

Designing Professional Development
around the Common Core
Standards for Mathematical Practice
A Mathematician-Teacher Collaboration

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The Mathematical Preparation of Teachers:
The Impact of the Common Core State Standards Initiative

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The Standards for Mathematical Practice

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

The Standards for Mathematical Practice

“varieties of expertise that mathematics educators at all levels should seek to develop in their students”

These have historical roots in the “processes and proficiencies” of, for eg.,

- ▶ NCTM Process Standards, 2000
- ▶ National Research Council’s Adding It Up Report, 2001
- ▶ National Math Advisory Panel Report, 2008

What do these Mathematical Practices mean...

to Teachers?

Are they Obvious?

- ▶ Aren't these just the usual "problem-solving strategies"?

Are they Ambiguous?

- ▶ Does #5 mean "tools" like protractors and calculators?
- ▶ Does "precision" in #6 mean "enough decimal places"?
- ▶ What's the difference between #7 and #8?
 - ▶ #7: Look for and make use of structure.
 - ▶ #8: Look for and express regularity in repeated reasoning

And just what do they look like *in practice*?

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What do Mathematical Practices mean...

to Mathematicians?

- ▶ Un-codified, highly individual, not limited to a single list
- ▶ Usually not explicitly taught (no 'Math Research Methods' course)
- ▶ Second nature or learned habits of mind?

How did those of us trained as mathematicians learn 'mathematical practices'?

Apprenticing Mathematical Practices

- ▶ In well-taught lectures and classes?
- ▶ Through 'reading' texts and papers on our own?
- ▶ Through research in undergrad, grad and postgrad?
- ▶ Through hands-on learning of unfamiliar mathematics?
- ▶ Through collaboration with advisors, peers and colleagues?

How can this inform *teacher* professional development around mathematical practices?

Designing Professional Development around Mathematical Practices

- ▶ Focused on mathematical content
(not pedagogy, initially)
- ▶ Extended investigation into familiar and unfamiliar math
(“low threshold, high ceiling”)
- ▶ Explicit Reflection
 - ▶ Mathematical (on content and practices)
 - ▶ Pedagogical (how to translate to classroom)

Why should Mathematicians be involved?

- ▶ Our experience with ‘mathematical practices’
in practice
- ▶ Our viewpoint of unifying concepts in math
(bridging, connecting)
- ▶ Our role in K-16 spectrum as college and
university instructors

An Example: Focus on Math (FoM) Summer Institute

Focus on Math (an NSF-funded MSP)

“a unique partnership ... to improve student achievement by providing mathematics teachers with the content knowledge and skills valuable in their profession.”

- ▶ Greater Boston School Districts (7 Districts)
- ▶ Boston University, UMass Lowell, Lesley University
- ▶ Educational Development Center, Inc. (EDC)

FoM Summer Institute: "Transformation and Number"

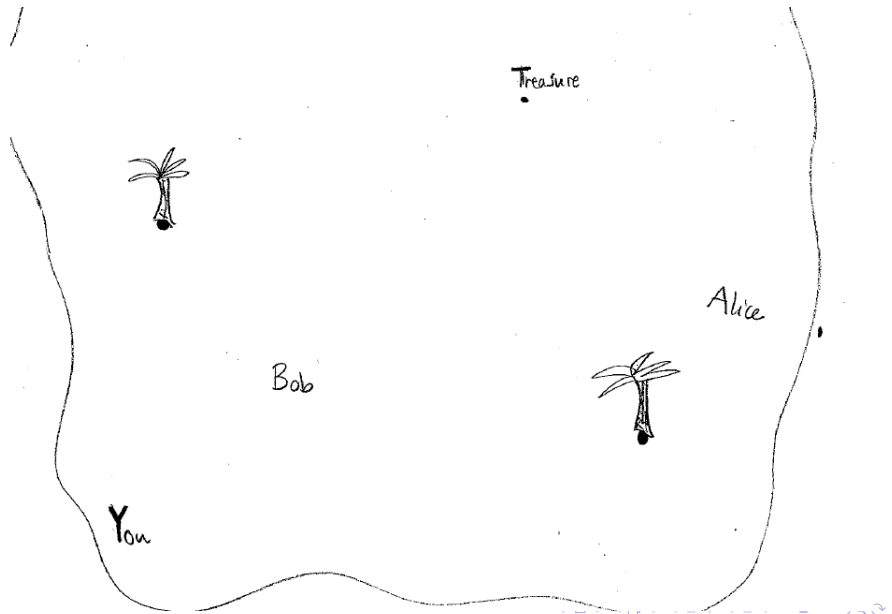
- ▶ **The Audience:** Middle and High-School Teachers (5 MS/12 HS)
- ▶ **The Math:** Geometric Transformations and Arithmetic *from plane w/o coordinates to plane w/ coordinates to complex numbers*
- ▶ **The Goals:**
 - ▶ **Vertical content connections** related to geometry, congruence, similarity, functions and complex number arithmetic
 - ▶ Exploration of the CCSS **Standards for Mathematical Practice**

