Cryptographic Hash Functions
Message Authentication
Digital Signatures

Abstract

We will discuss

- Cryptographic hash functions
- Message authentication codes
  - HMAC and CBC-MAC
- Digital signatures

Encryption/Decryption

- Provides message confidentiality.
- Does it provide message authentication?

Message Authentication

- Bob receives a message *m* from Alice, he wants to know
  - (Data origin authentication) whether the message was really sent by Alice;
  - (Data integrity) whether the message has been modified.
- Solutions:
  - Alice attaches a message authentication code (MAC) to the message.
  - Or she attaches a digital signature to the message.

Hash function

- A hash function maps from a domain to a smaller range, typically many-to-one.
- Properties required of a hash function depend on its applications.
- Applications:
  - Fast lookup (hash tables)
  - Error detection/correction
  - Cryptography: cryptographic hash functions
  - Others

Cryptographic hash function

- Hash functions: $h: X \to Y, \quad |X| > |Y|.$
- For example, $h: \{0,1\}^* \to \{0,1\}^n$

  $h: \{0,1\}^* \to Z_n$

  $h: \{0,1\}^k \to \{0,1\}^l, \ k > l.$
- If X is finite, h is also called a compression function.
- A classical application: users/clients passwords are stored in a file

  not as $(\text{username, password})$,

  but as $(\text{username, } h(\text{password}))$ using some cryptographic hash function h.

Security requirements

- Pre-image: if $h(m) = y$, m is a pre-image of y.
- Each hash value typically has multiple pre-images.
- Collision: a pair of (m, m'), $m \neq m'$, s.t. $h(m) = h(m')$.

A hash function is said to be:

- Pre-image resistant if it is computationally infeasible to find a pre-image of a hash value.
- Collision resistant if it is computationally infeasible to find a collision.
- A hash function is a cryptographic hash function if it is collision resistant.

- Collision-resistant hash functions can be built from collision-resistant compression functions using Merkle-Damgard construction.

Merkle-Damgard construction

- Construct a cryptographic hash function $h:\{0,1\}^* \to \{0,1\}^n$ from a compression function $f:\{0,1\}^{n+b} \to \{0,1\}^n$.

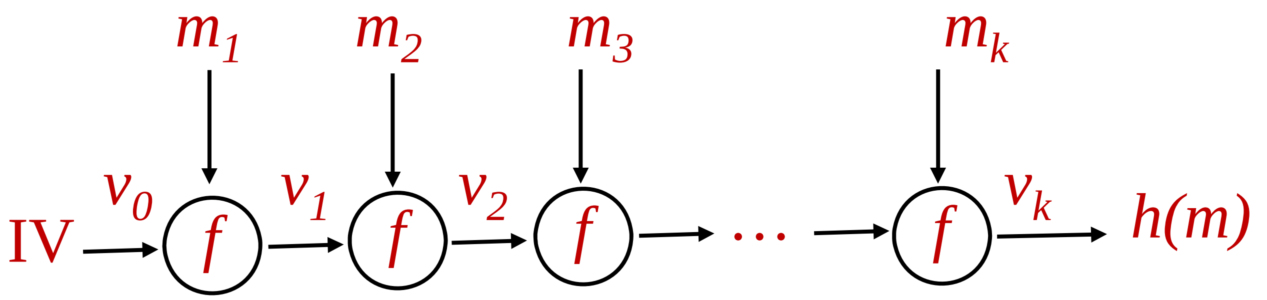
1. For $m \in \{0,1\}^*$, add padding to m so that $|m'|$ is a multiple of b.
   Let padded $m' = m_1 m_2 \ldots m_k$, each m_i of length b.
   (padding = 10...0 $|m|$, where $|m|$ is the length of m)

3. Let $v_0 = \text{IV}$ and $v_i = f(v_{i-1} \,||\, m_i)$ for $1 \le i \le k$.

4. The hash value $h(m) = v_k$.

Theorem. If f is collision-resistant, then h is collision-resistant.

Merkle-Damgard Construction



Compression function $f : \{0,1\}^{n+b} \rightarrow \{0,1\}^{n}$

The Secure Hash Algorithm (SHA-1)

- an NIST standard.
- using Merkle-Damgard construction.
- input message m is divided into blocks with padding.
- padding = 10...0ℓ, where $\ell \in \{0,1\}^{64}$ indicates $|m|$ in binary.
- thus, message length limited to $|m| \le 2^{64} - 1$.
- block = 512 bits = 16 words = $W_0 \| \ldots \| W_{15}$.
- IV = a constant of 160 bits = 5 words = $H_0 \| \ldots \| H_4$.
- resulting hash value: 160 bits.
- underlying compression function $f : \{0,1\}^{160+512} \to \{0,1\}^{160}$, a series (80 rounds) of $\wedge$, $\vee$, $\oplus$, $\neg$, +, and Rotate on words W_i's & H_i's.

