

# Generalized Utility Metrics for Supercomputers

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- The TOP500 is often criticized for its simplicity.
- What influences the usefulness of our Supercomputers these days?
  - Performance (P)
  - Memory size (M)
  - Power Consumption (Power)
  - Floor Space (maybe later)
  - \$: Best, but way too ‘touchy’!
- Nobody can select an appropriate performance measure for you!

- We want an “*extensive*” “Utility Metric” for Supercomputers to rank them.
- We should ask ourselves:  
*When is a Supercomputer  $x$  times as powerful as my current one?*
  - Let’s focus on:
    - Performance (P)
      - (Computational, I/O, etc)
    - Memory size (M)
      - Disk size etc
    - Power Consumption (Power)
    - Floor Space (Space)

- Let's construct a Utility Metric (UM).
- The influence of these extra factors depend on your situation.
  - Include Weights for them.
  - The size of these weights will (to some extent) depend on you.
- Relative ranks have to independent of scales:

$$UM(a * x) = f(a) * UM(x)$$

- Simplest functional form is a weighted geometric mean:

$$UM(x_i) = \prod x_i^{\alpha_i}$$

- We want something which scales in first order like *performance* (extensive) with corrections from the other features (intensive):

$$UM(SC) = P^\alpha * \left( \frac{P}{Peak} \right)^\beta * \left( \frac{M}{Peak} \right)^\gamma * \left( \frac{Peak}{Power} \right)^\delta * \left( \frac{Peak}{Space} \right)^\epsilon$$

$$\alpha, \beta, \gamma, \delta, \epsilon \geq 0$$

- **Scale Peak is arbitrary and UM has to be independent from it:**

$$\beta + \gamma = \delta + \epsilon$$

$$UM(SC) = P^\alpha * \left(\frac{M}{P}\right)^\gamma * \left(\frac{P}{Power}\right)^{\delta+\epsilon} * \left(\frac{Power}{Space}\right)^\epsilon$$

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$P$	Sustained Performance	$GFlop/s$	$\alpha$
$\frac{M}{P}$	Effective Byte/Flop Ratio	$\frac{GByte}{GFlop/s}$	$\gamma$
$\frac{P}{Power}$	Power-efficiency	$\frac{GFlop/s}{kW}$	$\delta$
$\frac{Power}{Space}$	Power-density	$\frac{kW}{m^2}$	$\epsilon$

Which system upgrade doubles your utility?

**A:**  $2^*P$ ,  $2^*M$ ,  $2^*Power$  ('double system')

**B:**  $2^*P$ ,  $2^*M$ ,  $1^*Power$  (Capability based – larger problems)

**C:**  $2^*P$ ,  $1^*M$ ,  $1^*Power$  (Capacity based – same problems)

**D:**  $2^*P$ ,  $2^k *M$ ,  $1^*Power$  (Workload based – same time)

– The largest jobs possible complete in **equal times**.

–  $k$  represents workload scaling

– Linpack:  $M \sim N^2$ ,  $Flop \sim N^3$

•  $k=2/3$ :  $2^{2/3} *M = 1.59 *M$

–  $0 \leq k \leq 1$ :

$k=0$ : C, capacity;  $k=1$ : B, 'extreme' capability

- Three (four) variables and (only) one condition means at least two (three) free parameters.

$$UM(SC) = P^\alpha * \left(\frac{M}{P}\right)^\gamma * \left(\frac{P}{Power}\right)^{\delta+\epsilon} * \left(\frac{Power}{Space}\right)^\epsilon$$

### Resulting conditions:

**A:** 2\*P, 2\*M, 2\*Power  $\Rightarrow \alpha = 1$

**B:** 2\*P, 2\*M, 1\*Power  $\Rightarrow \alpha + \delta + \epsilon = 1$

**C:** 2\*P, 1\*M, 1\*Power  $\Rightarrow \alpha + \delta + \epsilon - \gamma = 1$

**D:** 2\*P, 2<sup>k</sup>\*M, 1\*Power  $\Rightarrow \alpha + \delta + \epsilon - (1-k)\gamma = 1$

**D: Eliminate  $\alpha$ ; No 'space' (for now)  $\Rightarrow \epsilon=0$ ;**

$$UM_{ps}(SC) = P^{1-\delta+(1-k)\gamma} * \left(\frac{M}{P}\right)^\gamma * \left(\frac{P}{Power}\right)^\delta$$

- **For  $k=1$  (extreme capability):**
  - Memory exponent  $\gamma$  decouples!
  - Weighted geometric mean of Performance and Power-Efficiency.
- **For  $k=0$  (extreme capacity):**
  - Almost (but not quite!) a weighted geometric mean between three quantities with 2 free weights.

