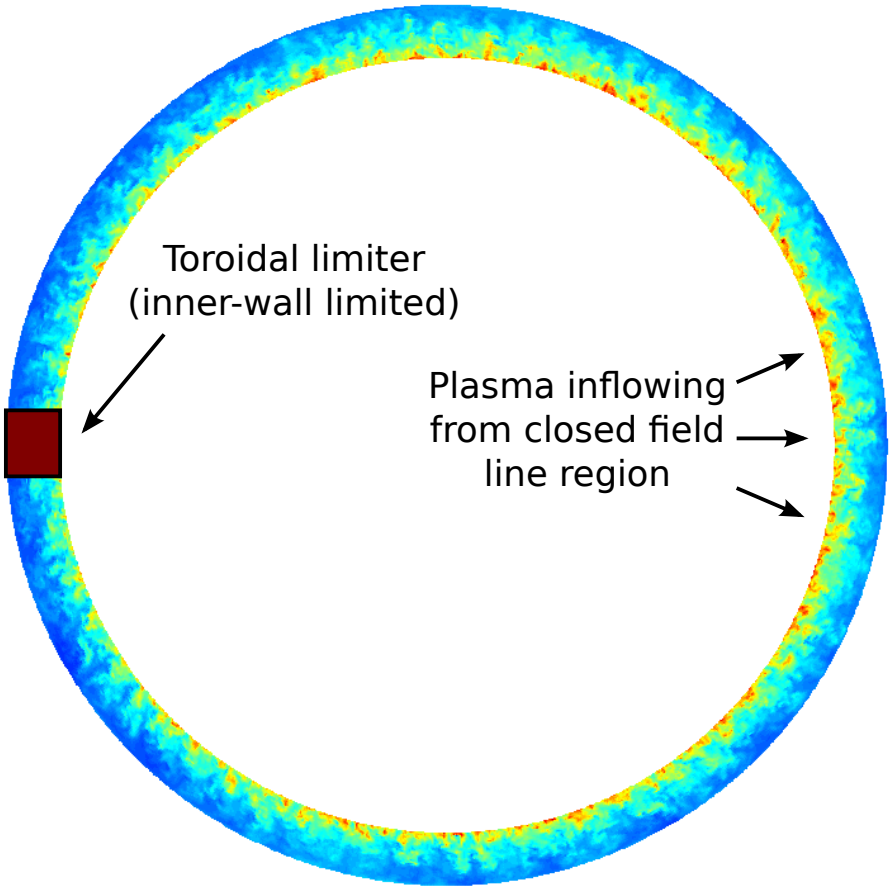
# Scrape-off layer physics crucial for magnetic fusion

Heat load to PFCs, rotation, impurities, L-H transition...

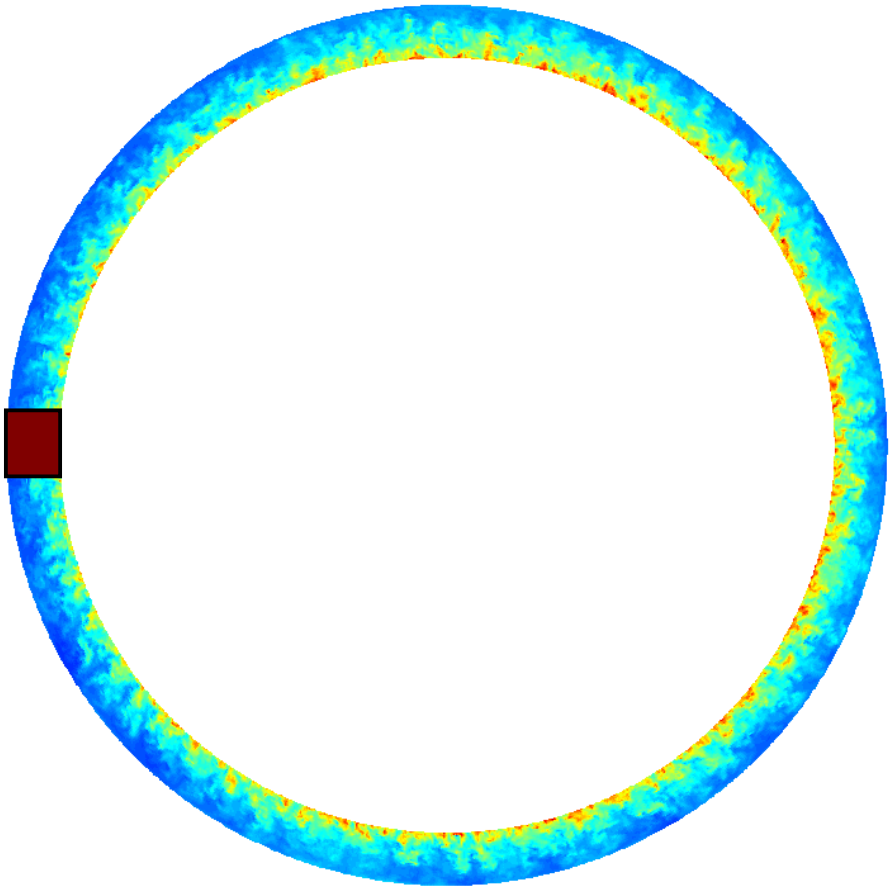


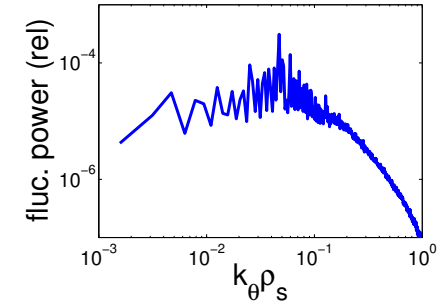

How do we develop 1st principles understanding of SOL dynamics?

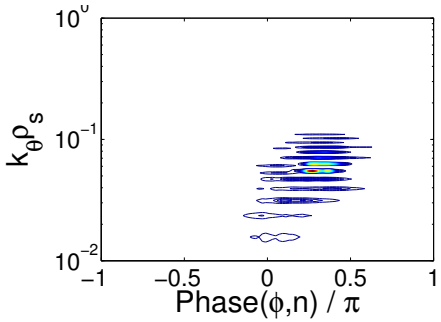
# Simple problem: inner wall limited (pol. ×-section)