Is SHA-1 secure?

- An attack is to produce a collision.
- Birthday attack: randomly generate a set of messages $\{m_1,\ m_2,\ \ldots,\ m_k\}$, hoping to produce a collision.
- $n = 160$ is big enough to resist birthday attacks for now.
- There is no mathematical proof for its collision resistancy.
- In 2004, a collision for a "58 rounds" SHA-1 was produced. (The compression function of SHA-1 has 80 rounds.)
- Newer SHA's have been included in the standard: SHA-256, SHA-384, SHA-512.

- Birthday problem: In a group of k people, what is the probability that at least two people have the same birthday?
  - Having the same birthday is a collision?
- Birthday paradox: $p \geq 1/2$ with k as small as 23.

- Consider a hash function $h:\{0,1\}^* \rightarrow \{0,1\}^n$.
- If we randomly generate k messages, the probability of having a collision depends on n.
- To resist birthday attack, we choose n to be sufficiently large that it will take an infeasibly large k to have a non-negligible probability of collision.

Applications of cryptographic hash functions

- Storing passwords
- Used to produce modification detection codes (MDC)
  - $h(m)$, called an MDC, is stored in a secure place;
  - if m is modified, we can detect it;
  - protecting the integrity of m.
- We will see some other applications.

Message Authentication

- Bob receives a message *m* from Alice, he wants to know
  - (Data origin authentication) whether the message was really sent by Alice;
  - (Data integrity) whether the message has been modified.
- Solutions:
  - Alice attaches a message authentication code (MAC) to the message.
  - Or she attaches a digital signature to the message.

MAC

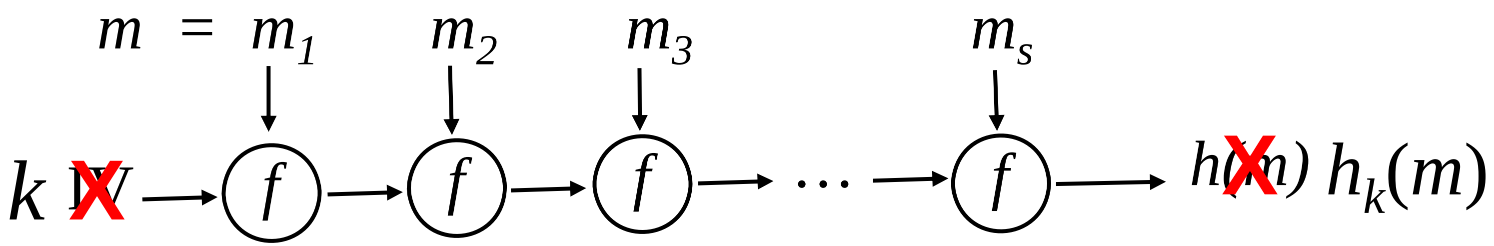
- Message authentication protocol:
  1. Alice and Bob share a secret key k.
  2. Alice sends $m \| \text{MAC}_k(m)$ to Bob.
  3. Bob authenticates the received $m' \| \text{MAC}'$ by checking if $\text{MAC}' = \text{MAC}_k(m')$?
- $\text{MAC}_k(m)$ is called a message authentication code.
- Security requirement: infeasible to produce a valid pair $(x, \text{MAC}_k(x))$ without knowing the key k.

Constructing MAC from a hash

- A common way to construct a MAC is to incorporate a secret key k into a fixed hash function h (e.g. SHA-1).
-
    - $\mathrm{MAC}_k(m) = h_k(m) = h(m)$ with $\mathrm{IV} = k$
    - $\mathrm{MAC}_k(m) = h_k(m) = h(k \,||\, m)$

- Insecure: $MAC_k(m) = h(m)$ with IV $= k$.

