

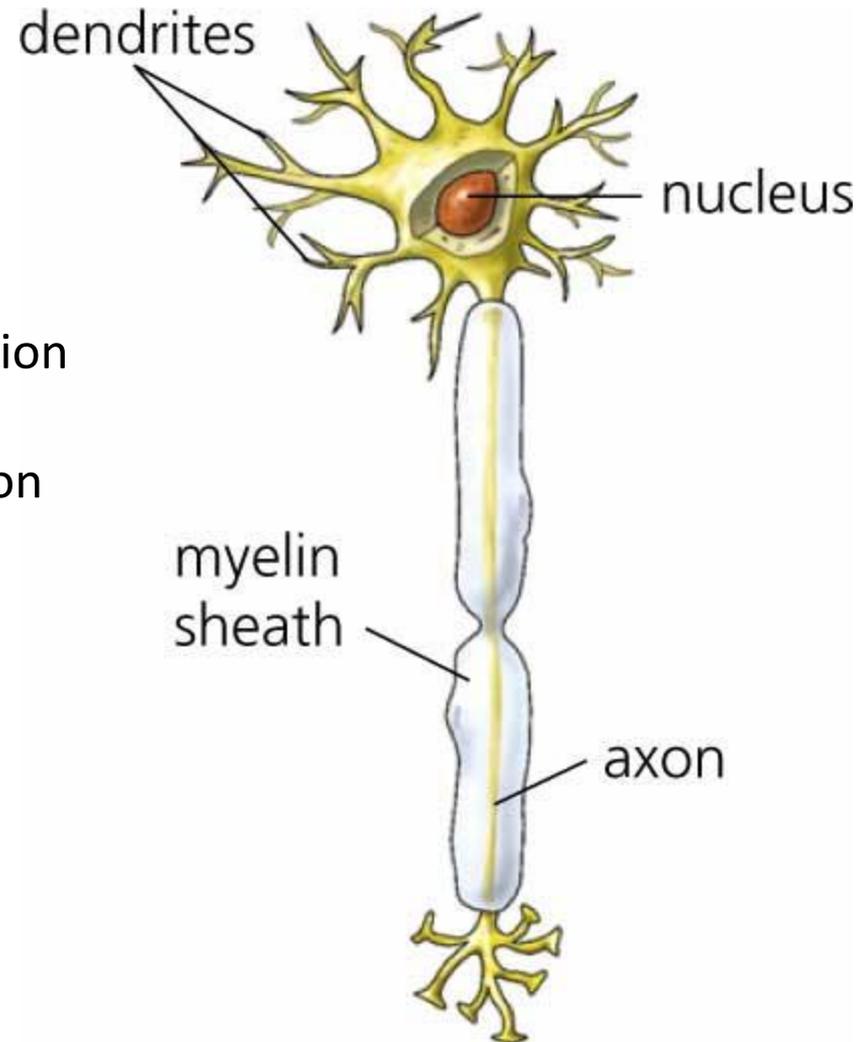
# Dendritic computation

Dendrites as computational elements:

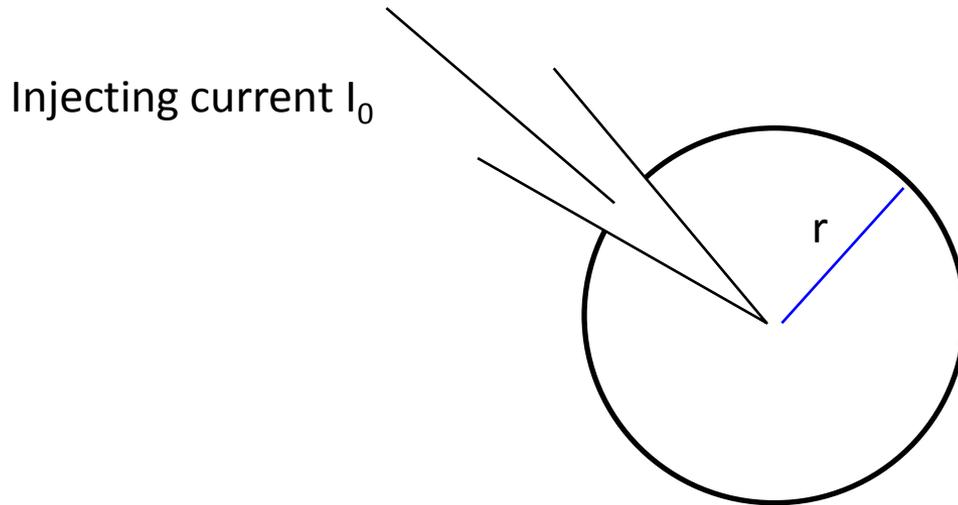
Passive contributions to computation

Active contributions to computation

Examples



## Geometry matters: the isopotential cell



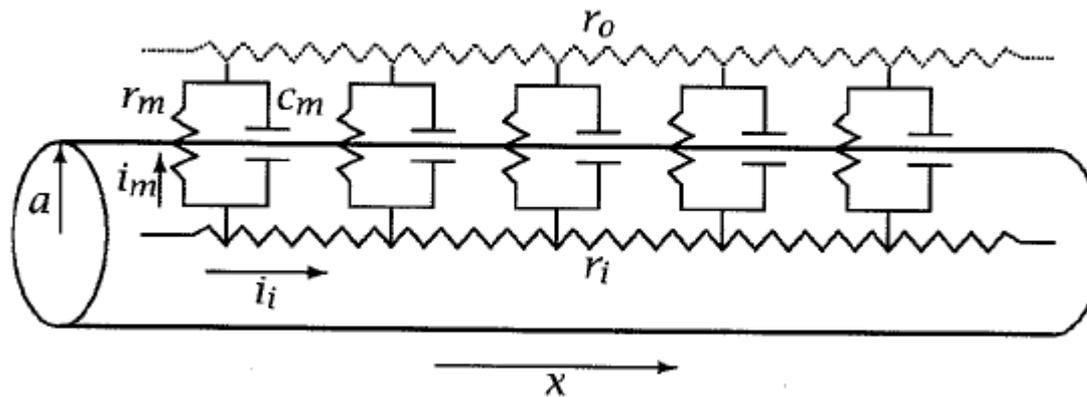
$$V_m = I_m R_m$$

Current flows uniformly out through the cell:  $I_m = I_0/4\pi r^2$

Input resistance is defined as  $R_N = V_m(t \rightarrow \infty)/I_0$

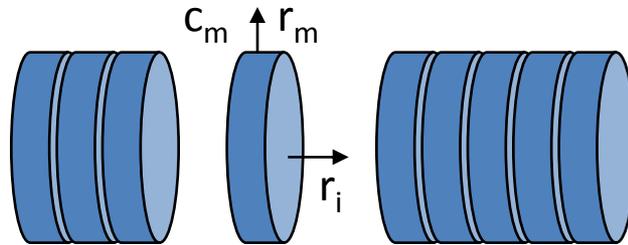
$$= R_m/4\pi r^2$$

## Linear cable theory



$r_m$  and  $r_i$  are the membrane and axial resistances, i.e. the resistances of a thin slice of the cylinder

## Axial and membrane resistance



For a length  $L$  of membrane cable:

$$r_i \rightarrow r_i L$$

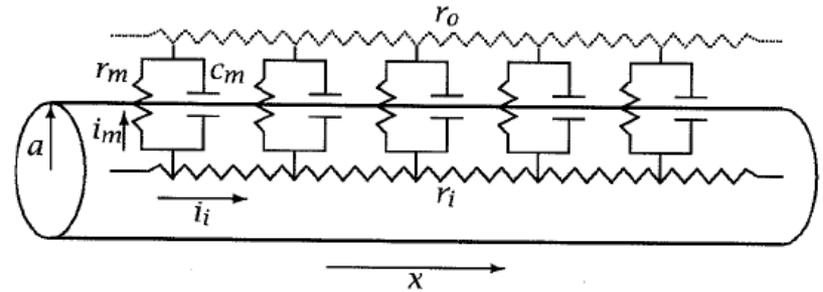
$$r_m \rightarrow r_m / L$$

$$C_m \rightarrow C_m L$$

## The cable equation

$$(1) \quad \frac{\partial V_m}{\partial x} = -r_i i_i$$

$$(2) \quad \frac{\partial i_i(x)}{\partial x} = -i_m$$



$$\frac{\partial}{\partial x} (1) \rightarrow \frac{\partial^2 V_m}{\partial x^2} = -r_i \frac{\partial i_i}{\partial x} = r_i i_m.$$

$$i_m = i_C + i_{\text{ionic}} = c_m \frac{\partial V_m}{\partial t} + \frac{V_m}{r_m}$$

$$\frac{1}{r_i} \frac{\partial^2 V_m(x, t)}{\partial x^2} = c_m \frac{\partial V}{\partial t} + \frac{V_m}{r_m}.$$

or

$$\lambda^2 \frac{\partial^2 V_m}{\partial x^2} = \tau_m \frac{\partial V_m}{\partial t} + V_m$$

where

$$\tau_m = r_m c_m$$

Time constant

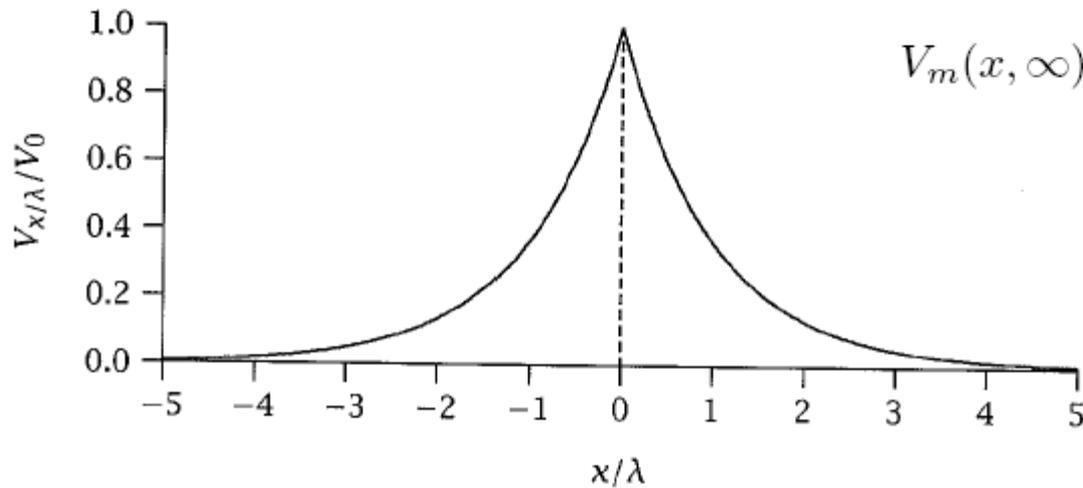
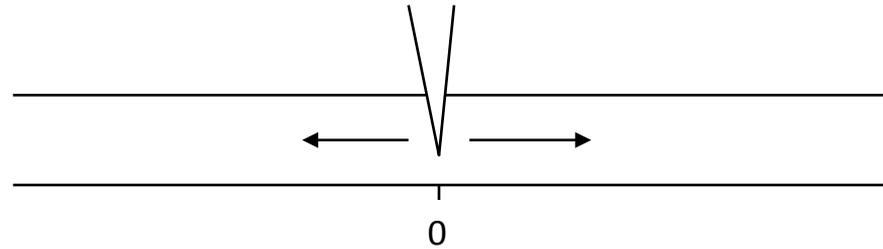
$$\lambda = \sqrt{\frac{r_m}{r_i}}$$