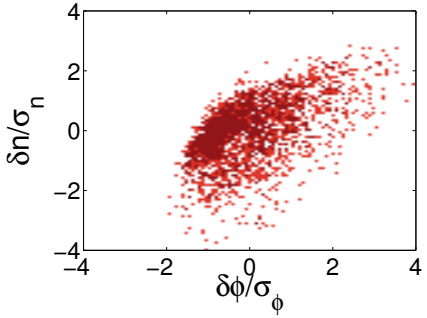


# Ballooning turbulence with $k_\theta \rho_s \approx 0.1 \sim 1\text{cm}^{-1}$

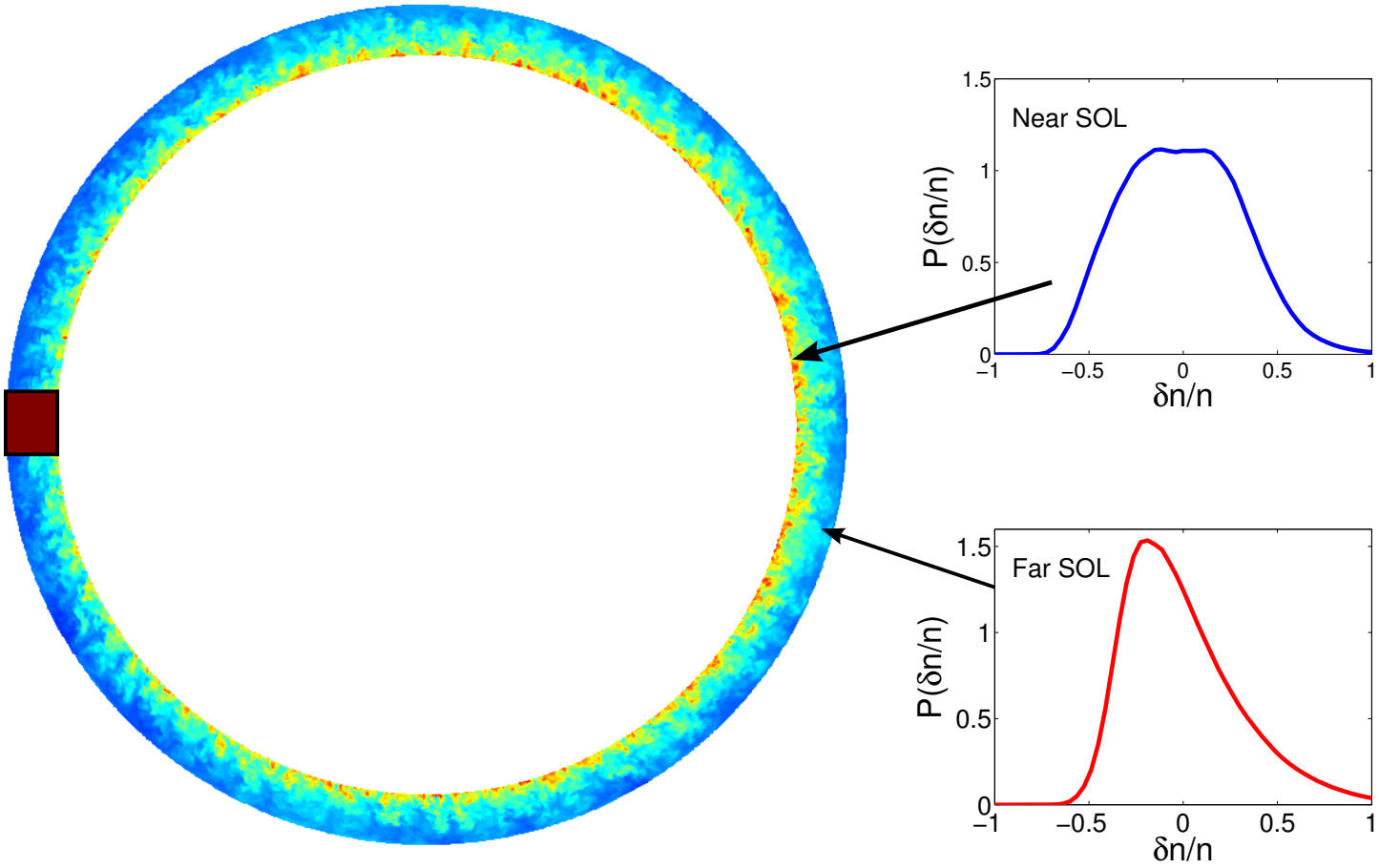




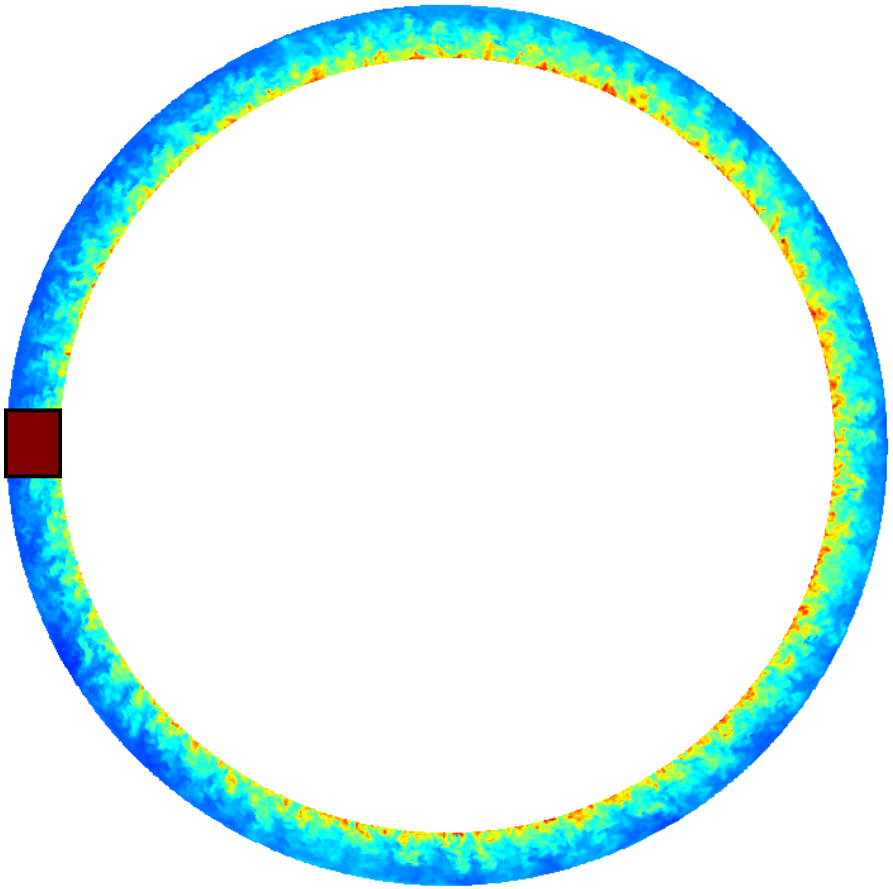



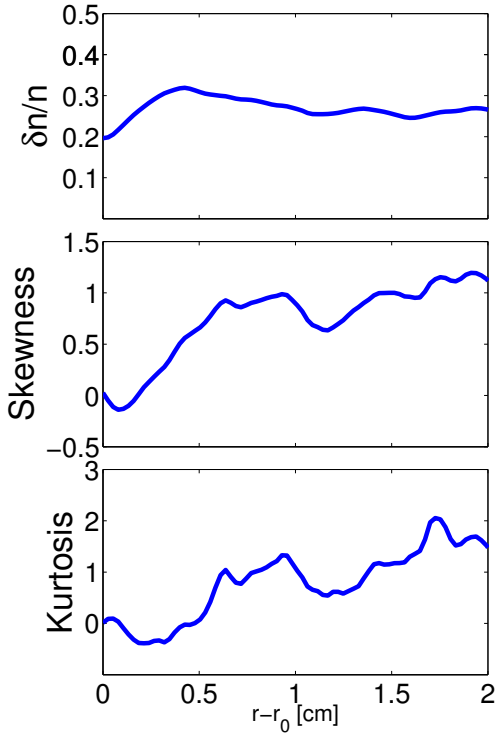



# Gaussian in near SOL, intermittent in far SOL

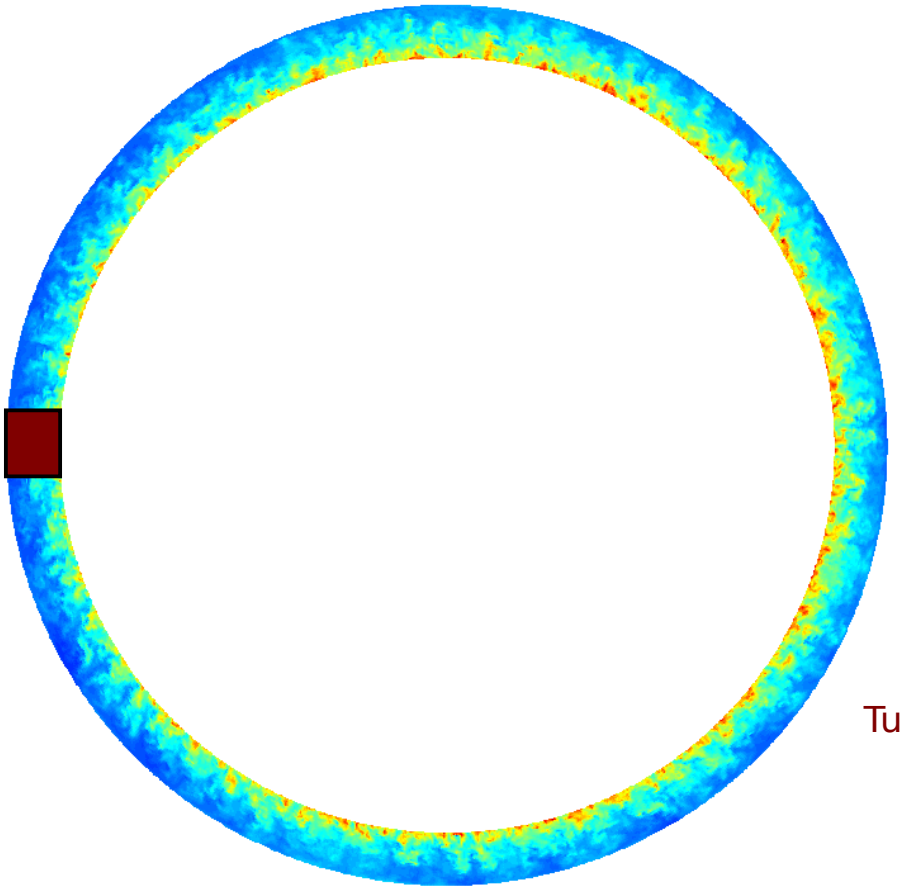


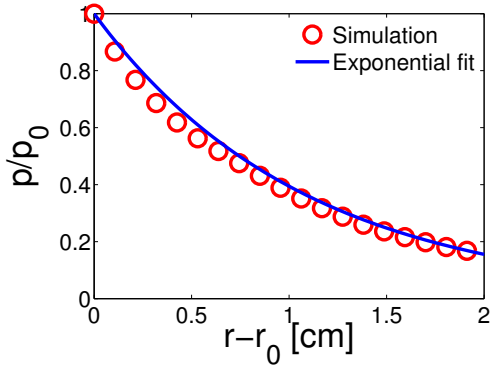
# Fluctuation level $\mathcal{O}(1)$, skewed PDF





# Power balance → exponentially decaying profiles






$$\nabla \cdot \Gamma_\perp + \nabla_\parallel \cdot \Gamma_\parallel = 0$$

Turbulence

Sonic flows towards PFCs

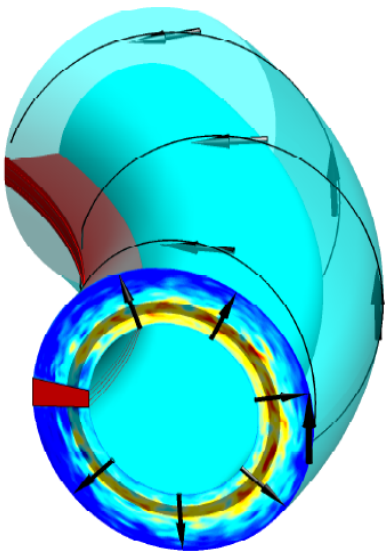
# Some of the questions that must be addressed...

- ✓ What mechanism sets the turbulence levels?
- ✓ What instability drives the perpendicular transport?
- ✓ What is the qualitative effect of finite $T_i$?
- ✓ How does the SOL width change with parameters?
- ✓ Can we reconcile theory, simulations, and experiments?
- ✓ What are the effects of neutrals? [C. Wersal, P-22 Thursday]
- ✓ ~~How is toroidal rotation generated in the SOL?~~ [Loizu, PoP 2014]
- × ~~Is SOL transport related to the density limit?~~ [LaBombard, NF 2005/08]
- × ~~How is the SOL coupled with the closed flux surface region?~~

# A tool to simulate SOL turbulence

Global Braginskii Solver (GBS) [Ricci, PPCF (2012)]

- Drift-reduced Braginskii equations $d/dt \ll \omega_{ci}$, $k_\perp^2 \gg k_\parallel^2$
- Evolves $n$, $\phi$, $V_{||e}$, $V_{||i}$, $T_e$, $T_i$ in 3D
- Global, flux-driven, no separation between equilibrium and fluctuations
- Power balance between plasma outflow from the core, turbulent transport, and parallel losses
- Scalable $\rho_\star$ up to medium size tokamak (e.g. TCV, C-Mod)



# Drift-reduced Braginskii equations to describe the SOL

$$\frac{\partial n}{\partial t} = -\frac{\rho_\star^{-1}}{B}[\phi, n] + \frac{2}{B}\left[nC(T_e) + T_e C(n) - nC(\phi)\right] - n\nabla_\parallel v_{\parallel e} - v_{\parallel e}\nabla_\parallel n$$

$$\frac{\partial \tilde{\omega}}{\partial t} = -\frac{\rho_\star^{-1}}{B}[\phi, \tilde{\omega}] - v_{\parallel i}\nabla_\parallel \tilde{\omega} + \frac{B^2}{n}\nabla_\parallel j_\parallel + \frac{2B}{n}C(p) + \frac{B}{3n}C(G_i), \quad \tilde{\omega} = \nabla_\perp^2(\phi + \tau T_i)$$

$$\frac{\partial}{\partial t}\left(v_{\parallel e} + \frac{m_i}{m_e}\frac{\beta_e}{2}\psi\right) = -\frac{\rho_\star^{-1}}{B}[\phi, v_{\parallel e}] - v_{\parallel e}\nabla_\parallel v_{\parallel e} + \frac{m_i}{m_e}\left[\nu j_\parallel/n + \nabla_\parallel \phi - \frac{\nabla_\parallel p_e}{n} - 0.71\nabla_\parallel T_e - \frac{2}{3n}\nabla_\parallel G_e\right]$$

$$\frac{\partial v_{\parallel i}}{\partial t} = -\frac{\rho_\star^{-1}}{B}[\phi, v_{\parallel i}] - v_{\parallel i}\nabla_\parallel v_{\parallel i} - \frac{2}{3}\nabla_\parallel G_i - \frac{1}{n}\nabla_\parallel p$$

$$\frac{\partial T_e}{\partial t} = -\frac{\rho_\star^{-1}}{B}[\phi, T_e] - v_{\parallel e}\nabla_\parallel T_e + \frac{4}{3}\frac{T_e}{B}\left[\frac{7}{2}C(T_e) + \frac{T_e}{n}C(n) - C(\phi)\right] +$$

$$+\frac{2}{3}\left\{T_e\left[0.71\nabla_\parallel v_{\parallel i} - 1.71\nabla_\parallel v_{\parallel e}\right] + 0.71\,T_e(v_{\parallel i} - v_{\parallel e})\frac{\nabla_\parallel n}{n}\right\} + \mathcal{D}_{T_e}^{\parallel}(T_e)$$

$$\frac{\partial T_i}{\partial t} = -\frac{\rho_\star^{-1}}{B}[\phi, T_i] - v_{\parallel i}\nabla_\parallel T_i + \frac{4}{3}\frac{T_i}{B}\left[C(T_e) + \frac{T_e}{n}C(n) - C(\phi)\right] +$$

$$+\frac{2}{3}T_i\left(v_{\parallel i} - v_{\parallel e}\right)\frac{\nabla_\parallel n}{n} - \frac{2}{3}T_i\nabla_\parallel v_{\parallel e} - \frac{10}{3}\frac{T_i}{B}C(T_i) + \mathcal{D}_{T_i}^{\parallel}(T_i)$$