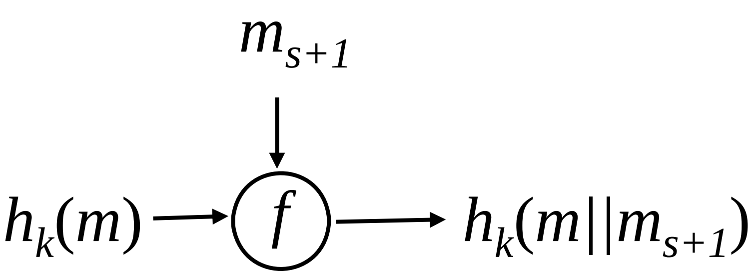
  (For simplicity, without padding)



- Easy to forge:

  $(m',\ h_k(m'))$,

  where $m' = m \,||\, m_{s+1}$



HMAC (Hash-based MAC)

- A FIPS standard for constructing MAC from a hash function h. Conceptually,

$$\text{HMAC}_k(m) = h\left(k_2 \,\|\, h(k_1 \,\|\, m)\right)$$

where k_1 and k_2 are two keys generated from k.

- Various hash functions (e.g., SHA-1, MD5) may be used for h.
- If we use SHA-1, then HMAC is as follows:

$$\text{HMAC}_k(m) = \text{SHA-1}\left(k \oplus opad \,\|\, \text{SHA-1}(k \oplus ipad \,\|\, m)\right)$$

where

  - k is padded with 0's to 512 bits
  - $ipad = 3636\cdots36$ (x036 repeated 64 times)
  - $opad = 5\text{c}5\text{c}\cdots5\text{c}$ (x05c repeated 64 times)

CBC-MAC

- A FIPS and ISO standard.
- One of the most popular MACs in use.
- Use a block cipher in CBC mode with a fixed, public IV.
- Called DES CBC-MAC if the block cipher is DES.
- Let $E : \{0,1\}^n \rightarrow \{0,1\}^n$ be a block cipher.
- CBC-MAC(m, k)

$m = m_1 \,||\, m_2 \,||\, \ldots \,||\, m_l$, where $|m_i| = n$.

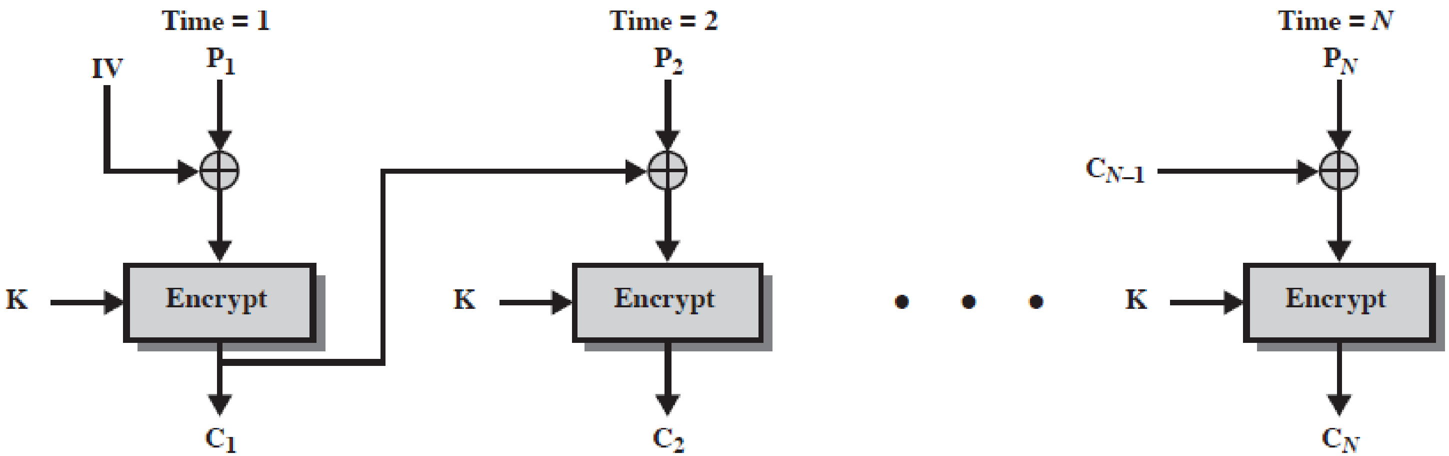
$c_0 \leftarrow$ IV (typically 0^n)

for $i \leftarrow 1$ to l do

$c_i \leftarrow E_k(c_{i-1} \oplus m_i)$

return(c_l)

Cipher Block Chaining (CBC)



(a) Encryption

CMAC (Cipher-based MAC)