**Pick D:** What are the ‘values’ of other upgrades?

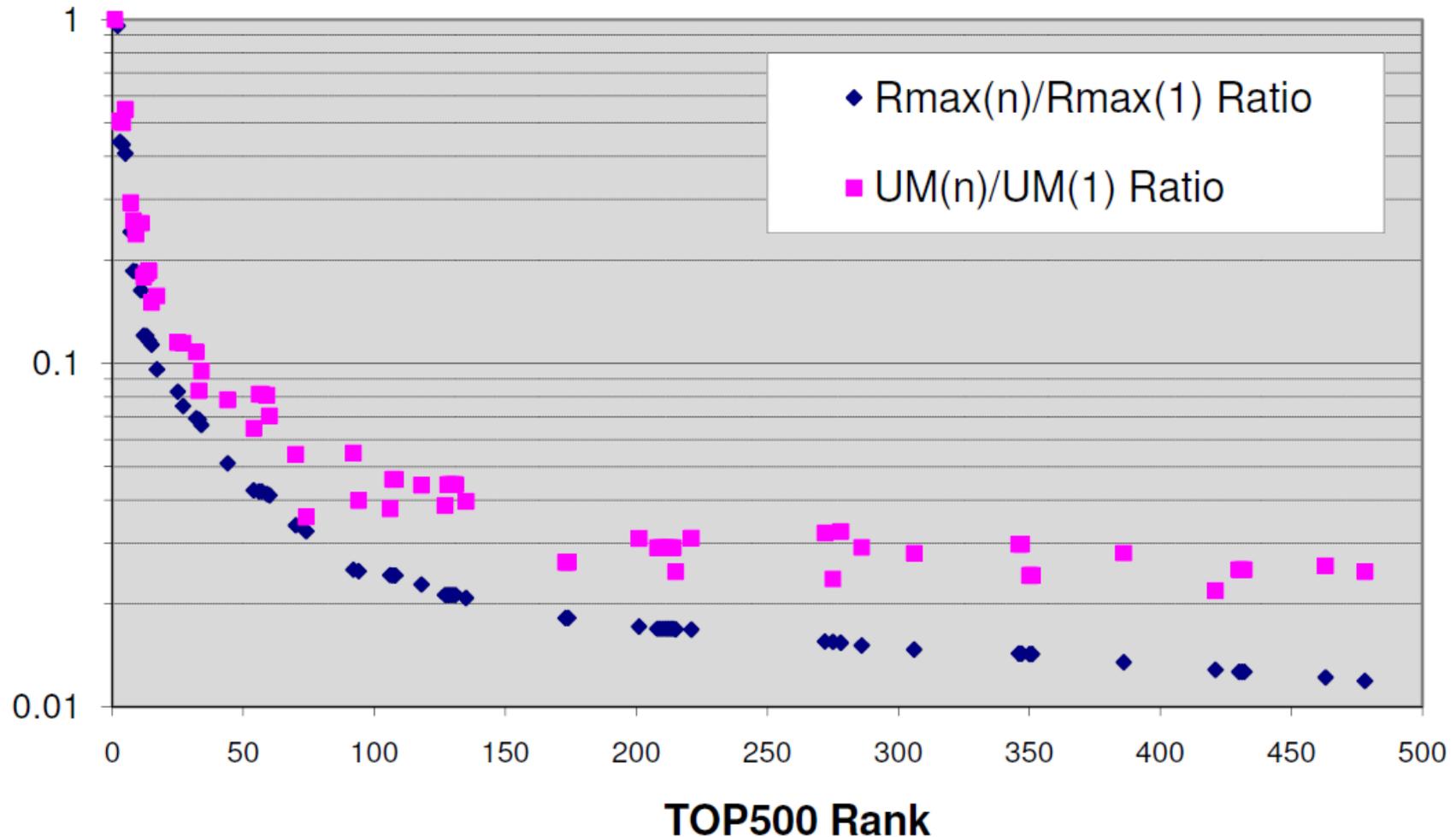
$$UM_{ps}(SC) = P^{1-\delta+(1-k)\gamma} * \left(\frac{M}{P}\right)^\gamma * \left(\frac{P}{Power}\right)^\delta$$

