Sovereign default and public debt sustainability.

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Sustainability and default

The standard definition of sustainability : based on the transversality condition.

$\Longrightarrow$ Ruling out default.

Yet:

1. defaults exist.
2. there is life after default.

The need to go beyong the standard theory of sustainability.

In search for an empirically relevant notion of sustainability

Default, rule of default and new debt: the need to modify the theory of sustainability.

- *Default as proof of the unsustainability of current public debt*
- *Default is decided by lenders on the market.*
  $\Rightarrow$ *In search of a* ***market-based theory of public debt sustainability****.*
- *A public debt strategy is sustainable if, in the case of default, the post-default debt is expected to converge toward a stable steady-state level.*

The macroeconomics of default

- Here we ask:
  - How to take into account the risk of sovereign default in a "standard" macroeconomic model?
  - How to relate default to fiscal policy? The issue of (un-)sustainability.

The contribution of this paper

- A macromodel where the dynamics of public debt is studied when the possibility of default is incorporated,
- analytically solved and allowing
  - a formal redefinition of the sustainability condition,
  - an understanding of the interplay between financial markets and public debt issuer,
  - an understanding of the role of the risk premium and the refinancing in the dynamics of public debt,
  - an analysis of the macroeconomic impact of debt reduction after default.

The model

A simple macro model

- A representative agent:

$$U_0 = \sum_{t=0}^{+\infty} \beta^t \left[ln(c_t) - \delta^{-1} \ell_t^{1+1/\sigma} / (1 + 1/\sigma) \right] \tag{1}$$

- Budget constraint:

$$C_t + q_t B_t + E_t Q_{t,t+1} D_{t+1} \leq (1 - \tau_t)(W_t L_t + \Pi_t) + h_t B_{t-1} + D_t. \tag{2}$$

Two assets: a AD contingent asset (D_t: quantity bought in $t-1$);

a public bond (maturity 1).
Two interest rates, defining a risk premium.

Debt

Intertemporal private wealth constraint :

$$h_{t+1}B_t + D_{t+1} \geq -E_{t+1} \sum_{s=t+1}^{\infty} Q_{t+1,s} \left(1 - \tau_s\right) \left(W_s \ell_t + \Gamma_s\right) \qquad \forall t+1 \tag{3}$$

using transversality condition:

$$\lim_{T \to \infty} E_t Q_{t,T} \left[h_T B_T + D_T\right] = 0$$

Production

- A_t: productivity. Production given by:
  $Y_t \leq A_t \ell_t$
- Therefore:
  $W_t/P_t = A_t$, $\Gamma_t = 0$; $Y_t = A_t \ell_t$.
- A_t: random shock with persistence (and assumptions on the distribution).

Fiscal Policy

- Here, the government does not choose to default or when to default.
- Nothing strategic.
- Public debt issued on the market is the result of fiscal rules automatically followed by the government.
- If it finds no buyer (at any price), there is default.

Fiscal rules

Budget constraint

$$\frac{B_t}{R_t} = h_t B_{t-1} + (g - \tau_t) Y_t \tag{4}$$

Fiscal rule with a maximum tax rate!

$$\tau_t = \Upsilon(\omega_t) = \min(\bar{\tau} + \theta \cdot (\omega_t - \bar{\omega}); \hat{\tau}) \tag{5}$$

Debt/GDP Define the *redeemed* debt $\omega_t = h_t B_{t-1}/Y_t$.
Difference with $b_t \equiv B_t/Y_t$, *contractual* debt.

Target ratio $\bar{\omega}$

Maximum Define a level $\hat{\omega}$ at which the tax rate reaches its maximum:

$$\tau_t = \hat{\tau} \Longleftrightarrow \omega_t \geq \bar{\omega} + \frac{\hat{\tau} - \bar{\tau}}{\theta} \equiv \hat{\omega}. \tag{6}$$

Kink in the behavior of debt.