Space constant

### Full solution for current step in infinite cable

$$V_m(t, x) = \frac{r_i I_0 \lambda}{4} \left[ e^{-x/\lambda} \operatorname{erfc} \left( \frac{x/\lambda}{2\sqrt{t/\tau_m}} - \sqrt{t/\tau_m} \right) - e^{x/\lambda} \operatorname{erfc} \left( \frac{x/\lambda}{2\sqrt{t/\tau_m}} + \sqrt{t/\tau_m} \right) \right]$$

Decay of voltage in space for current injection at  $x = 0$ ,  $T \rightarrow \infty$

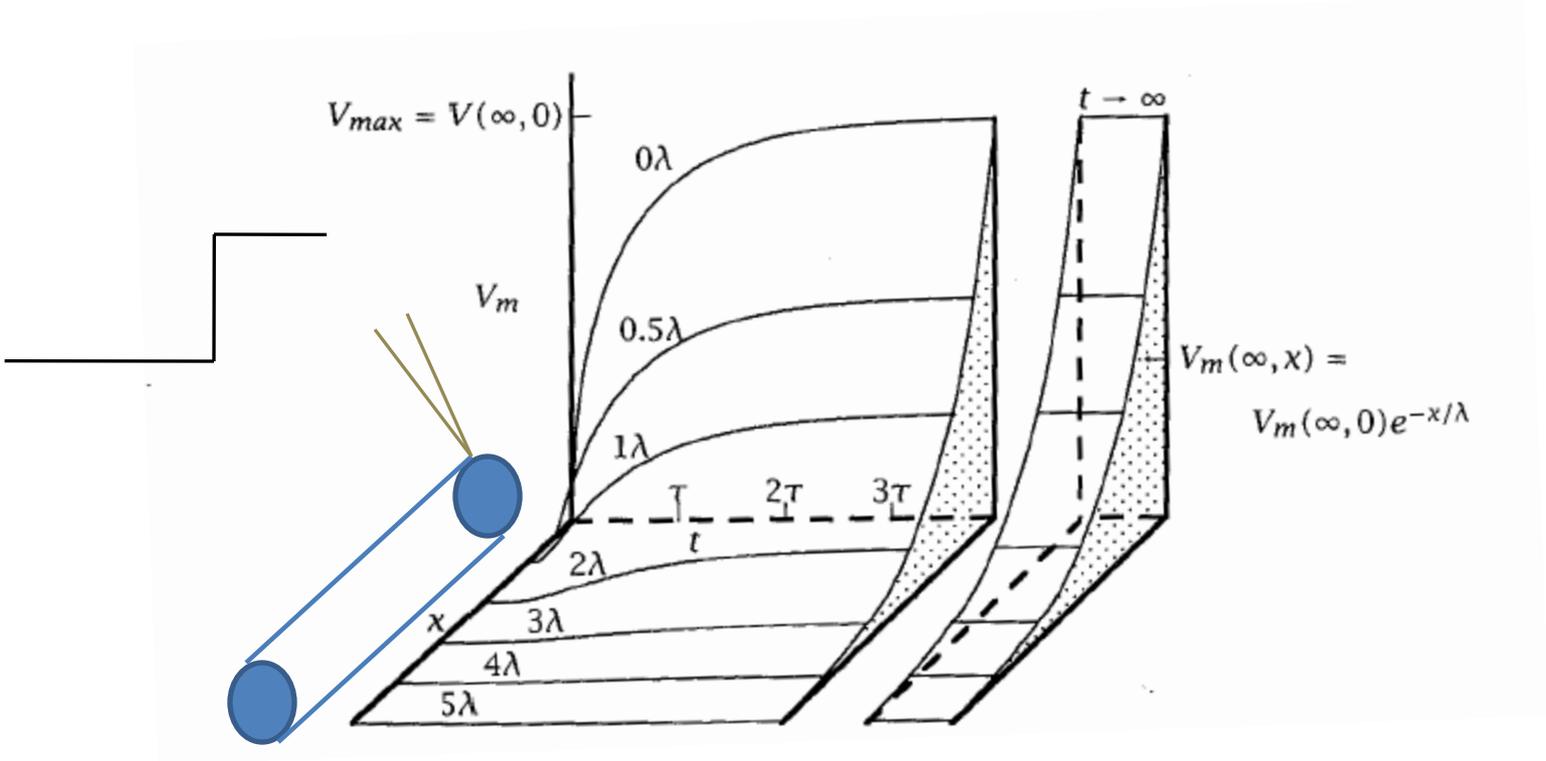


$$V_m(x, \infty) = \frac{r_i I_0 \lambda}{2} e^{-x/\lambda}$$

## Properties of passive cables

→ Electrotonic length  $\lambda = \sqrt{\frac{r_m}{r_i}}$

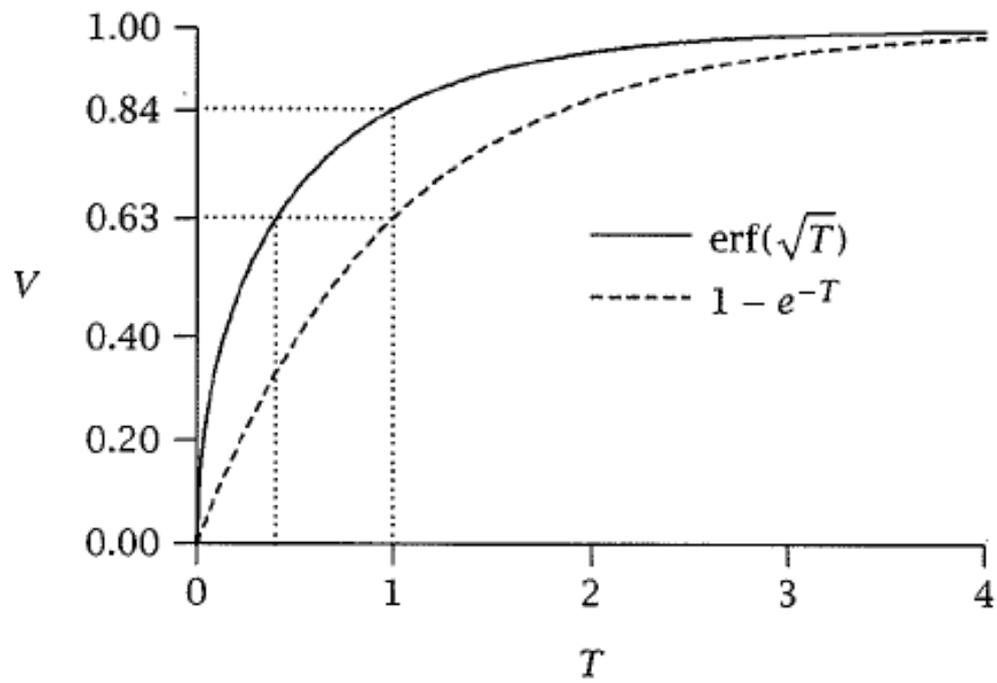
# Electrotonic length



## Properties of passive cables

→ Electrotonic length  $\lambda = \sqrt{\frac{r_m}{r_i}}$

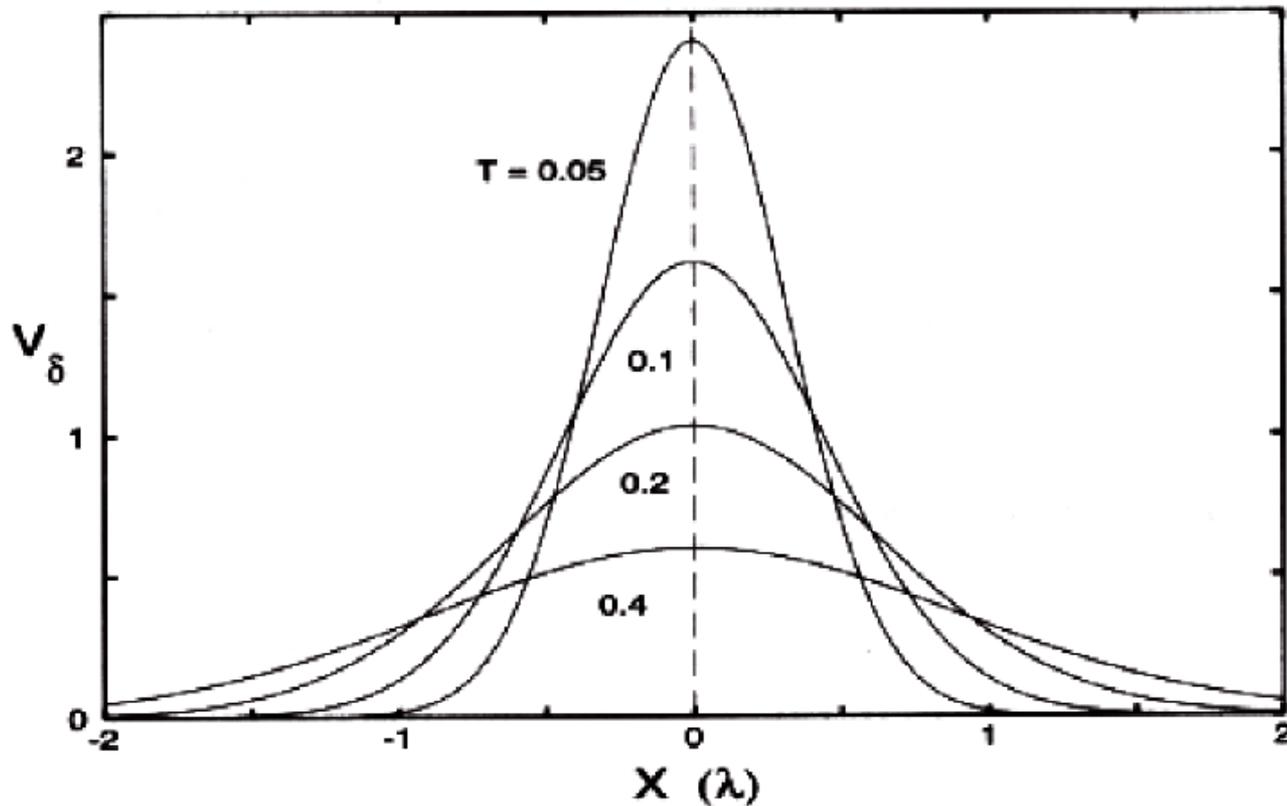
→ Current can escape through additional pathways: speeds up decay