**A (Double system):** 2\*P, 2\*M, 2\*Power  $\Rightarrow 2^{1-\delta+(1-k)\gamma}$

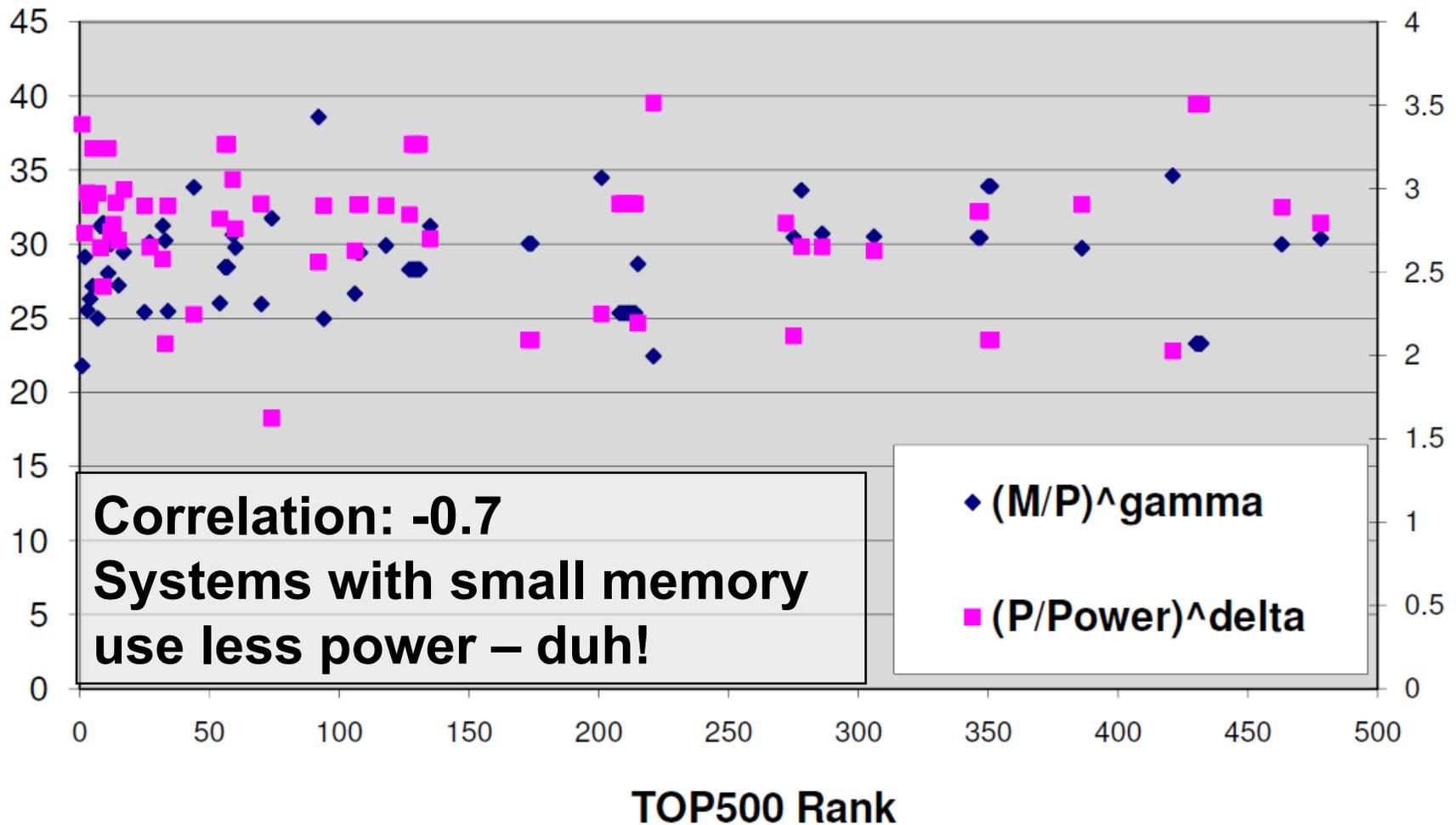
**B (Replace system):** 2\*P, 2\*M, 1\*Power  $\Rightarrow 2^{1+(1-k)\gamma} \geq 2$

**C (Upgrade processor):** 2\*P, 1\*M, 1\*Power  $\Rightarrow 2^{1-k\gamma} \leq 2$

$$k=2/3; \quad \gamma=0.2; \quad \delta=0.2; \quad \alpha=1-\delta+(1-k)\gamma=0.8667$$



$$k=2/3; \gamma=0.2; \delta=0.2; \alpha=1-\delta+(1-k)\gamma=0.8667$$



- **Maximum of UM(SC)**
  - **Power (and Space) grow linear with size**

$$UM(n * SC) = P(n)^\alpha * \left( \frac{M(n)}{P(n)} \right)^\gamma * \left( \frac{P(n)}{Power(n)} \right)^\delta$$