+Sheath BCs consistent with PIC simulations [Loizu, PoP (2012)]

# Parameters, normalizations, coordinates

- Coordinate system: $(\theta, r, \varphi) \rightarrow (poloidal, radial, toroidal)$
- Equations expressed in normalized units:
  - $L_\perp \rightarrow \rho_s$
  - $v \rightarrow c_s$
  - $L_\parallel \rightarrow R$
  - $t \sim \gamma^{-1} \rightarrow R/c_s$
- The dimensionless code parameters are as follows:
  - $\rho_\star = \rho_s/R$
  - $\beta_e = 2\mu_0 p_e/B^2$
  - $\nu = e^2 nR/(m_i \sigma_\parallel c_s)$
  - $q \approx (r/R)B_\varphi/B_\theta$
- Simplified notation in analytical expressions:
  - $p_0 = \langle p \rangle_t,\ t \gg \gamma^{-1}$
  - $L_p = -\langle p/\partial_r p \rangle_t$

# Poloidal cross sections showing SOL turbulence

Play

Play

# Modes saturate due to pressure non-linearity

We observe in simulations [Ricci, PoP (2013)]:

- Mode saturation caused by local pressure non-linearity

$$\boxed{\partial_r p_1 \sim \partial_r p_0 \rightarrow \frac{p_1}{p_0} \sim \frac{\sigma_r}{L_p}}$$

- Radial eddy length is mesoscopic [Ricci, PRL (2008)]

$$\boxed{\sigma_r \approx \sqrt{L_p / k_\theta}}$$

- Turbulent flux dominated by radial $\mathbf{E} \times \mathbf{B}$ convection

$$\boxed{\Gamma_1 = \rho_\star^{-1} \left\langle p_1 \frac{\partial \phi_1}{\partial \theta} \right\rangle}$$

# Saturation model yields $\mathbf{E} \times \mathbf{B}$ turbulent flux

Gradient removal hypothesis

$$\frac{p_1}{p_0} \approx \frac{\sigma_r}{L_p}$$

$$\partial_t p = -\rho_\star^{-1} [\phi, p]$$

$$\partial_\theta \phi_1 = \gamma \left(p_1/p_0\right) \left(\rho_\star L_p\right)$$

$$\Gamma_1 \approx \rho_\star^{-1} \langle p_1 \partial_\theta \phi_1 \rangle$$

$$\boxed{\Gamma_1 \sim p_0 \left(\frac{\gamma}{k_\theta}\right)_{\max}}$$

# Self-consistent prediction of pressure gradient length

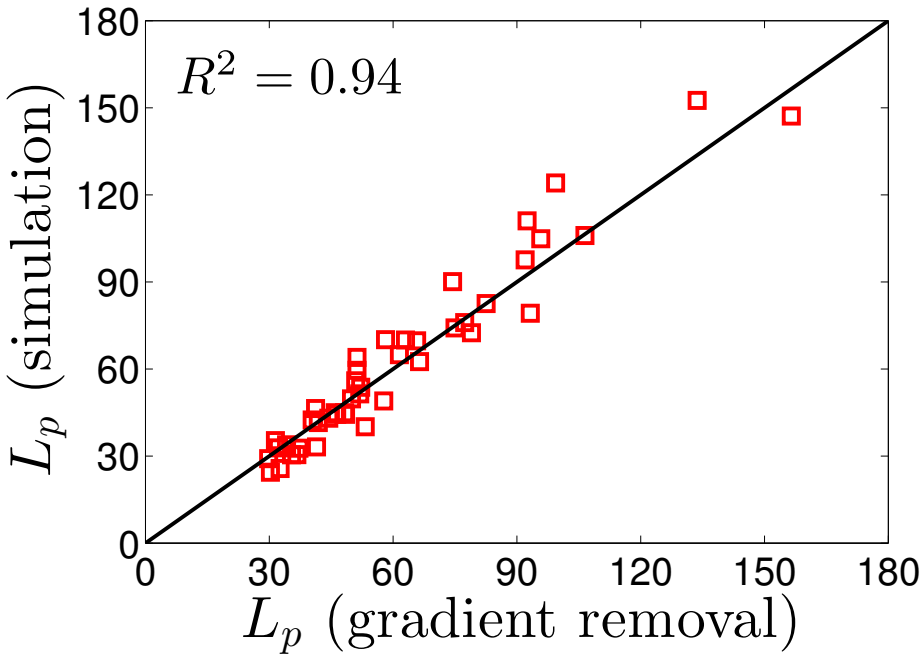
In steady state, $\nabla \cdot \Gamma_1$ balances parallel losses $\sim \nabla_{\|} \cdot (p v_{\|e})$, hence

$$\boxed{L_p \approx \frac{q}{c_s} \left( \frac{\gamma}{k_\theta} \right)_{\max}}$$

- Results in iterative scheme to <u>predict</u> $L_p$ self-consistently:
  - Compute $\gamma = f(\underbrace{L_p}_{vary}, \underbrace{k_\theta}_{scan}, \underbrace{\rho_\star, q, \nu, \hat{s}, m_i/m_e}_{plasma\ parameters})$
  - Vary $L_p$ until LHS = RHS using secant method

# Excellent agreement between theory and simulations

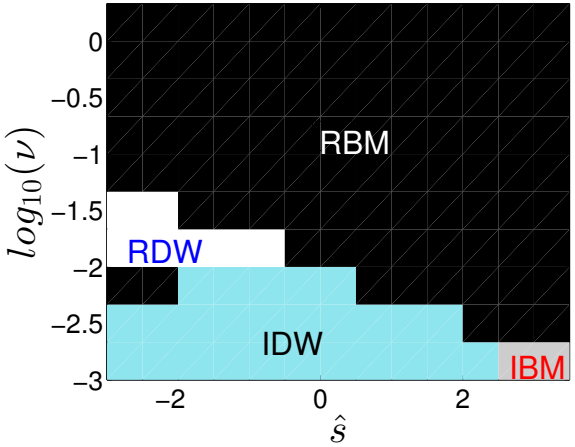
$L_p$ predicted using self-consistent procedure [Halpern, NF (2014)]




GBS sims.: $\rho_\star^{-1} = 500$–$2000$, $q = 3$–$6$, $\nu = 0.01$–$1$, $\beta = 0$–$3 \times 10^{-3}$

# Dominant instability depends principally on $q$, $\nu$, $\hat{s}$, $T_i/T_e$

- Build instability parameter space using reduced models
  $\rightarrow$ gradient removal theory, linear dispersion relations
- Verify results using GBS non-linear simulations [Mosetto, PoP (2013)]




- Which instability drives $\perp$ transport?
  - Inertial/Resistive Ballooning modes/Drift Waves?

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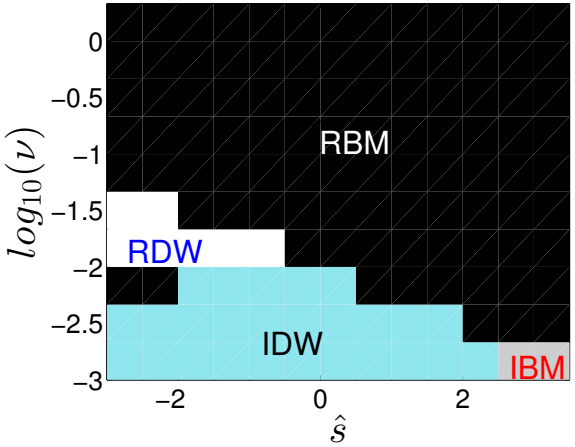


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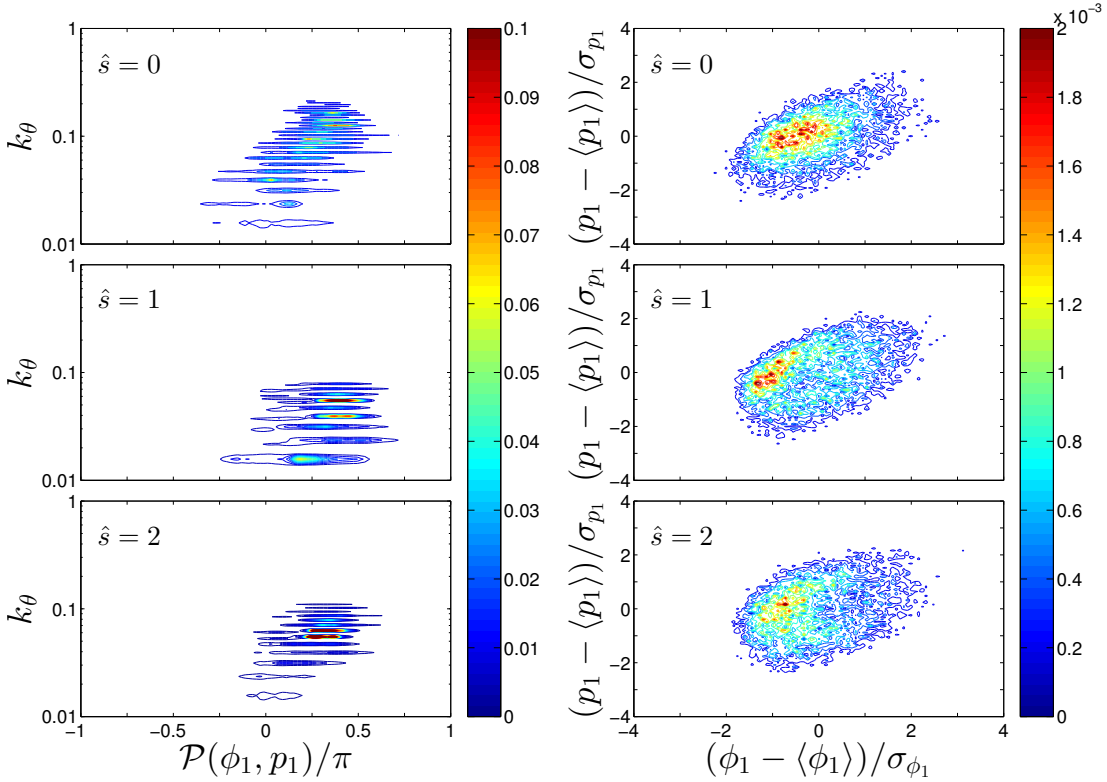
  - ~~Inertial~~/Resistive Ballooning modes/~~Drift Waves~~?