## Pulse response

$$V(x, t) \propto \sqrt{\frac{\tau}{4\pi\lambda^2 t}} e^{-\frac{t}{\tau} - \frac{\tau x^2}{4\lambda^2 t}}$$

$$V(x, t) = V(0) e^{-\frac{1}{2} \ln t/\tau - \frac{t}{\tau} - \frac{x^2 \tau}{4\lambda^2 t}}$$

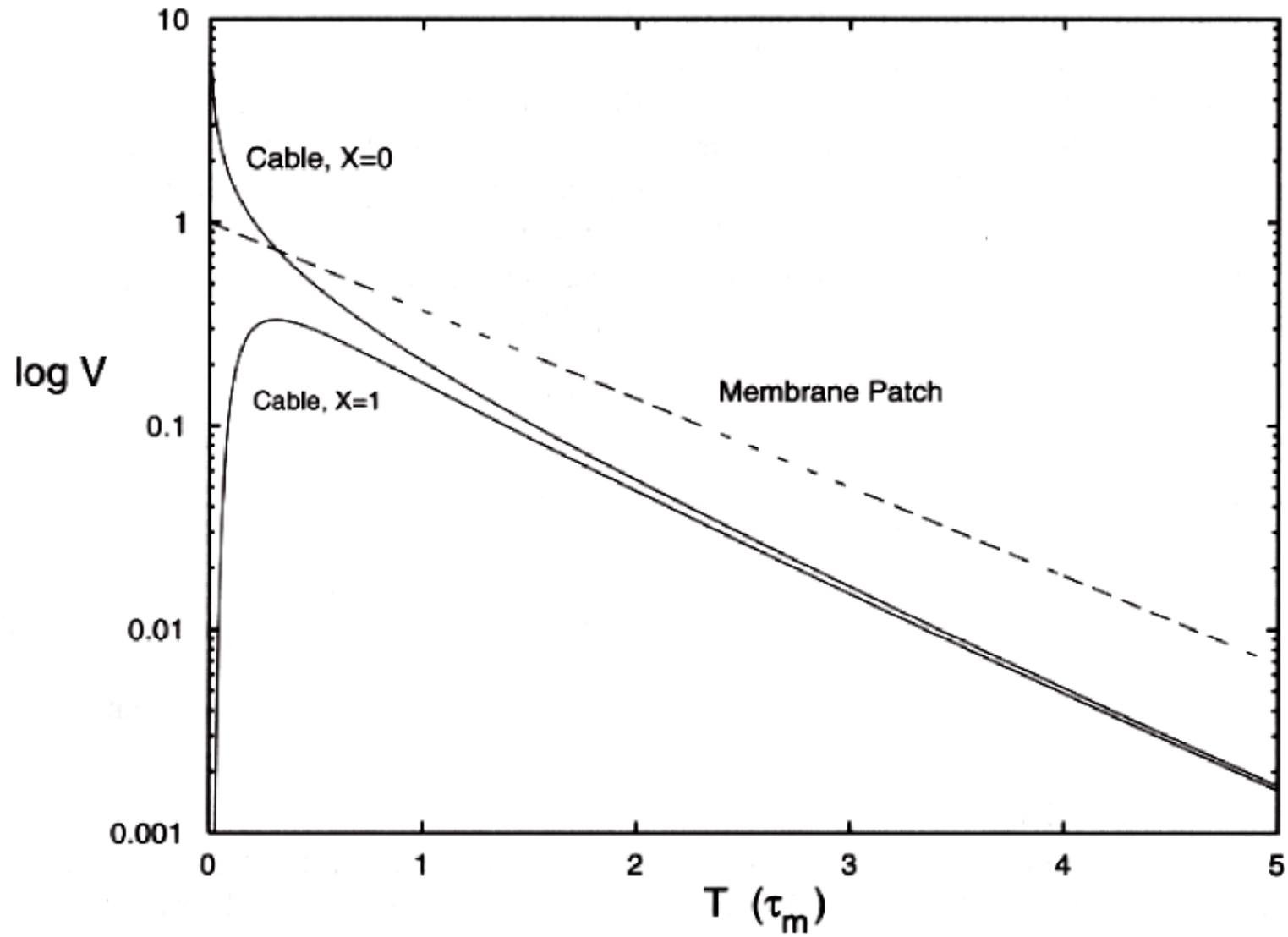


## Pulse response

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$$V(x, t) = V(0) e^{-\frac{1}{2} \ln t/\tau - \frac{t}{\tau} - \frac{s^2 \tau}{4\lambda^2 t}}$$

Dendrites as *filters*



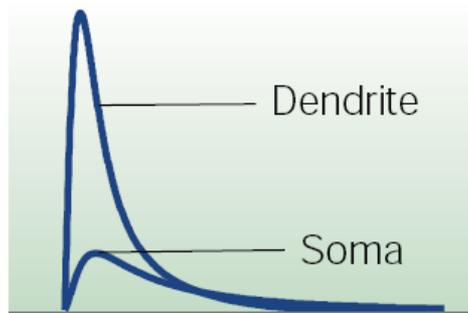
## Properties of passive cables

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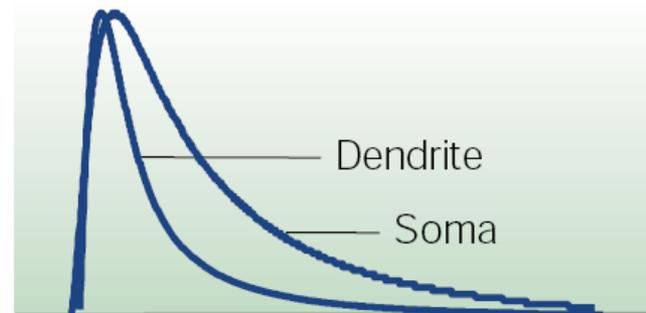
→ Current can escape through additional pathways: speeds up decay

→ Cable diameter affects input resistance  $R_N = \frac{\sqrt{R_m R_i / 2}}{2\pi a^{3/2}}$

