An Abductive Reasoning Approach to the Belief-Bias Effect

Luís Moniz Pereira[1] **Emmanuelle Dietz**[1,2] **Steffen Hölldobler**[2]

[1]Centro de Inteligência Artificial (CENTRIA),
Universidade Nova de Lisboa, Portugal

[2]International Center for Computational Logic (ICCL)
TU Dresden, Germany

A Syllogistic Reasoning Task by Evans, Barston, and Pollard [1983]

Let's consider S_{rich}

PREMISE 1	*No millionaires are hard workers.*
PREMISE 2	*Some rich people are hard workers.*
CONCLUSION	*Therefore, some millionaires are not rich people.*

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Evans, Barston, and Pollard [1983] call this phenomenon the belief-bias effect

> It occurs when we think to judge something based on our reasoning,
> but are actually influenced by our beliefs and our prior knowledge.

Motivation

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- logic programs
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Can we also adequately model the syllogistic reasoning task under weak completion semantics?

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Stenning & van Lambalgen (2008) propose to model conditionals with *ab* predicates.

PREMISE 1 of S_{add} can be represented as

If something is inexpensive and not abnormal, then it is not addictive. (3)
Nothing (as a rule) is abnormal (wrt (3)).

The belief in (1) and (2) generates an exception for cigarettes

If something is a cigarette, then it is abnormal (wrt (3)). (4)

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1. we know that there are addictive things, let's say *b*.

   b is addictive.

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   b is addictive.

2. Given PREMISE 1 we infer that these addictive things are not inexpensive.

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3. By the background knowledge generated by PREMISE 2, we abduce, because *b* is not inexpensive, that these cannot be cigarettes.

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Rich people are hard workers (compared to other milionaires). (2)

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Premise 1 of S_{rich} would then be represented as

If someone is a hard worker and not abnormal, then this person is not a milionaire. (3)
Nobody is abnormal (wrt (3)).

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If someone is rich, then this person is abnormal (wrt (3)). (4)

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Even though not tested yet, our hypothesis is, while checking S_{rich}, participants did not make these assumptions and thus, had not been influenced by the belief-bias effect.

Thank you very much for your attention!

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Logic Programs

We restrict ourselves to datalog programs. A logic program $\mathcal{P}$ is a finite set of clauses

$$A \leftarrow A_1 \wedge \ldots \wedge A_n \wedge \neg B_1 \wedge \ldots \wedge \neg B_m, \tag{1}$$

$$A \leftarrow \perp, \tag{2}$$

- where A and A_i, $0 \leq i \leq n$, are atoms and $\neg B_j$, $1 \leq j \leq m$, are negated atoms.
- If $i = 0$, then we write $A \leftarrow \top$, which is called a positive fact.
- A clause of the form (2) is called a negative fact.
- A is undefined if it is not the head of any clause.
- $\mathrm{g}\,\mathcal{P}$ denotes ground $\mathcal{P}$, that is, it contains all ground instances of its clauses.
- $\mathrm{undef}(\mathcal{P})$ is the set of all undefined atoms in $\mathrm{g}\,\mathcal{P}$.

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The following transformation is the weak completion of $\mathcal{P}$

1. Replace all clauses in $\mathrm{g}\,\mathcal{P}$ with the same head $A \leftarrow body_1, \ldots, A \leftarrow body_n$ by the single expression $A \leftarrow body_1 \vee \ldots \vee body_n$.
2. Replace all occurrences of $\leftarrow$ by $\leftrightarrow$.

Three-Valued Łukasiewicz Logic

	$\neg$
$\top$	$\bot$
$\bot$	$\top$
U	U

$\wedge$	$\top$	U	$\bot$
$\top$	$\top$	U	$\bot$
U	U	U	$\bot$
$\bot$	$\bot$	$\bot$	$\bot$

$\vee$	$\top$	U	$\bot$
$\top$	$\top$	$\top$	$\top$
U	$\top$	U	U
$\bot$	$\top$	U	$\bot$

$\leftarrow_L$	$\top$	U	$\bot$
$\top$	$\top$	$\top$	$\top$
U	U	$\top$	$\top$
$\bot$	$\bot$	U	$\top$

$\leftrightarrow_L$	$\top$	U	$\bot$
$\top$	$\top$	U	$\bot$
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Table: $\top$, $\bot$, and U denote *true*, *false*, and *unknown*, respectively.

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An interpretation I of g$\mathcal{P}$ is a mapping of the Herbrand base $\mathcal{B}_\mathcal{P}$ to $\{\top, \bot, \mathsf{U}\}$ and is represented by an unique pair, $\langle I^\top, I^\bot \rangle$, where

$$I^\top = \{A \in \mathcal{B}_\mathcal{P} \mid A \text{ is mapped to } \top\} \text{ and } I^\bot = \{A \in \mathcal{B}_\mathcal{P} \mid A \text{ is mapped to } \bot\}.$$

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- For every I it holds that $I^\top \cap I^\bot = \emptyset$.
- A model of a formula F is an interpretation I such that F is true under I.
- A model of g$\mathcal{P}$ is an interpretation that is a model of each clause in g$\mathcal{P}$.