# Presence of RBMs verified in TCV SOL sims

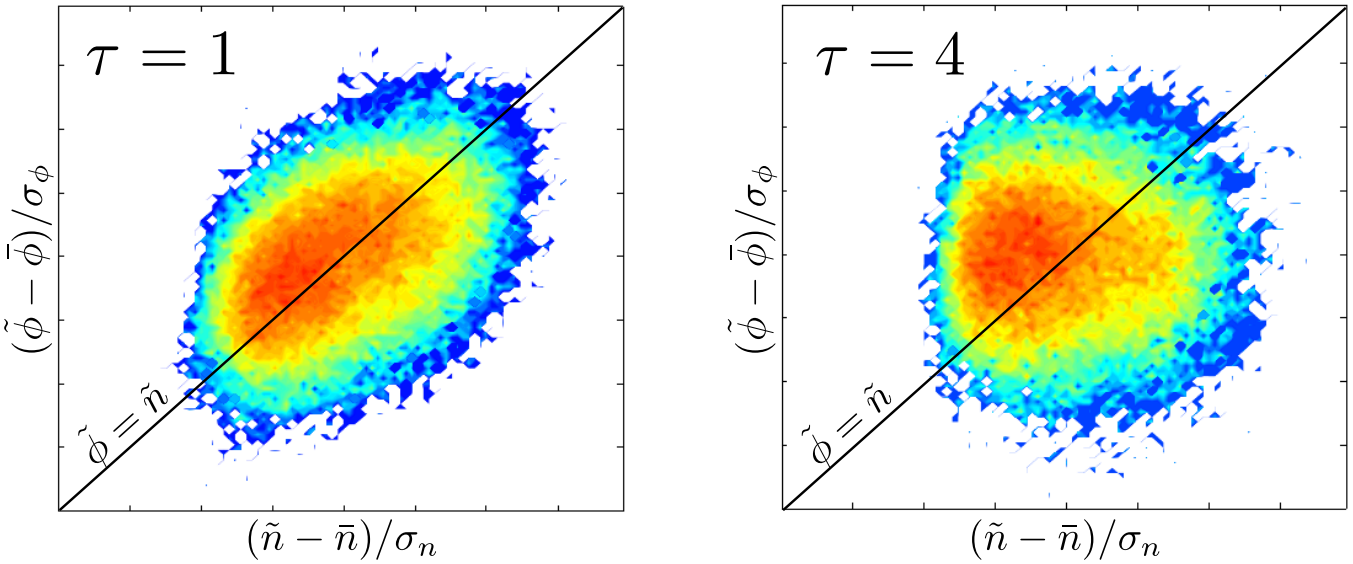
- $(\tilde{n}, \tilde{\phi})$ phase difference, joint $(\tilde{n}, \tilde{\phi})$ pdf [Halpern, NF (2014)]




Curvature-driven, non-adiabatic mode → RBMs

# Addition of finite $T_i$ weakens adiabatic coupling

- Analysis extended to include $T_i$ effects [Mosetto, PoP (submitted)]
- Joint $(\tilde{n}, \tilde{\phi})$ pdf in GBS sims with $\tau = 1$, $\tau = 4$




RBM component is enhanced by finite $T_i$

# SOL width in RBM regime scales with $\rho_\star$, $q$

- SOL width obtained analytically with RBMs [Halpern, NF 2013/14]:

$$L_p = q\left(\frac{\gamma}{k_\theta}\right)_{\max}$$

$$\gamma_b = \sqrt{2/(\rho_\star L_p)}$$

$$k_b = \sqrt{(1-\alpha)/(\nu\gamma_b)}/q$$

- Our simple model leads to a dimensionless scaling:

$$L_p = \left[2\pi\rho_\star\,(1-\alpha)^{1/2}\,\alpha_d/q\right]^{-1/2}$$

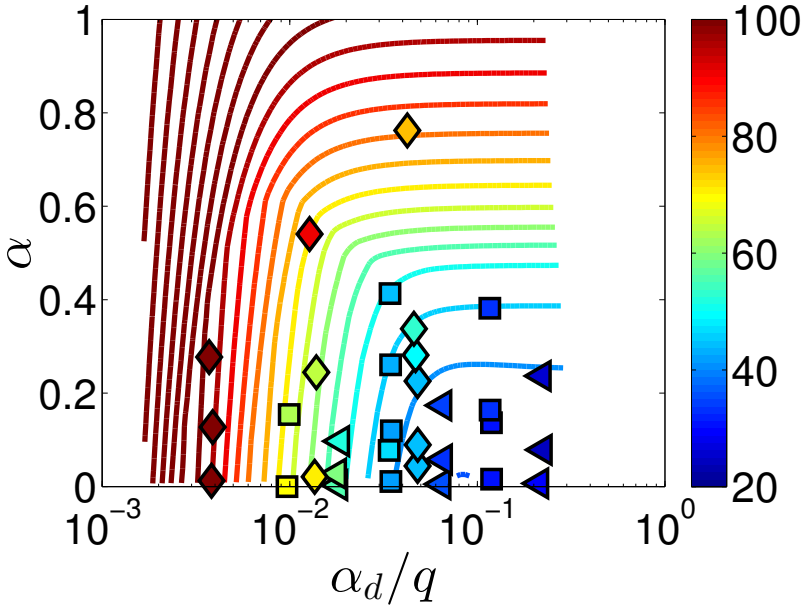
Machine size

$\alpha = q^2\beta/(\rho_\star L_p)$
Electromagnetic effects

$\alpha_d = \nu^{-1/2}(\rho_\star L_p)^{1/4}/q$
Collisionality vs connection length

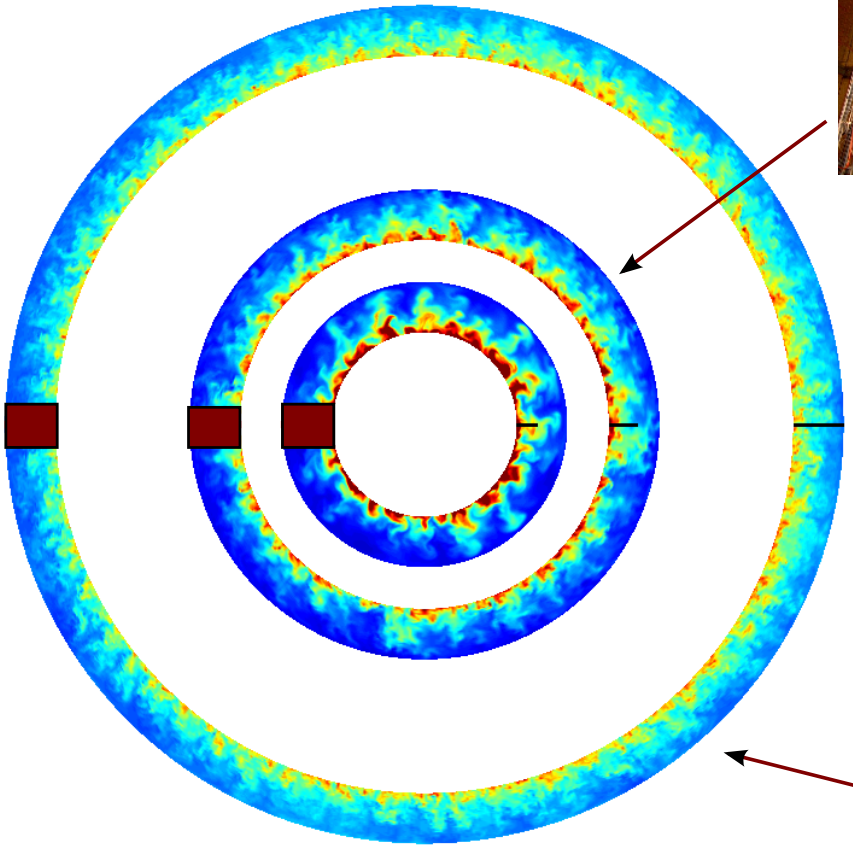
# Parallel dynamics physics in agreement with simulations

- Verify saturated RBM theory with GBS EM simulations
  - $\rho_\star^{-1} = 500$, $\beta_e = 0\text{–}3 \times 10^{-3}$, $\nu = 0.01\text{–}1$, $q = 3, 4, 6$

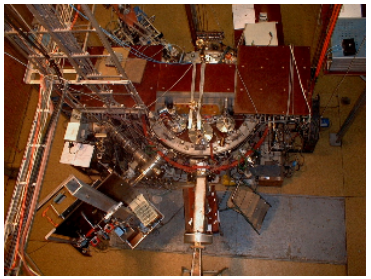



(Contours of $L_p$ given by theory, symbols are GBS simulations)