: Amplitude



Time course



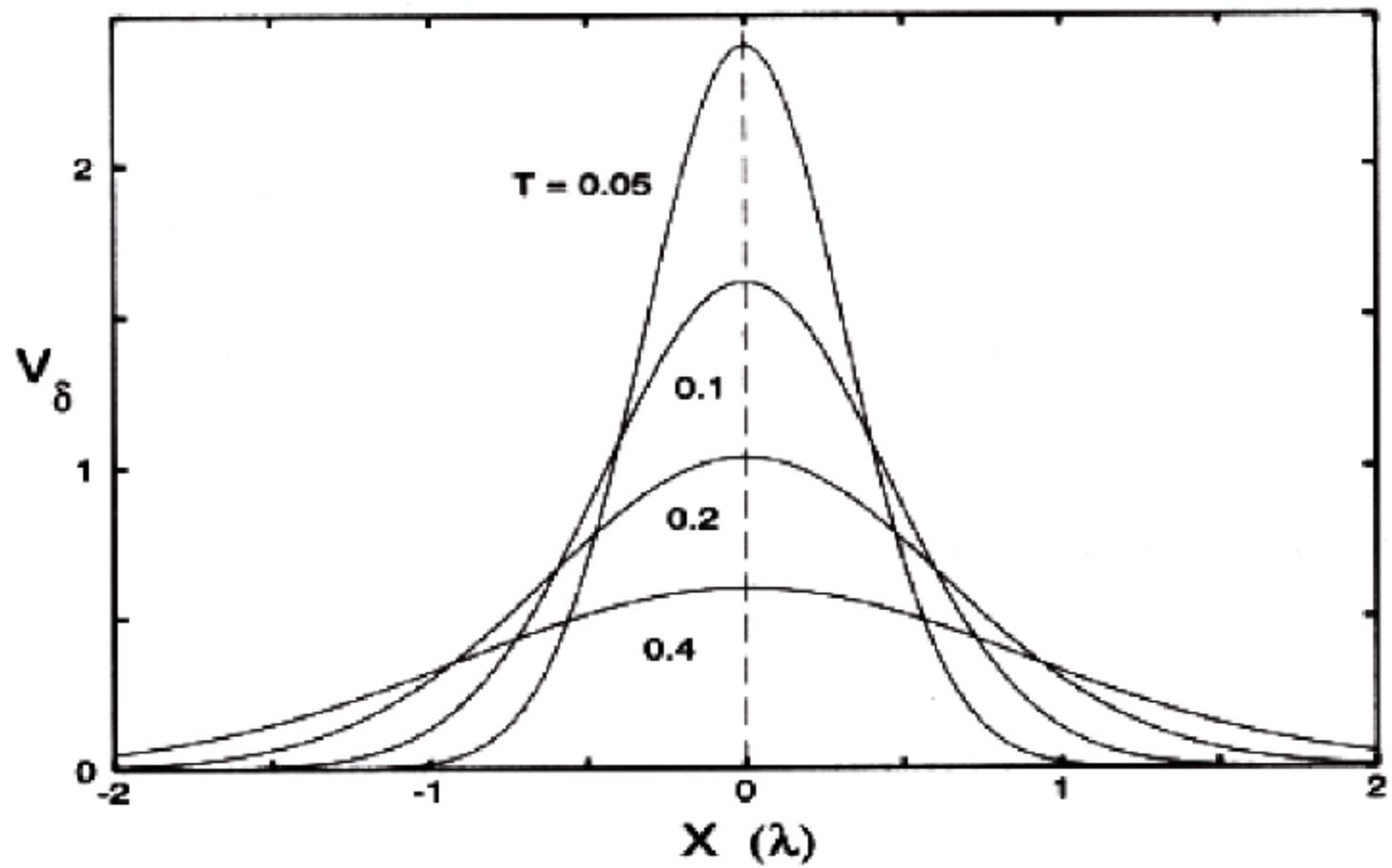
## Properties of passive cables

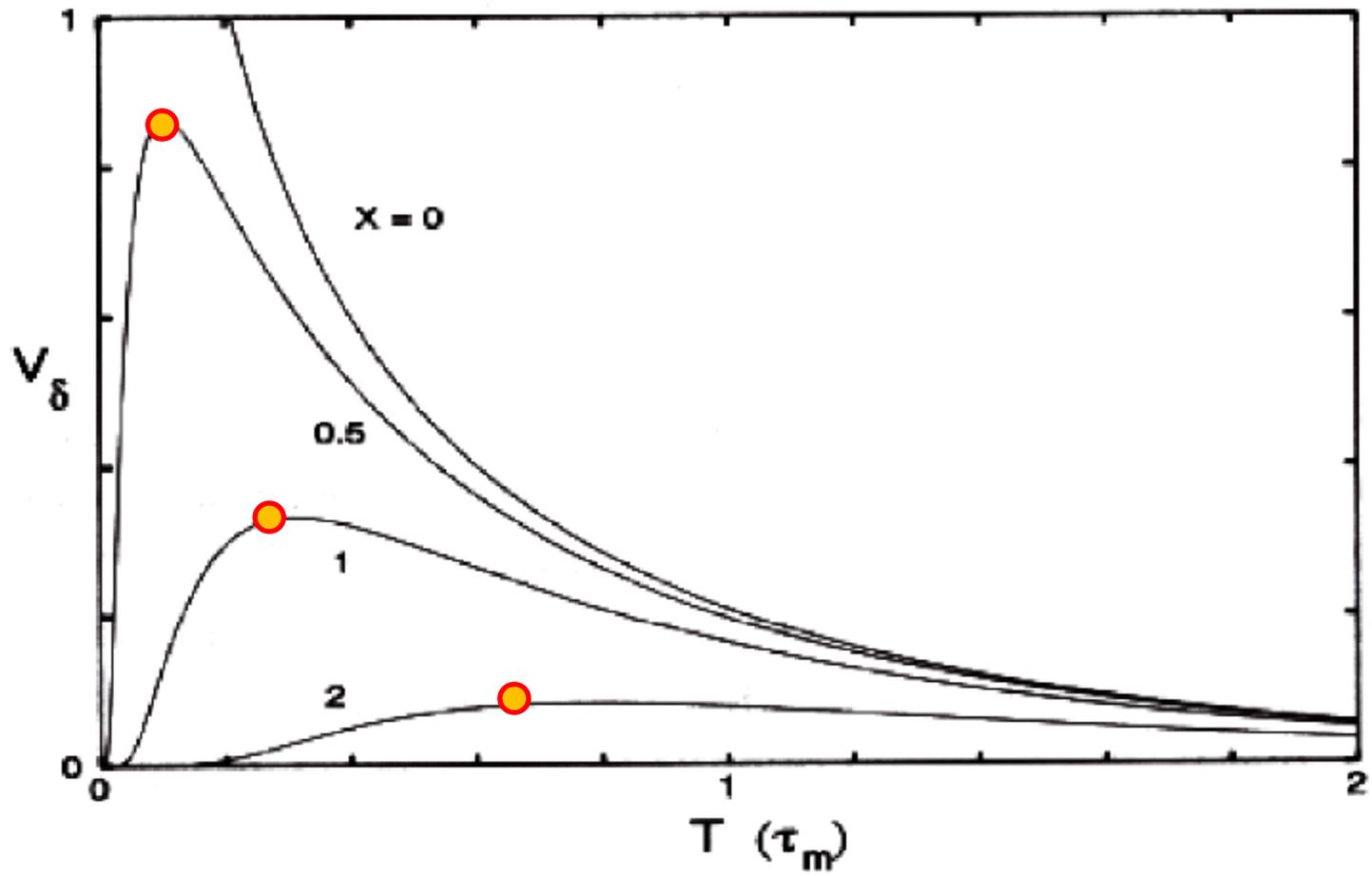
→ Electrotonic length  $\lambda = \sqrt{\frac{r_m}{r_i}}$

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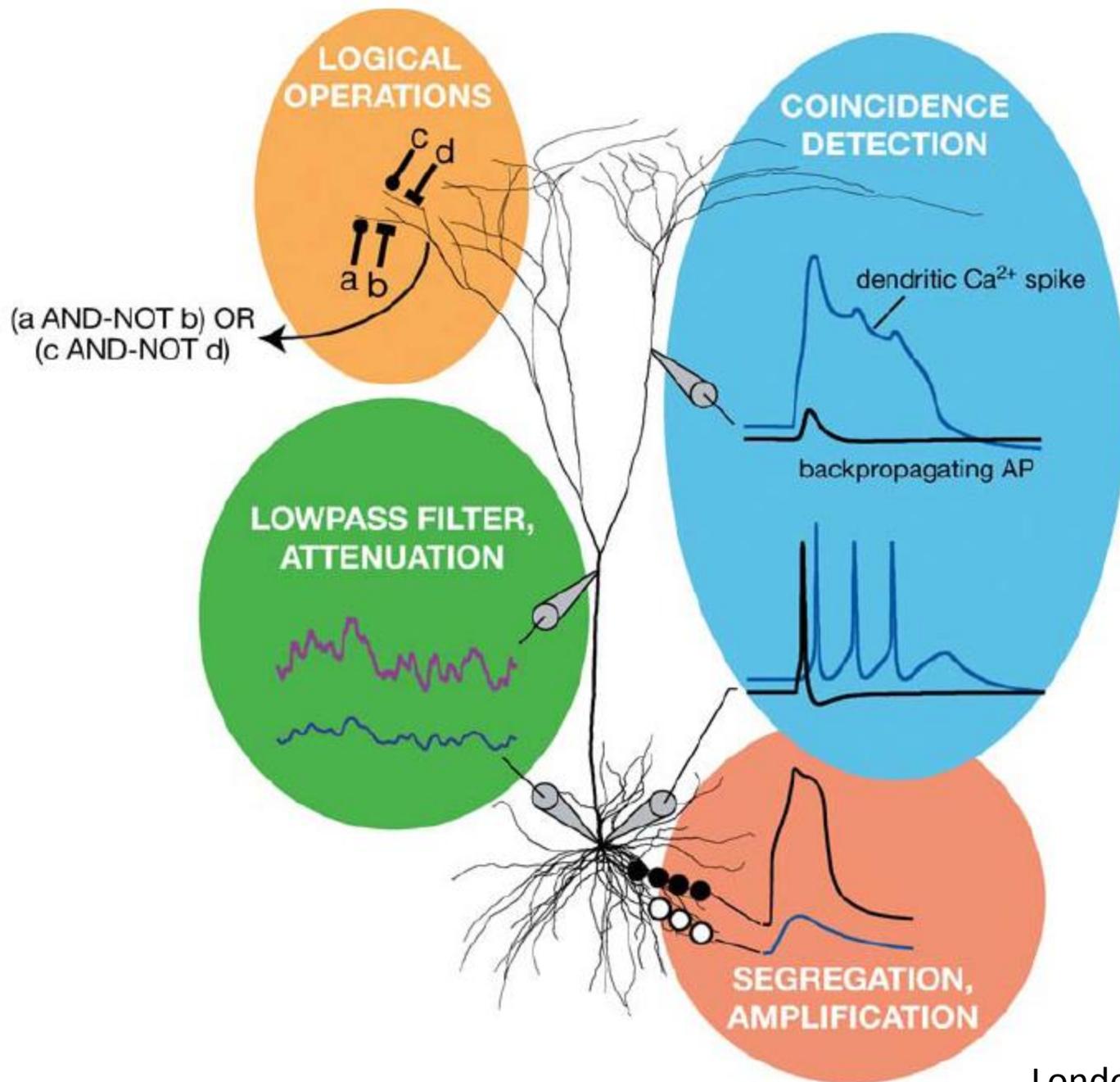
→ Cable diameter affects input resistance  $R_N = \frac{\sqrt{R_m R_i / 2}}{2\pi a^{3/2}}$

→ Cable diameter affects transmission velocity



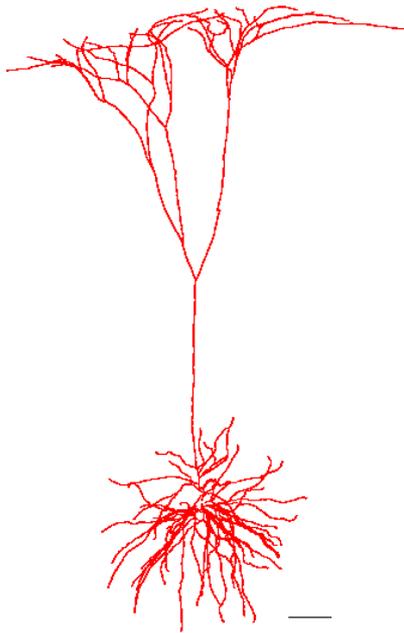


$$\theta = \frac{2\lambda}{\tau_m} = \sqrt{\frac{2a}{R_m R_i C_m^2}}$$



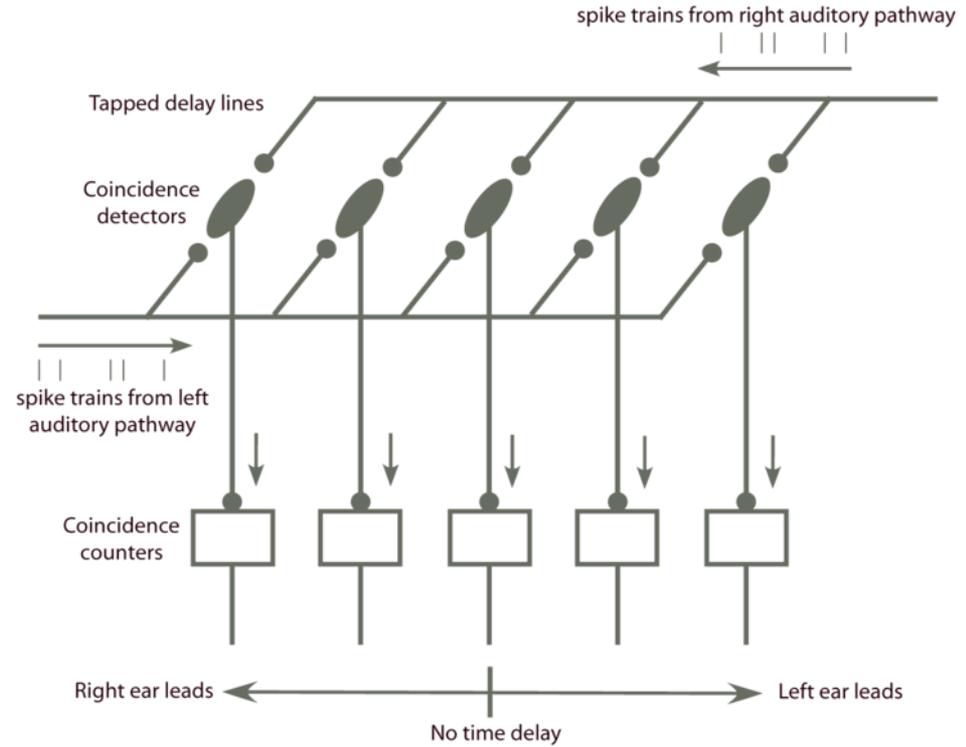
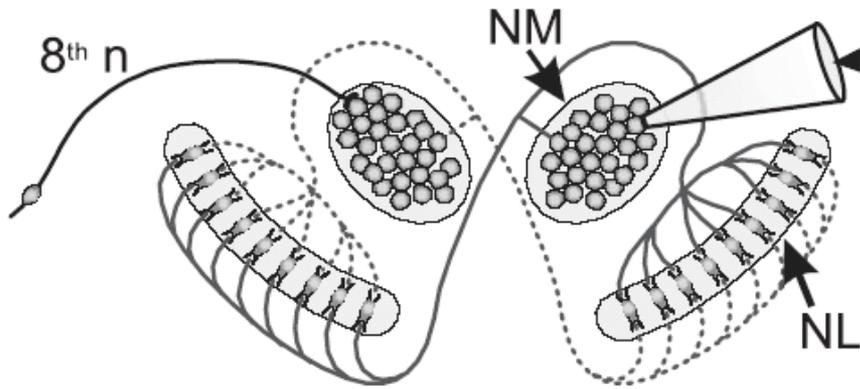
# Passive computations