- **Introduce Speedup S:**  $S(n) = P(n)/P$
- **Differentiate:**  
d UM(n\*SC) / d n

$$\begin{aligned}
 UM(n * SC) &= \left( \frac{P(n)}{P} \right)^{\alpha - \gamma + \delta} * n^{\gamma - \delta} \\
 &* P^\alpha * \left( \frac{M}{P} \right)^\gamma * \left( \frac{P}{Power} \right)^\delta \\
 &= S(n)^{\alpha - \gamma + \delta} * n^{\gamma - \delta} * UM(SC)
 \end{aligned}$$

- **UM(n\*SC) is given by Speedup and weights.**

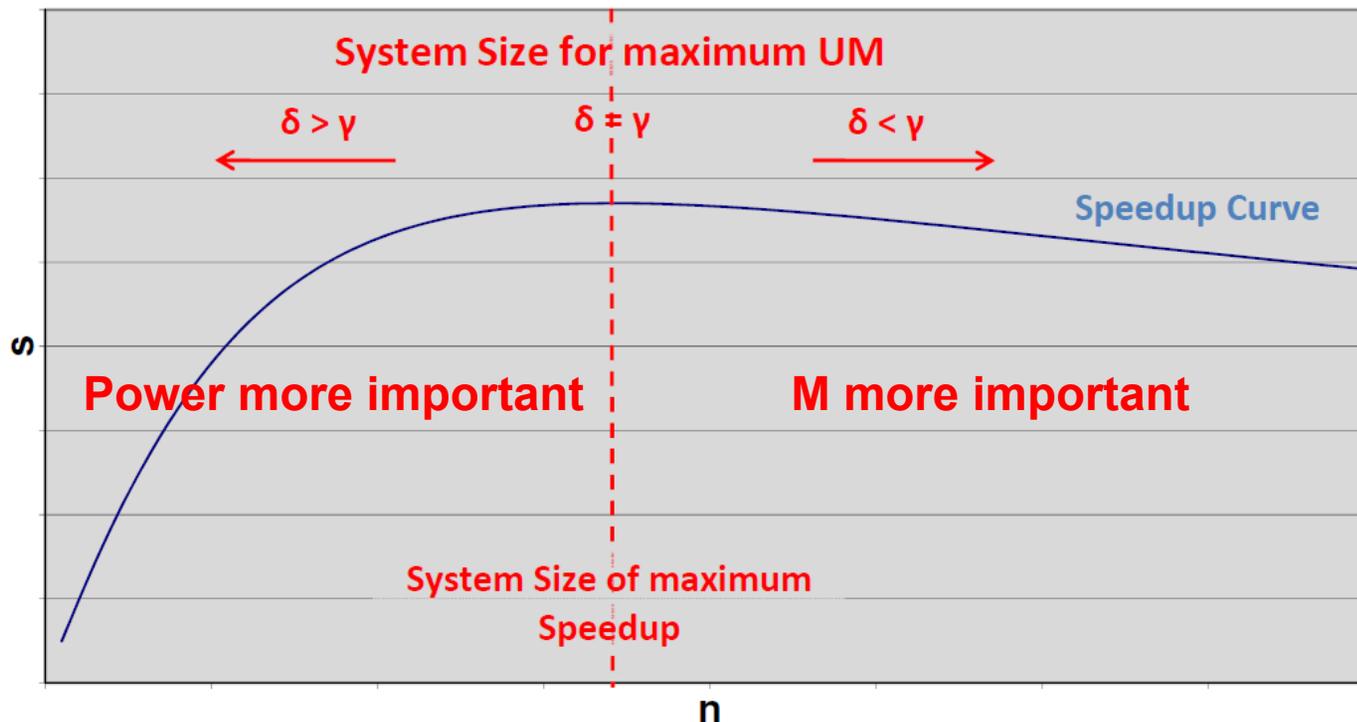
- We differentiate  $UM(n * SC)$

$$\frac{dUM(n * SC)}{dn} = \left( (\alpha - \gamma + \delta) \frac{\frac{dS(n)}{dn}}{S(n)} + (\gamma - \delta)/n \right) * UM(n * SC)$$

- Condition for optimum system size:

$$(\alpha - \gamma + \delta) * \frac{dS(n)}{dn} * \frac{n}{S(n)} = \delta - \gamma$$

Maximum of UM relative to Speedup



- **Utility Metrics (UM) for ranking Supercomputers can be defined to include effects of memory sizes, power consumptions, and other factors.**
  - You have to decide how important these factors are.
- **Definition of an appropriate Performance metric still critical.**
- **UM is naturally formulated with factors: Performance, Byte to Flop Ratio, Power efficiency, Power density.**
- **(Partial) Calibration of parameters possible by normalizing the increase-rate of UM.**