# GBS simulations confirm size-scaling up to TCV size

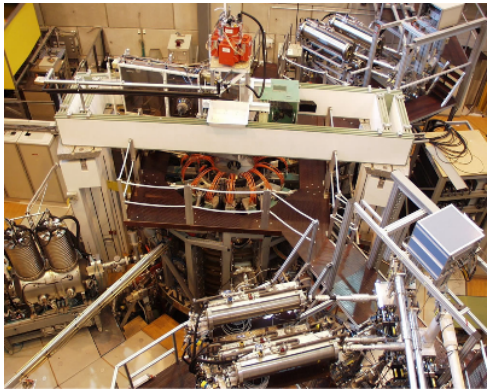


CASTOR



$\rho_\star^{-1} \approx 1000$

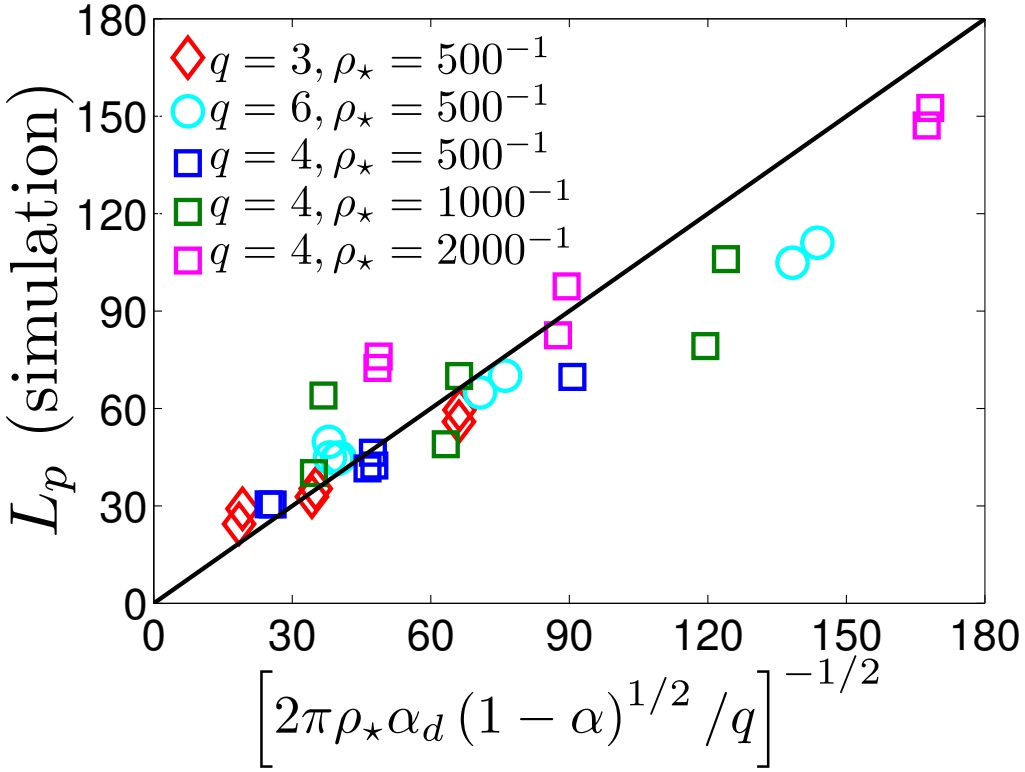
TCV



$\rho_\star^{-1} \approx 2000$

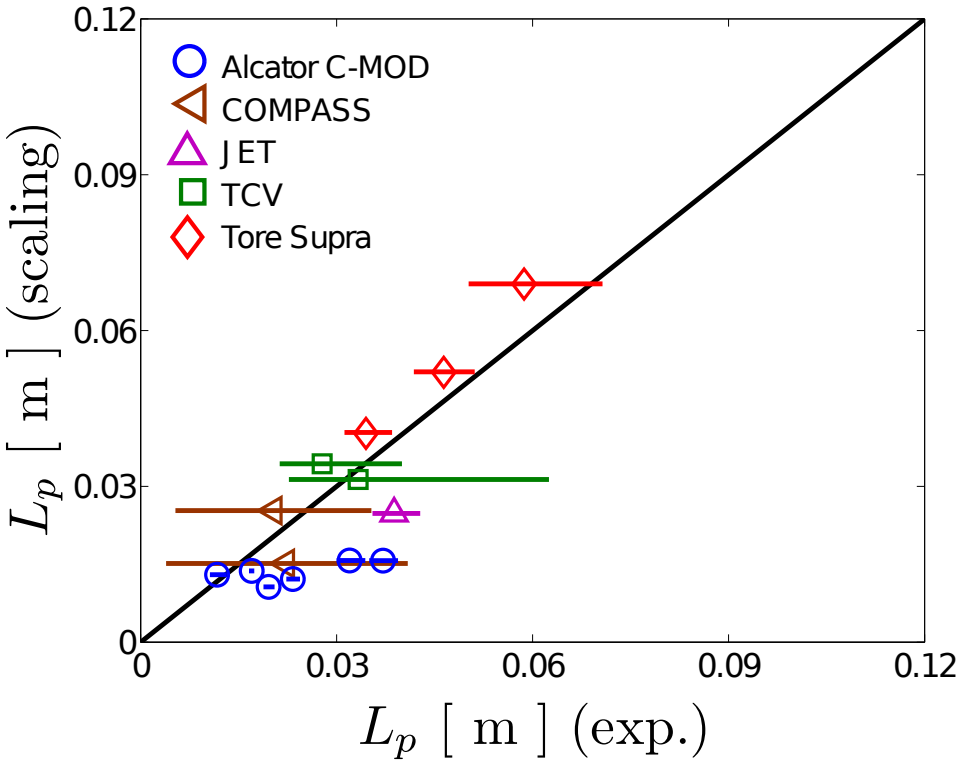
# Dimensionless scaling follows GBS simulation data

Comparison carried out over wide range of parameters ($\rho_\star$, $q$, $\beta$, $\nu$)



# Good agreement with SOL width measurements

$$L_p \approx 7.2 \times 10^{-8} q^{8/7} R^{5/7} B_\phi^{-4/7} T_{e0}^{-2/7} n_{e0}^{2/7} (1 + T_i/T_e)^{1/7} \ [\ \mathrm{m}\ ]$$




Exp. data:

*G. Arnoux*

*I. Furno*

*J.P. Gunn*

*J. Horacek*

*M. Kočan*

*B. Labit*

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*C. Silva*

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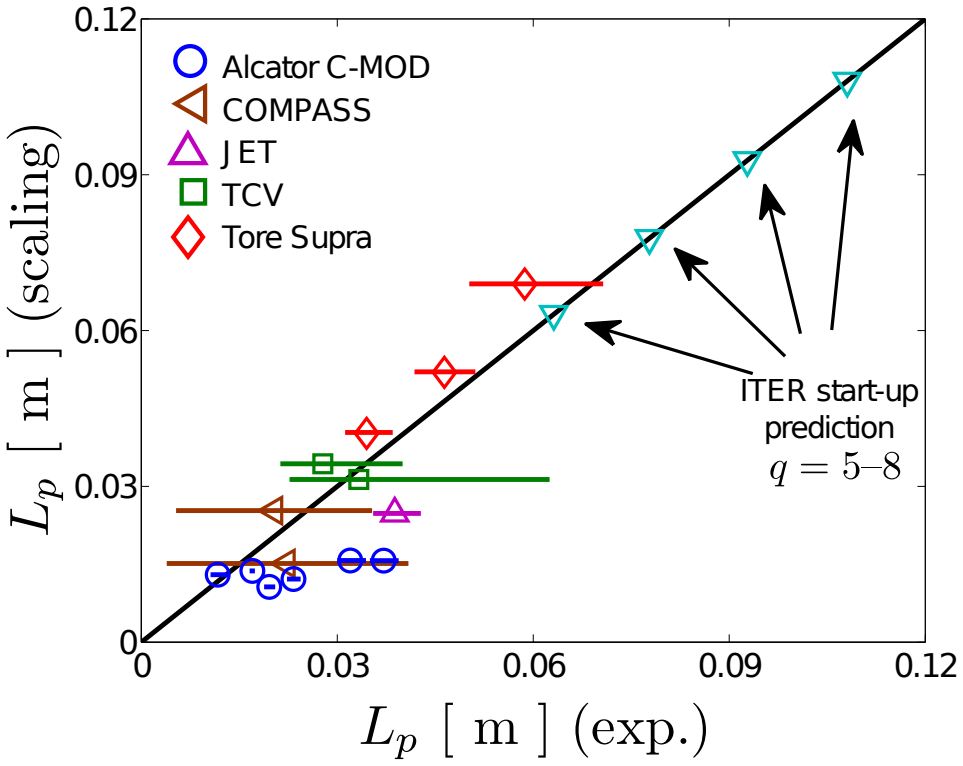


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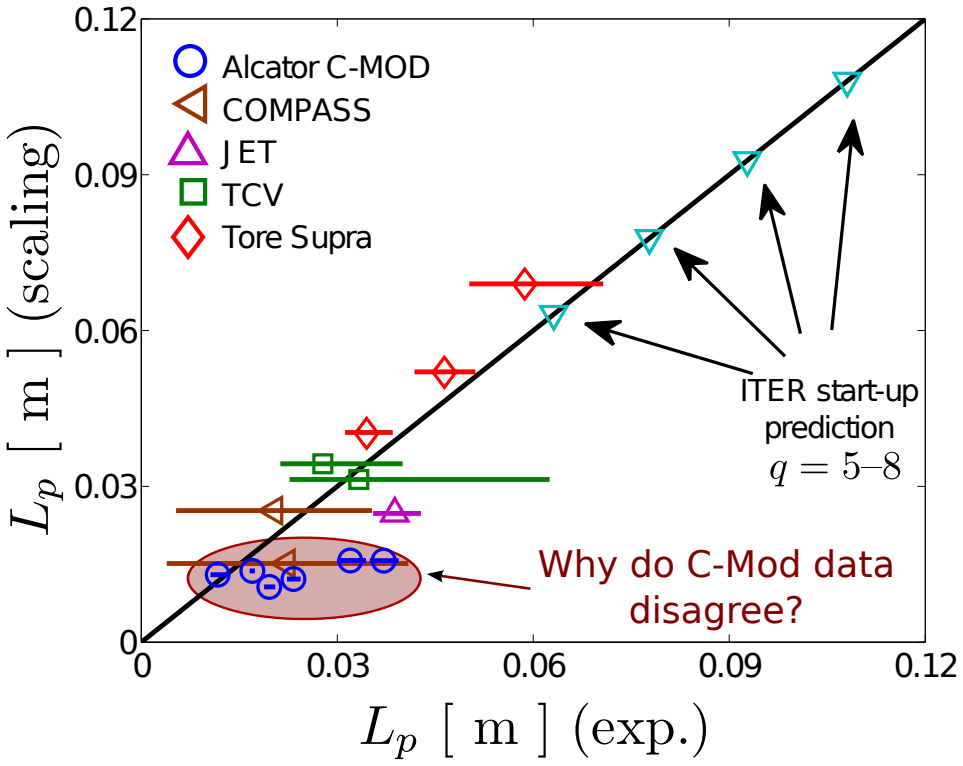


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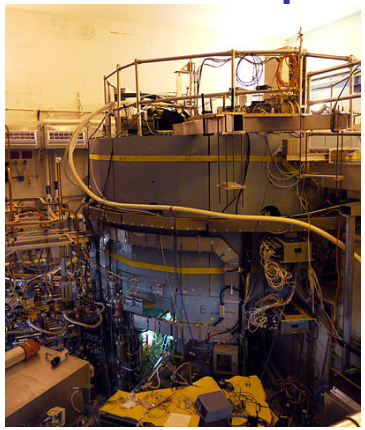
# Intermission

- We discussed a theory describing SOL turbulent dynamics
  - ✓ Turbulent saturation mechanism
  - ✓ Non-linear instability driving $\perp$ transport
  - ✓ SOL width scaling with plasma parameters
  - ✓ Verified with non-linear simulations
  - ✓ Compared against data from several machines
  - × Some experimental data disagrees with theory