The Opening Problem: Treasure Island

The two largest palm trees on the island serve as the markers for the treasure. You must first start at the secret location. Count your paces to the western most palm tree. Once you arrive at the tree take a sharp right turn and walk the same number of paces. Mark this spot. Return to the secret location and walk to the eastern most palm tree, again counting your paces. Take a sharp left turn and walk the same number of paces and the mark this spot. The treasure is located directly in the middle of the two spots that you have marked. Good luck.

Does the location of the treasure depend on the secret location?

The Island



FoM Summer Institute: "Transformation and Number"

- ▶ **The Origin** Developed in Spring 2011 by
 - ▶ 9 MS/HS 'master teachers' (Math for America-Boston and Boston Univ. Mathematics Teaching Scholars)
 - ▶ + 4 math educators/mathematicians (from BU, Harvard)as part of a new "Mathematical Practices Seminar"

- ▶ **The Overview** Co-led by two HS teachers in August 2011
4 days, each with
 - ▶ Problem Sets (series of scaffolded questions)
 - ▶ Geometer's Sketchpad Lab Exploration
 - ▶ Summary of Mathematics from Day
 - ▶ **Reflection on "problem-solving strategies/mathematical habits/practices"**

Articulating Mathematical Practices: Day 1 and 2

- ▶ Day 1:
 - ▶ **Working together/Mathematical discussion:**
'Draw, graph and talk'
 - ▶ **Trial and error**
 - ▶ **From concrete to abstract:**
'Start w concrete (start with numbers and then generalize with variables)'
- ▶ Day 2: Highly problem-specific, for eg:
'translate to origin, rotate and translate back'

Articulating Mathematical Practices: Day 3

- ▶ More generalization of the specific strategies, including:
 - ▶ **Experiment First:** *'simplify problems by adding constraints', 'use specific numbers then generalize', 'chunking, breaking down into manageable parts'*
 - ▶ **Collaboration helps:** *'divide and conquer', 'keep us from quitting', '[your work] reminds someone else of something'*
 - ▶ **Learn from prior knowledge:** *'firm up understanding of each step before going onto new step, stop and summarize'*
 - ▶ **Use alternate representations:** *'look at geometric/algebraic representation'*

Articulating Mathematical Practices: Day 4

► Our Strategies and the Standards for Mathematical Practice

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“What examples of these practices have we seen in action?”

Back to the Classroom: “The Kids”

In three groups (MS, HS Geometry, Advanced HS):

*How would you **scaffold the opening problem** so your students could develop the tools to solve it?*

*Which **content standards** would arise in this context?*

*Which **practice standards** would arise in this context and **why**?*

Designing Professional Development around Mathematical Practices: Some Principles

- ▶ Focused on Mathematical Content (familiar and less familiar)
- ▶ Extended investigation into the Mathematics
- ▶ Explicit Reflection on Practices (as well as the Math) throughout
- ▶ Translation Back to the Classroom
 - ▶ During: look at student work, textbooks, video...
 - ▶ Follow-up: school-year workshop, classroom visits, ...

Thanks to...

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Chelsea MS
Lawrence HS
Watertown HS
Framingham HS
Lawrence HS
City on a Hill HS

Mathematicians/Math Educators

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Steve Rosenberg
Glenn Stevens

Math for America, Boston
BU Math Dept.
BU Math Dept.

Questions?

- ▶ **Common Core State Standards**
www.corestandards.org
- ▶ **The 'Focus on Math' Math Science Partnership**
www.focusonmath.org
- ▶ **Contact: jbelding@math.harvard.edu**