What's inside the stars?

We have found a plausible power source - nuclear fusion. But in discussing it we have already made some assumptions about the thermal conditions in the cores of stars. How do we know?

We have only 2 ways to look inside of stars:

Asteroseismology: the surfaces of stars oscillate with characteristic frequencies that provide some information about their interiors and bulk properties.

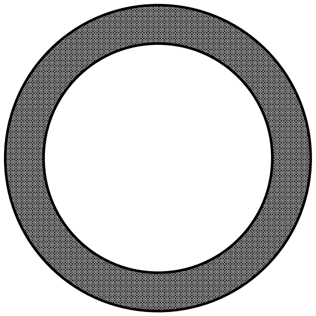
Solar neutrinos: assure us that nuclear reactions occur in the Sun's core, but can't observe them from other stars (unless they go supernova!).

However, most stars provide us with more important clue - they appear to change little over very long periods of time (especially the main sequence stars). This suggests that they are governed by the laws of physics for self-gravitating gas spheres. With sufficient input data those laws determine models for the entire internal structure.

Thus, we know what's inside of stars because physics realized in computer models tells. How?

Physical Principles of Stellar Structure Models

Consider a thin shell of mass in a star.



1) Mass continuity:

No sources or sinks of mass. Eq. relates ρ, r, M_r

2) Total energy conservation:

Energy in + nuclear energy produced = energy out.

3) Hydrostatic equilibrium:

Pressure balances gravity.

4) Radiation transport:

Radiative or convective transport equations.

Not total energy, just the photon part of it.

Hydrostatic Equilibrium

Pressure balances gravity.

Gravity = $(-GM(r)/r^2)$ dm.

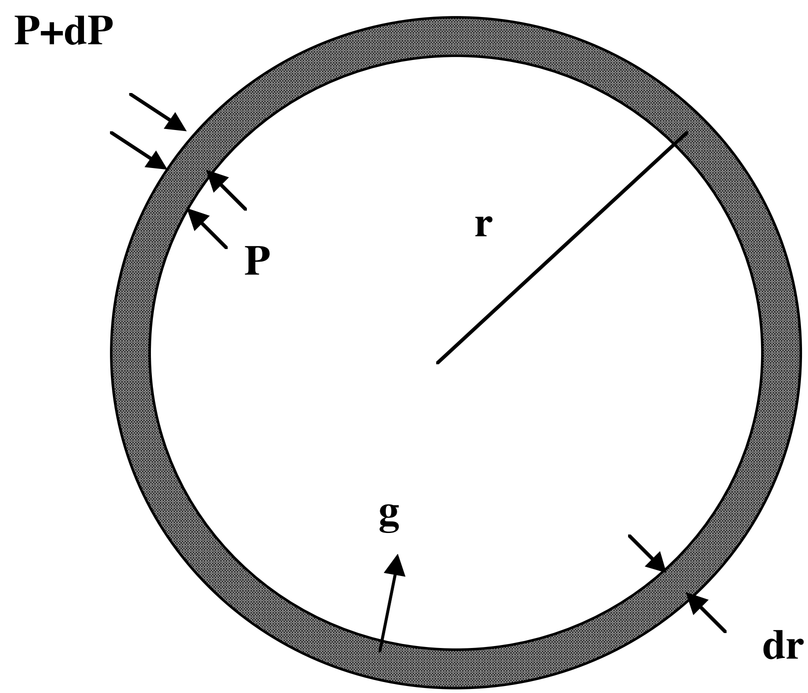
Net pressure = P - (P+dP) = -dP.

Also, mass continuity: $dm = \rho(4\pi r^2\, dr)$

Combine to get:

$$4\pi r^2 dP = \frac{-GM(r)}{r^2}\rho(r)\,4\pi r^2 dr,$$

$$\frac{1}{\rho}\frac{dP}{dr} = \frac{-GM(r)}{r^2}.$$



Thermal Equilibrium = conservation of energy = 1st law of thermo.