Computing Least Models

Hölldobler and Kencana Ramli [2009] propose to compute the least model of the weak completion of $\mathcal{P}$ ($\mathrm{lm}_{Ł}\mathrm{wc}\,\mathcal{P}$) which is identical to the least fixed point of $\Phi_{\mathcal{P}}$, by an operator defined by Stenning and van Lambalgen [2008].

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In (Dietz, Hölldobler, and Wernhard [2014]) we show that weak completion semantics corresponds to well-founded semantics for modified tight logic programs.

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- $\mathcal{E}$ is an explanation and a consistent subset of $\mathcal{A}$,
- logical consequence relation $\models_{Ł}^{\mathsf{lmwc}}$, where $\mathcal{P} \models_{Ł}^{\mathsf{lmwc}} F$ iff $\mathrm{lm}_{Ł}\mathrm{wc}\,\mathcal{P}(F) = \top$, and
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F **follows skeptically from** $\mathcal{P}$**,and** $\mathcal{O}$ iff $\mathcal{O}$ can be explained given $\mathcal{P}$, and for all minimal explanations $\mathcal{E}$ we find that $\mathcal{P} \cup \mathcal{E} \models_{Ł}^{\mathrm{lmwc}} \mathcal{O}$.

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Abducing the CONCLUSION

Given our background knowledge we know, there are addictive things, let's say about b

$$\mathcal{O}_{add(b)} = \{add(b)\}$$

We have two minimal explanations for $\mathcal{O}_{add(b)}$

$$\begin{array}{lcll} \mathrm{Im}_{Ł}\mathrm{wc}\,(\mathcal{P}_{add} \cup \mathcal{E}_{cig(b)}) & = & \langle\{add(b), cig(b), inex(b), ...\}, & \{...\}\rangle \\ \mathrm{Im}_{Ł}\mathrm{wc}\,(\mathcal{P}_{add} \cup \mathcal{E}_{\neg cig(b)}) & = & \langle\{add(b), ...\}, & \{cig(b), inex(b), ...\}\rangle \end{array}$$

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\end{aligned}$$

Recall $\mathcal{P}_{add}$. Together with $\mathcal{E}_{cig(b)}$ it contains

$$\begin{aligned}
add'(X) &\leftarrow inex(X) \wedge \neg ab_{add'}(X), & add(X) &\leftarrow \neg add'(X), \\
inex(X) &\leftarrow cig(X) \wedge \neg ab_{inex}(X), & ab_{add'}(X) &\leftarrow cig(X), \\
ab_{add'}(X) &\leftarrow \bot, & ab_{inex}(X) &\leftarrow \bot, \\
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Credulously, we validate some addictive things are not cigarettes.

Contextual Abductive Reasoning

How to express that PREMISE 1 describes the usual and PREMISE 2 the exceptional case? Inexpensive cigarette should be the exception in the context of addictive things.

Contextual Abductive Reasoning

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Introduce for every A, two reserved (meta-) predicates (Pereira and Pinto [2011]).

$$\text{inspect}(A) \qquad \text{and} \qquad \text{inspect}_{\text{not}}(A)$$

These are special abducibles only to be abduced if A or $\neg A$ are abduced somewhere else already.

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$$\begin{array}{rclrcl}
add'(X) & \leftarrow & inex(X) \wedge \neg ab_{add'}(X), & add(X) & \leftarrow & \neg add'(X), \\
inex(X) & \leftarrow & cig(X) \wedge \neg ab_{inex}(X), & ab_{add'}(X) & \leftarrow & \text{inspect}(cig(X)), \\
ab_{add'}(X) & \leftarrow & \bot, & ab_{inex}(X) & \leftarrow & \bot,
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Suppose again b is addictive , i.e. $\mathcal{O}_{add(b)} = \{add(b)\}$. $\mathcal{E}_{cig} = \{cig(b) \leftarrow \top\}$ cannot be abduced anymore to explain $\mathcal{O}_{add(b)}$. Its only minimal explanation is $\mathcal{E}_{\neg cig(b)}$.

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Skeptically, we validate some addictive things are not cigarettes.

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Suppose that we observe that b is addictive and inexpensive

$$\mathcal{O}_{add,inex} \quad = \quad \{add(b), inex(b)\}$$

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- Because b is inexpensive, $cig(b) \leftarrow \top$ can be directly abduced to explain $inex(b)$.
- As $inex(b)$ is explained, $\text{inspect}(cig(b)) \leftarrow \top$ can be abduced to explain $add(b)$.

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Recall the two clauses in $g\,\mathcal{P}^{\text{cons}}_{add_{\text{insp}}}$ representing PREMISE 1 are

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