Default

- Default : when $h_t < 1$.
  Depending on debt burden.
- Debt recovery rule (DRR):
  DRR specified as:

$$h_t = \mathcal{H}\left(B_{t-1}, \Omega_t^{\max}\right) = \begin{cases} \mathbf{h} \cdot \Omega_t^{\max} / B_{t-1} < 1 & \text{if } \Omega_t^{\max} < B_{t-1}, \\ 1 & \text{if not.} \end{cases} \tag{7}$$

  with $0 \leq \mathbf{h} \leq 1$.
- $\Omega_t^{\max}$: the default threshold at t (set on the market).

The complete model and issues

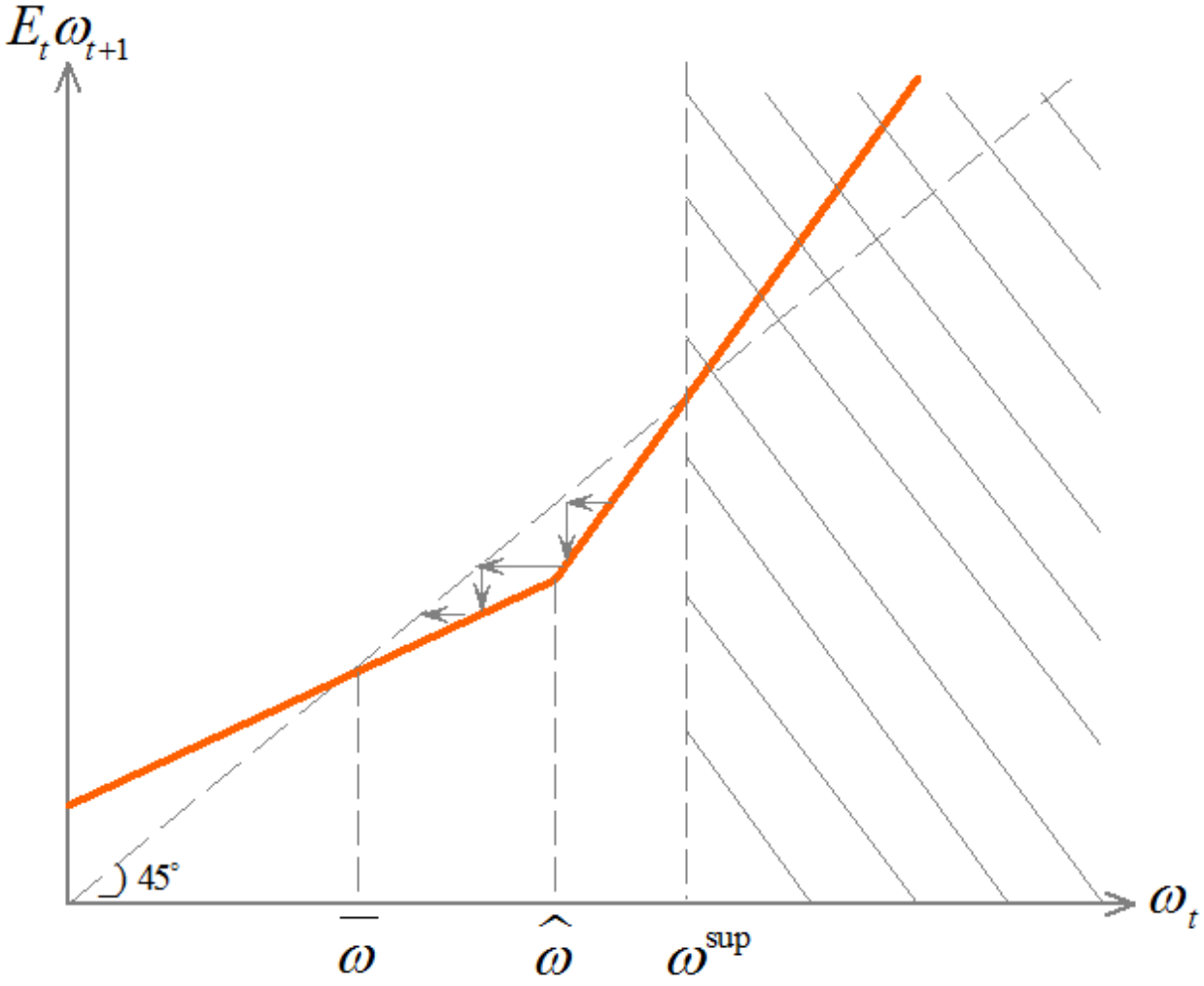
A stochastic general equilibrium model contingent on $\Omega_t^{\max}$.