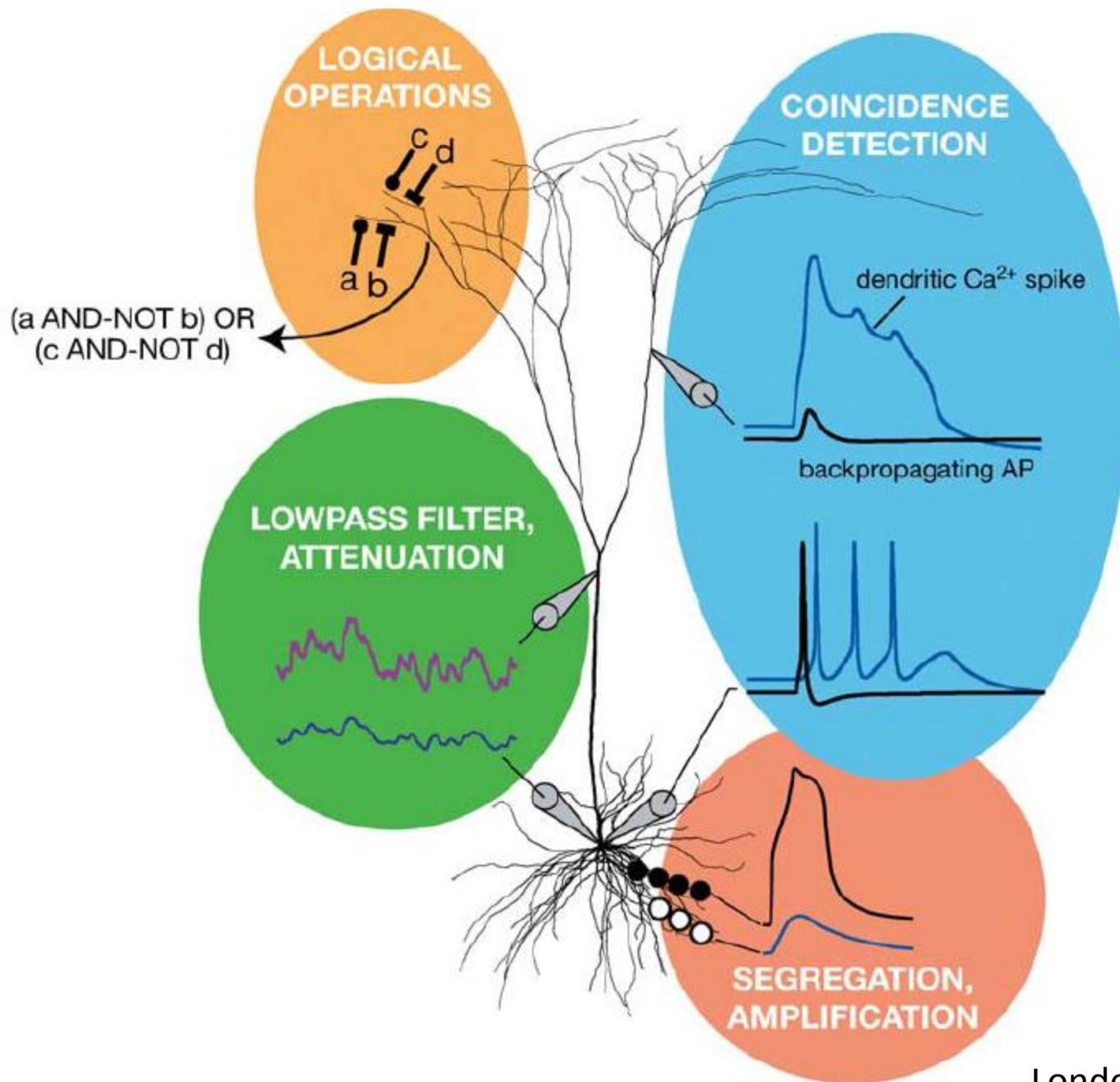
Linear filtering:



- Inputs from dendrites are broadened and delayed
- Alters summation properties..  
    coincidence detection to temporal integration
- Delay lines
- Segregation of inputs
- Nonlinear interactions within a dendrite
  - sublinear summation
  - shunting inhibition
- Dendritic inputs “labelled”

# Delay lines: the sound localization circuit





# Active dendrites

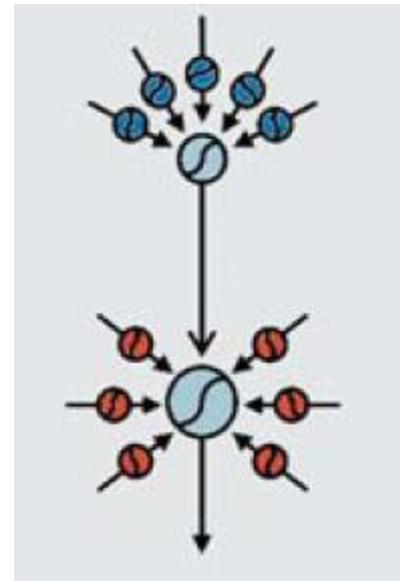
Mechanisms to deal with the distance dependence of PSP size

→ Subthreshold boosting: inward currents with reversal near rest  
Eg persistent  $\text{Na}^+$

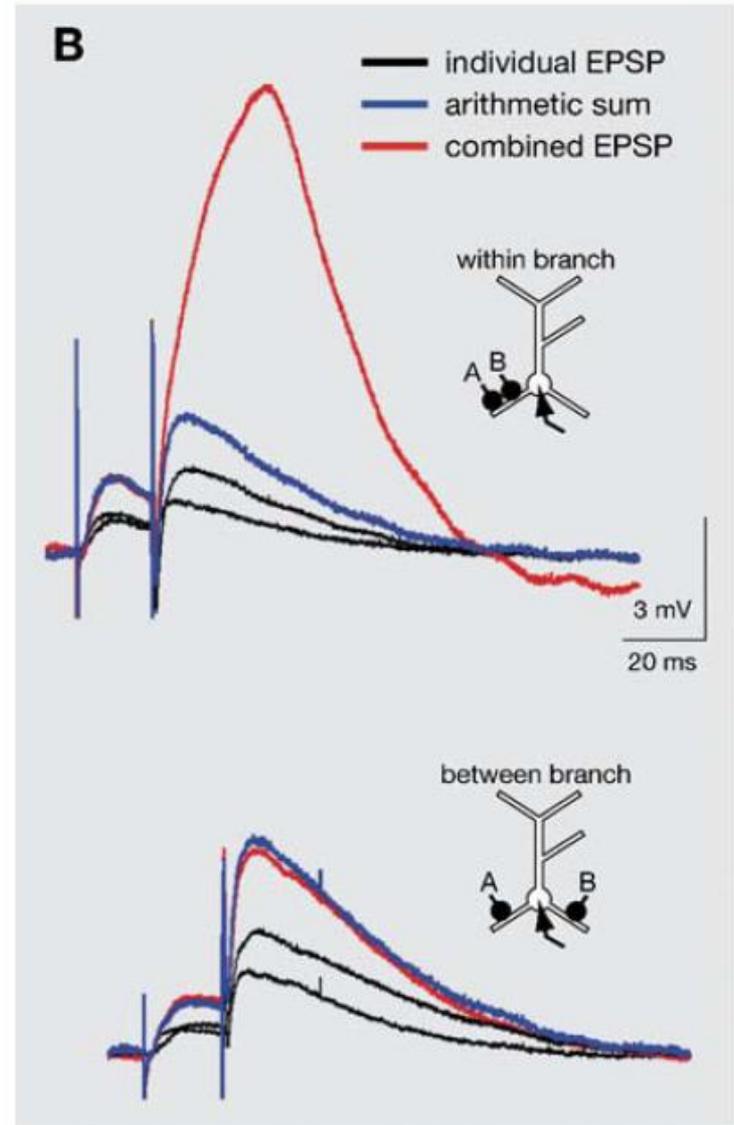
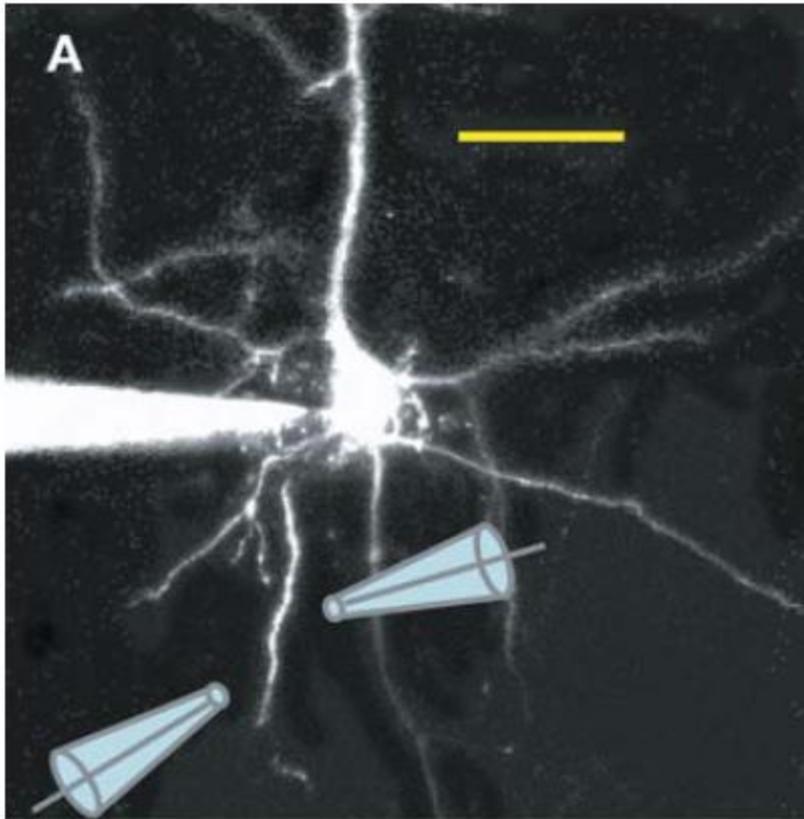
→ Synaptic scaling

→ Dendritic spikes  
 $\text{Na}^+$ ,  $\text{Ca}^{2+}$  and NMDA  
Dendritic branches as  
mini computational units

