



Symmetry & Transformations

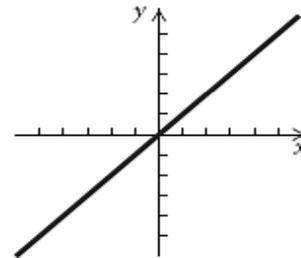
Transformation of Functions

- Recognize graphs of common functions
- Use vertical shifts to graph functions
- Use horizontal shifts to graph functions
- Use reflections to graph functions
- Use vertical stretching & shrinking to graph functions
- Use horizontal stretching & shrinking to graph functions
- Graph functions w/ sequence of transformations

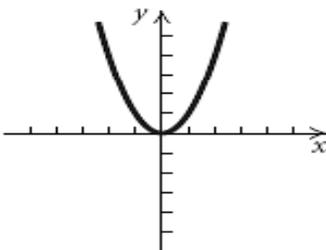
Basic Functions

- You should _____ the following basic functions. They are vital to understanding upper level math courses.
 - See the back section of your book, Algebra's Common Functions, for all of the basic functions you should _____. (See inside back cover and turn back one page.)

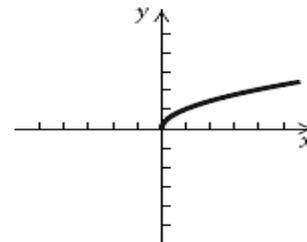
Identity function:
 $y = x$

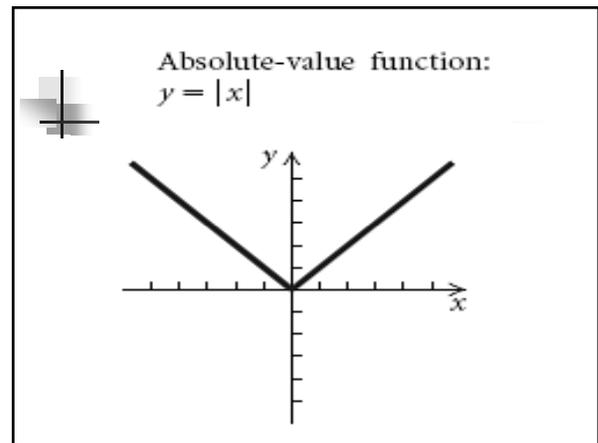
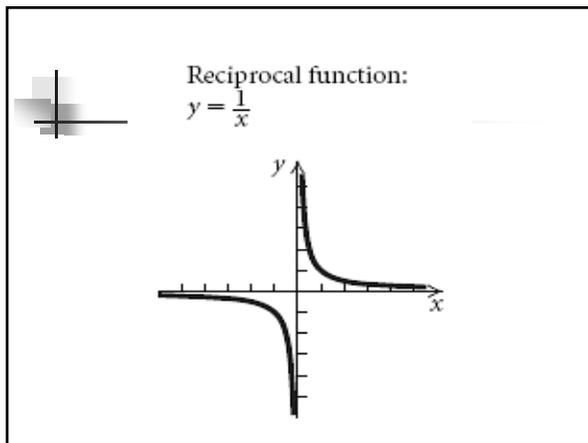
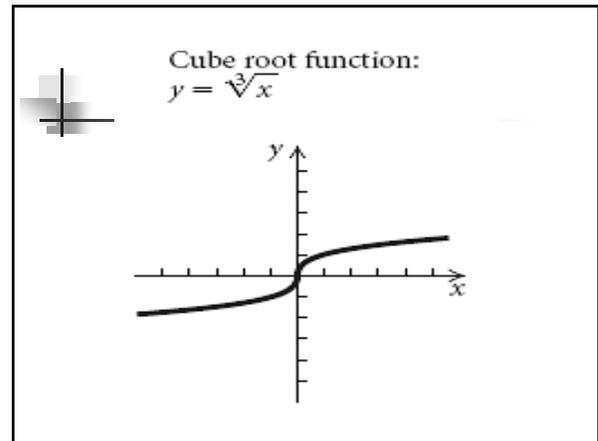
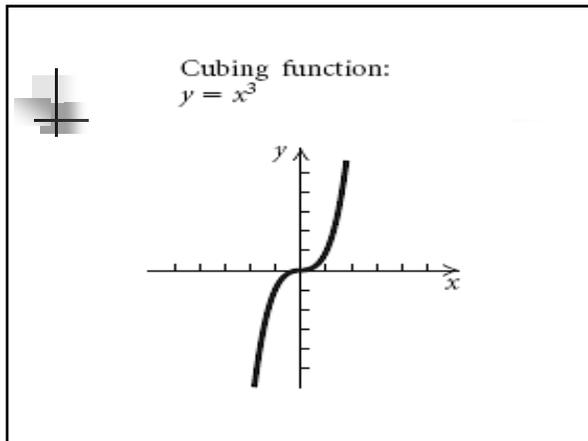