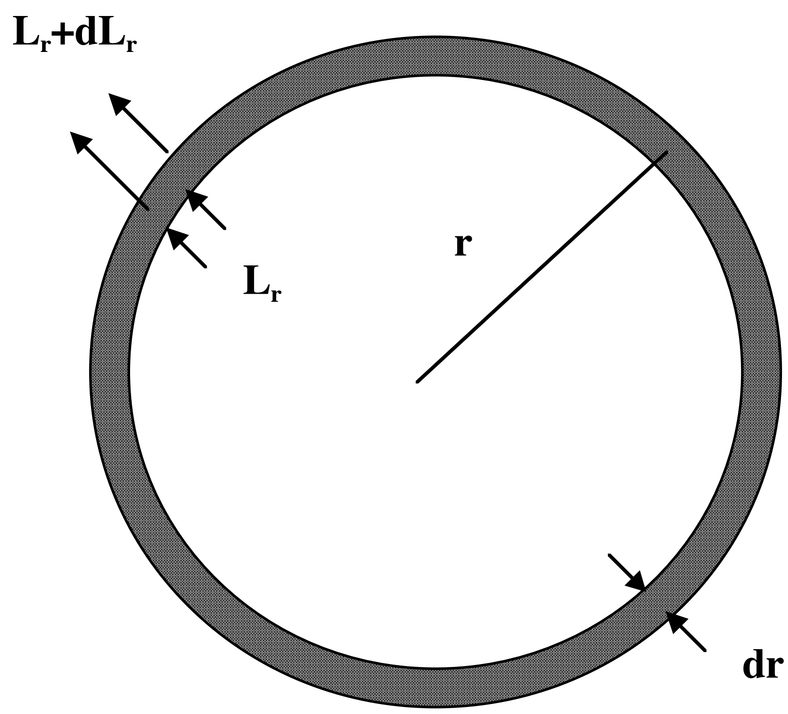
Assume that in a shell of radius r, thickness dr: the sources of energy are balanced by the sinks.

L_r = total energy/sec. entering shell from below.

$L_r + d\,L_r$ = energy/sec. leaving the shell.

ε = rate of energy generation per gram per sec.

$\rho\varepsilon(4\pi r^2 dr)$ = rate of energy production over the whole shell.



Sources - Sinks = 0 $\longrightarrow$

$$4\pi r^2 \rho\varepsilon\, dr - (L_r + dL_r) + L_r = 0,$$

$$\frac{dL_r}{dr} = 4\pi r^2 \rho\varepsilon.$$

“Stellar” Energy Transport

3 ways to move energy (photons): conduction, convection, & radiative diffusion.

Conduction: not important in normal stars, but is in stellar remnants.

Radiative diffusion: photons random walk out of the stars. This is the most common mode of transport.

Convection:

- if the gas/plasma is very opaque, or,

- if the luminosity is high so there is a large amount of energy to transport.

In either case, it can be more efficient to move the gas (containing the photons), rather than wait for the photons to scatter out of it.

Opacity

The rate of radiative diffusion is determined by the "opacity." More precisely, we define the absorption coefficient per gram, κ, such that -

$\kappa\rho\, dl$ = the fraction of energy in a beam of radiation lost over a distance dl,

or the probability that any photon in the beam is absorbed or scattered while traversing the distance dl.

The units of κ are m^2/kg, so κ can be viewed as an effective cross section of 1 kg of absorbing material.

Some sources of opacity in stars:

1) Photoionization

2) Bremsstrahlung ('braking radiation' or free-free) electron-electron scattering assisted by a nearby nucleus.

3) Electron or Thomson scattering.

Radiative transfer

is the complete, detailed theory, which we will not explore. (A bit too technical for this class!) A result that can be derived from this theory, and which is adequate for our purposes is the radiative diffusion eq.: -

$$L_r = -4\pi r^2 \frac{4ac}{3} \frac{T^3}{\kappa\rho} \frac{dT}{dr},$$

where 'a' is a radiation constant (consisting of other fundamental constants).

This eq., together with the thermal equilibrium eq. Gives the temperature structure in terms of ρ and ε.

Stellar Structure Equations

We now have the complete set of equations governing the internal structure of a star, assuming we know ρ, ε, and the equation of state (e.g., the ideal gas law).

Hydrostatic equil.

$$\frac{1}{\rho}\frac{dP}{dr} = \frac{-GM(r)}{r^2}.$$

Continuity

$$\frac{dM}{dr} = 4\pi r^2 \rho,$$

Thermal Eq.

$$\frac{dL_r}{dr} = 4\pi r^2 \rho\varepsilon.$$

Radiative Diffusion

(or equivalent eq. describing convection)

$$L_r = -4\pi r^2 \frac{4ac}{3}\frac{T^3}{\kappa\rho}\frac{dT}{dr},$$

Of course, for this set of equations we also need integration constants, or boundary conditions. E.g.,

$M(r=0)$, $L(r=0) \longrightarrow 0$, and $\rho(r=R) = 0$.

As you can see, this gets rather complex… so we'll mostly leave it to the computers.

A couple of side notes:

- In smaller, cooler stars strong opacity sources (e.g., the H^- molecule) are found deeper in, so it's harder for the radiation to get out, and convection is found deeper in. Opacity determines the convection zone.

- In more massive stars, there are fewer opacity sources, so the envelope is radiative. However, the CNO cycle in the core burns hotter, and produces more energy, than the pp cycle. This is more energy than radiative transfer 'can handle,' so the core is convective in massive stars. Temperature gradients determine the convection zone.