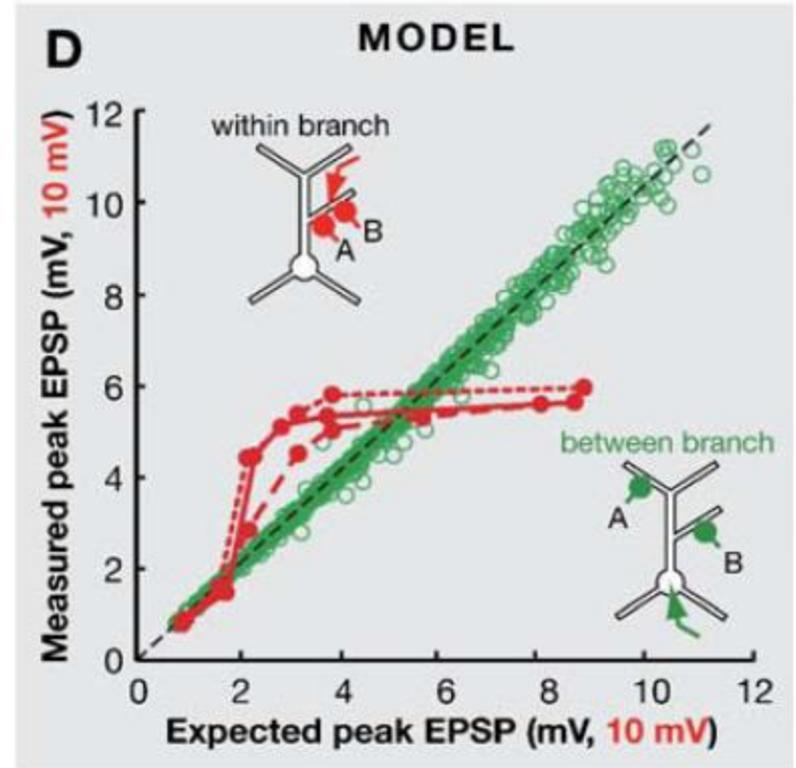
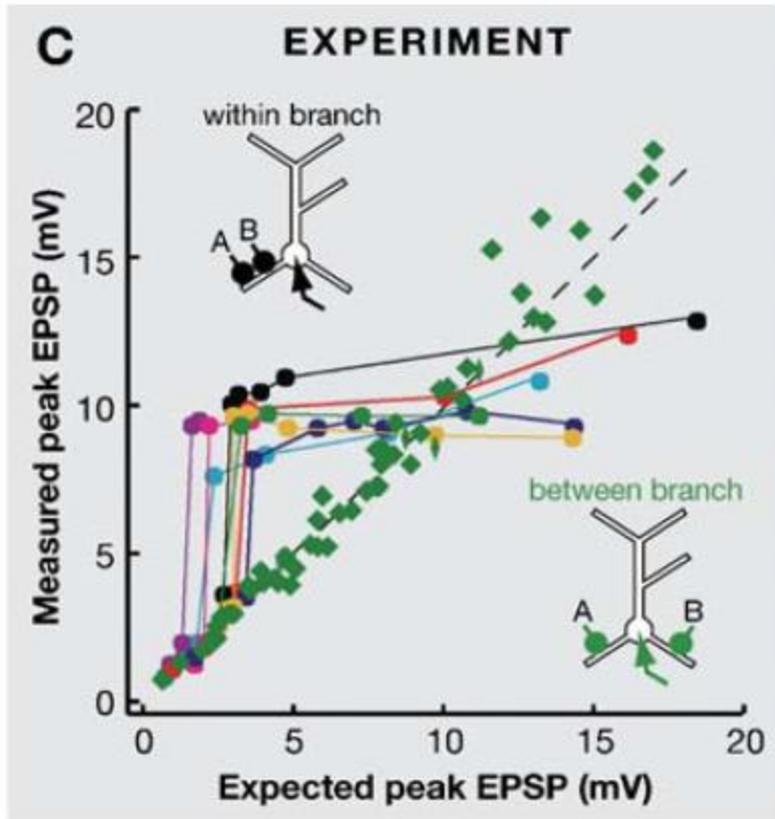
→ backpropagation:  
feedback circuit  
Hebbian learning through  
supralinear interaction of backprop spikes with inputs



# Segregation and amplification

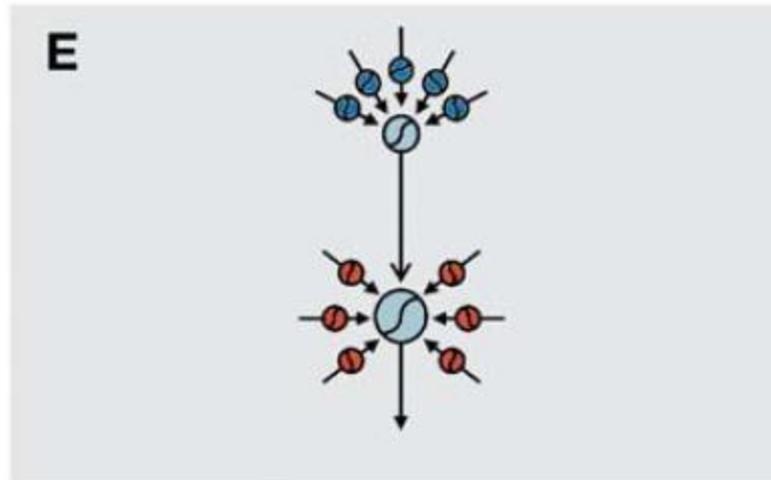


# Segregation and amplification

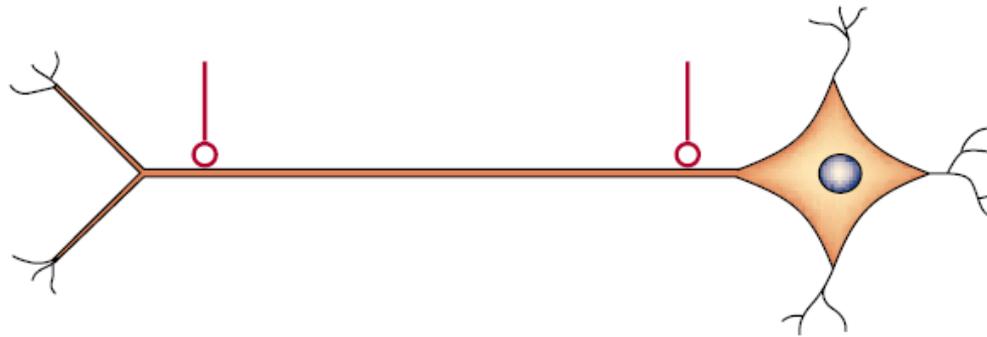


# Segregation and amplification

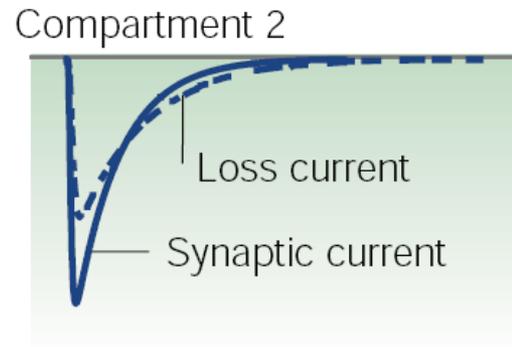
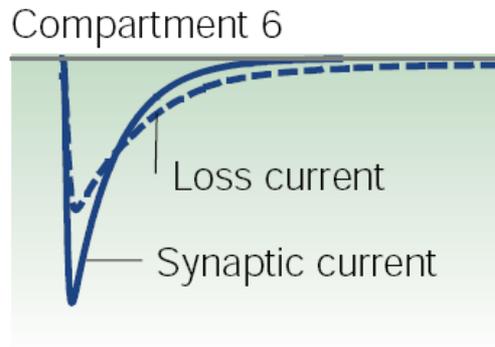
The single neuron as a neural network