Carry out detailed comparison with these experiments

# An ideal testbed for simulation-experiment comparison

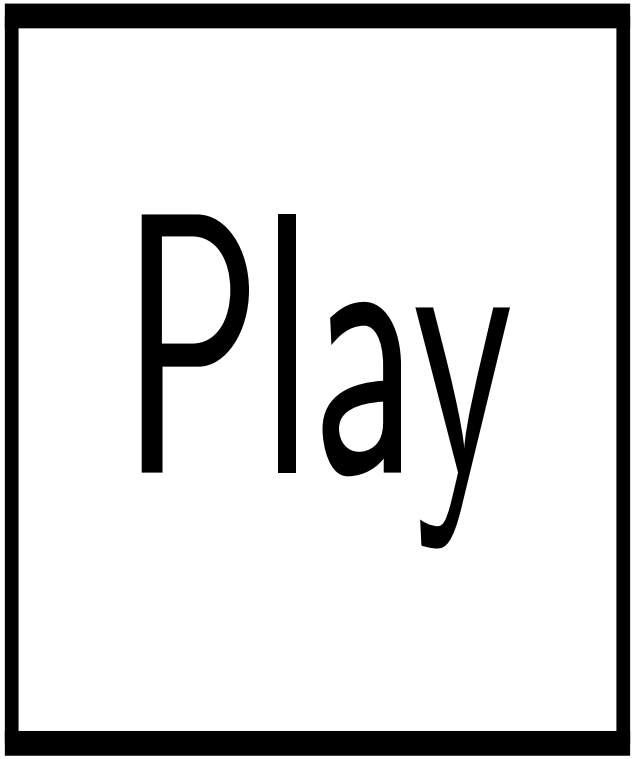


- Inner-wall limited Ohmic C-Mod discharges [Zweben, PoP (2009)]
- $R = 0.67$m, $a = 0.20$m, $B = 2.7, 3.8$T, $\kappa = 1.2$
- Density scan at each value of $B$
- Characterize C-Mod SOL turbulence using GPI diagnostic, and compare with GBS results
  - Low $\beta$, no $T_i$ or $\tilde{B}$ diagnostics $\rightarrow$ simple electrostatic, cold ion model
  - $\delta D_\alpha D_\alpha$, pdf moments ,$\tau_{auto}$, $L_r$, $L_\theta$, $v_r$, $v_\theta$, $\mathcal{P}(k_\theta)$, $\mathcal{P}(\omega)$

Very stringent test!

# Gas-puff imaging of C-Mod SOL

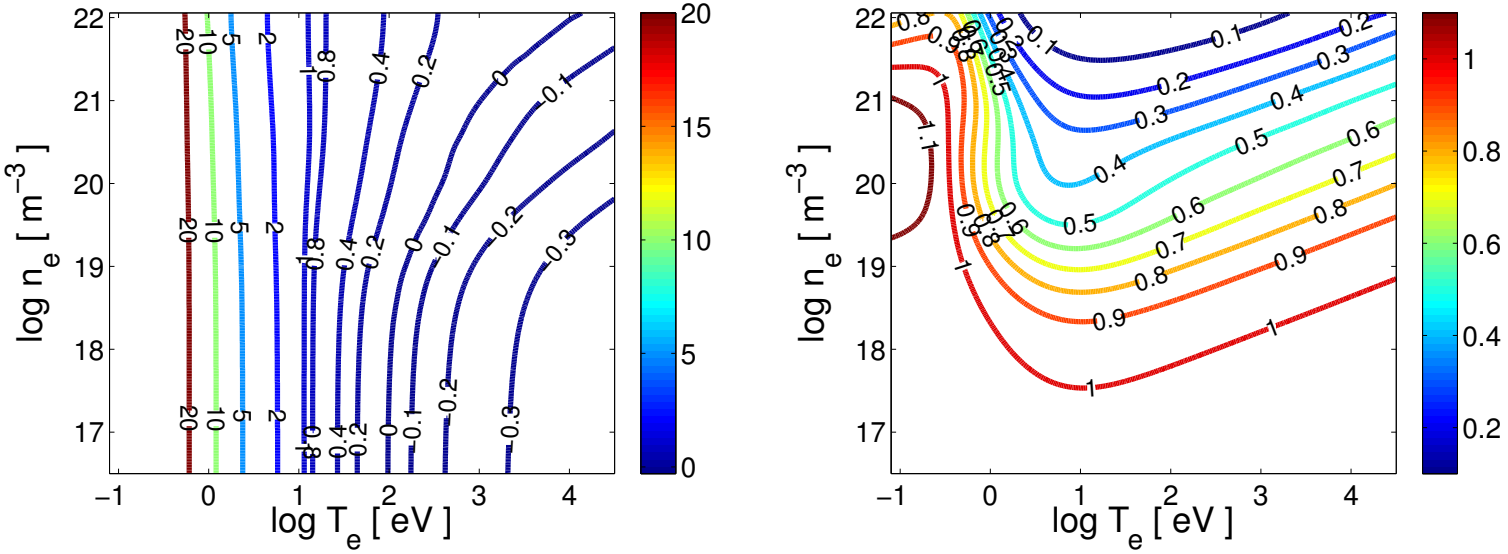
Phantom 710 high-speed camera at 400'000fps [S.Zweben, J.Terry]



# $\delta D_\alpha / D_\alpha$ diagnostic for GBS

Using DEGAS modeling of GPI emissivity, model $D_\alpha$ fluctuations

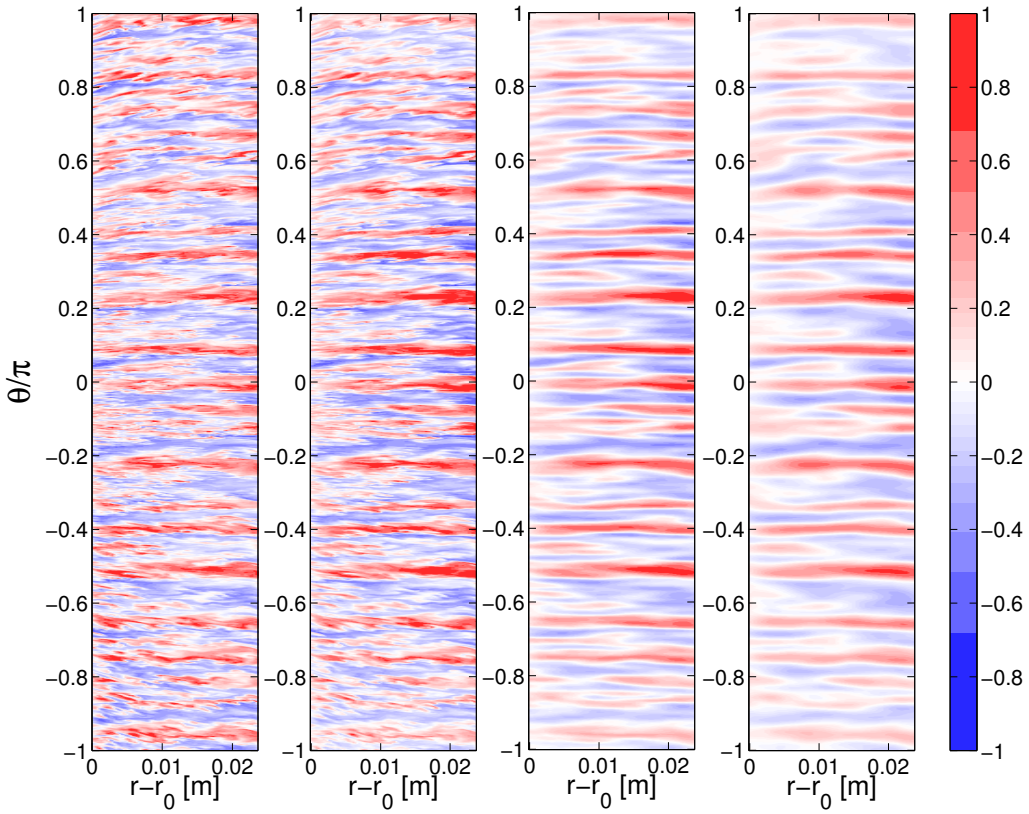
- Emissivity locally parametrized as $E \propto T_e^\alpha n_e^\beta$, use H656 line
- Fluctuations modelled as $\delta D_\alpha / D_\alpha \approx \alpha(T_e, n_e)\tilde{T}_e + \beta(T_e, n_e)\tilde{n}$




- Simulate finite GPI resolution ($3 \times 3$mm $+$ 2.5$\mu$s smoothing), B-field tilt respect to sensors (8mm poloidal smoothing)

# $\delta D_\alpha / D_\alpha$ synthetic diagnostic results

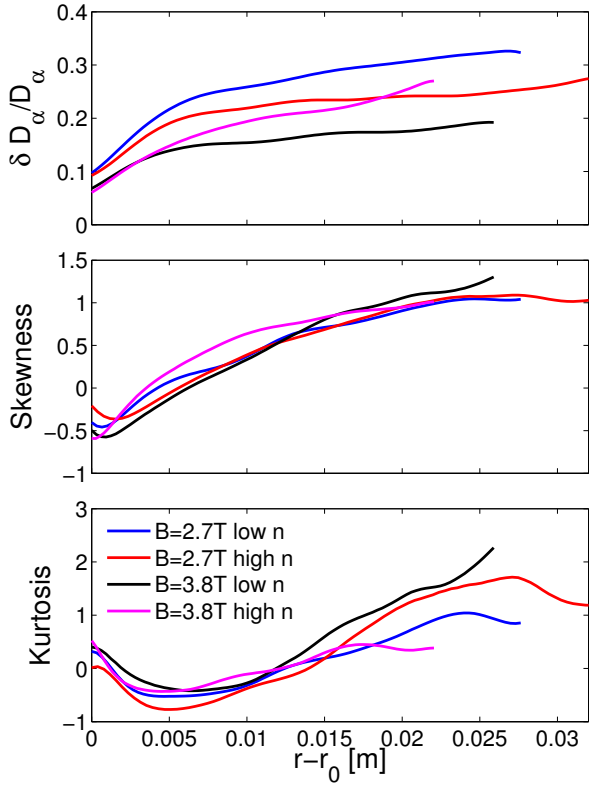
- Left to right: $\tilde{n}$, $\delta D_\alpha / D_\alpha$, $\delta D_\alpha / D_\alpha$ (diode), $\delta D_\alpha / D_\alpha$ (full)




High $k_\theta$ modes strongly damped by smoothing

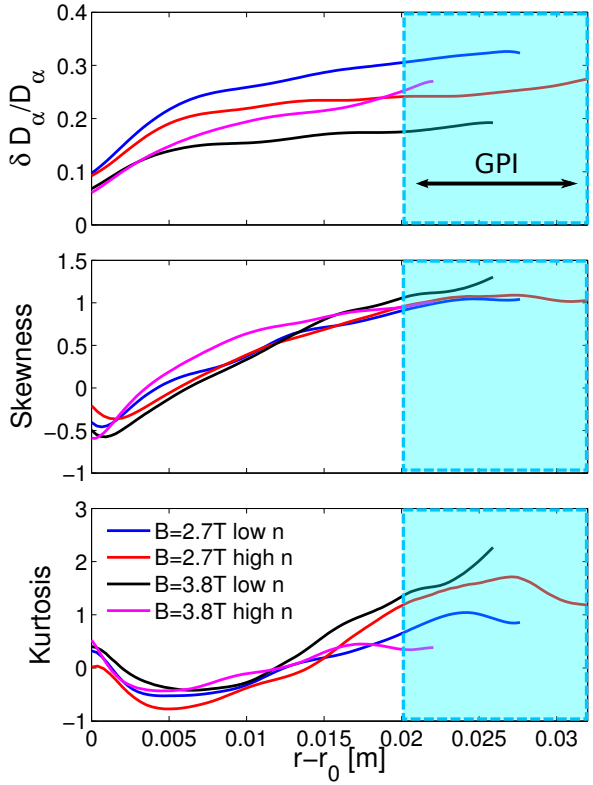
# Large $\delta D_\alpha / D_\alpha$ fluctuations, skewed PDF