- A refined version of CBC-MAC.
- Adopted by NIST for use with AES and 3DES.
- Use two keys: k, k' (assuming $|m|$ is a multiple of n).
- Let $E:\{0,1\}^n \rightarrow \{0,1\}^n$ be a block cipher.
- $\text{CMAC}(m, k)$

$$m = m_1 \,||\, m_2 \,||\, \ldots \,||\, m_l, \text{ where } |m_i| = n.$$

$$c_0 \leftarrow \text{IV (typically } 0^n)$$

$$\text{for } i \leftarrow 1 \text{ to } l-1 \text{ do}$$

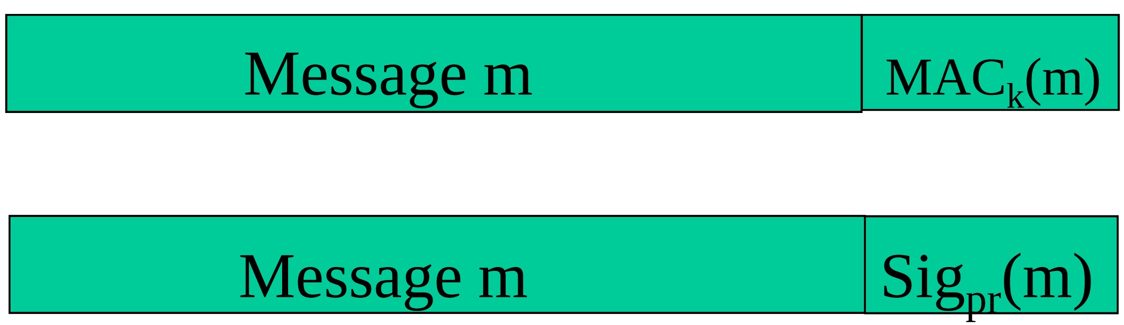
$$c_i \leftarrow E_k(c_{i-1} \oplus m_i)$$

$$c_l \leftarrow E_k(c_{l-1} \oplus m_l \qquad)$$

$$\text{return}(c_l)$$

Digital Signatures

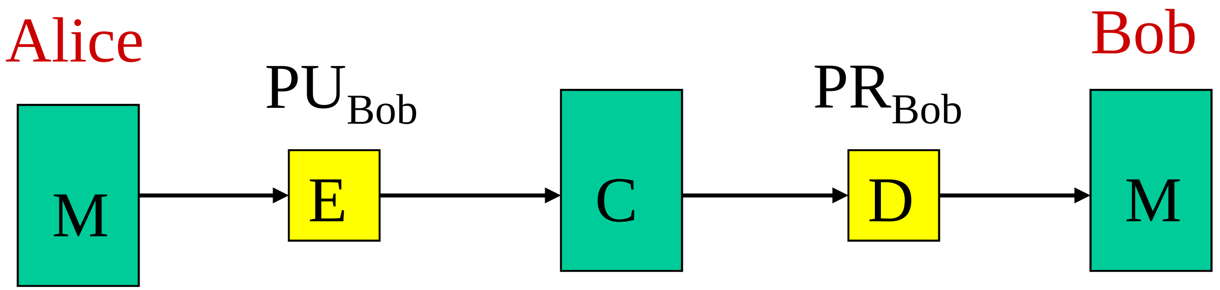
- RSA can be used for digital signatures.
- A digital signature is the same as a MAC except that the tag (signature) is produced using a public-key cryptosystem.
- Digital signatures are used to provide message authentication and non-repudiation.



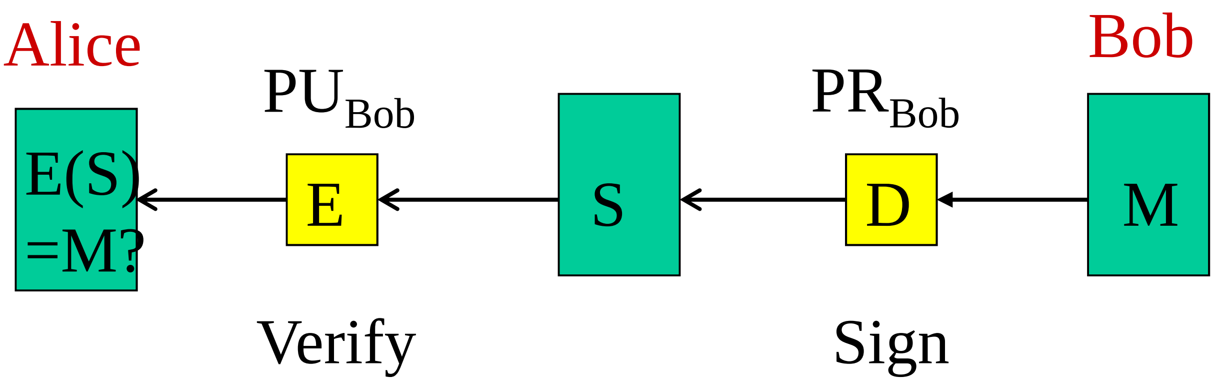
- Digital signature protocol:
  1. Bob has a key pair (pr, pu).
  2. Bob sends $m \,||\, \mathrm{Sig}_{pr}(m)$ to Alice.
  3. Alice verifies the received $m' \,||\, s'$ by checking if $s' = \mathrm{Verify}_{pu}(m')$.