# Synaptic scaling



Currents

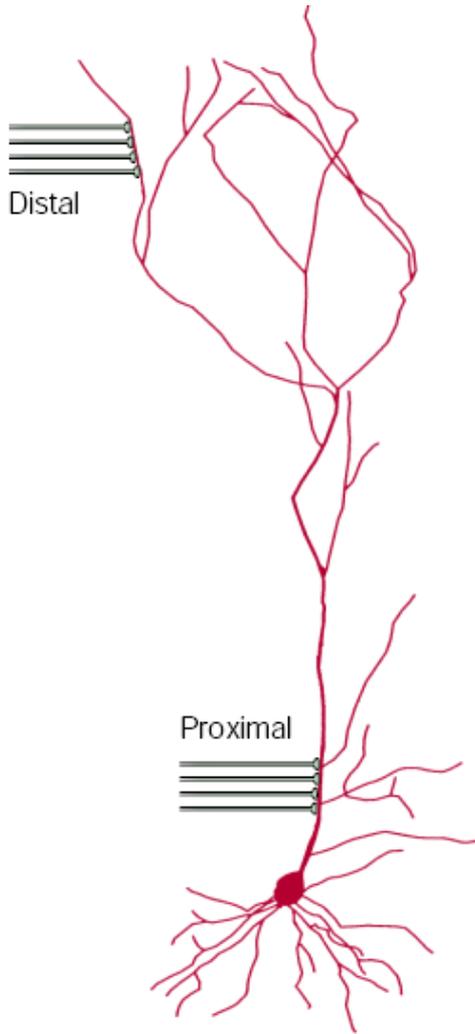


Potential

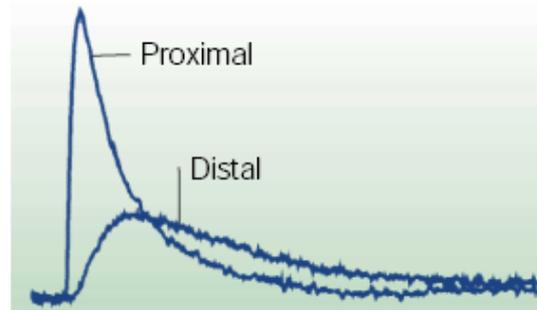


Distal: integration  
Proximal: coincidence

# Expected distance dependence

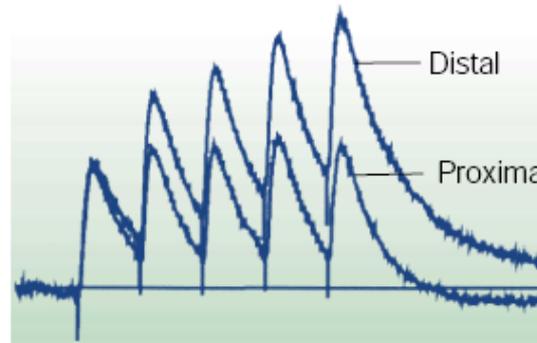


Localized output

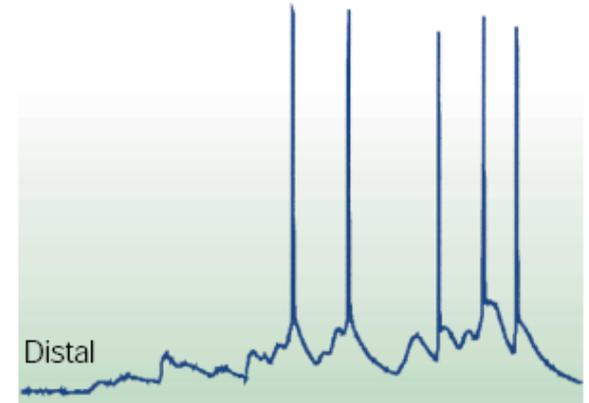


Amplitude and kinetics

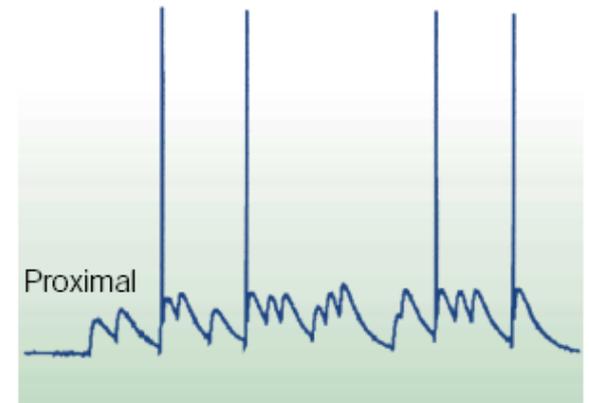
## Synaptic potentials



Temporal summation

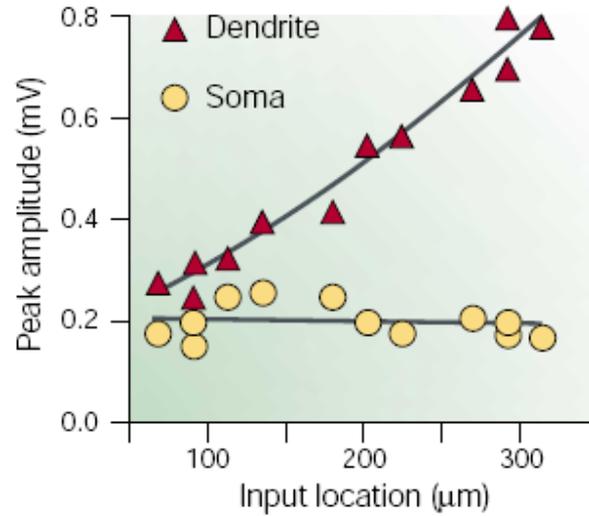
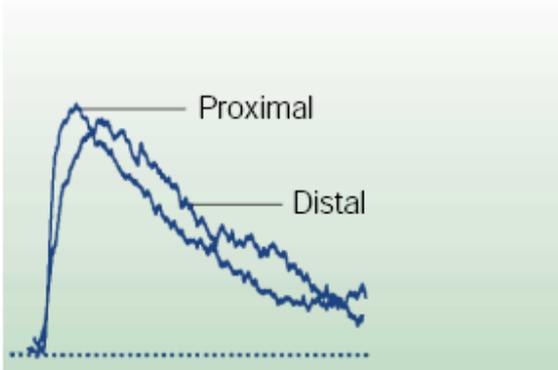


## Somatic action potentials

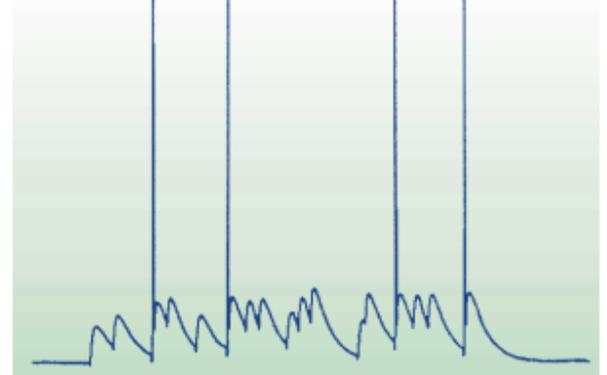


# CA1 pyramidal neurons

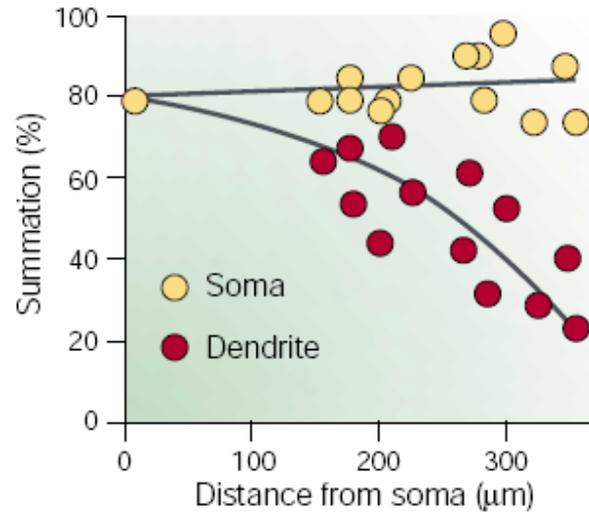
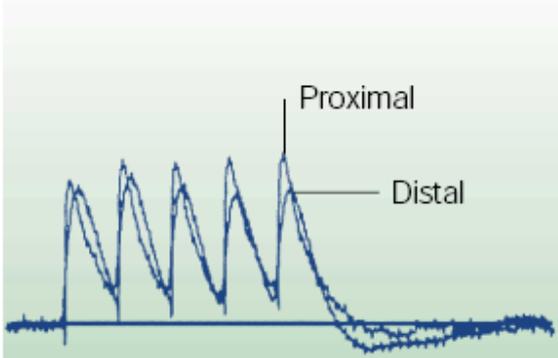
Amplitude and kinetics



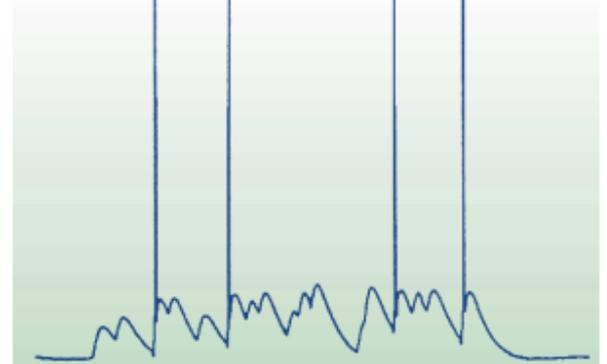
Proximal



Temporal summation

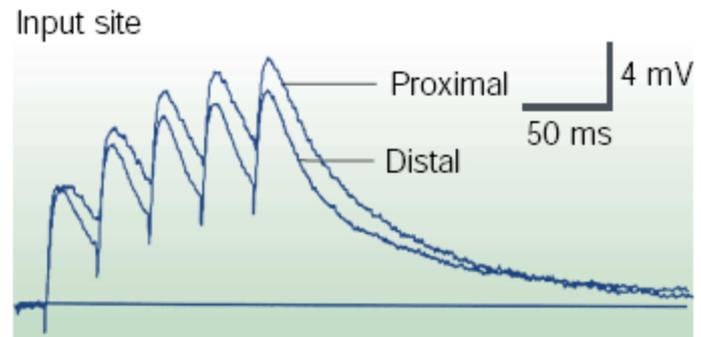
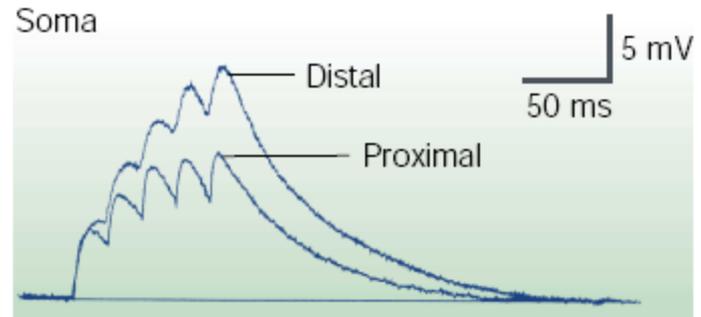
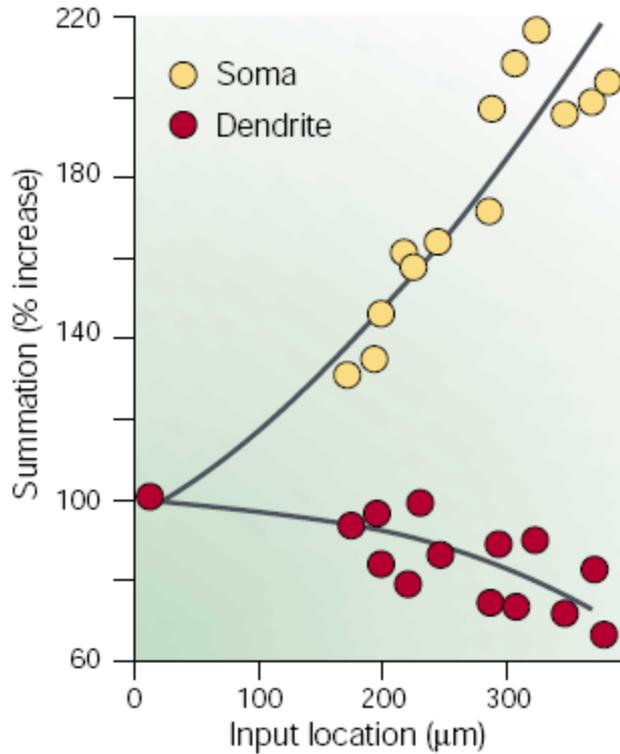


Distal



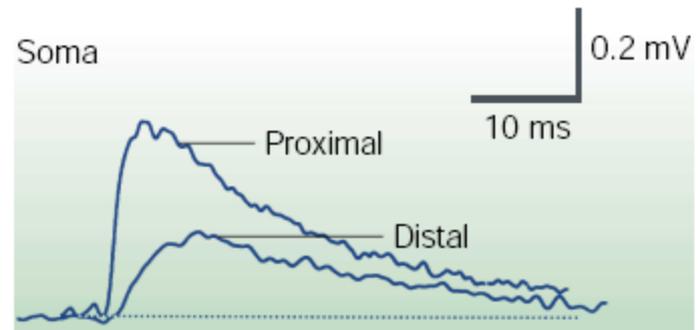
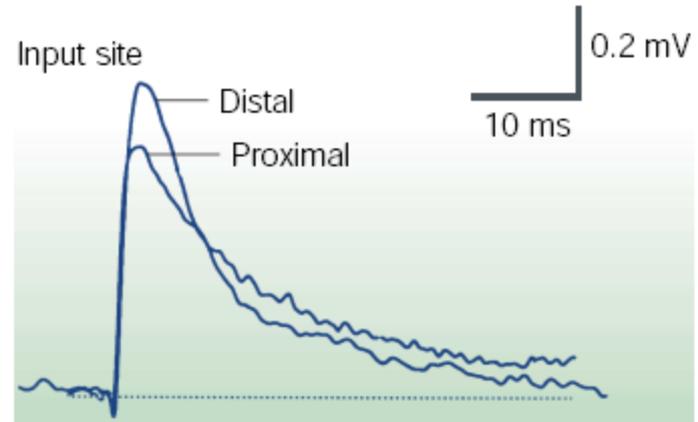
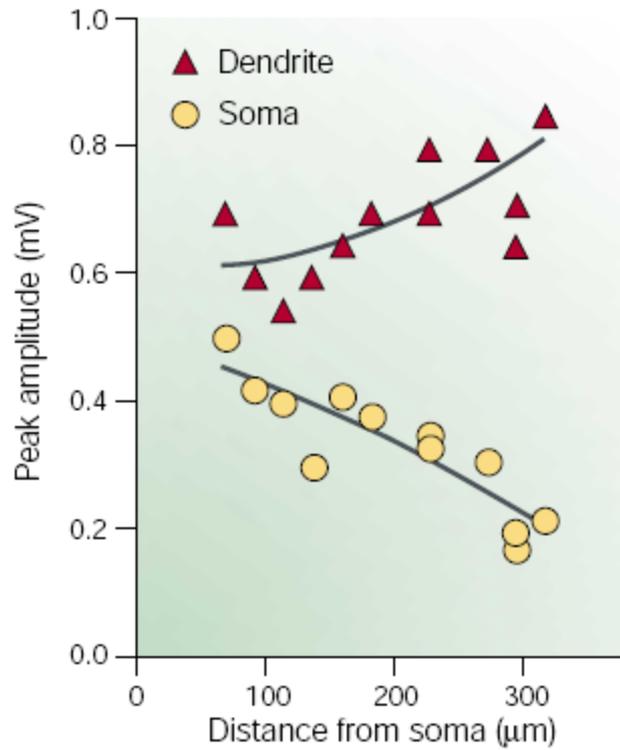
# Passive properties