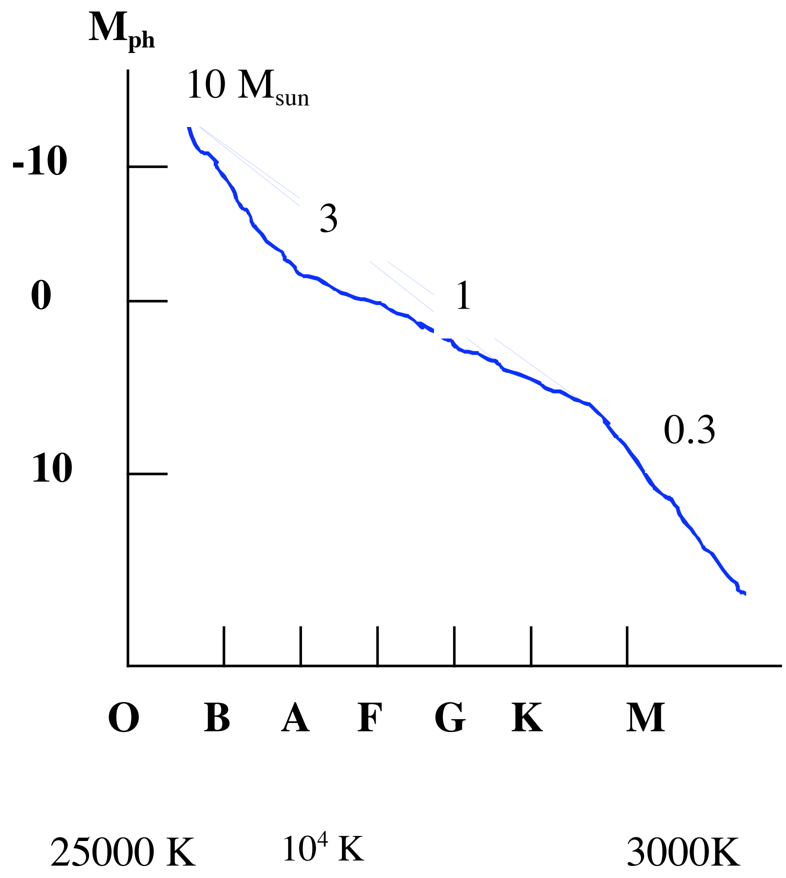
The Main Sequence

As the culmination of our study of stellar structure, for these equations and boundary conditions we have the following result.

Russell-Vogt theorem (1926): The structure of a star is (almost) uniquely determined by the chemical composition and total mass M (or equivalently R, L, or T_c).

Thus, for a given chemical composition the stars form a one parameter family called the main sequence. With this thm. H. N. Russell proved the existence (and uniqueness) of the main sequence, which he had discovered empirically in 1913 (also found by Ejnar Hertzsprung in 1911).

The Zero-Age Main Sequence (ZAMS)



The Mass-Luminosity Relation

The Russell-Vogt theorem implies the existence of a mass-luminosity relation (also a M-R relation, an L-R relation, etc.). Next we derive this scaling from the structure equations.

First - $\rho \sim M/R^3$.

Also from the hydrostatic equation - $P \sim M\rho/R \sim M^2/R^4$.

From the ideal gas law - $T \sim P/\rho \sim (M^2/R^4)\,(R^3/M) \sim M/R$.

And finally, the radiative diffusion eq. scales as,

$$L \sim \frac{RT^4}{\rho} \sim R\left(\frac{M}{R}\right)^4\left(\frac{R^3}{M}\right),$$

Therefore, $L \sim M^3$.

Notice the "fortuitous" cancellation of the R terms in the last step. (Not really!)

Note: in deriving this estimate, we assumed that the opacity in the radiative diffusion equation was constant. We can refine this estimate by considering the opacity term's dependences on ρ, T more carefully, $L \sim (RT^4)/(\kappa\rho)$.

For high mass stars, where electron scattering opacity is dominant, $\kappa \sim$ constant, so $L \sim M^3$ as above.

For low mass stars, bound-free opacity dominates, and $\kappa \sim \rho T^{-3}$,

$$\therefore \quad \kappa \sim \frac{M}{R^3}\left(\frac{M}{R}\right)^{-3} \sim M^{-2},$$

$$\text{Then,} \quad L \sim \frac{R\left(\frac{M}{R}\right)^4}{\left(\frac{M}{R^3}\right)M^{-2}} \sim \frac{R^4 M^5}{R^4},$$

So for low mass stars - $L \sim M^5$.

For very high mass stars, radiation pressure dominates gas pressure, so,

$$P \sim T^4 \sim \frac{M^2}{R^4}, \qquad (\text{since } T \sim \sqrt{M/R^2}),$$

$$Then, \quad L \sim \frac{RT^4}{\kappa\rho} \sim R\left(\frac{M^2}{R^4}\right)\left(\frac{R^3}{M}\right).$$

So for these stars, $L \sim M$.

Empirically:

$\mathrm{Log}(L/L_{sun}) \approx 4 \log(M/M_{sun})$, for $L > L_{sun}$,

$\mathrm{Log}(L/L_{sun}) \approx 2.8 \log(M/M_{sun})$, for $L < L_{sun}$.