- $\mathrm{Sig}_{pr}(m)$ is called a signature for m.
- Security requirement: infeasible to forge a valid pair $(m,\ \mathrm{Sig}_{pr}(m))$ without knowing pr.

Encryption (using RSA):



Digital signature (using RSA^{-1}):



RSA Signature

- Keys are generated as for RSA encryption:

  Public key: $PU = (n, e)$. Private key: $PR = (n, d)$.

- Signing a message $m \in Z_n^*$: $\sigma = D_{PR}(m) = m^d \bmod n$.

  That is, $\sigma = \mathrm{RSA}^{-1}(m)$.

- Verifying a signature (m, σ):

  check if $m = E_{PU}(\sigma) = \sigma^e \bmod n$, or $m = \mathrm{RSA}(\sigma)$.

- Only the key's owner can sign, but anybody can verify.

Security of RSA Signature

- Existential forgeries:

  1. Every message $m \in Z_n^*$ is a valid signature for its ciphertext $c := \mathrm{RSA}(m)$.

     Encryption (using Bob's public key): $m \xrightarrow{\mathrm{RSA}} c$

     Sign (if using Bob's private key): $m \xleftarrow{\mathrm{RSA}^{-1}} c$

  2. If Bob signed m_1 and m_2, then the signature for $m_1 m_2$ can be easily forged: $\sigma(m_1 m_2) = \sigma(m_1)\sigma(m_2)$.

- Countermeasure: hash and sign: $\sigma = \mathrm{Sign}_{PR}(h(m))$, using some collision resistant hash function h.

- Question:

  Does hash-then-sign make RSA signature secure against all chosen-message attacks?

- Answer:

  Yes, if h is a full-domain random oracle, i.e.,

  - h is a random oracle mapping $\{0,1\}^* \rightarrow Z_n$
  - (Z_n is the full domain of RSA)

- Problem with full-domain hash:

  In practice, h is not full-domain.

  For instance, the range of SHA-1 is $\{0,1\}^{160}$,

  while $Z_n = \{0, 1, \ldots, 2^n - 1\}$, with $n \geq 1024$.

- Desired: a secure signature scheme that does not require a full-domain hash.

Probabilistic signature scheme

- Hash function $h : \{0,1\}^* \rightarrow \{0,1\}^l \subset Z_N$ (not full domain).

  $l < n = |N|$. (E.g., SHA-1, $l = 160$; RSA, $n = 1024$.)

- Idea: $m \xrightarrow{\text{pad}} m \,||\, r \quad \in \{0,1\}^*$

$$\xrightarrow{\text{hash}} w = h(m \,||\, r) \quad \in \{0,1\}^l$$

$$\xrightarrow{\text{expand}} y = w \,||\, (r \,||\, 0^{n-1-l-k}) \oplus G(w) \quad \in \{0,1\}^{n-1}$$

$$\xrightarrow{\text{sign}} \sigma = \text{RSA}^{-1}(y) \quad \in Z_N$$

where $r \in \{0,1\}^k$

$G : \{0,1\}^l \rightarrow \{0,1\}^{n-1-l}$ (pseudorandom generator)

- Signing a message $m \in \{0,1\}^*$:

1. choose a random $r \in \{0,1\}^k$; compute $w = h(m \| r)$;

2. compute $y = w \| r \oplus G_1(w) \| G_2(w)$; // $G = G_1 \| G_2$ //

3. The signature is $\sigma = \mathrm{RSA}^{-1}(y)$.

Remarks

- PSS is secure against chosen-message attacks in the random oracle model (i.e., if h and G are random oracles).
- PSS is adopted in PKCS #1 v.2.1.
- Hash functions such as SHA-1 are used for h and G.
- For instance,

  let $n = 1024$, and $l = k = 160$

  let h = SHA-1

  $(G_1, G_2)(w) = G(w) = h(w \| 0) \;\|\; h(w \| 1) \;\|\; h(w \| 2), \ldots$