## a Temporal summation



# Passive properties

## b EPSP amplitude



## Active properties: voltage-gated channels

For short intervals (0-5ms), summation is linear or slightly supralinear

For longer intervals (5-100ms), summation is sublinear

Na<sup>+</sup>, Ca<sup>2+</sup> or NMDA receptor block eliminates supralinearity

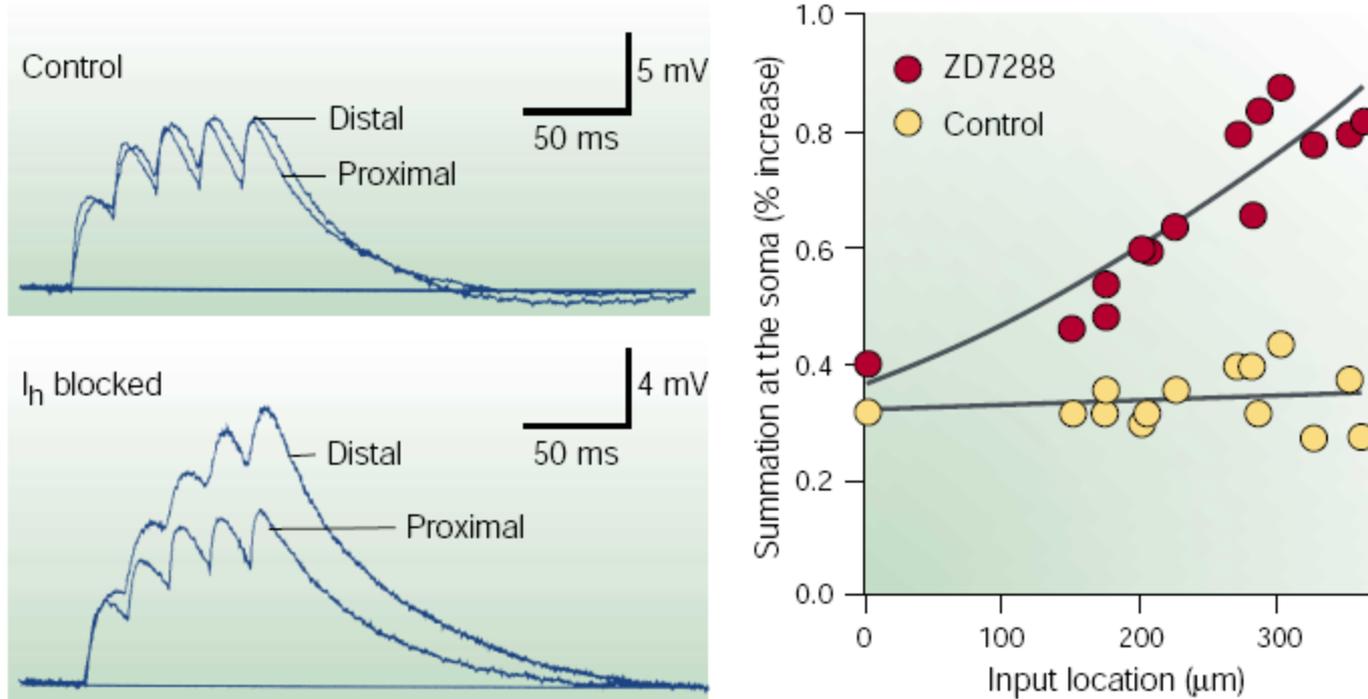
I<sub>h</sub> and K<sup>+</sup> block eliminates supralinearity

Major player in synaptic scaling: hyperpolarization activated K current, I<sub>h</sub>

Increases in density down the dendrite

Effectively outward current due to deactivation during EPSP hyperpolarizes, shortens EPSP duration, reduces local summation

## Active properties: voltage-gated channels



Major player in synaptic scaling: hyperpolarization activated K current,  $I_h$

Increases in density down the dendrite

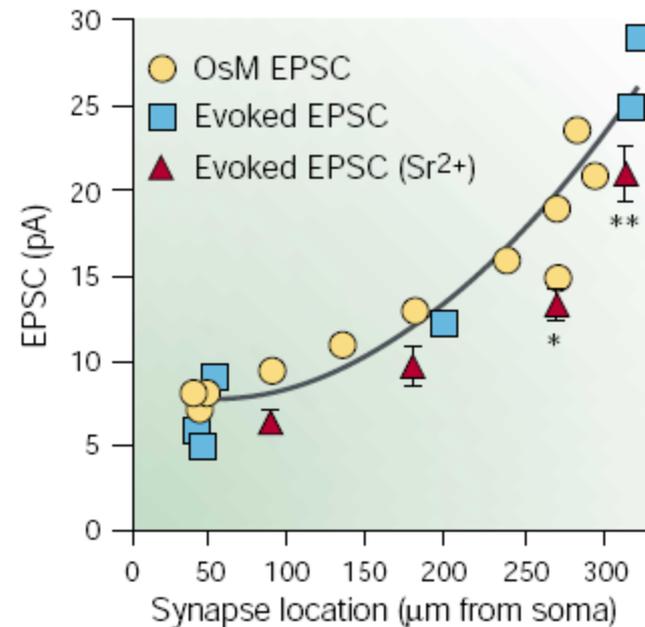
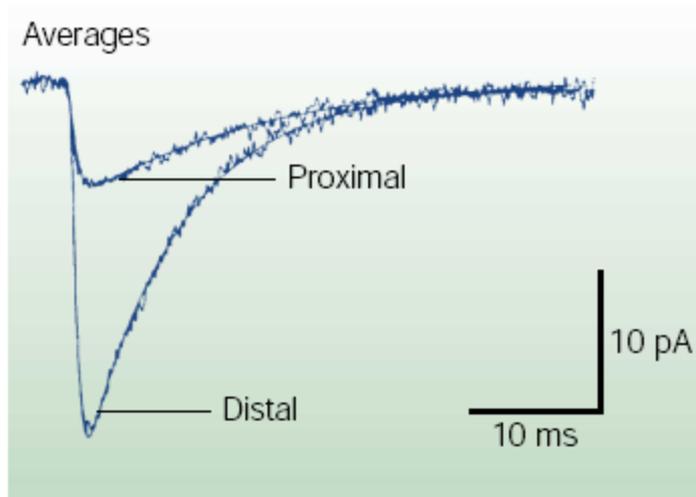
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# Synaptic properties

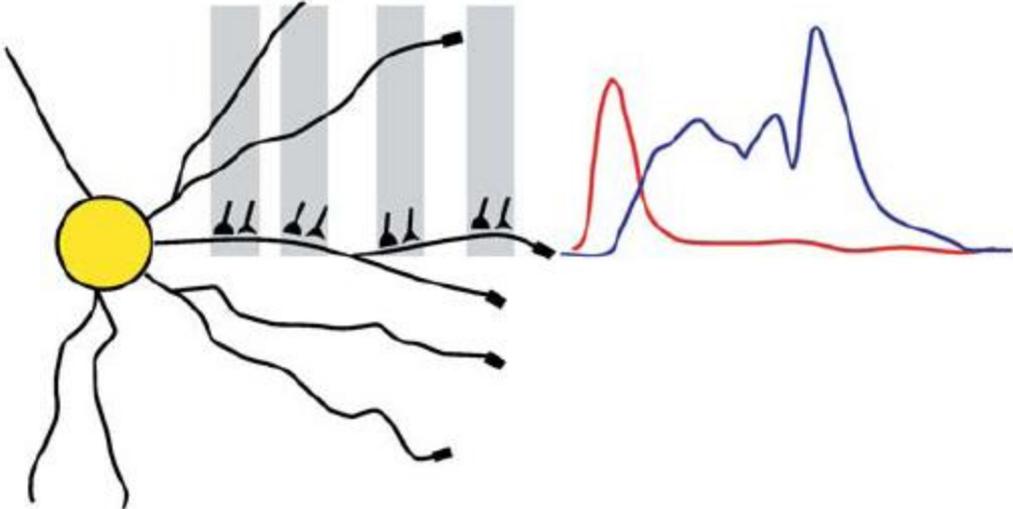
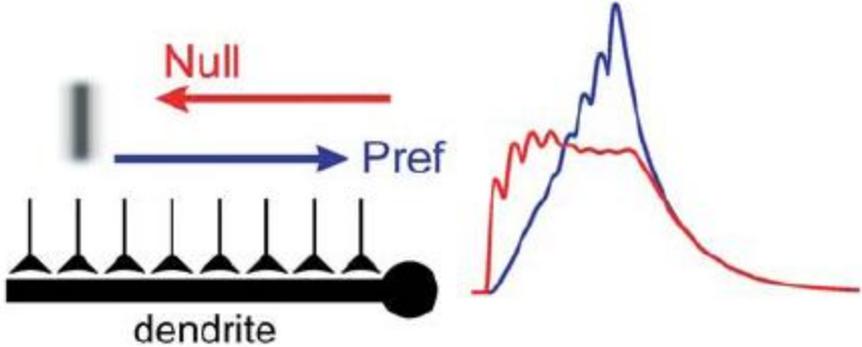
While active properties contribute to summation, don't explain normalized amplitude

Shape of EPSC determines how it is filtered .. Adjust ratio of AMPA/NMDA receptors

Eliminate role of  $I_h$



# Direction selectivity



## References:

Johnson and Wu, *Foundations of Cellular Physiology*, Chap 4

Koch, *Biophysics of Computation*

Magee, *Dendritic integration of excitatory synaptic input*, Nature Reviews Neuroscience, 2000

London and Hausser, *Dendritic Computation*, Annual Reviews in Neuroscience, 2005