- $\delta D_\alpha / D_\alpha$ level increases with SOL, $\sim 30\%$ in far SOL
- Skewness $\sim 1 \rightarrow$ blobs (?)
- Moment profiles robust with plasma parameters



# Large $\delta D_\alpha / D_\alpha$ fluctuations, skewed PDF

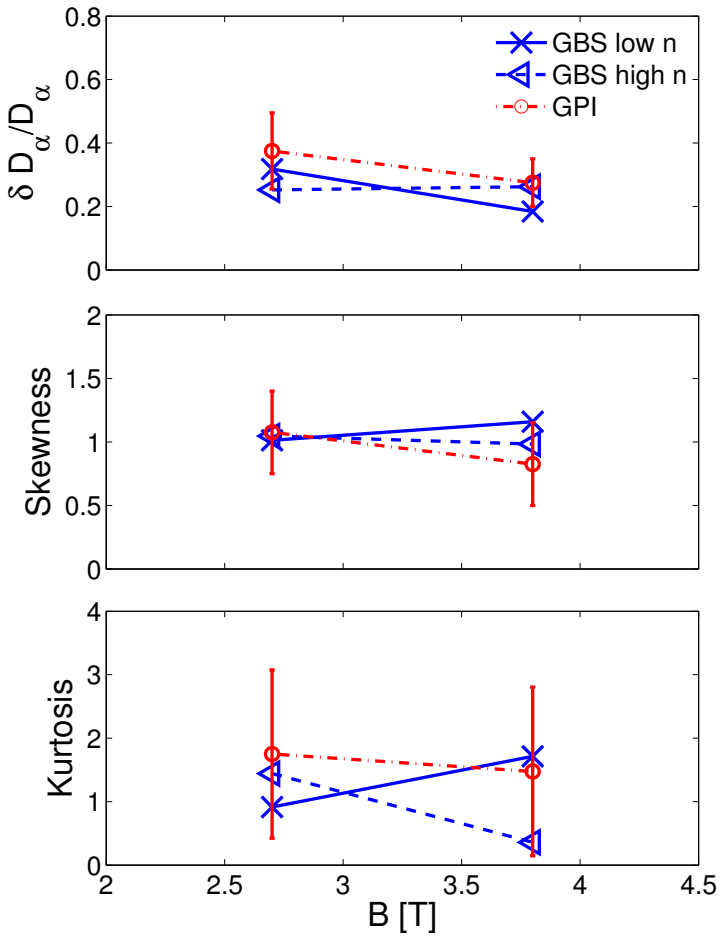
- $\delta D_\alpha / D_\alpha$ level increases with SOL, $\sim 30\%$ in far SOL
- Skewness $\sim 1 \rightarrow$ blobs (?)
- Moment profiles robust with plasma parameters




Quantitative comparison using shaded area (GPI sensors)

# GBS agrees with [Zweben PoP 2009] within error bars

- Compare GBS radial/poloidal average against GPI data
- Shot-to-shot variation indicated with error bars
- GBS gives good match for $\delta D_\alpha / D_\alpha$ and higher moments
- Previous gyrofluid simulations gave $\delta D_\alpha / D_\alpha \approx 5$–$10\%$



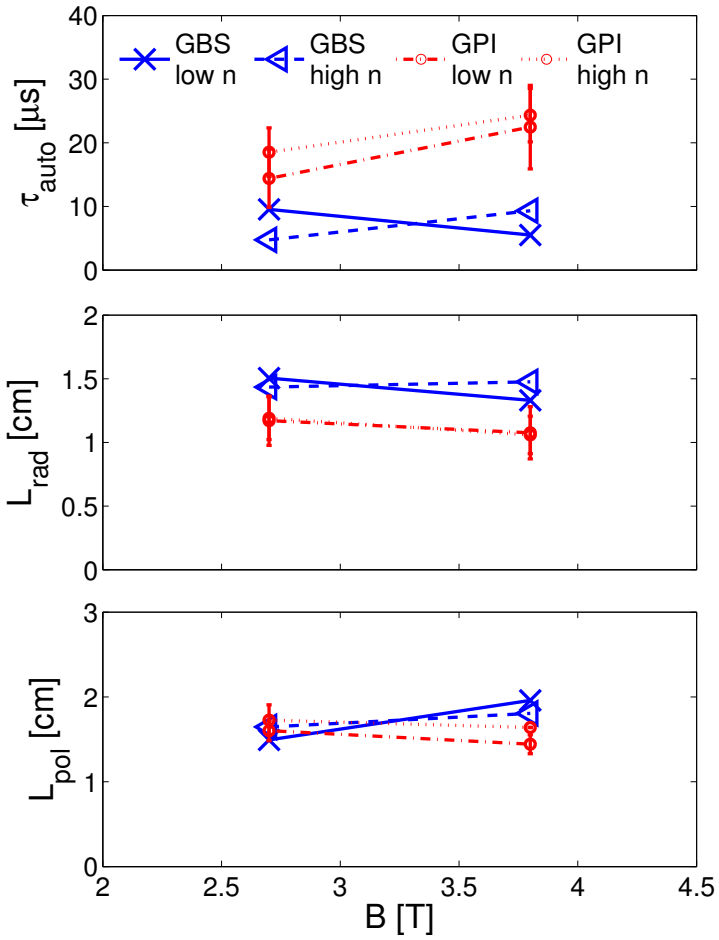
# Typical spatial, temporal turbulent scales give reasonable agreement

- Compute $\tau_{auto}$, $L_{rad}$, $L_{pol}$ using 2 point correlations functions $C_{ij}$

$$C_{ii}(\tau_{auto}) = \frac{1}{2}$$

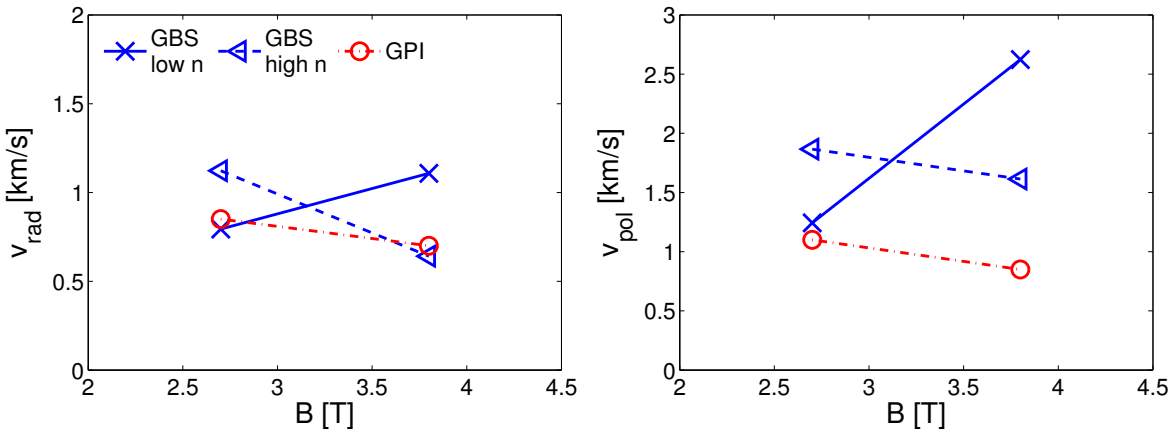
$$L = 1.66 \frac{\delta x}{\sqrt{-\ln C_{ij}(t=0)}}$$

- Good match for $L \sim 1.5$cm, $\tau_{auto}$ underpredicted by $\sim$2



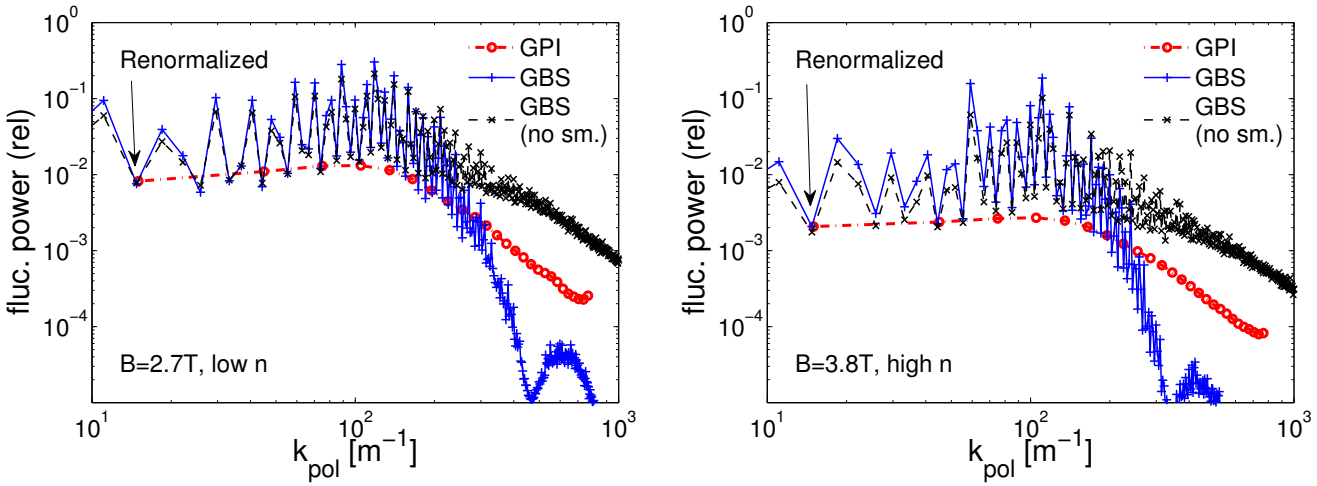
# Propagation velocities

- Obtain $v_{rad}$, $v_{pol}$ from time lag that maximizes correlation between two neighboring points separated by $\delta_x \rightarrow v = \delta_x/\tau$
- Good agreement in $v_{rad} \rightarrow$ poloidal mode structure
- Large mismatch in $v_{pol} \rightarrow$ resolution smoothing in GBS data?