- Can we endogenize $\Omega_t^{\max}$? Does it depend on the DRR? How? What is the role of the risk premium?
- If so, what is the debt dynamics? What is the impact of shocks?
- How to define the sustainability of public debt in the presence of shocks and the existence of a default threshold?

Equilibrium

Dynamics of debt burden

Two steady states because of the kink in the fiscal rule.



The possibility of default

- A "high" steady-state associated with the second equation.

$$\frac{\hat{\tau} - \gamma}{1 - \beta} \equiv \omega^{\text{sup}} \tag{8}$$

- Because of transversality, impossibility that $\omega_t > \omega^{\text{sup}}$.
  $\Longrightarrow$ In this economy, default is a possibility. But ω^{sup} not necessarily the default threshold, as it is not determined by the market.

Debt pricing

- $v_t \equiv q_t b_t$: total value of public bonds relative to GDP, that is - loosely speaking - the debt market value at date t:

$$v_t \equiv q_t b_t = h_t \frac{b_{t-1}}{a_t} + g - \hat{\tau}, \tag{9}$$

  with $a_t \equiv A_t / A_{t-1}$.
- It depends on the future of shocks (the prospect of default) and therefore the DRR, through the risk premium. How ?

The market value function

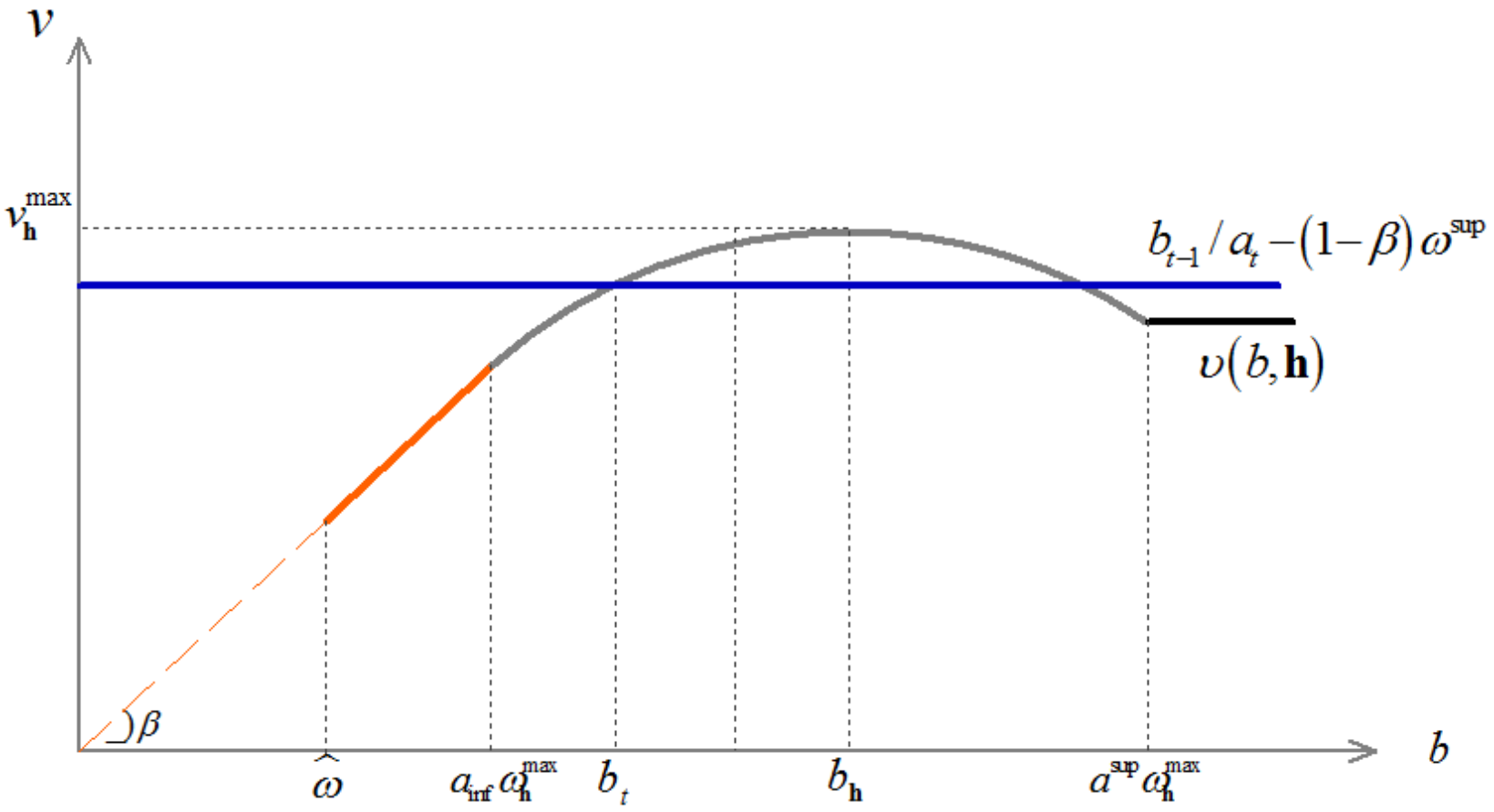


Figure : $\upsilon(b;\mathbf{h})$

Default threshold

- Two results
  - the maximum market value $v_{\mathbf{h}}^{\max}$
  - the stationary default threshold $\omega_{\mathbf{h}}^{\max}$

  both depending on the DRR.
- Proposition:

  The equilibrium default threshold $\omega_t^{\max}$ is locally unique and equal to:

$$\omega_t^{\max} = \omega_{\mathbf{h}}^{\max} \equiv \frac{1-\beta}{1-\beta x_{\mathbf{h}}} \omega^{\sup}, \forall t. \tag{10}$$

Debt and DRR

- One lesson: DRR affects debt at any period, not just the after-default debt.
- The snowball effect of the risk premium: fed by the DRR. $\Longrightarrow$the DRR impacts on default !

Debt dynamics

- Back to the issue of sustainability.
- What is the dynamics of b_t?
  - What is the contribution of shocks to this dynamics?
  - What is the contribution of the DRR?

Risky steady state

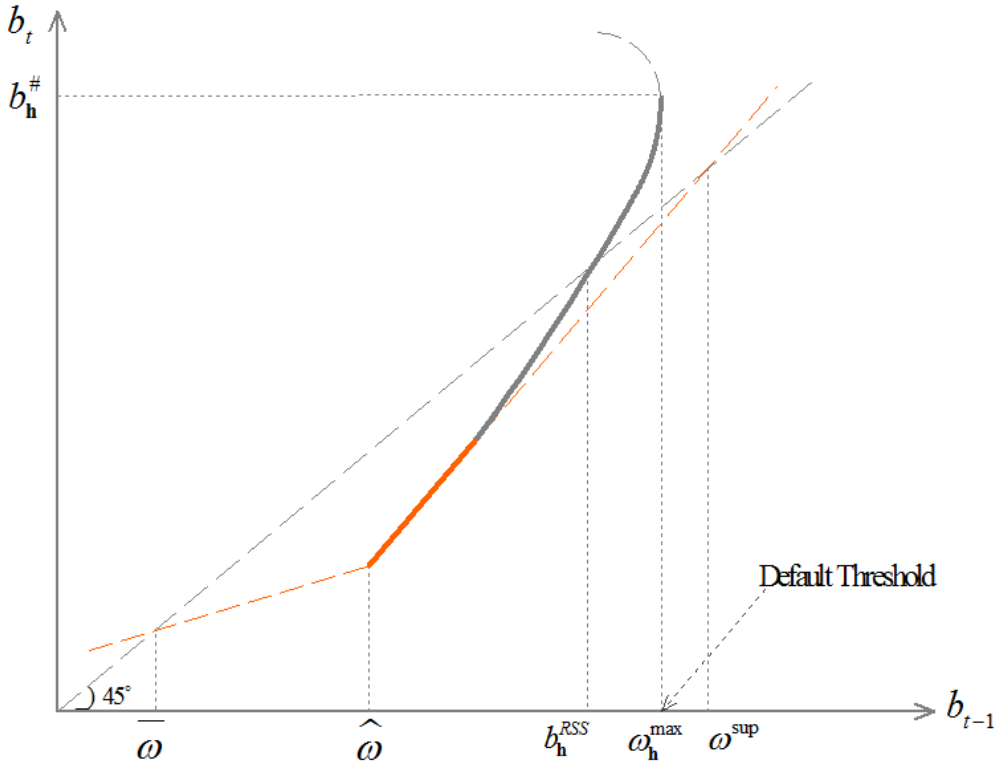
- What is the contribution of shocks to this dynamics?
- To study shocks, a simple case:

  H Assume that the realized values of shocks are equal to their mean value at all dates.

- What do we get?
- Can we define a RSS?

Default and RSS

The dynamics of debt under H.



- Why backward-bending? Because of the snowball effect of risk-premium. The closer to default, the higher the risk-premium, the closer to default,.... up to the default threshold.

Generalization

Definition 1: *A public debt is said to be "$\underline{\varphi}$-sustainable" at date t when its future trajectory never reaches the default threshold, assuming that there is no growth rate realization a_{t+s} lower than $\underline{\varphi} \leq 1$. A public debt is said to be "$\overline{\varphi}$-unsustainable" at date t when its future trajectory reaches at some finite date the default threshold, assuming that there is no growth rate realization a_{t+s} higher than $\overline{\varphi} \geq 1$.*

- Simple RSS when $\underline{\varphi} = \overline{\varphi} = 1$

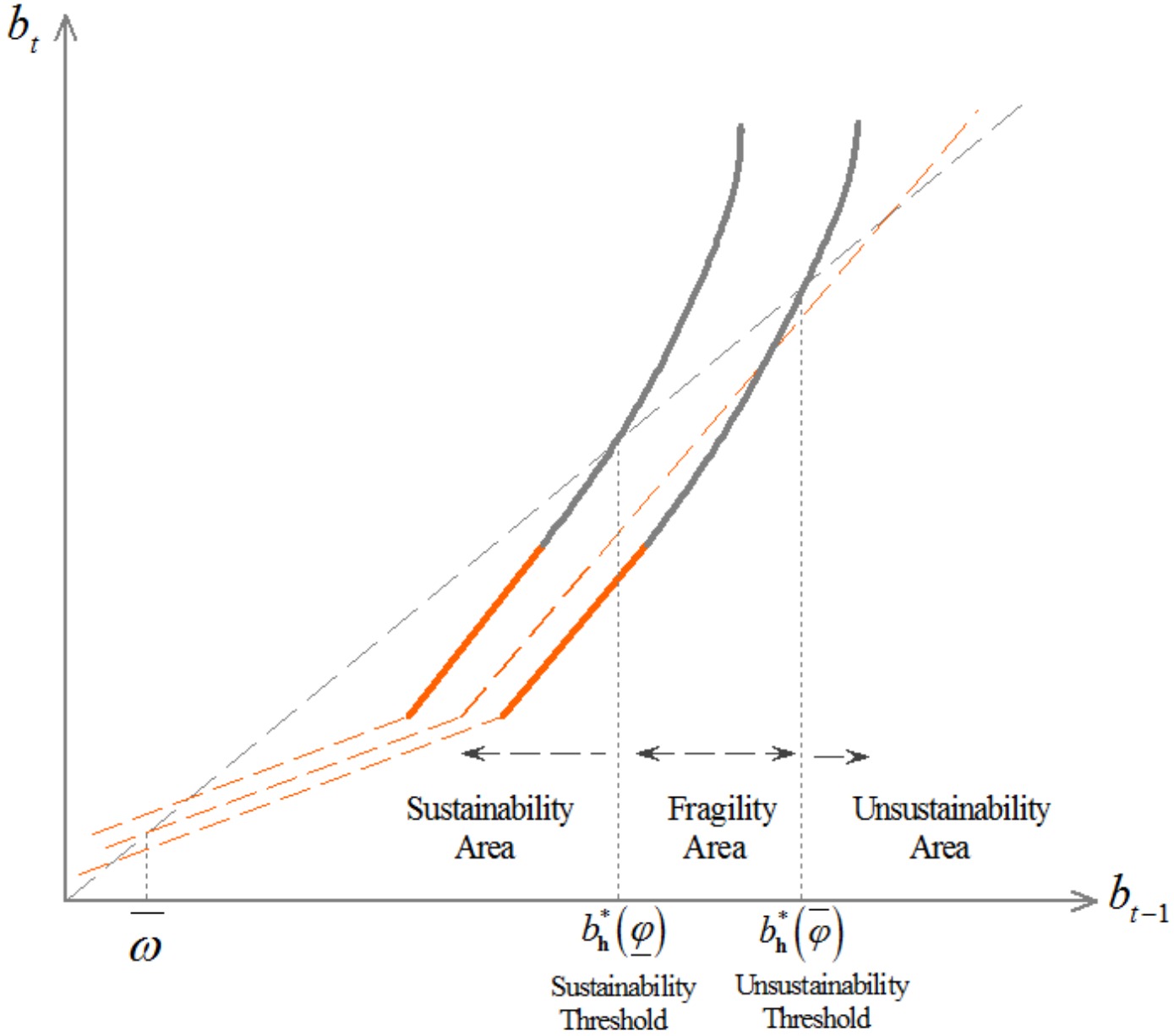
Thresholds

Proposition 2: for any admissible $(\underline{\varphi}, \overline{\varphi})$

1. The public debt due at t, B_{t-1}, is "*$\underline{\varphi}$-sustainable*" *if* $b_{t-1} \equiv B_{t-1}/Y_{t-1} < b^*_{\mathbf{h}}(\underline{\varphi})$.
2. The public debt due at t, B_{t-1}, is "*$\overline{\varphi}$-unsustainable*" *if* $b_{t-1} \equiv B_{t-1}/Y_{t-1} > b^*_{\mathbf{h}}(\overline{\varphi})$.

- $b^*_{\mathbf{h}}(\underline{\varphi})$: "sustainability threshold".
- $b^*_{\mathbf{h}}(\overline{\varphi})$: "unsustainability threshold".

(Un-)sustainability



DRR and sovereign default dynamics

- What is the contribution of the DRR?
