Logarithmic Functions

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Logarithm

If $b^y = x$, then $\log_b x = y$. The expression $\log_b x$ is read "the logarithm base b of x" or "log base b of x."

Example 1.

Write the following exponential equations as logarithmic equations.

(a) $2^4 = 16$

(b) $5^3 = 125$

(c) $81^{\frac{1}{2}} = 9$

(d) $4^{-3} = \dfrac{1}{64}$

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(d) $4^{-3} = \dfrac{1}{64}$

Solution.

(a) $\log_2 16 = 4$

(b) $\log_5 125 = 3$

(c) $\log_{81} 9 = \dfrac{1}{2}$

(d) $\log_4 \dfrac{1}{64} = -3$

Example 2.

Write the following logarithmic equations as exponential equations.

(a) $\log_{10} 1000 = 3$

(b) $\log_3 243 = 5$

(c) $\log_{27} 3 = \frac{1}{3}$

(d) $-3 = \log_2 \frac{1}{8}$

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Solution.

(a) $10^3 = 1000$

(b) $3^5 = 243$

(c) $27^{\frac{1}{3}} = 3$

(d) $2^{-3} = \frac{1}{8}$

Example 3.

Evaluate the following logarithms.

(a) $\log_5 25$

(b) $\log_{10} 10000$

(c) $\log_3 1$

(d) $\log_2 64$

(e) $\log_{36} \frac{1}{6}$

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Solution.

(a) $\log_5 25 = 2$

(b) $\log_{10} 10000 = 4$

(c) $\log_3 1 = 0$

(d) $\log_2 64 = 6$

(e) $\log_{36} \frac{1}{6} = -\frac{1}{2}$

Example 4.

Find the inverse function of $f(x) = 2^x$.

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Solution. Recall the inverse function interchanges the x and y values.

$$f(x) = 2^x$$

$$y = 2^x$$

$$x = 2^y$$

$$\log_2 x = y$$

$$f^{-1}(x) = \log_2 x$$

Example 5.a.

Graph the function $f(x) = 2^x$ and $g(x) = \log_2 x$.

Example 6.

Graph the function $f(x) = \log_3 x$. State the domain and range.

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x	$f(x)$
	-2
	-1
	0
	1
	2

Natural and Common Logarithms

The natural logaritm of x is denoted $\ln x$ and $\ln x = \log_e x$.

The common logaritm of x is denoted $\log x$ and $\log x = \log_{10} x$.

Example 7.

Graph the function $f(x) = \log x$. State the domain and range.

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x	$f(x)$
	-2
	-1
	0
	1
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x	$f(x)$
	-2
	-1
	0
	1
	2

Example 9.

Graph the function $f(x) = \log_2(x - 3)$. State the domain and range.

Example 10.

Graph the function $f(x) = \log_2 x - 3$. State the domain and range.