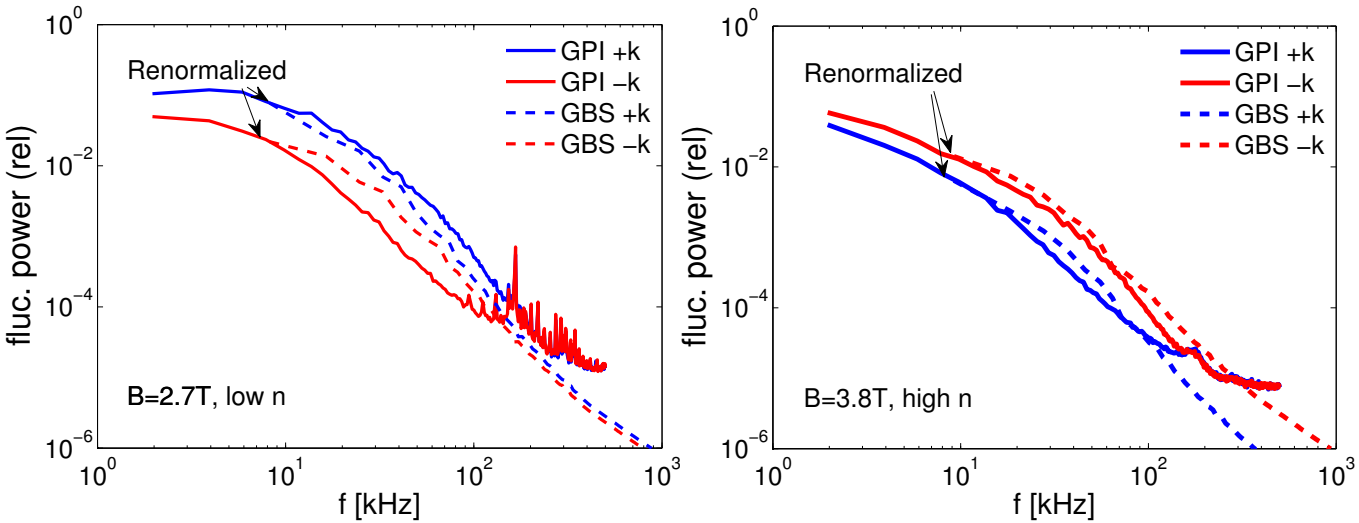
# Spectral power vs wavenumber of $\delta D_\alpha / D_\alpha$

- From FFT of $\delta D_\alpha / D_\alpha$ in $\theta$, then average over $r$, $t$
- Significant drop at $k_{pol} = 125\text{m}^{-1}$ high $k$ due to smoothing
- Unsmoothed $\delta D_\alpha / D_\alpha$ has same power law scaling as GPI



# Spectral power vs frequency of $\delta D_\alpha / D_\alpha$

- From FFT of $\delta D_\alpha / D_\alpha$ in $t$, then average over $t$, $r = 2 \pm 0.2$cm
- GPI measurements and GBS show same asymptotic behavior

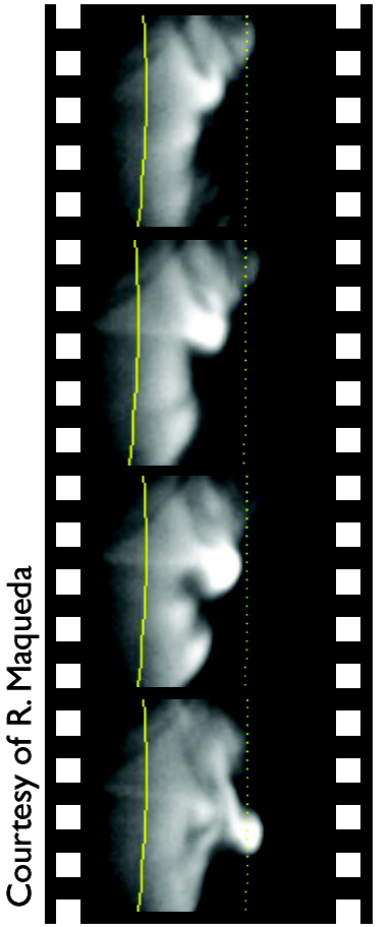


# Summary and outlook

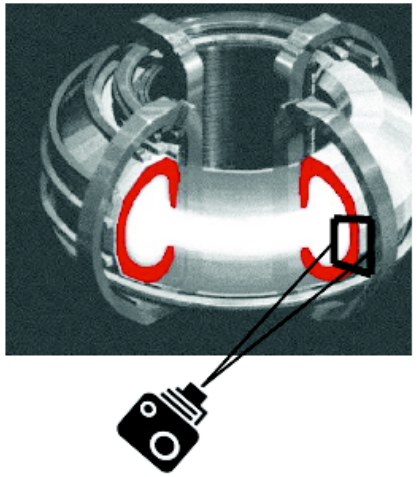
- Towards first principles understanding of SOL width:
  - ✓ Non-linearly saturated RBMs, enhanced with $T_i$ effects
  - ✓ SOL width scales with $\rho_\star$, $q$, collisionality
  - ✓ Simple analytical scaling agrees with experimental data
- Detailed comparison between GBS and C-Mod discharges
  - ✓ $L_p$, $\delta D_\alpha / D_\alpha$ pdf moments, $L_{rad}$, $L_{pol}$, $v_{rad}$, $\mathcal{P}(\omega)$, $\mathcal{P}(k_{pol})$
  - × $\tau_{auto}$, $v_{pol}$ $\rightarrow$ under/overpredicted by factor $\sim 2$
- Next: 2 $L_p$'s profile structure using 2014 C-Mod discharges
  - More advanced simulation model $\rightarrow$ $T_i$, shaping
  - Mirror langmuir probe $\rightarrow$ high res. profiles, $(n, \phi)$ phase

# Thank you for your attention!

# Properties of the SOL



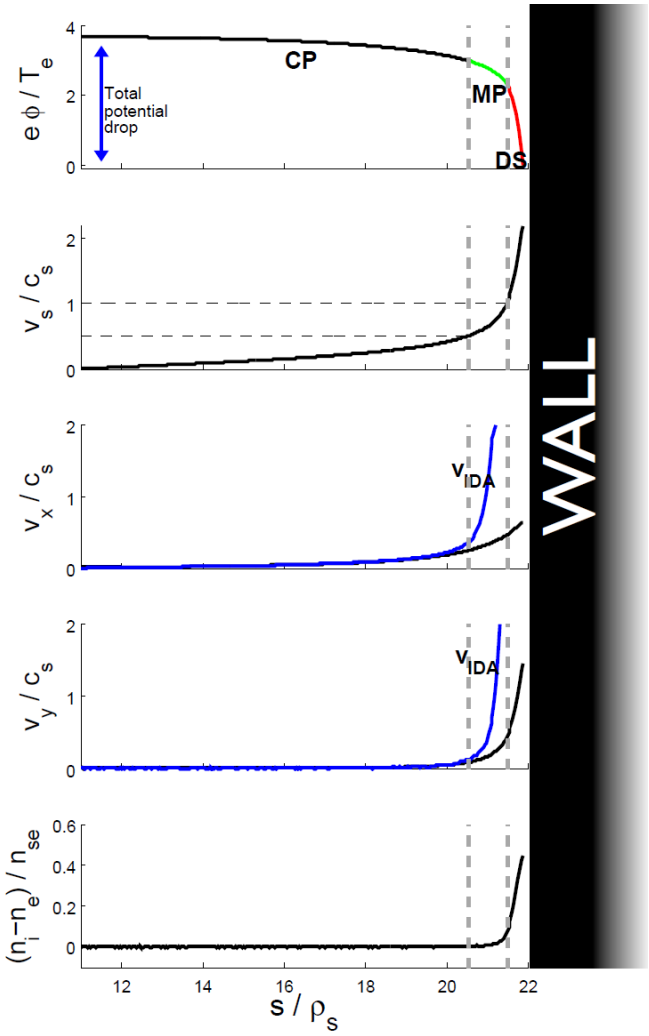
Courtesy of R. Maqueda



- $L_{fluc} \sim \langle L \rangle_t$
- $n_{fluc} \sim \langle n \rangle_t$
- Collisional magnetized plasma
- Low frequency modes $\omega \ll \omega_{ci}$
- Open field lines

# Sheath BCs from kinetic approach [Loizu, PoP (2012)]

- COLLISIONAL PRESHEATH (**CP**)
  - Quasi-neutral, IDA holds
  - Potential drop $\sim 0.5\,T_e$ over $\sim$ L
  - Ions accelerated to $v_s = c_s \sin\alpha$
- MAGNETIC PRESHEATH (**MP**)
  - Quasi-neutral, IDA breaks
  - Potential drop $\sim 0.5\,T_e$ over $\sim \rho_s$
  - Ions accelerated to $v_s = c_s$
- DEBYE SHEATH (**DS**)
  - Non-neutral, IDA breaks
  - Potential drop $\sim 3\,T_e$ over $\sim 10\lambda_D$
  - Ions accelerated to $v_s > c_s$



# Extra slides: Summary of the BC

$$v_{||i} = c_s\left(1+\theta_n - \frac{1}{2}\theta_{T_e} - \frac{2\phi}{T_e}\theta_\phi\right)$$

$$v_{||e} = c_s\left(\exp\left(\Lambda - \eta_m\right) - \frac{2\phi}{T_e}\theta_\phi + 2(\theta_n + \theta_{T_e})\right)$$

$$\frac{\partial\phi}{\partial s} = -c_s\left(1+\theta_n + \frac{1}{2}\theta_{T_e}\right)\frac{\partial v_{||i}}{\partial s}$$

$$\frac{\partial n}{\partial s} = -\frac{n}{c_s}\left(1+\theta_n + \frac{1}{2}\theta_{T_e}\right)\frac{\partial v_{||i}}{\partial s}$$

$$\frac{\partial T_e}{\partial s} \simeq 0$$

$$\omega = -\cos^2\alpha\left[\left(1+\theta_{T_e}\right)\left(\frac{\partial v_{||i}}{\partial s}\right)^2 + c_s\left(1+\theta_n+\theta_{T_e}/2\right)\frac{\partial^2 v_{||i}}{\partial s^2}\right]$$

where $\theta_A = \frac{\rho_s}{2\tan\alpha}\frac{\partial_x A}{A}$, and $\eta_m = e(\phi_{mpe} - \phi_{wall})/T_e$. [Loizu et al PoP 2012]

# Resistive ballooning modes destabilized by EM effects

- Starting from reduced MHD, obtain simple dispersion relation

$$\gamma^2\left(\nu+\frac{\beta_{e0}}{2}\frac{\gamma}{k_\perp^2}\right)=2\frac{R}{L_p}\left(\nu+\frac{\beta_{e0}}{2}\frac{\gamma}{k_\perp^2}\right)-\frac{k_\parallel^2}{k_\perp^2}\gamma$$

- Neglecting ideal ballooning mode, the resistive branch gives

$$\left(\gamma^2-\gamma_b^2\right)k_\perp^2=-\gamma\left(\frac{1-\alpha}{q^2\nu}\right)$$

and we identify $\gamma\sim\gamma_b=\sqrt{2R/L_p}$ and $k_b\sim\sqrt{(1-\alpha)/(\nu\gamma_b)}/q$