- After default, if a high **h**: much to recover after default (therefore little contribution to the snowball effect), but little debt reduction, therefore still close to the default threshold.
- A low **h**: the reverse.
  - What is best? What saves us from recurring default?

Solving the dilemma

Proposition 3 (simplified):

1. For a given admissible φ , there exists a critical value $\mathbf{H}(\varphi)$ satisfying $0 < \mathbf{H}(\varphi) < \varphi$ such that, in case of default, the post-default debt-to-GDP ratio $\mathbf{h}\omega_{\mathbf{h}}^{\max}$ is φ-sustainable.
2. $\mathbf{H}(\varphi)$ is an increasing function of φ.

Meaning: Better to have a high haircut. Beware the "too little" effect.

Conclusion

Why defaulting?

- Why default on sovereign debt?
- For a combination of reasons:

1. the existence of a fiscal limit, such that once reached further outlays must be solely financed by new debt,
2. an "active" monetary policy which does not give up on its inflation stabilization objective despite the prospect of default and thus does not provide enough seignorage income,
3. a series of bad shocks which deteriorate the macroeconomic situation and worsen the financial plight of the sovereign,
4. a rule of default such that risk premia are high and concur to the burdening of public debt.

When defaulting?

- Why defaults happen sometimes but not always, nor never?

1. Because there are multiple steady-state equilibria, one corresponding to default.
2. And shocks put the sovereign closer to one or the other.

Defaulting with success

- Is it possible to succeed a default?

1. Yes, when the default rule is such that you converge to the no-default.
2. But it does not mean no future default! Beware of future shocks.