Squaring function:
 $y = x^2$



Square root function:
 $y = \sqrt{x}$





Types of Transformations

- **Vertical translation:** shifts basic graph up or down; _____ if $y = f(x) + b$ or _____ if $y = f(x) - b$
- **Horizontal translation:** shifts basic graph left or right; _____ if $y = f(x - d)$ or _____ if $y = f(x + d)$
- **Reflection:** crosses an axis; across x axis if $y = -f(x)$ & across y axis if $y = f(-x)$
- **Vertical stretching and shrinking:** $y = af(x)$
 _____ if $|a| > 1$; _____ if $0 < |a| < 1$
- **Horizontal stretching and shrinking:** $y = f(cx)$
 stretch if $0 < |c| < 1$; shrink if $|c| > 1$

- **Vertical shifts**
 - Moves the graph up or down
 - Impacts only the "y" values of the function
 - No changes are made to the "x" values
- **Horizontal shifts**
 - Moves the graph left or right
 - Impacts only the "x" values of the function
 - No changes are made to the "y" values

Recognizing the shift from the equation, look at examples of shifting the function $f(x)=x^2$.

- Vertical shift of 3 units up

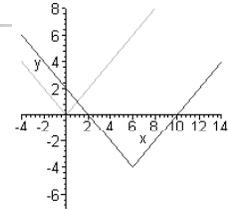
$$f(x) = x^2, h(x) = x^2 + 3$$

- Horizontal shift of 3 units left (HINT: x's go the _____ direction that you might think.)

$$f(x) = x^2, g(x) = (x + 3)^2$$

Combining a vertical & horizontal shift

- Example of function that is shifted down 4 units and right 6 units from the original function.



$$f(x) = |x|, g(x) = |x - 6| - 4$$

Types of Symmetry

Symmetry with respect to the

- y-axis** (x, y) & $(\text{---}, y)$ are reflections across the y-axis
- Origin** (x, y) & $(\text{---}, \text{---})$ are reflections across the origin
- x-axis** (x, y) & $(x, \text{---})$ are reflections across the x-axis

Tests of Symmetry

- $f(x) = f(-x)$ symmetric to _____
even function
- $-f(x) = f(-x)$ symmetric to _____
_____ function
- $f(x) = -f(x)$ symmetric to _____
neither even nor odd

Example – Determine any/all symmetry of $6x + 7y = 0$.

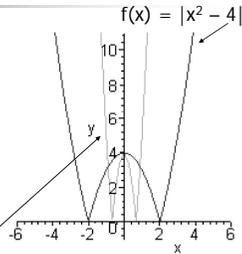
- a) y-axis
- b) Origin
- c) x-axis
- Is the function even, odd, or neither?

Example – Determine any/all symmetry of $f(x) = \sqrt{x^2 + 1}$

- d) y-axis
- e) origin
- f) x-axis
- Is the function even, odd, or neither?

Horizontal stretch & shrink

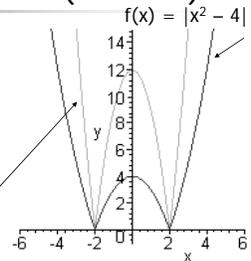
- We're MULTIPLYING by an integer (not 1 or 0).
- x's do the opposite of what we think they should. (If you see $3x$ in the equation where it used to be an x , you DIVIDE all x 's by 3, thus it's compressed or shrunk horizontally.)



$$g(x) = |(3x)^2 - 4|$$

VERTICAL STRETCH (SHRINK)

- y's do what we think they should: If you see $3(f(x))$, all y's are MULTIPLIED by 3 (it's now 3 times as high or low!)



$$h(x) = 3|x^2 - 4|$$

Sequence of transformations

- Follow the _____ of operations.
- Select two points (or more) from the original function and _____ that point one step at a time.

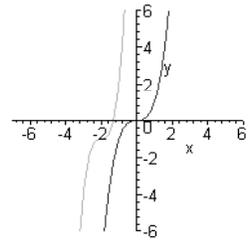
$$f(x) = x^3$$

$$3f(x+2) - 1 = 3(x+2)^3 - 1$$

Graph of Example

$$f(x) = x^3$$

$$g(x) = 3f(x+2) - 1 = 3(x+2)^3 - 1$$



Transformations with the Squaring Function

$$f(x) = x^2$$

Function

- $g(x) = x^2 + 4$
- $h(x) = x^2 - 5$
- $j(x) = (x - 3)^2$
- $k(x) = (x + 1)^2$
- $q(x) = -x^2$
- $r(x) = 2x^2$
- $s(x) = \frac{1}{4}x^2$
- $t(x) = (5x)^2$

Transformation

Your turn. Describe these transformations with the Absolute Value Function.

$$f(x) = |x|$$

Function

- $g(x) = -|x|$
- $h(x) = |2x|$
- $j(x) = 3|x|$
- $k(x) = |x + 4|$
- $q(x) = |x - 5|$
- $r(x) = |x| - 1$
- $s(x) = -\frac{1}{2}|x|$
- $t(x) = |x| + 2$

Transformation

Summary of Transformations

- See instructor webpage.